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When to Align and When to Contract: Technology Shocks, Optimal Policies, and Exchange Rate Regimes

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When to Align and When to Contract:
Technology Shocks, Optimal Policies, and Exchange Rate Regimes

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Abstract

This paper investigates the design of optimal monetary policy responses to technology shocks in a two-country model framework featuring sticky prices and local currency pricing, where technology shocks propagate internationally. We demonstrate that technology shocks originating in the tradable sector, regardless of their country of origin, elicit monetary policy responses that are symmetric and closely aligned across countries, thereby providing a rationale for a fixed exchange rate regime. In contrast, technology shocks in the nontradable sector generate asymmetric policy reactions and weaken the source country's currency, supporting the case for exchange rate flexibility. In addition, the international transmission of technology shocks amplifies real-sector dynamics through news effects, prompting central banks to adopt contractionary policies, starkly contrasting with the findings of previous literature.

Keywords: Sticky Price; Local Currency Pricing; Exchange Rate Regimes; Technology Diffusion; Interest Rate Rules

JEL Classification: F31; F41; O0; E52

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1 Introduction

In his seminal work, Friedman (1953) argues that, since nominal goods prices tend to adjust sluggishly, flexible nominal exchange rates are optimal as they facilitate the necessary relative price adjustments across countries in response to country-specific productivity shocks. A central element in the case for flexible exchange rates made by Friedman and many later studies is based on the "producer currency pricing" paradigm (PCP) whereby exporters use the currency of their home country for invoicing (see, among others, Mundell 1963, Marcus 1962, Svensson and Wijnbergen 1989, and Obstfeld and Rogoff 1995,). Under PCP, the exchange rate pass through (ERPT) to consumer prices is immediate, implying that optimal monetary policy under sticky prices relies on nominal exchange rate adjustments in the presence of country-specific real shocks.

However, PCP is seemingly at odds with empirical evidence showing that import prices are rather stable in local currency. The evidence of low ERPT has led to a shift toward another type of invoicing paradigm under which prices are set in the currency of the destination market (see, e.g., Bacchetta and van Wincoop 2000, Betts and Devereux 2000 and Chari et al. 2002). Under this "local currency pricing" paradigm (LCP), nominal exchange rate changes do not alter relative import prices and thus play no expenditure-switching role between domestic and foreign goods. In Devereux and Engel (2003), DE hereafter, they challenge the validity of the classical claim in favor of flexible exchange rates and conclude that, under LCP, optimal monetary policy should keep nominal exchange rates fixed to accommodate country-specific productivity shocks.

This rather surprising prescription of a fixed exchange rate regime, however, is upset by Duarte and Obstfeld (2008), DO hereafter, who add nontradable goods to the DE model. DO demonstrate that the optimality of fixed exchange rates is primarily due to DE's model structure where international consumptions are perfectly synchronized. However, with nontradable goods, consumptions across countries cannot move in a synchronized fashion even with complete international asset markets, since domestic nontraded goods cannot be shipped abroad to augment foreign consumption. Put it differently, when there are nontradable goods, a country's consumption responds disproportionately to local productivity shocks, requiring asymmetric responses of monetary policy. Therefore, the case for flexible exchange rates is restored even in the absence of expenditure-switching effects of exchange rate fluctuations.

In contrast to the DO model, which assumes that productivity shocks in the tradable and nontradable sectors within a country are perfectly correlated, we introduce not only country-specific but also sector-specific technology shocks. Furthermore, we allow technology to diffuse to the same sector in the other country with a one-period lag, generating additional distortions to which central banks may respond.[1,2]

Our model provides novel insights, demonstrating that the sectoral sources of technology shocks and their international diffusion critically shape optimal monetary policy and exchange rate dynamics. Specifically, a positive productivity shock originating in the tradable sector, regardless of where it occurs, generates two distinct distortions under LCP: price stickiness and news effects. We show that welfare-maximizing central banks raise the nominal interest rate when the news effect dominates the price stickiness effect.

This finding contrasts sharply with the results of DE and DO, where only the sticky-price effect is present, and monetary authorities systematically respond to positive productivity shocks with expansionary policies. Our results are consistent with empirical evidence showing that nominal interest rates are procyclical, typically rising during economic expansions (see, among others, Friedman 1986, Konstantakopoulou et al. 2009, and Forbes et al. 2024). We also observe that, in response to tradable sector shocks, central banks align their policy responses, eliminating the need for exchange rate adjustments, which is consistent with the conclusions of DE.

In contrast to shocks in the tradable sector, productivity shocks in the nontradable goods sector generate only a sticky-price effect in the source country, while inducing a purely informational (news) effect abroad under LCP. This asymmetry leads home and foreign central banks to adopt differing monetary policy responses, producing an interest rate differential that necessitates exchange rate adjustment under optimal policies, consistent with the conclusions of DO. We further show that, under a rule-based policy targeting the domestic price, central banks do not need to respond to tradable-sector productivity shocks. However, they must still adjust their policies in response to nontradable-sector shocks to stabilize the economy.

The remainder of the paper is organized as follows. Section 2 presents the main building blocks of the model. Section 3 examines the model's properties under flexible prices. Section 4 provides analytical solutions under LCP-type price stickiness. Section 5 derives the optimal interest rate rule implied by the central bank's welfare-maximization

[1] Aysun (2024) also shows that synchronized fluctuations in open economies can be driven not only by technology shocks but also by their propagation through cross-country technology diffusion.

[2] Beyond PCP and LCP, recent studies propose alternative invoicing paradigms. For example, Gopinath et al. (2020) develop the dominant currency paradigm (DCP), under which firms set export prices in a third-country dominant currency, most notably the U.S. dollar, and adjust them infrequently. Amiti et al. (2022) further emphasize that invoicing is an endogenous firm-level choice shaped by import intensity and strategic complementarities in price setting across firms. We abstract from these frameworks and leave their incorporation to future research.

problem. Section 6 analyzes the resulting exchange rate dynamics. Section 7 concludes.

2 The Model

2.1 Preferences

There are two countries, each populated by a continuum of identical households: $h \in [0,1]$ in the home country and by $f \in [0,1]$ in the foreign country. Each household produces both tradable and nontradable goods, with only tradable goods being exported. The foreign country's problem mirrors that of the home country, with all foreign variables denoted by a superscript asterisk (*) throughout the paper.

The representative household h maximizes the following utility function by choosing consumption (C_t) and labor (L_t).

$$U = \mathbb{E}_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[\frac{C_t^{1-\rho}(h)}{1-\rho} - \kappa L_t(h) \right] \right\}, \ \beta \in (0,1), \ \rho, \kappa > 0, \tag{1}$$

where ρ denotes the inverse of the intertemporal elasticity of substitution in consumption, and κ is the parameter governing the disutility of labor. Facing the aggregate price P_t, households purchase C_t and hold nominal marketable wealth (D_t), which yields the gross nominal interest rate (R_t), set by the monetary authority. Each household earns wage income ($W_t L_t$), given the nominal wage (W_t), receives profits (Π_t) from the ownership of domestic firms, as defined below. The flow budget constraint for household h is as follows.

$$P_t C_t(h) + D_t(h) = W_t(h) L_t(h) + \Pi_t + R_{t-1} D_{t-1}(h) \tag{2}$$

Aggregate consumption (C) is a composite of tradable goods (C_T) and nontradable goods (C_N), where $\gamma = 1$ in the absence of nontradable goods. That is,

$$C = \frac{C_T^{\gamma} C_N^{1-\gamma}}{\gamma^{\gamma}(1-\gamma)^{1-\gamma}},$$

Consumption of tradable goods (C_T) is determined by domestically produced tradable consumption goods (C_H) and foreign (imported) tradable consumption goods (C_F) as follows.

$$C_T = \frac{C_H^{\xi} C_F^{1-\xi}}{\xi^{\xi}(1-\xi)^{1-\xi}}, \tag{3}$$

For simplicity, ξ ia assumed to be $1/2$, implying no home bias. Then, (3) can be rewritten as follows.[3]

$$C_T = 2C_H^{1/2} C_F^{1/2} \tag{4}$$

Consumption of home tradable goods (C_H), nontradable goods (C_N), and foreign tradable goods (C_F) is represented by the following constant elasticity of substitution (CES) function defined over the quantities consumed of all varieties within each category.

$$C_j = \begin{cases} \left[\int_0^1 C_j(h)^{\frac{\theta-1}{\theta}}\, \mathrm{d}h\right]^{\frac{\theta}{\theta-1}}, & j = \mathrm{H, N} \\ \left[\int_0^1 C_j(f)^{\frac{\theta-1}{\theta}}\, \mathrm{d}f\right]^{\frac{\theta}{\theta-1}}, & j = \mathrm{F}, \end{cases} \tag{5}$$

where $\theta > 1$ is the elasticity of substitution across goods in each sector. An aggregator identical to (5) is also assumed for foreign country's consumption.

Profits from domestic firms at time t are defined as follows.

$$\Pi_t = P_{H,t}(h)Y_{H,t}(h) + S_t P^*_{H,t}(h)Y^*_{H,t}(h) + P_{N,t}(h)Y_{N,t}(h) - W_t(h)L_t(h),$$

where $Y_{H,t}$ and $Y^*_{H,t}$ denote domestically produced tradable goods supplied to the home and foreign country, respectively, and $Y_{N,t}$ denotes the production of nontradable goods. $P_{H,t}$, $P^*_{H,t}$, and $P_{N,t}$ are the corresponding prices of these goods expressed in local currencies. S_t is the nominal exchange rate, defined as the domestic currency price of foreign currency. Note that producers employ price-discrimination by setting a separate price for tradable goods sold in the foreign country.[4]

Given market prices, solving the consumer's optimization problem gives the following consumer demand functions for domestically produced goods

$$C_{H,t}(h) = \frac{\gamma}{2}\left(\frac{P_{H,t}(h)}{P_{H,t}}\right)^{-\theta}\left(\frac{P_t}{P_{H,t}}\right)C_t, \tag{6}$$

$$C_{N,t}(h) = (1-\gamma)\left(\frac{P_{N,t}(h)}{P_{N,t}}\right)^{-\theta}\left(\frac{P_t}{P_{N,t}}\right)C_t, \tag{7}$$

$$C^*_{H,t}(h) = \frac{\gamma}{2}\left(\frac{P^*_{H,t}(h)}{P^*_{H,t}}\right)^{-\theta}\left(\frac{P^*_t}{P^*_{H,t}}\right)C^*_t, \tag{8}$$

[3] We follow both DE and DO in assuming that consumers' preferences for tradable goods are identical across countries. That is, as long as $\xi = \xi^*$, any value for $0 < \xi < 1$ supports our results.

