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Total Factor Productivity with Energy as a Factor of Production

Nazif Durmaz[*], Farhad Rassekh[†], and Henry Thompson[‡]

[*]Kean University; [†]University of Hartford; [‡]Auburn University

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Nazif Durmaz* Farhad Rassekh† Henry Thompson‡

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Abstract

The present paper estimates total factor productivity TFP for the United States with 74 years of data including primary energy as a factor of production along with fixed capital assets and the labor force. The inclusion of energy improves the empirics of the neoclassical production function. In differences of natural logs, energy doubles the explanatory power and reduces residual correlation as well as heteroscedasticity. In addition, energy reduces the mean and variance of the Solow residual leading to a slower cumulative effect. In the growth accounting literature, to calculate TFP, the weight of 0.3 is commonly assigned to capital and 0.7 to labor. Our estimation suggests that these weights should be adjusted to make room for an energy weight of 0.07.

Keywords: economic growth, TFP, Energy

JEL Codes: D24, E23, O47

*Department of Marketing, Global Business & Economics, Kean University, Union NJ 07083, ndurmaz@kean.edu

†Department of Business Insight, University of Hartford, West Hartford CT 06117, rassekh@hartford.edu

‡Department of Economics, 125 Miller Hall, Auburn University AL 36849 henry.thompson@auburn.edu

The neoclassical Solow (1957) growth model based on capital and labor accounts for only a portion of economic growth; the remining portion is captured by the Solow residual or total factor productivity TFP. Gordon (2016) (p. 707) explains TFP as:

> *...any source of growth not captured by the measured quantity and quality of labor and capital, including not only innovation and technological change, but such other factors as the movements of workers from the farm to the city and the shift of production from industries having low productivity to those having higher productivity.*

The US Bureau of Labor Statistics (BLS, 2024, p. 9) notes that TFP is "designed to measure the joint influences of technological change, efficiency improvements, return to scale, reallocation of resources, and other factors on economic growth, allowing for the effects of capital and labor." Technology is singled out as the most important component of TFP. Gordon (2016) (p. 546) calls TFP "the best available measure of innovation and technological advance." Aghion and Howitt (2009) (p. 7) make the point that "innovation is a vital source of long-run growth."

The role of technology is apparent reflecting back to the industrial revolution and scientific advances over the past century. Hulten (2001) (p. 40) lists "organizational and institutional changes, shifts in societal attitudes, fluctuations in demand, changes in factor shares, omitted variables, and measurement errors. The residual should *not* be equated with technical change, although it often is" (emphasis original). A change in the capital/labor ratio causes movement along the growth function while other influences shift the curve. Hulten further notes that "the residual ... generally understates the importance of productivity change in stimulating the growth of output because the shift in the function generally induces further movement along the function as capital increases.

The present paper shows that including primary energy as a factor of production alters TFP. The estimates in first differences of natural logs with 74 years of US data treat total Btu energy as a factor of production along with fixed capital assets and the full time equivalent labor force. The familiar time series issues of residual correlation and heteroskedasticity subside including energy input. Energy doubles explanatory power of the estimates and lowers the mean and variance of the Solow residual series. Energy as the third factor of production leads to a more reliable measure of TFP and a white noise residual.

Section 1 reviews the related literature on TFP. Section 2 presents the neoclassical theory of production and growth including energy with capital and labor. Section 3 reports the estimates of TFP and the Solow residual series followed by the Conclusion 4.

1 A Review of the TFP Literature

The neoclassical production function $Y = Af(K, L)$ includes output Y, capital K, and labor L all implicitly at time t. The term $A = Y/f(K, L)$, referred to as total factor productivity (TFP), is the inverse of the share of output not due to the inputs. Gordon (2016) (p. 546) refers to a "mechanical" calculation of TFP as "output divided by a weighted average of labor and capital input with standard weights of 0.7 for labor and 0.3 for capital," assuming Cobb-Douglas production. The share of labor in US income is now closer to 0.6. The present results suggest the energy share of 0.07 should be included as well.