[4] Under PCP, firms set a single price for tradable goods across markets. That is, once the domestic price $P_{H,t}(h)$ is chosen, the foreign price $P^*_{H,t}(h)$ is automatically set by $P_{H,t}(h)/S_t$.

where $C^*_{H,t}$ denotes foreign consumption of domestically produced tradable goods.[5,6]

The aggregate price index and the price index for tradable goods are derived from an expenditure minimization problem as follows.[7]

$$P = P_T^{\gamma} P_N^{1-\gamma}, \tag{9}$$

$$P_T = P_H^{\frac{1}{2}} P_F^{\frac{1}{2}}, \tag{10}$$

where

$$P_j = \begin{cases} \left[\int_0^1 P_j(h)^{1-\theta}\,\mathrm{d}h\right]^{\frac{1}{1-\theta}}, & j = \mathrm{H, N} \\ \left[\int_0^1 P_j(f)^{1-\theta}\,\mathrm{d}f\right]^{\frac{1}{1-\theta}}, & j = \mathrm{F}. \end{cases} \tag{11}$$

2.2 Risk Sharing

Following the standard literature, we assume the existence of complete international asset markets. This implies that consumers in each country equalize the marginal consumption value of one unit of nominal bonds across countries. Therefore, the international risk-sharing condition of Backus and Smith (1993) holds and can be expressed as follows.

$$\frac{C_t^{-\rho}}{P_t} = \frac{C_t^{*\,-\rho}}{S_t P_t^*}, \tag{12}$$

Note that Purchasing Power Parity (PPP), i.e., $P_t = S_t P_t^*$, does not hold *ex post* in our model, although it remains valid *ex ante* for tradable goods. Therefore, consumption is not necessarily equalized across countries in general.

2.3 Production Technologies

The production functions for each type of goods by domestic firms are given by,

$$Y_{H,t}(h) = A_t L_{H,t}(h),\ Y^*_{H,t}(h) = A_t L^*_{H,t}(h),\ Y_{N,t}(h) = B_t L_{N,t}(h), \tag{13}$$

[5]The demand function for foreign goods can be derived analogously.
[6]Derivations are provided in Online Appendix A.
[7]Derivations are provided in Online Appendix B.

where A_t and B_t represent the sector-specific productivity levels in the tradable and nontradable sectors, respectively, at time t. Note also that the following constraint holds,

$$L_t(h) = L_{H,t}(h) + L^*_{H,t}(h) + L_{N,t}(h), \tag{14}$$

for each domestic household h. The foreign country's production functions and technology variables are defined analogously. These specifications imply that productivity shocks in our model are both country- and sector-specific, in contrast to the economy-wide productivity shocks assumed by DE and DO.[8]

Denoting the logarithms of variables by lowercase letters, the stochastic processes for log technologies are given by,

$$\begin{aligned} a_t &= \lambda a_{t-1} + u_t + u^*_{t-1},\ a^*_t = \lambda a^*_{t-1} + u^*_t + u_{t-1},\ u \sim N(0, \sigma_u^2), \\ b_t &= \lambda b_{t-1} + v_t + v^*_{t-1},\ b^*_t = \lambda b^*_{t-1} + v^*_t + v_{t-1},\ v \sim N(0, \sigma_v^2) \end{aligned} \tag{15}$$

where u_t and u^*_t denote tradable-sector productivity shocks in the home and foreign countries, respectively. Similarly, v_t and v^*_t denote nontradable-sector productivity shocks. The persistence parameter λ satisfies $\lambda \in [0, 1)$.[9]

Following Kim (2008), our model incorporates technology diffusion of productivity shocks across corresponding sectors in the other country, with a one-period lag. Specifically, a productivity shock in the home tradable sector at time t immediately increases domestic tradable-sector productivity but has a muted impact on the same sector at time $t+1$. Simultaneously, the shock u_t diffuses to the foreign country and is fully incorporated into the foreign tradable sector's productivity at time $t+1$. While u_t is unanticipated by domestic agents at time t, its impact on foreign productivity at time $t+1$ is perfectly anticipated by agents in both countries.[10]

We also allow productivity shocks in the nontradable sector to diffuse across countries, even though nontradable goods and services cannot be exported. For example, while a haircut service is nontradable, new skills and techniques can be transferred across countries. As a result, a foreign (home) nontradable-sector shock can prompt a policy response from the domestic (foreign) central bank, even before the shock fully

[8]That is, DE and DO assume $A_t = B_t$ and $A^*_t = B^*_t$ for all t.

[9]This is a standard assumption when an interest rate rule is used to close the model. See also Obstfeld (2006) and Itskhoki and Mukhin (2021). On the other hand, DE and DO specify log technology processes as a random walk, as their models rely on a money demand function.

[10]Conceptually, the productivity shocks in our model capture both surprise and anticipated technology shocks, as identified in the empirical literature. See, among others, Beaudry and Portier (2006), Barsky and Sims (2011), and Nam and Wang (2015).

materializes in the home (foreign) country.

2.4 Interest Rate Rules

We assume that the monetary authority commits to a state-contingent monetary policy feedback rule, which is a log-linear function of productivity shocks in both the tradable and nontradable sectors. Following Obstfeld (2006), the nominal interest rate rule for each country is given by,

$$\begin{aligned} i_t &= \iota + \psi p_t - \alpha_1 u_t - \alpha_2 v_t - \alpha_3 u_t^* - \alpha_4 v_t^*, \\ i_t^* &= \iota + \psi p_t^* - \alpha_1^* u_t^* - \alpha_2^* v_t^* - \alpha_3^* u_t - \alpha_4^* v_t, \end{aligned} \tag{16}$$

where $i_t = \log R_t$ and $i_t^* = \log R_t^*$ denote the nominal (net) interest rates in the home and foreign countries, respectively.

In this specification, the coefficient $\psi > 0$ governs the response of the nominal interest rate to p_t and p_t^*, ensuring the *determinacy* of the price level (see the Online Appendix C for the details). In contrast, the coefficient $\alpha_j, j = 1, 2, 3, 4$ determines the policy responses to productivity shocks in the home and foreign tradable and nontradable sectors. That is, central banks choose the optimal α_j given $\psi > 0$. In what follows, we also discuss optimal policies through the choice of ψ.

3 Flexible Price Equilibrium as a Benchmark

This section characterizes the fully flexible-price equilibrium under the assumption that central banks do not respond to productivity shocks (i.e., all policy parameters α's are set to zero), which serves as the benchmark solution. With flexible prices, firms set prices each period as a constant markup, $\frac{\theta}{1-\theta}$, over nominal marginal cost. Labor markets are assumed to be perfectly competitive, implying that nominal marginal costs in the home country are $\frac{W_t}{A_t}$ and $\frac{W_t}{B_t}$, while in the foreign country they are $\frac{W_t^*}{A_t^*}$ and $\frac{W_t^*}{B_t^*}$ in the tradable and nontradable sectors, respectively.

Applying the first-order conditions for labor-consumption optimization together with the risk-sharing condition, we obtain the following flexible-price equilibrium consumption levels for both the home and foreign countries. Detailed derivations are provided in Online Appendix D.

$$C_t = \left[\left(\frac{\theta - 1}{\theta \kappa} \right) A_t^{\frac{\gamma}{2}} B_t^{1-\gamma} A_t^{*\frac{\gamma}{2}} \right]^{\frac{1}{\rho}}, \tag{17}$$
$$C_t^* = \left[\left(\frac{\theta - 1}{\theta \kappa} \right) A_t^{*\frac{\gamma}{2}} B_t^{*1-\gamma} A_t^{\frac{\gamma}{2}} \right]^{\frac{1}{\rho}},$$

Combining the law of motion for technology (15) with (17), the innovations to log consumption can be expressed as follows.

$$c_t - \mathbb{E}_{t-1} c_t = \frac{\gamma}{2\rho} (u_t + u_t^*) + \frac{1-\gamma}{\rho} v_t, \tag{18}$$
$$c_t^* - \mathbb{E}_{t-1} c_t^* = \frac{\gamma}{2\rho} (u_t + u_t^*) + \frac{1-\gamma}{\rho} v_t^*, \tag{19}$$

Our results show that consumption responses are equalized across countries following either home or foreign tradable-sector productivity shocks. The resulting synchronization of international consumption under flexible prices suggests that, even with sticky prices, central banks in both countries would respond symmetrically to tradable-sector productivity shocks, regardless of their country of origin, as long as LCP is assumed. This conjecture is consistent with the prediction of the DE model, which supports a fixed exchange rate regime.

By contrast, consumption in each country depends solely on its own nontradable-sector productivity shocks, implying that consumption dynamics need not be synchronized internationally. Such asymmetric consumption responses may give rise to distinct optimal interest rate rules across countries. In turn, this requires nominal exchange rates to adjust flexibly, as in the DO model. We therefore emphasize that the sectoral origin of productivity shocks has a direct bearing on optimal monetary policy design and the appropriate exchange rate regime. It is also worth noting that, under flexible prices, cross-country technology diffusion has no effect on consumption innovations in either country.

Note also that nominal interest rates affect the economy primarily through the intertemporal Euler equation for nominal bonds,

$$\frac{C_t^{-\rho}}{P_t} = R_t \beta \mathbb{E}_t \left(\frac{C_{t+1}^{-\rho}}{P_{t+1}} \right) \tag{20}$$

And analogously for the foreign consumers,

$$\frac{C_t^{*-\rho}}{P_t^*} = R_t^* \beta \mathbb{E}_t \left(\frac{C_{t+1}^{*-\rho}}{P_{t+1}^*} \right) \tag{21}$$

4 Equilibrium with Local Currency Pricing

This section highlights how market equilibrium deviates from the fully flexible-price benchmark when prices are predetermined in the local currency of consumers. Specifically, firms set home-currency prices for domestic consumers and foreign-currency prices for foreign consumers one period in advance, leading to low ERPT to consumer prices, consistent with empirical evidence (see, among others, Devereux and Yetman 2010, Forbes et al. 2018, Jašová et al. 2019, and Forbes et al. 2020).

In the home country, the representative producer h sets the prices $P_{H,t}(h)$, $P_{H,t}^*(h)$, and $P_{N,t}(h)$ at time $t-1$ based on all available information and keeps them fixed for one period. The first-order conditions with respect to these price variables yield the following optimal pricing rules.

$$P_{H,t} = \frac{\theta\kappa}{\theta - 1} \frac{P_t \mathbb{E}_{t-1}\left(C_t / A_t\right)}{\mathbb{E}_{t-1} C_t^{1-\rho}}, \tag{22}$$

$$P_{N,t} = \frac{\theta\kappa}{\theta - 1} \frac{P_t \mathbb{E}_{t-1}\left(C_t / B_t\right)}{\mathbb{E}_{t-1} C_t^{1-\rho}}, \tag{23}$$

$$P_{H,t}^* = \frac{\theta\kappa}{\theta - 1} \frac{P_t^* \mathbb{E}_{t-1}\left(C_t^* / A_t\right)}{\mathbb{E}_{t-1} C_t^{*1-\rho}}, \tag{24}$$

Similarly, the home-currency price of foreign tradable goods is given by,

$$P_{F,t} = \frac{\theta\kappa}{\theta - 1} \frac{P_t \mathbb{E}_{t-1}\left(C_t / A_t^*\right)}{\mathbb{E}_{t-1} C_t^{1-\rho}}, \tag{25}$$

Full derivations of these pricing equations are provided in Online Appendix E.1.

Combining the Euler equation (20), the stochastic processes for technology (15), the interest rate rule (16), and the optimal pricing conditions above, log-normality yields the following solution for the home price index p_t.[11]

$$p_t = -\frac{\gamma\lambda(1-\lambda)}{2(1+\psi-\lambda)}(a_{t-1}+a^*_{t-1}) - \frac{(1-\gamma)\lambda(1-\lambda)}{1+\psi-\lambda}b_{t-1} - \frac{\gamma(1-\lambda)}{2(1+\psi-\lambda)}(u_{t-1}+u^*_{t-1}) - \frac{(1-\gamma)(1-\lambda)}{1+\psi-\lambda}v^*_{t-1} - \frac{1}{\psi}\left(\log\beta + \iota + \frac{\rho^2}{2}\sigma_c^2\right) \tag{26}$$

As shown in equation (26), the home price index p_t decreases following a positive tradable-sector productivity shock at time $t-1$, regardless of whether the shock originates domestically (u_{t-1}) or abroad (u^*_{t-1}). This decline operates through two channels: the domestic price component $p_{H,t}$ and the foreign price component $p_{F,t}$. For instance, when u_{t-1} occurs, the domestic producer h sets $p_{H,t}$ at a lower level, anticipating that home tradable-sector productivity will increase by λu_{t-1} at time t. Simultaneously, $p_{F,t}$ also falls, as the foreign producer f incorporates u_{t-1} into foreign tradable-sector productivity at time t via technology diffusion. An analogous mechanism applies to the foreign price index p^*_t.

Substituting equation (26) into the Euler equation (20), and using the law of motion for technology (15) together with the interest rate rule (16), yields the following expression for realized consumption c_t.[12]

$$c_t = \frac{\gamma}{2\rho}\left(\frac{\lambda\psi}{1+\psi-\lambda}\right)(a_t+a^*_t) + \frac{1-\gamma}{\rho}\left(\frac{\lambda\psi}{1+\psi-\lambda}\right)b_t + \frac{\gamma}{2\rho}\left(\frac{\psi}{1+\psi-\lambda}\right)(u_t+u^*_t) + \frac{1-\gamma}{\rho}\left(\frac{\psi}{1+\psi-\lambda}\right)v^*_t + \frac{1}{\rho}(\alpha_1 u_t + \alpha_2 v_t + \alpha_3 u^*_t + \alpha_4 v^*_t) + \tilde{\nabla}, \tag{27}$$

where $\tilde{\nabla}$ denotes a function of parameters, unconditional moments, and lagged variables dated $t-1$.

[11] The full derivation of equation (26) is provided in Online Appendix E.2.
[12] Full derivations are provided in Online Appendix E.2.

Foreign consumption is similarly obtained as,

$$
\begin{aligned}
c_t^* = {} & \frac{\gamma}{2\rho}\left(\frac{\lambda\psi}{1+\psi-\lambda}\right)(a_t + a_t^*) + \frac{1-\gamma}{\rho}\left(\frac{\lambda\psi}{1+\psi-\lambda}\right) b_t^* \\
& + \frac{\gamma}{2\rho}\left(\frac{\psi}{1+\psi-\lambda}\right)(u_t + u_t^*) + \frac{1-\gamma}{\rho}\left(\frac{\psi}{1+\psi-\lambda}\right) v_t \\
& + \frac{1}{\rho}(\alpha_1^* u_t^* + \alpha_2^* v_t^* + \alpha_3^* u_t + \alpha_4^* v_t) + \tilde{\nabla}^*,
\end{aligned} \tag{28}
$$

where $\tilde{\nabla}^*$ is defined analogously.

Combining the law of motion for technology (15) with equations (27) and (28), and assuming no monetary policy responses (i.e., setting all policy parameters α and α^* to zero) as in the flexible-price benchmark, the innovations to consumption can be expressed as follows.

$$
\begin{aligned}
c_t - \mathbb{E}_{t-1}c_t = {} & [\underbrace{\frac{\gamma}{2\rho}\left(\frac{\lambda\psi}{1+\psi-\lambda}\right)}_{\text{Sticky Price Effect T}} + \underbrace{\frac{\gamma}{2\rho}\left(\frac{\psi}{1+\psi-\lambda}\right)}_{\text{News Effect T}}](u_t + u_t^*), \\
& + \underbrace{\frac{1-\gamma}{\rho}\left(\frac{\lambda\psi}{1+\psi-\lambda}\right)}_{\text{Sticky Price Effect N}} v_t + \underbrace{\frac{1-\gamma}{\rho}\left(\frac{\psi}{1+\psi-\lambda}\right)}_{\text{News Effect N}} v_t^*
\end{aligned} \tag{29}
$$

for the home country, and

$$
\begin{aligned}
c_t^* - \mathbb{E}_{t-1}c_t^* = {} & [\underbrace{\frac{\gamma}{2\rho}\left(\frac{\lambda\psi}{1+\psi-\lambda}\right)}_{\text{Sticky Price Effect T}} + \underbrace{\frac{\gamma}{2\rho}\left(\frac{\psi}{1+\psi-\lambda}\right)}_{\text{News Effect T}}](u_t + u_t^*), \\
& + \underbrace{\frac{1-\gamma}{\rho}\left(\frac{\lambda\psi}{1+\psi-\lambda}\right)}_{\text{Sticky Price Effect N}} v_t^* + \underbrace{\frac{1-\gamma}{\rho}\left(\frac{\psi}{1+\psi-\lambda}\right)}_{\text{News Effect N}} v_t
\end{aligned} \tag{30}
$$

for the foreign country.

Comparing the solutions for consumption innovations in equations (18) and (19) with those in (29) and (30) highlights the key difference between the flexible-price and LCP environments. Under LCP, productivity shocks in the tradable sector generate two distinct distortions: the Sticky Price Effect T, given by $\frac{\gamma}{2\rho}\left(\frac{\lambda\psi}{1+\psi-\lambda}\right)$, and the News Effect T, given by $\frac{\gamma}{2\rho}\left(\frac{\psi}{1+\psi-\lambda}\right)$. Note that these effects originate in the tradable sector and arise irrespective of the country where the shocks occur.

The sticky price effect arises because prices are predetermined at $t-1$, preventing consumption from fully adjusting to u_t or u_t^* as it would under flexible prices (with response $\frac{\gamma}{2\rho}$). Consequently, the impact of shocks is attenuated by the factor of $\frac{\lambda\psi}{1+\psi-\lambda}$, which is strictly less than one whenever $\lambda < 1$.

By contrast, the news effect, a novel feature of our framework, stems from agents' perfect foresight about the cross-country diffusion of tradable-sector productivity shocks. For instance, when a domestic shock u_t occurs, agents anticipate that it will raise foreign productivity in the following period. Anticipating a decline in p_{t+1} as shown in equation (26), agents expect higher future consumption c_{t+1}, which in turn induces an immediate rise in c_t through consumption smoothing.

This response, however, is distortionary because the foreign productivity level A_t^* has not yet changed at time t. In this sense, technology diffusion generates consumption *overreactions* under LCP. The total impact of tradable-sector shocks on consumption is given by $\frac{\gamma}{2\rho}\left(\frac{\psi(1+\lambda)}{1+\psi-\lambda}\right)$, which exceeds the flexible-price benchmark $\frac{\gamma}{2\rho}$ whenever $\psi > \frac{1-\lambda}{\lambda}$, thereby inducing contractionary monetary policies in both countries.[13] This result stands in stark contrast to previous claims of DE and DO. When $0 < \psi < \frac{1-\lambda}{\lambda}$, central banks should respond by uniformly cutting interest rates, whereas a negative ψ leads to indeterminacy of p_t as shown in Online Appendix C.

Nontradable-sector productivity shocks, on the other hand, generate only a sticky price effect domestically (Sticky Price Effect N) and a pure news effect abroad (News Effect N). Although nontradable goods cannot be traded internationally, their technology is mobile across countries. Thus, a positive home nontradable-sector shock enhances foreign nontradable production, raising foreign consumption demand without any feedback to home demand. This contrasts with the flexible-price equilibrium, in which foreign (home) consumption is entirely insulated from home (foreign) nontradable productivity shocks.

5 Optimal Monetary Policy and Welfare Outcomes

5.1 Optimal Monetary Policy Responses to Productivity Shocks

We now turn to the implications of technology diffusion and the sectoral origin of productivity shocks for optimal monetary policy. To evaluate welfare, we also derive the endogenous covariances. For algebraic simplicity, we assume that all shocks are mutu-

[13]Since λ typically reflects high persistence (e.g., $\lambda = 0.9$), this condition is not restrictive for strictly positive ψ.

ally uncorrelated. From equation (27), we obtain the following.

$$\begin{aligned} \sigma_c^2 &= A_1^2\sigma_u^2 + A_2^2\sigma_v^2 + A_3^2\sigma_{u^*}^2 + A_4^2\sigma_{v^*}^2 \\ \sigma_{cu} &= A_1\sigma_u^2,\ \sigma_{cv} = A_2\sigma_v^2,\ \sigma_{cu^*} = A_3\sigma_{u^*}^2,\ \sigma_{cv^*} = A_4\sigma_{v^*}^2, \end{aligned} \tag{31}$$

where

$$\begin{aligned} A_1 &= \frac{\gamma}{2\rho}\left(\frac{\psi(1+\lambda)}{1+\psi-\lambda}\right) + \frac{\alpha_1}{\rho},\ A_2 = \frac{1-\gamma}{\rho}\left(\frac{\lambda\psi}{1+\psi-\lambda}\right) + \frac{\alpha_2}{\rho} \\ A_3 &= \frac{\gamma}{2\rho}\left(\frac{\psi(1+\lambda)}{1+\psi-\lambda}\right) + \frac{\alpha_3}{\rho},\ A_4 = \frac{1-\gamma}{\rho}\left(\frac{\psi}{1+\psi-\lambda}\right) + \frac{\alpha_4}{\rho} \end{aligned}$$

The foreign second moments are defined analogously.