Solow (1957) pioneered measuring TFP for the US, examining the period 1910–1950 and finding capital per worker accounts for only 13% of growth in output per worker (income per capita). Gordon (2016) (p. 16) adds the level of education over three periods (1890–1920, 1920–1970, 1970–2014) commenting:

> Because the contributions of education and capital deepening were roughly the same in each of the three intervals, all the faster growth of labor productivity in the middle period (1920–1970) is the result of more rapid innovation and technological change.

Gordon calculates TFP for every decade over 1900–2014, presenting a bar chart with a steadily rising TFP from 1910 that peaks at nearly 3.5% annual growth during the 1940s. Gordon (2016) (p. 547) notes that "Labor productivity and TFP soared during World War II, and the cessation of defense production did not prevent the wartime productivity from becoming permanent."

Recently, the BLS (2024) calculated TFP for 1990–2024, accounting for the growth of capital with hours worked and labor composition. Subtracting the contribution of the inputs yields a measure of TFP during different periods. During 1990–2000, output grew annually at 4% with about a quarter of that attributed to TFP. This low value is surprising given the rise of information technology during the 1990s. In three other periods, the contribution of TFP was higher: 46% during 2000–2007, 27% for 2007–2019, and 35% during 2019–2024. Gordon (2016) reports similar TFP measures over these decades.

Aghion and Howitt (2009) calculate the contributions of TFP to output growth for individual OECD countries over the period 1960–2000. They find the share of growth due to TFP ranges from 55% in Spain to 86% in Greece. OECD countries had 68% of output growth due to TFP. Human capital contributes to growth, diminishing TFP as found by Jorgenson and Fraumeni (1981) for the US during 1948–86.

Abramovitz (1956) refers to TFP as a "measure of our ignorance." Jorgenson and Griliches (1967) make the point that TFP would disappear if we could identify and account for all influences on growth. Some variables are difficult or impossible to quantify. Phelps (2013) (p. ix) refers to "indigenous innovations" as "adoption of new methods or goods stemming from homegrown ideas originating in the national economy itself." Lipsey and Carlaw (2004) argue that TFP measures "only the super-normal returns to investing" or "returns that exceed the full opportunity cost of the activity." They also uncover issues for measuring TFP that include output response timing, R&D as a component of national income, and omitted resource inputs.

Analysis of the importance of energy to economic growth starts with Jorgenson and Griliches (1967), highlighting input quality. Berndt and Wood (1979) analyze the factor price elasticities, including energy with capital and labor. Jorgenson and Fraumeni (1981) discuss how productivity adjusted during the energy crisis when energy prices tripled after decades of monopsony price control. Saunders (1992) finds increased energy input improves efficiency, promoting investment that in turn raises energy input.

In the first of three studies relating energy to economic growth, Stern (1993) finds a direct relationship between energy input and TFP in US manufacturing. Stern (2004) notes energy is a key input affecting the efficiency of other inputs in US manufacturing. Stern (2011) argues that improvements in energy accessibility, technology, and fuel quality improving energy efficiency ease the constraint of energy resources on growth.

Popp (2001) finds investment in renewable energy leads to faster economic growth in a study of 450 US manufacturing industries during 1958–91. Zhang and Cheng (2009) compare energy productivity across regions in panel data for China during 1960–2007, finding regional differences and improved energy efficiency in coastal regions. Acemoglu et al. (2012) highlight enhanced productivity due to green energy in world energy data during 2002–06. Mulder and de Groot (2012) analyze national accounts data on energy intensity for 18 OECD countries and 50 sectors during 1970–

2005, finding energy intensity declined more in manufacturing than services. In OECD data for five energy-intensive manufacturing sectors, Liddle (2012) applies panel cointegration with Cobb-Douglas production to show energy quality plays a role. In panel data from 30 OECD countries during 2000–09, Chou et al. (2014) examine the impact of information technology (IT), finding a positive link to TFP. Huang et al. (2019) study the effects of technological factors on TFP in China in a panel of 30 provinces during 2000–14, finding domestic R&D is the main driver of TFP growth along with technology spillovers due to trade openness. Wang et al. (2020) find similar results in 41 major economies during 2005–14, finding capital-energy substitution and technological progress stimulate TFP.