The monetary authority in the home country chooses the parameters of the interest rate rule in equation (16) to maximize the expected utility of the representative household, given in equation (1). Combining the consumer's demand functions (6)-(8), the production functions (13), and the law of motion for technologies (15), the home labor supply is given by the following.

$$\begin{aligned} \mathbb{E}_t L_{t+1} = &\frac{\gamma}{2}\left(\frac{P_{t+1}}{P_{H,t+1}}\right)\mathbb{E}_t\left(\frac{C_{t+1}}{A_{t+1}}\right) + \frac{\gamma}{2}\left(\frac{P^*_{t+1}}{P^*_{H,t+1}}\right)\mathbb{E}_t\left(\frac{C^*_{t+1}}{A_{t+1}}\right) \\ &+ (1-\gamma)\left(\frac{P_{t+1}}{P_{N,t+1}}\right)\mathbb{E}_t\left(\frac{C_{t+1}}{B_{t+1}}\right), \end{aligned} \tag{32}$$

Plugging the pricing equations (22)-(24) and the labor supply condition (32) into the home country's expected utility at time t, we obtain the following.

$$\mathbb{E}_t U_{t+1} = \mathbb{E}_t\left[\left(\frac{\theta\gamma + 2 - \gamma}{2\theta(1-\rho)}\right)\mathbb{E}_t C_{t+1}^{1-\rho} - \frac{\gamma(\theta-1)}{2\theta}\mathbb{E}_t C_{t+1}^{*1-\rho}\right] \tag{33}$$

As shown in the foreign consumption equation (28), C^* is independent of the home country's interest rate rule parameters, α's. Therefore, the policy problem reduces to maximizing the term involving C_{t+1}. Also, under the log-normality assumption, $\mathbb{E}_t C_{t+1}^{-\rho} = \exp\left[(1-\rho)\mathbb{E}_t c_{t+1} + \frac{(1-\rho)^2}{2}\sigma_c^2\right]$. Hence, maximizing (33) is equivalent to the following problem.

$$\max_{\alpha}\left\{\mathbb{E}_t c_{t+1} + \frac{1-\rho}{2}\sigma_c^2\right\} \tag{34}$$

Substituting the covariances from equation (31) into (34), the maximization problem can be expressed in terms of the policy coefficients and the unconditional moments of the shocks. Solving for the optimal policy parameters yields the following results, with the complete derivation provided in Online Appendix F.

$$\begin{aligned}
\alpha_1 &= \frac{\gamma}{2}\left[1 - \frac{\psi(1+\lambda)}{1+\psi-\lambda}\right] = \alpha_3, \\
\alpha_2 &= (1-\gamma)\left(1 - \frac{\lambda\psi}{1+\psi-\lambda}\right) \\
\alpha_4 &= -(1-\gamma)\frac{\psi}{1+\psi-\lambda}
\end{aligned} \tag{35}$$

The corresponding foreign response coefficients in equation (16) take the following anlagous form.

$$\begin{aligned}
\alpha_1^* &= \frac{\gamma}{2}\left[1 - \frac{\psi(1+\lambda)}{1+\psi-\lambda}\right] = \alpha_3^* \\
\alpha_2^* &= (1-\gamma)\left(1 - \frac{\lambda\psi}{1+\psi-\lambda}\right) \\
\alpha_4^* &= -(1-\gamma)\frac{\psi}{1+\psi-\lambda}
\end{aligned} \tag{36}$$

An important implication of these solutions is that the optimal monetary policy responses replicate the consumption variances obtained under flexible prices. Specifically,

$$\begin{aligned}
\sigma_c^2 &= \left(\frac{\gamma}{2\rho}\right)^2\left(\sigma_u^2 + \sigma_{u^*}^2\right) + \left(\frac{1-\gamma}{\rho}\right)^2 \sigma_v^2, \\
\sigma_{cu} &= \left(\frac{\gamma}{2\rho}\right)\sigma_u^2, \quad \sigma_{cv} = \left(\frac{1-\gamma}{\rho}\right)\sigma_v^2, \quad \sigma_{cu^*} = \left(\frac{\gamma}{2\rho}\right)\sigma_{u^*}^2, \quad \sigma_{cv^*} = 0.
\end{aligned}$$

5.2 Policy Rules and Discretionary Responses to Shocks

5.2.1 Three Cases of Optimal Monetary Policy to Productivity Shocks

As shown in the previous section, the optimal interest rate responses (35) and (36) replicate the fully flexible-price consumption responses to productivity shocks. In what follows, we examine the optimal policy responses under the following three cases: (1) $0 < \psi < \frac{1-\lambda}{\lambda}$; (2) $\psi > \frac{1-\lambda}{\lambda}$; (3) $\psi = \frac{1-\lambda}{\lambda}$.

(1) $0 < \psi < \frac{1-\lambda}{\lambda}$: This case corresponds to the conclusions in DE and DO in the sense that central banks respond to tradable-sector technology shocks by lowering interest

rates, regardless of their country of origin. From (35) and (36),

$$\begin{aligned}
\alpha_1 &= \alpha_1^* = \alpha_3 = \alpha_3^* > 0 \\
\alpha_2 &= \alpha_2^* > 0 \\
\alpha_4 &= \alpha_4^* < 0,
\end{aligned} \tag{37}$$

That is, both central banks cut interest rates in response to u_t and u_t^* shocks, leading to symmetrically aligned expansionary policies that boost consumption in each country. In this case, the sticky price effect dominates the news effect.

On the other hand, central banks respond asymmetrically to nontradable-sector technology shocks. Specifically, each central bank lowers its interest rate in response to a domestic shock, while raising it in response to a foreign shock. This asymmetry arises because domestic shocks generate a sticky price effect, whereas foreign shocks give rise to a news effect. This result holds across all three cases.

(2) $\psi > \frac{1-\lambda}{\lambda}$: This case arises when central banks commit to sufficiently aggressive price responses. In such a setting, optimal monetary policy requires the central banks of both countries to symmetrically raise nominal interest rates in response to tradable-sector productivity shocks, regardless of their country of origin, in order to dampen the excessive consumption responses driven by the news effect.

$$\begin{aligned}
\alpha_1 &= \alpha_1^* = \alpha_3 = \alpha_3^* < 0 \\
\alpha_2 &= \alpha_2^* > 0 \\
\alpha_4 &= \alpha_4^* < 0,
\end{aligned} \tag{38}$$

This result stands in sharp contrast to the predictions of DE and DO in the first case. The same asymmetric policy responses also emerge in the presence of nontradable-sector technology shocks.

(3) $\psi = \frac{1-\lambda}{\lambda}$: The final case involves a fixed policy rule, which yields the following outcome.

$$\begin{aligned}
\alpha_1 &= \alpha_1^* = \alpha_3 = \alpha_3^* = 0 \\
\alpha_2 &= \alpha_2^* = \frac{1-\gamma}{1+\lambda} > 0 \\
\alpha_4 &= \alpha_4^* = -\frac{1-\gamma}{1+\lambda} < 0
\end{aligned} \tag{39}$$

That is, when central banks commit to this rule, they do not respond to tradable-sector productivity shocks, while continuing to respond asymmetrically to nontradable-sector productivity shocks. Rather than implementing discretionary responses to tradable-sector shocks, central banks follow a fixed rule tied to domestic prices ($\psi = \frac{1-\lambda}{\lambda}$), thereby replicating the fully flexible-price equilibrium consumption responses.

One intuition behind the first and second cases is as follows. As shown in equation (26), technology shocks u_t or u_t^* lower p_{t+1} under LCP, which, via the interest rate rule (16), reduces the nominal interest rate by ψ.[14] The magnitude of this response depends on the interaction between price stickiness and the news effect. When ψ is sufficiently high, the nominal interest rate can fall substantially, leading to an overreaction in consumption. To correct this, a welfare-maximizing central bank implements a contractionary policy. This result is consistent with Galí et al. (2003), who document that the Federal Reserve's response to technology shocks during the Volcker–Greenspan era aligns with an optimal monetary policy rule.

5.2.2 Some Simulation Exercise

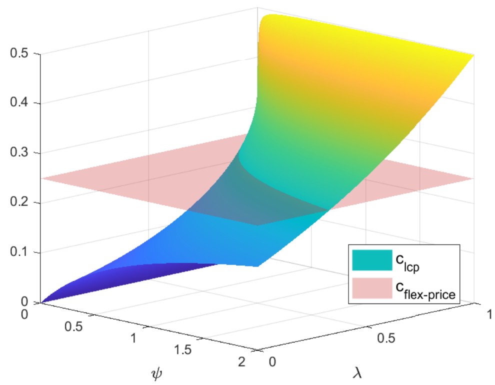
Figure 1 presents the consumption responses to technology shocks under given values of ψ and λ in the LCP setting, alongside the benchmark flexible-price equilibrium in the absence of discretionary monetary policy (i.e., when all α coefficients are set to zero).

Panels (a) and (b) illustrate the responses when technology shocks occur in the tradable goods sector. Panel (a) reports consumption responses under each regime as a function of two key parameters, (ψ, λ), where $\psi > 0$ denotes the central bank's response coefficient to the price level and $\lambda \in [0, 1)$ represents the persistence of technology shocks. Under the flexible-price regime, consumption responses are independent of both coefficients and thus appear as a flat surface. By contrast, under LCP, responses increase with either parameter and eventually surpass the optimal responses under the flexible-price benchmark.

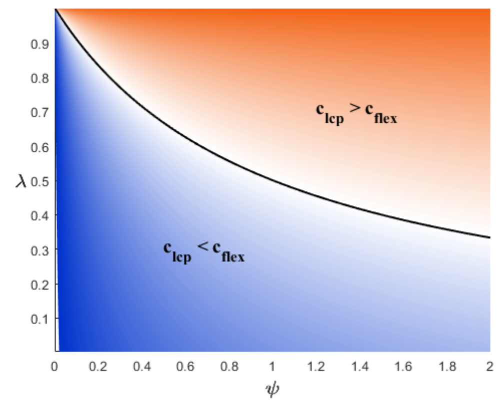
Panel (b) presents this flat-surface flexible-price equilibrium along with (ψ, λ), which determines the boundary condition equating flexible-price and LCP responses in the absence of monetary policy intervention. The northeast region indicates cases where the LCP responses exceed those under the flexible-price regime, that is, when the news effect outweighs the sticky price effect, thereby calling for a contractionary monetary

[14] When a shock u_t occurs, $p_{H,t+1}$ is set at a lower level because u_{t+1} is expected to increase by λu_t at time $t+1$. Likewise, $p_{F,t+1}$ also declines, as the foreign producer f anticipates higher productivity at time $t+1$ through technology diffusion. Consequently, the aggregate price p_{t+1} decreases. Similarly, p_{t+1} declines when a foreign shock u_t^* occurs, because home producers lower $p_{H,t+1}$ in anticipation of technology diffusion, whereas foreign producers also set a lower $p_{F,t+1}$ reflecting λu_t^* at time $t+1$.

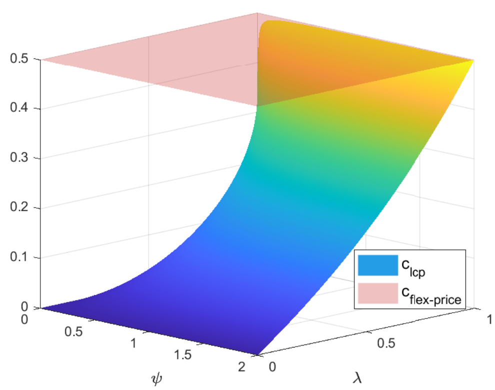
Figure 1: Consumption Responses to Technology Shocks



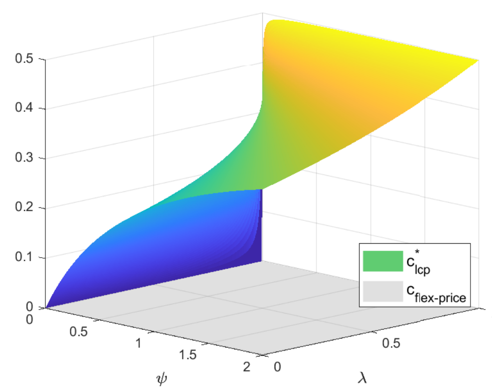
(a) Responses of c_t to u_t and u_t^*



(b) Responses of c_t: Flexible-Price vs. LCP



(c) Responses of c_t to v_t



(d) Responses of c_t to v_t^*

policy. The opposite region corresponds to cases where the sticky price effect dominates the news effect, resulting in an expansionary policy response. Note that these results are independent of γ with respect to this boundary condition.

Panels (c) and (d) present consumption responses to nontradable-sector shocks originating from the home and foreign country, respectively. It should be noted that nontradable-sector shocks generate opposite consumption responses depending on the country of origin of the shocks, in sharp contrast to the responses to tradable-sector productivity shocks. As shown in equations (29) and (30), a domestic nontradable-sector shock gives rise to the sticky price effect, whereas a foreign nontradable-sector shock triggers the news effect.

Because the sticky price effect must be countered with expansionary monetary policy at home, while the news effect requires contractionary policy abroad, nontradable-sector shocks necessarily call for asymmetric policy responses between the home and foreign central banks. As shown in the optimal policy coefficients (35)–(39), a positive home nontradable-sector productivity shock requires a cut in the home nominal interest rate, while the foreign central bank must raise its policy rate. Consequently, domestic nontradable-sector shocks lead to a depreciation of the home currency, whereas foreign nontradable-sector shocks result in its appreciation. By contrast, tradable-sector shocks have no effect on the exchange rate, as both countries align their policy responses symmetrically. This mechanism is examined in more detail in the next section.

6 Exchange Rate Regimes

The preceding optimal interest rate rules have important implications for exchange rate dynamics. When monetary policy responds symmetrically to tradable-sector productivity shocks, the nominal exchange rate remains constant, consistent with the findings of DE. In contrast, nontradable-sector shocks trigger asymmetric policy responses from the home and foreign central banks, leading to divergent interest rate movements that, in turn, generate exchange rate fluctuations, in line with DO's conclusions.

In what follows, we further illustrate these mechanisms by deriving the dynamics of the exchange rate implied by the optimal policy rules. Combining the home and foreign Euler equations, (20) and (21), respectively, with the risk-sharing condition (12), yields the following.

$$S_t = \frac{R_t^*}{R_t} \frac{\mathbb{E}_t(S_{t+1}C_{t+1}^{-\rho})}{\mathbb{E}_t(C_{t+1}^{-\rho})}, \tag{40}$$

where P_{t+1} is known at time t, and therefore cancels out.

By log-linearizing equation (40), replacing i_t and i_t^* with the optimal interest rate responses in (35) and (36), and subsequently applying the risk-sharing condition (12), and realized consumptions (27) and (28), we derive the following expression for the exchange rate.

$$\begin{aligned} s_t = {} & \frac{1}{1+\psi}\mathbb{E}_t s_{t+1} + (1-\gamma)\left[1 + \frac{\psi}{1+\psi}\left(1 - \frac{\psi(1+\lambda)}{1+\psi-\lambda}\right)\right](v_t - v_t^*) \\ & - \frac{(1-\gamma)\psi}{1+\psi}\frac{\lambda\psi}{1+\psi-\lambda}(b_t^* - b_t) \end{aligned} \tag{41}$$

Equation (41) demonstrates our earlier findings, indicating that the nominal exchange rate responds exclusively to productivity shocks in the nontradable sector, whereas shocks in the tradable sector play no role in its determination. Specifically, technological advancement in the nontradable sector causes a depreciation of the originating country's currency, as the other country implements contractionary policies to offset the excessive expansion of its consumption triggered by the news effect.

Note also that when the central banks choose the price responses according to $\psi = \frac{1-\lambda}{\lambda}$, the exchange rate dynamics (41) takes the form,

$$s_t = \lambda \mathbb{E}_t s_{t+1} + (1-\gamma)(v_t - v_t^*) - (1-\gamma)\frac{\lambda(1-\lambda)}{1+\lambda}(b_t^* - b_t), \tag{42}$$

which can be solved forward as follows.

$$s_t = (1-\gamma)\sum_{j=0}^{\infty} \lambda^j \mathbb{E}_t \left[\left(v_{t+j} - v_{t+j}^* \right) - \frac{\lambda(1-\lambda)}{1+\lambda} \left(b_{t+j} - b_{t+j}^* \right) \right] \tag{43}$$

Here, the productivity parameters of the nontradable sector, rather than the tradable sector, enter as the fundamental driving variables.

Within the current framework, exchange rate movements primarily facilitate independent monetary policies. Although exchange rates do not directly serve an expenditure-switching role under LCP, their fluctuations accommodate expenditure-changing interest rate policies in response to nontradable-sector shocks. Specifically, a positive home nontradable-sector shock leads to a home currency depreciation, caused by a decrease in interest rate spread between home and foreign economies. Conversely, a positive foreign nontradable-sector shock leads to a home currency appreciation, consistent with an increase in the home–foreign interest rate differential. Thus, the extent to which optimal monetary policy requires exchange rate flexibility depends critically on the sectoral origin of productivity shocks. If nontradable-sector shocks are infrequent, the benefits of exchange rate adjustment may be limited.

It is also worth noting that to examine the source of asymmetric responses, Obstfeld (2006) decomposes productivity shocks into global and idiosyncratic components, defined respectively as the average of the sum and the average of the difference of the two countries' productivity shocks. He shows that both countries respond identically to global shocks but in opposite directions to idiosyncratic shocks, implying that only the latter drive exchange rate movements. While this decomposition is analytically elegant, it lacks a structural interpretation of shock origins. Moreover, the distinction between global and idiosyncratic shocks becomes blurred when the foreign shock is shut down. In contrast, we define productivity shocks by both country and sector of origin, allowing for clearer identification of the sources of asymmetry and the resulting exchange rate dynamics.

7 Conclusion

This paper develops a sticky price, local currency pricing (LCP) model that allows technology shocks to diffuse across borders. Rather than assuming perfect correlation between technology shocks in the tradable and nontradable sectors within a country, we introduce country- and sector-specific productivity shocks.

Allowing these shocks to diffuse to the corresponding sector in the other country enables consumption to respond even in the absence of changes in fundamentals, generating a news effect that amplifies consumption fluctuations. When this effect outweighs the influence of price stickiness, it triggers a contractionary monetary policy response. This mechanism constitutes a novel feature that is contrary to the predictions of Devereux and Engel (2003) and Duarte and Obstfeld (2008).

We further show that central banks respond identically to shocks originating in the tradable sector, even in the presence of nontraded goods. In contrast, shocks arising in the nontradable sector elicit opposite policy responses across countries, generating interest rate differentials that necessitate exchange rate flexibility. In this way, our model nests the two distinct cases of Devereux and Engel (2003) (DE) and Duarte and Obstfeld (2008) (DO) within a unified framework.

We abstract from the effects of technology shocks on monetary policy and the ensuing exchange rate adjustments under dominant currency paradigm (DCP), a mechanism that has received considerable attention since Gopinath et al. (2020). Under DCP, all export and import prices are sticky in a common dominant currency, such as the U.S. dollar, leading to low pass-through for exchange rate movements against non-dominant currencies and high pass-through for movements against the dominant currency. This asymmetry arises independently of the productivity shocks analyzed in this paper, opening a distinct and rich avenue for further study. We therefore leave its incorporation to future research.

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Online Appendix to "When to Align and When to Contract"

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A Consumer Demand Function

Given market prices, the representative domestic consumer solves the following optimization problem with respect to consumption of domestically produced tradable goods,

$$\underset{C_{H,t}(h)}{\text{Max}} \left(\int_0^1 C_{H,t}(h)^{\frac{\theta-1}{\theta}} dh \right)^{\frac{\theta}{\theta-1}}$$
$$\text{s.t.} \int_0^1 C_{H,t}(h) P_{H,t}(h) dh = Z_{H,t},$$

where $Z_{H,t}$ denotes total nominal expenditure allocated to home produced tradable goods in period t. The first order condition with respect to $C_{H,t}(h)$ gives

$$\lambda_t P_{H,t}(h) = \left(\int_0^1 C_{H,t}(h)^{\frac{\theta-1}{\theta}} dh \right)^{\frac{1}{\theta-1}} C_{H,t}(h)^{-\frac{1}{\theta}}$$

An analogous first order condition holds for the consumption of home tradable goods by another representative household h', denoted $C_{H,t}(h')$.

Combining the first order conditions of consumption of households h and h' gives

$$\left(\frac{P_{H,t}(h)}{P_{H,t}(h')} \right)^{\theta-1} = \left(\frac{C_{H,t}(h')}{C_{H,t}(h)} \right)^{\frac{\theta-1}{\theta}}$$

Rearranging this,

$$C_{H,t}(h')^{\frac{\theta-1}{\theta}} P_{H,t}(h)^{1-\theta} = C_{H,t}(h)^{\frac{\theta-1}{\theta}} P_{H,t}(h')^{1-\theta}$$

Taking integration over h',

$$P_{H,t}(h)^{1-\theta}\int_0^1 C_{H,t}(h')^{\frac{\theta-1}{\theta}}dh' = C_{H,t}(h)^{\frac{\theta-1}{\theta}}\int_0^1 P_{H,t}(h')^{1-\theta}dh'$$
$$\Rightarrow P_{H,t}(h)^{-\theta}C_{H,t} = C_{H,t}(h)P_{H,t}^{-\theta}$$

Therefore,

$$C_{H,t}(h) = \left(\frac{P_{H,t}(h)}{P_{H,t}}\right)^{-\theta} C_{H,t}$$

Likewise,

$$C_{N,t}(h) = \left(\frac{P_{N,t}(h)}{P_{N,t}}\right)^{-\theta} C_{N,t}$$

and

$$C_{F,t}(f) = \left(\frac{P_{F,t}(f)}{P_{F,t}}\right)^{-\theta} C_{F,t}$$

Note that both $C_{T,t}$ and C_t are Armington forms so that

$$C_{H,t} = \frac{1}{2}\frac{P_{T,t}}{P_{H,t}}C_{T,t}, \quad C_{F,t} = \frac{1}{2}\frac{P_{T,t}}{P_{F,t}}C_{T,t}$$

and

$$C_{T,t} = \gamma\frac{P_t}{P_{T,t}}C_t, \; C_{N,t} = (1-\gamma)\frac{P_t}{P_{N,t}}C_T$$

Combining these equations, we get

$$C_{H,t}(h) = \frac{\gamma}{2}\left(\frac{P_{H,t}(h)}{P_{H,t}}\right)^{-\theta}\left(\frac{P_t}{P_{H,t}}\right)C_t$$

$$C_{N,t}(h) = (1-\gamma)\left(\frac{P_{N,t}(h)}{P_{N,t}}\right)^{-\theta}\left(\frac{P_t}{P_{N,t}}\right)C_t$$

and

$$C_{F,t}(f) = \frac{\gamma}{2}\left(\frac{P_{F,t}(f)}{P_{F,t}}\right)^{-\theta}\left(\frac{P_t}{P_{F,t}}\right)C_t$$

Similarly,

$$C^*_{H,t}(h) = \frac{\gamma}{2}\left(\frac{P^*_{H,t}(h)}{P^*_{H,t}}\right)^{-\theta}\left(\frac{P^*_t}{P^*_{H,t}}\right)C^*_t$$

B Price Index for Consumption Goods

Given a consumption index, the consumption-based price index for domestically produced tradable consumption goods, C_H, can be derived from the following minimization problem:

$$\min_{C_H(h)} \int_0^1 P_H(h) C_H(h) dh$$
$$\text{s.t.} \quad \left[\int_0^1 C_H(h)^{\frac{\theta-1}{\theta}} dh \right]^{\frac{\theta}{\theta-1}} = 1$$

The first order condition with respect to $C_H(h)$ gives

$$P_H(h) = \lambda \left[\int_0^1 C_H(h)^{\frac{\theta-1}{\theta}} dh \right]^{\frac{1}{\theta-1}} C_H(h)^{-\frac{1}{\theta}}$$

where λ is the shadow price of one unit of the composite goods C_H. From this,

$$\int_0^1 P_H(h)^{1-\theta} dh = \lambda^{1-\theta} \left[\int_0^1 C_H(h)^{\frac{\theta-1}{\theta}} dh \right]^{-1} \int_0^1 C_H(h)^{\frac{\theta-1}{\theta}} dh = \lambda^{1-\theta}$$

Therefore,

$$\left(\int_0^1 P_H(h)^{1-\theta} dh \right)^{\frac{1}{1-\theta}} = \lambda$$

Hence, the price index for the home tradable goods is,

$$P_H = \left(\int_0^1 P_H(h)^{1-\theta} dh \right)^{\frac{1}{1-\theta}}$$

Similarly, the price index for home non-tradable consumption goods and foreign (imported) tradable consumption goods are

$$P_N = \left(\int_0^1 P_N(h)^{1-\theta} dh \right)^{\frac{1}{1-\theta}}, \; P_F = \left(\int_0^1 P_F(f)^{1-\theta} df \right)^{\frac{1}{1-\theta}}$$

The minimum expenditure problem for tradable goods is

$$\min_{c_H(h), c_F(f)} \int_0^1 P_H(h)C_H(h)dh + \int_0^1 P_F(f)C_F(f)dh$$
$$\text{s.t. } 2\left[\int_0^1 C_H(h)^{\frac{\theta-1}{\theta}}dh\right]^{\frac{\theta}{2(\theta-1)}}\left[\int_0^1 C_F(f)^{\frac{\theta-1}{\theta}}df\right]^{\frac{\theta}{2(\theta-1)}} = 1$$

The first order condition with respect to $C_H(h)$ gives

$$P_H(h) = \lambda\left[\int_0^1 C_H(h)^{\frac{\theta-1}{\theta}}dh\right]^{\frac{-\theta+2}{2(\theta-1)}}\left[\int_0^1 C_F(f)^{\frac{\theta-1}{\theta}}dz\right]^{\frac{\theta}{2(\theta-1)}} C_H(h)^{-\frac{1}{\theta}},$$

From this,

$$\int_0^1 P_H(h)^{1-\theta}dh = \lambda^{1-\theta}\left[\int_0^1 C_H(z)^{\frac{\theta-1}{\theta}}dz\right]^{\frac{\theta}{2}}\left[\int_0^1 C_F(z)^{\frac{\theta-1}{\theta}}dz\right]^{-\frac{\theta}{2}}$$

Similarly, the first order condition with respect to $C_F(f)$ gives

$$\int_0^1 P_F(f)^{1-\theta}df = \lambda^{1-\theta}\left[\int_0^1 C_H(z)^{\frac{\theta-1}{\theta}}dz\right]^{-\frac{\theta}{2}}\left[\int_0^1 C_F(z)^{\frac{\theta-1}{\theta}}dz\right]^{\frac{\theta}{2}}$$

Multiplying these two equations, we obtain

$$\lambda^{2(1-\theta)} = \left(\int_0^1 P_H(h)^{1-\theta}dh\right)\left(\int_0^1 P_F(f)^{1-\theta}df\right)$$

Therefore,

$$\begin{aligned} P_T &= \left(\int_0^1 P_H(h)^{1-\theta}dh\right)^{\frac{1}{2(1-\theta)}}\left(\int_0^1 P_F(f)^{1-\theta}df\right)^{\frac{1}{2(1-\theta)}} \\ &= P_H^{1/2}P_F^{1/2} \end{aligned}$$

Solving a similar minimum expenditure problem for aggregate home consumption goods, we obtain

$$P = P_T^{\gamma}P_N^{1-\gamma}$$

C Determinacy of the Price Level

The intertemporal Euler equation for the home country is given by

$$\frac{C_t^{-\rho}}{P_t} = R_t \beta \mathbb{E}_t \left(\frac{C_{t+1}^{-\rho}}{P_{t+1}} \right)$$

Taking logs of the preceding equation and noting that consumption is lognormally distributed, we have

$$-\rho c_t - p_t = i_t + \log \beta - \rho \mathbb{E}_t c_{t+1} - \mathbb{E}_t p_{t+1} + \frac{\rho^2}{2}\sigma_c^2 + \frac{1}{2}\sigma_p^2 + \rho \sigma_\phi.$$

Substituting the interest rate rule for i_t into the Euler equation, we derive a difference equation with the price level solution:

$$p_t = \sum_{s=t}^{\infty} \left(\frac{1}{1+\psi} \right)^{s+1-t} \rho \left(\mathbb{E}_t \{ c_{s+1} - c_s \} \right) - \frac{1}{\psi} \left(\log \beta + \bar{\imath} + \frac{\rho^2}{2}\sigma_c^2 + \frac{1}{2}\sigma_p^2 + \rho \sigma_{cp} \right).$$

The above equation clearly indicates that ψ must be strictly positive to ensure a unique and stable price-level solution. The same condition applies when the overall price level is predetermined one period in advance.

D Equilibrium Consumption with Flexible Price

$$\frac{W_t}{P_t} C_t^{-\rho} = \kappa = \frac{W_t^*}{P_t^*} C_t^{*-\rho}$$

Using the definitions of the consumption price indices, the first equality in the optimal labor-consumption trade-off condition can be written as

$$C_t^\rho = \frac{W_t}{\kappa P_t} = \frac{W_t}{\kappa \left(\frac{\theta}{\theta-1} \frac{W_t}{A_t} \right)^{\frac{\gamma}{2}} \left(\frac{\theta}{\theta-1} \frac{S_t W_t^*}{A_t^*} \right)^{\frac{\gamma}{2}} \left(\frac{\theta}{\theta-1} \frac{W_t}{B_t} \right)^{1-\gamma}}$$

The last equality holds from the markup pricing rule. Rearranging it gives

$$C_t^\rho = \frac{\theta - 1}{\theta \kappa} \left(\frac{W_t}{S_t W_t^*} \right)^{\frac{\gamma}{2}} A_t^{\frac{\gamma}{2}} B_t^{1-\gamma} A_t^{*\frac{\gamma}{2}}$$

It is straightforward to show $\frac{W_t}{S_t W_t^*} = 1$ by combining the labor-consumption condition with the risk-sharing condition. Therefore,

$$C_t = \left[\left(\frac{\theta - 1}{\theta\kappa} \right) A_t^{\frac{\gamma}{2}} B_t^{1-\gamma} A_t^{*\frac{\gamma}{2}} \right]^{\frac{1}{\rho}}$$

Similarly,

$$C_t^* = \left[\left(\frac{\theta - 1}{\theta\kappa} \right) A_t^{*\frac{\gamma}{2}} B_t^{*1-\gamma} A_t^{\frac{\gamma}{2}} \right]^{\frac{1}{\rho}}$$

E Equilibrium with Sticky Price and Local Currency Pricing Rule

E.1 Pricing equations

We assume that producers set their nominal prices for their goods in local currency one period in advance. For example, the representative home producer h sets the prices $P_{H,t}(h), P_{H,t}^*(h)$, and $P_{N,t}(h)$ at time $t-1$ using all available information, and maintains them for one period.

Taking all aggregate prices and quantities as given, the home agent h solves,

$$\underset{P_{H,t}(h), P_{H,t}^*(h), P_{N,t}(h)}{Max} \; E_{t-1} \left\{ \frac{C_t(h)^{1-\rho}}{1-\rho} - \kappa L_t(h) \right\}$$

subject to the household budget constraint, the consumption demand equations, and the labor demand function,

$$L_t(h) = \frac{Y_{H,t}(h) + Y_{H,t}^*(h)}{A_t} + \frac{Y_{N,t}(h)}{B_t}$$

Under market clearing, plugging the consumption demand functions into household's flow budget constraint gives

$$
\begin{aligned}
C_t(h) &= \frac{P_{H,t}(h)Y_{H,t}(h)}{P_t} + \frac{S_t P^*_{H,t}(h) Y^*_{H,t}(h)}{P_t} + \frac{P_{N,t}(h)Y_{N,t}(h)}{P_t} - \frac{D_{t+1}(h)}{P_t} + \frac{(1+R_{t+1})D_t(h)}{P_t} \\
&= \frac{\gamma}{2}\frac{P_{H,t}(h)}{P_t}\left(\frac{P_{H,t}(h)}{P_{H,t}}\right)^{-\theta}\left(\frac{P_t}{P_{H,t}}\right)C_t + \frac{\gamma}{2}\frac{S_t P^*_{H,t}(h)}{P_t}\left(\frac{P^*_{H,t}(h)}{P^*_{H,t}}\right)^{-\theta}\left(\frac{P^*_t}{P^*_{H,t}}\right)C^*_t \\
&\quad + (1-\gamma)\frac{P_{N,t}(h)}{P_t}\left(\frac{P_{N,t}(h)}{P_{N,t}}\right)^{-\theta}\left(\frac{P_t}{P_{N,t}}\right)C_t - \frac{D_{t+1}(h)}{P_t} + \frac{(1+R_{t+1})D_t(h)}{P_t} \\
&= \frac{\gamma}{2}\frac{C_t}{P^{1-\theta}_{H,t}}P_{H,t}(h)^{1-\theta} + \frac{\gamma}{2}\frac{S_t P^*_t C^*_t}{P_t P^{*1-\theta}_{H,t}}P^*_{H,t}(h)^{1-\theta} + (1-\gamma)\frac{C_t}{P^{1-\theta}_{N,t}}P_{N,t}(h)^{1-\theta} \\
&\quad - \frac{D_{t+1}(h)}{P_t} + \frac{(1+R_{t+1})D_t(h)}{P_t}
\end{aligned}
$$

From the labor demand function,

$$
\begin{aligned}
L_t(h) &= \frac{\gamma}{2}\frac{1}{A_t}\left(\frac{P_{H,t}(h)}{P_{H,t}}\right)^{-\theta}\left(\frac{P_t}{P_{H,t}}\right)C_t + \frac{\gamma}{2}\frac{1}{A_t}\left(\frac{P^*_{H,t}(h)}{P^*_{H,t}}\right)^{-\theta}\left(\frac{P^*_t}{P^*_{H,t}}\right)C^*_t \\
&\quad + (1-\gamma)\frac{1}{t}\left(\frac{P_{N,t}(h)}{P_{N,t}}\right)^{-\theta}\left(\frac{P_t}{P_{N,t}}\right)C_t \\
&= \frac{\gamma}{2}\frac{1}{A_t}\frac{P_t C_t}{P^{1-\theta}_{H,t}}P_{H,t}(h)^{-\theta} + \frac{\gamma}{2}\frac{1}{A_t}\frac{P^*_t C^*_t}{P^{*1-\theta}_{H,t}}P^*_{H,t}(h)^{-\theta} + (1-\gamma)\frac{1}{B_t}\frac{P_t C_t}{P^{1-\theta}_{N,t}}P_{N,t}(h)^{-\theta}
\end{aligned}
$$

Plugging the above two equations into the objective function, the first order conditions with respect to $P_{H,t}(h)$, $P^*_{H,t}(h)$, and $P_{N,t}(h)$ imply

$$
\begin{aligned}
P_{H,t}(h) &: \frac{\gamma(1-\theta)}{2P^{1-\theta}_{H,t}}E_{t-1}\left[C_t(h)^{-\rho}C_t\right]P_{H,t}(h)^{-\theta} = -\frac{\gamma\theta\kappa P_t}{2P^{1-\theta}_{H,t}}E_{t-1}\left[C_t/A_t\right]P_{H,t}(h)^{-\theta-1} \\
P^*_{H,t}(h) &: \frac{\gamma(1-\theta)P^*_t}{2P_t P^{*1-\theta}_{H,t}}E_{t-1}\left[S_t C_t(h)^{-\rho}C^*_t\right]P^*_{H,t}(h)^{-\theta} = -\frac{\gamma\theta\kappa P^*_t}{2P^{*1-\theta}_{H,t}}E_{t-1}\left[C^*_t/A_t\right]P^*_{H,t}(h)^{-\theta-1} \\
P_{N,t}(h) &: \frac{(1-\gamma)(1-\theta)}{P^{1-\theta}_{N,t}}E_{t-1}\left[C_t(h)^{-\rho}C_t\right]P_{N,t}(h)^{-\theta} = -\frac{(1-\gamma)\theta\kappa P_t}{P^{1-\theta}_{N,t}}E_{t-1}\left[C_t/B_t\right]P_{N,t}(h)^{-\theta-1}
\end{aligned}
$$

Finally,

$$P_{H,t}(h) = \frac{\theta\kappa}{\theta-1}\frac{P_t E_{t-1}\left[C_t/A_t\right]}{E_{t-1}\left[C_t(h)^{-\rho}C_t\right]} \tag{E1.1}$$

$$P^*_{H,t}(h) = \frac{\theta\kappa}{\theta-1}\frac{P_t E_{t-1}\left[C^*_t/A_t\right]}{E_{t-1}\left[S_t C_t(h)^{-\rho}C^*_t\right]} \tag{E1.2}$$

$$P_{N,t}(h) = \frac{\theta\kappa}{\theta-1}\frac{P_t E_{t-1}\left[C_t/B_t\right]}{E_{t-1}\left[C_t(h)^{-\rho}C_t\right]} \tag{E1.3}$$

Assuming a symmetric equilibrium gives pricing equations (22)-(24) in section 4.

E.2 Equilibrium consumption

Using price indexes definitions (6) and (7), equation (22) for $P_{H,t}$ can be rewritten as

$$\frac{P_{H,t}^{1-\frac{\gamma}{2}}}{P_{F,t}^{\frac{\gamma}{2}}} = \frac{\theta\kappa}{\theta-1}\frac{P_{N,t}^{1-\gamma}E_{t-1}\left[C_t/A_t\right]}{E_{t-1}\left[{C_t}^{1-\rho}\right]} \tag{E2.1}$$

Taking the ratio of $P_{H,t}$ and $P_{N,t}$ gives

$$\frac{P_{H,t}}{P_{N,t}} = \frac{E_{t-1}\left[C_t/A_t\right]}{E_{t-1}\left[C_t/B_t\right]} \tag{E2.2}$$

Note that, unlike Obstfeld (2004) and others, the relative price of the home tradable goods to nontradable goods is not one in general. Using (E2.2), (E2.1) can be rewritten as,

$$\frac{P_{H,t}}{P_{F,t}} = \left(\frac{\theta\kappa}{\theta-1}\right)^{\frac{2}{\gamma}}\frac{\left(E_{t-1}\left[C_t/A_t\right]\right)^2\left(E_{t-1}\left[C_t/B_t\right]\right)^{\frac{2(1-\gamma)}{\gamma}}}{\left(E_{t-1}\left[{C_t}^{1-\rho}\right]\right)^{\frac{2}{\gamma}}}$$

Taking the ratio of $P_{F,t}$ and $P_{H,t}$ gives

$$\frac{P_{F,t}}{P_{H,t}} = \left(\frac{\theta\kappa}{\theta-1}\right)^{\frac{2}{2-\gamma}}\frac{\left(E_{t-1}\left[C_t/A^*_t\right]\right)^{\frac{2}{2-\gamma}}\left(E_{t-1}\left[C_t/B_t\right]\right)^{\frac{2(1-\gamma)}{2-\gamma}}}{\left(E_{t-1}\left[C_t/A_t\right]\right)^{\frac{2(1-\gamma)}{2-\gamma}}\left(E_{t-1}\left[{C_t}^{1-\rho}\right]\right)^{\frac{2}{2-\gamma}}}$$

$\frac{P_{H,t}}{P_{F,t}} \times \frac{P_{F,t}}{P_{H,t}}$ gives

$$1 = \left(\frac{\theta\kappa}{\theta-1}\right)^{\frac{4}{\gamma(2-\gamma)}}\frac{\left(E_{t-1}\left[C_t/A_t\right]\right)^{\frac{2}{2-\gamma}}\left(E_{t-1}\left[C_t/B_t\right]\right)^{\frac{4(1-\gamma)}{\gamma(2-\gamma)}}\left(E_{t-1}\left[C_t/A^*_t\right]\right)^{\frac{2}{2-\gamma}}}{\left(E_{t-1}\left[{C_t}^{1-\rho}\right]\right)^{\frac{4}{\gamma(2-\gamma)}}}$$

Log normality implies

$$E_{t-1}c_t = \frac{1}{\rho}\ln\left(\frac{\theta-1}{\theta\kappa}\right) + \frac{\gamma}{2\rho}\left(E_{t-1}a_t + E_{t-1}a_t^* + \sigma_{cu} + \sigma_{cu^*} - \frac{1}{2}\sigma_u^2 - \frac{1}{2}\sigma_{u^*}^2\right) \\ + \frac{1-\gamma}{\rho}\left(E_{t-1}b_t + \sigma_{cv} - \frac{1}{2}\sigma_v^2\right) - \frac{2-\rho}{2}\sigma_c^2 \tag{E2.3}$$

where σ_x^2 denotes the variance of shock x with $x \in \{u, u^*, v, v^*\}$ and σ_{cx} denotes the covariance between consumption and each respective shock.

The Euler equation is

$$\frac{C_t^{-\rho}}{P_t} = (1+i_t)\beta\mathbb{E}_t\left(\frac{C_{t+1}^{-\rho}}{P_{t+1}}\right)$$

Since P_{t+1} is known at time t, the log of the Euler equation is

$$c_t = \mathbb{E}_t c_{t+1} - \frac{1}{\rho}\left[\log\beta + i_t - (p_{t+1} - p_t) + \frac{\rho^2}{2}\sigma_c^2\right] \tag{E2.4}$$

Plug the interest rate rule into (E2.4), and take expectations at time $t-1$, we obtain

$$p_t = \frac{1}{1+\psi}E_{t-1}p_{t+1} + \frac{1}{1+\psi}\left[\rho(E_{t-1}c_{t+1} - E_{t-1}c_t) - \left(\log\beta + \iota + \frac{\rho^2}{2}\sigma_c^2\right)\right] \tag{E2.5}$$

Solving (E2.5) forward, we get

$$p_t = \rho\sum_{j=0}^{\infty}\left(\frac{1}{1+\psi}\right)^{j+1}E_{t-1}(c_{t+j+1} - c_{t+j}) - \frac{1}{\psi}\left(\log\beta + \iota + \frac{\rho^2}{2}\sigma_c^2\right) \tag{E2.6}$$

Using equation (E2.3), the difference between $E_{t-1}c_{t+j+1}$ and $E_{t-1}c_{t+j}$ is

$$E_{t-1}(c_{t+j+1} - c_{t+j}) = \frac{\gamma}{2\rho}E_{t-1}(a_{t+j+1} - a_{t+j} + a_{t+j+1}^* - a_{t+j}^*) + \frac{1-\gamma}{\rho}E_{t-1}(b_{t+j+1} - b_{t+j}) \tag{E2.7}$$

The stochastic processes of technologies imply the following conditional expectations at time $t1$.

$$E_{t-1}a_t = \lambda a_{t-1} + u_{t-1}^*,\ E_{t-1}a_t^* = \lambda a_{t-1}^* + u_{t-1} \\ E_{t-1}b_t = \lambda b_{t-1} + v_{t-1}^*,\ E_{t-1}b_t^* = \lambda b_{t-1}^* + v_{t-1} \tag{E2.8}$$

Plug (E2.8) into (E2.7) and combining the result with equation (E2.6) gives

$$p_t = \frac{\gamma}{2}\sum_{j=0}^{\infty}\left(\frac{\lambda}{1+\psi}\right)^{j+1}(\lambda-1)(a_{t-1}+a^*_{t-1}) + (1-\gamma)\sum_{j=0}^{\infty}\left(\frac{\lambda}{1+\psi}\right)^{j+1}(\lambda-1)b_{t-1}$$
$$+\frac{\gamma}{2}\sum_{j=0}^{\infty}\left(\frac{\lambda}{1+\psi}\right)^{j+1}\left(\frac{\lambda-1}{\lambda}\right)(u_{t-1}+u^*_{t-1}) + (1-\gamma)\sum_{j=0}^{\infty}\left(\frac{\lambda}{1+\psi}\right)^{j+1}\left(\frac{\lambda-1}{\lambda}\right)v^*_{t-1}$$
$$-\frac{1}{\psi}\left(\log\beta + \iota + \frac{\rho^2}{2}\sigma_c^2\right)$$

Solving the above equation gives

$$\begin{aligned} p_t = &-\frac{\gamma\lambda(1-\lambda)}{2(1+\psi-\lambda)}(a_{t-1}+a^*_{t-1}) - \frac{(1-\gamma)\lambda(1-\lambda)}{1+\psi-\lambda}b_{t-1} \\ &-\frac{\gamma(1-\lambda)}{2(1+\psi-\lambda)}(u_{t-1}+u^*_{t-1}) - \frac{(1-\gamma)(1-\lambda)}{1+\psi-\lambda}v^*_{t-1} \\ &-\frac{1}{\psi}\left(\log\beta + \iota + \frac{\rho^2}{2}\sigma_c^2\right) \end{aligned} \tag{E2.9}$$

Note that the home price index p_t will be lowered by a positive shock u_{t-1} at time $t-1$ via two channels, $p_{H,t}$ and $p_{F,t}$. First of all, $p_{H,t}$ will be set at a lower level because the home tradables-sector productivity will increase by λu_{t-1} at time t. At the same time, $p_{F,t}$ will decline because the foreign tradables-sector productivity will fully incorporate u_{t-1} at time t due to technology diffusion.

Next, plug the interest rate rule into (E2.4), and take expectations at time t, we obtain

$$c_t = \mathbb{E}_t c_{t+1} - \frac{1}{\rho}[\ln\beta + \iota - p_{t+1} + (1+\psi)p_t - \alpha_1 u_t - \alpha_2 v_t - \alpha_3 u^*_t - \alpha_4 v^*_t + \frac{\rho^2}{2}\sigma_c^2] \tag{E3.0}$$

Updating (E2.3) once gives

$$\begin{aligned} \mathbb{E}_t c_{t+1} = &\frac{1}{\rho}\ln\left(\frac{\theta-1}{\theta\kappa}\right) + \frac{\gamma}{2\rho}\left(\mathbb{E}_t a_{t+1} + \mathbb{E}_t a^*_{t+1} + \sigma_{cu} + \sigma_{cu^*} - \frac{1}{2}\sigma_u^2 - \frac{1}{2}\sigma_{u^*}^2\right) \\ &+\frac{1-\gamma}{\rho}\left(\mathbb{E}_t b_{t+1} + \sigma_{cv} - \frac{1}{2}\sigma_v^2\right) - \frac{2-\rho}{2}\sigma_c^2 \end{aligned} \tag{E3.1}$$

Plug (E3.1) into (E3.0) and rearrange it,

$$c_t = \frac{\gamma}{2\rho}(\mathbb{E}_t a_{t+1} + \mathbb{E}_t a^*_{t+1}) + \frac{1-\gamma}{\rho}\mathbb{E}_t b_{t+1} + \frac{1}{\rho}p_{t+1} - \frac{1+\psi}{\rho}p_t \\ + \frac{1}{\rho}(\alpha_1 u_t + \alpha_2 v_t + \alpha_3 u^*_t + \alpha_4 v^*_t) + \nabla \tag{E3.2}$$

where ∇ is a function of parameters and unconditional moments. Take expectations at time t, stochastic processes of technology become

$$\mathbb{E}_t a_{t+1} = \lambda a_t + u^*_t,\ \mathbb{E}_t a^*_{t+1} = \lambda a^*_t + u_t,\ \mathbb{E}_t b_{t+1} = \lambda b_t + v^*_t \tag{E3.3}$$

Plug (E3.3) and the equation for p_{t+1} (obtained by updating (E2.9) once) into (E3.2), we obtain the realized (log) equilibrium consumption in the home country can be expressed as a function of contemporaneous shocks:

$$c_t = \frac{\gamma\lambda\psi}{2\rho(1+\psi-\lambda)}(a_t + a^*_t) + \frac{(1-\gamma)\lambda\psi}{\rho(1+\psi-\lambda)}b_t, \\ + \frac{\gamma\psi}{2\rho(1+\psi-\lambda)}(u_t + u^*_t) + \frac{(1-\gamma)\psi}{\rho(1+\psi-\lambda)}v^*_t \\ + \frac{1}{\rho}(\alpha_1 u_t + \alpha_2 v_t + \alpha_3 u^*_t + \alpha_4 v^*_t) + \tilde{\nabla} \tag{E3.4}$$

where $\tilde{\nabla}$ denotes a function of parameters, unconditional moments, and variables dated $t-1$. Foreign consumption can be similarly obtained as

$$c^*_t = \frac{\gamma\lambda\psi}{2\rho(1+\psi-\lambda)}(a_t + a^*_t) + \frac{(1-\gamma)\lambda\psi}{\rho(1+\psi-\lambda)}b^*_t, \\ + \frac{\gamma\psi}{2\rho(1+\psi-\lambda)}(u_t + u^*_t) + \frac{(1-\gamma)\psi}{\rho(1+\psi-\lambda)}v_t \\ + \frac{1}{\rho}(\alpha^*_1 u^*_t + \alpha^*_2 v^*_t + \alpha^*_3 u_t + \alpha^*_4 v_t) + \tilde{\nabla}^* \tag{E3.5}$$

F Optimal Interest Rate Rule

Home labor supply must be consistent with the following condition

$$\mathbb{E}_t L_{t+1} = \frac{\gamma}{2}\left(\frac{P_{t+1}}{P_{H,t+1}}\right)\mathbb{E}_t\left(\frac{C_{t+1}}{A_{t+1}}\right) + \frac{\gamma}{2}\left(\frac{P^*_{t+1}}{P^*_{H,t+1}}\right)\mathbb{E}_t\left(\frac{C^*_{t+1}}{A_{t+1}}\right) + (1-\gamma)\left(\frac{P_{t+1}}{P_{N,t+1}}\right)\mathbb{E}_t\left(\frac{C_{t+1}}{B_{t+1}}\right) \tag{F1}$$

Plug pricing equations (E1.1)$\sim$(E1.3) into (F1), we obtain

$$\mathbb{E}_t L_{t+1} = \left(\frac{\theta - 1}{\theta\kappa}\right)\left\{\left(1 - \frac{\gamma}{2}\right)\mathbb{E}_t C_{t+1}^{1-\rho} + \frac{\gamma}{2}\mathbb{E}_t C_{t+1}^{*1-\rho}\right\} \tag{F2}$$

Plugging (F2) into the home expected utility at time t, and rearrange it, we get

$$\mathbb{E}_t\left[\frac{C_{t+1}^{1-\rho}}{1-\rho} - \kappa L_{t+1}\right] = \mathbb{E}_t\left[\left(\frac{\theta\gamma + 2 - \gamma}{2\theta(1-\rho)}\right)\mathbb{E}_t C_{t+1}^{1-\rho} - \frac{\gamma(\theta - 1)}{2\theta}\mathbb{E}_t C_{t+1}^{*1-\rho}\right] \tag{F3}$$

As we can see in consumption equation (E3.4), the foreign monetary intervention doesn't affect home consumption. Thus, it is sufficient to maximize the following

$$\mathbb{E}_t C_{t+1}^{-\rho} = \exp\left[(1-\rho)\mathbb{E}_t c_{t+1} + \frac{(1-\rho)^2}{2}\sigma_c^2\right]$$

where the last equality holds due to the log normality assumption. Or, more simply,

$$Max_\alpha\left\{\mathbb{E}_t c_{t+1} + \frac{1-\rho}{2}\sigma_c^2\right\}$$

Using (E3.1),

$$\mathbb{E}_t c_{t+1} + \frac{1-\rho}{2}\sigma_c^2 = \frac{\gamma}{2\rho}\left(\sigma_{cu} + \sigma_{cu^*}\right) + \frac{1-\gamma}{\rho}\sigma_{cv} - \frac{1}{2}\sigma_c^2 + \mathrm{NP} \tag{F4}$$

where NP denotes a function of non-policy variables. Using the covariance equations from Section 5.1 of the main draft, we can express (F4) as a function of policy coefficients and unconditional moments of shocks. So the maximization problem collapses down to the following

$$Max_\alpha\left\{A_1\left(\frac{\gamma}{2\rho} - \frac{1}{2}A_1\right)\sigma_u^2 + A_2\left(\frac{1-\gamma}{\rho} - \frac{1}{2}A_2\right)\sigma_v^2 + A_3\left(\frac{\gamma}{2\rho} - \frac{1}{2}A_3\right)\sigma_{u^*}^2 - \frac{1}{2}A_4^2\sigma_{v^*}^2\right\}$$

A straightforward optimization with respect to the policy parameters yields the optimal monetary policy coefficients.

G Exchange Rate with Optimal Interest Rate Rule

Combining the home and foreign Euler equations we get

$$\frac{C_t^{-\rho}}{C_t^{*-\rho}}\frac{P_t^*}{P_t} = \frac{R_t}{R_t^*}\frac{\mathbb{E}_t\left(C_{t+1}^{-\rho}/P_{t+1}\right)}{\mathbb{E}_t\left(C_{t+1}^{*-\rho}/P_{t+1}^*\right)} \tag{G1}$$

Using the risk sharing condition, equation (G1) can be rewritten as

$$\frac{1}{S_t} = \frac{R_t}{R_t^*}\frac{\mathbb{E}_t\left[C_{t+1}^{-\rho}/P_{t+1}\right]}{\mathbb{E}_t\left[S_{t+1}C_{t+1}^{-\rho}/P_{t+1}\right]} \tag{G2}$$

Given the fact that P_{t+1} and P_{t+1}^* are known at time t in our model, equation (G2) can be simplified to

$$S_t = \frac{R_t^*}{R_t}\frac{\mathbb{E}_t\left(S_{t+1}C_{t+1}^{-\rho}\right)}{\mathbb{E}_t\left(C_{t+1}^{-\rho}\right)} \tag{G3}$$

Log linearize equation (G3), we obtain

$$s_t = i_t^* - i_t + \mathbb{E}_t s_{t+1} \tag{G4}$$

According to the interest-rate rule, we have

$$i_t^* - i_t = \psi(p_t^* - p_t) + (\alpha_2 - \alpha_4^*)v_t + (\alpha_4 - \alpha_2^*)v_t^* \tag{G5}$$

Taking the log of the risk sharing condition and combining it with equations (G4) and (G5), we obtain

$$s_t = -\frac{\psi}{1+\psi}\rho(c_t^* - c_t) + \frac{1}{1+\psi}(\alpha_2 - \alpha_4^*)v_t + \frac{1}{1+\psi}(\alpha_4 - \alpha_2^*)v_t^* + \frac{1}{1+\psi}\mathbb{E}_t s_{t+1} \tag{G6}$$

Using the consumption innovations in (E3.4) and (E3.5), we have

$$c_t^* - c_t = \frac{(1-\gamma)\lambda\psi}{\rho(1+\psi-\lambda)}(b_t^* - b_t) + \frac{(1-\gamma)\psi}{\rho(1+\psi-\lambda)}(v_t - v_t^*) + \frac{1}{\rho}[(\alpha_2^* - \alpha_4)v_t^* + (\alpha_4^* - \alpha_2)v_t] \tag{G7}$$

Plugging (G7) into (G6), we obtain

$$s_t = -\frac{\psi}{1+\psi}\left[\frac{(1-\gamma)\lambda\psi}{1+\psi-\lambda}(b_t^* - b_t) + \frac{(1-\gamma)\psi}{1+\psi-\lambda}(v_t - v_t^*)\right] - [(\alpha_4^* - \alpha_2)v_t + (\alpha_2^* - \alpha_4)v_t^*] + \frac{1}{1+\psi}\mathbb{E}_t s_{t+1}$$

Finally, substituting the optimal monetary policy parameters for $\alpha_2, \alpha_2^*, \alpha_4, \alpha_4^*$, the exchange rate under optimal monetary policies is given by

$$\begin{aligned} s_t = & \frac{1}{1+\psi}\mathbb{E}_t s_{t+1} + (1-\gamma)\left[1 + \frac{\psi}{1+\psi}\left(1 - \frac{\psi(1+\lambda)}{1+\psi-\lambda}\right)\right](v_t - v_t^*) \\ & - \frac{(1-\gamma)\psi}{1+\psi}\frac{\lambda\psi}{1+\psi-\lambda}(b_t^* - b_t) \end{aligned}$$