Developing countries especially depend on energy for economic growth. Alam et al. (2016) find energy efficiency in India during 1971–2013 raises productivity. Santos et al. (2021) find energy efficiency in Portugal during 1960–2014 emerges as a unit elastic driver of economic growth. Hasanov and Mikayilov (2021) find TFP lowers energy consumption across 32 high-, 12 middle-, and 5 lower-income countries during 1990–2019. Rehman and Islam (2023) analyze the relationship between energy and TFP in 67 countries during 1990–2019, finding energy affects TFP more in higher-income countries.

2 Neoclassical Production and TFP

Economic growth based on Cobb-Douglas (CD) production $Y = AKL^{\alpha}$ has constant returns to scale (CRS) if $\alpha + \beta = 1$. Competitive factor markets imply marginal products equal factor prices: $Y_K = \frac{\partial Y}{\partial K} = \alpha A K^{\alpha-1} L^{\beta} = r$ for capital and $Y_L = w$ for labor. The coefficients are factor shares of income as $\frac{\partial \ln Y}{\partial \ln K} = \frac{Y_K K}{Y} = \alpha$ and β, with factor payments exhausting output $Y = rK + wL$. Empirically, $A = Y/K^{\alpha}L^{\beta}$, referred to as TFP, is typically greater than 1 due to unexplained output. The inverse $1/A$ is the share of output not captured by $K^{\alpha}L^{\beta}$. Estimation of the log-linear production function

$$\ln Y = \alpha_0 + \alpha \ln K + \beta \ln L + \epsilon$$

expands the production function to $Y = AK^{\alpha}L^{\beta}e^{\epsilon}$. Total factor productivity includes the residual

$$\ln TFP = \ln Y - \alpha \ln K - \beta \ln L = \alpha_0 + \epsilon$$

leading to the Solow residual $S = \exp(\alpha_0 + \epsilon) = Ae^{\epsilon}$. The present estimates of $\ln Y$ have high residual correlation, leading to estimation in first differences:

$$\Delta \ln Y = \alpha_0 + \alpha \Delta \ln K + \beta \Delta \ln L + \epsilon \tag{1}$$

Including energy E as a factor of production in $Y = AK^{\alpha}L^{\beta}E^{\gamma}e^{\epsilon}$ expands the estimate to

$$\Delta \ln Y = \alpha_0 + \alpha \Delta \ln K + \beta \Delta \ln L + \gamma \Delta \ln E + \epsilon \tag{2}$$

A competitive energy market implies $Y_E = \frac{\partial Y}{\partial E} = \gamma$ for energy price, leading to $\gamma = E$. Factor payments exhaust output in $rK + wL + p_E E = (\alpha + \beta + \gamma)Y$ given CRS. The physical production function $Y = A(KL)^{\delta}(KE)^{\delta}$ in Thompson (2016) is motivated by physics, with both labor and energy providing force for work. In log differences,

$$\Delta \ln Y = \alpha_0 + \delta(\Delta \ln K + \Delta \ln L) + \delta(\Delta \ln K + \Delta \ln E) + \epsilon \tag{3}$$

Factor payments exhaust output $rK + wL + p_E E = 2\delta + \epsilon\text{Y}$, assuming CRS and the condition $\alpha + \beta = 1/2$. Factor shares are $\alpha_K = \alpha + \delta$, $\alpha_L = \delta$, and $\alpha_E = \delta$. The present estimate of (3) provides a tighter fit than (1) or (2) and has the smallest mean and lowest variance in the derived Solow residual. Constant returns are not rejected as a null hypothesis in the present estimates of (1)–(3). Constraining the estimates to CRS weakens the results somewhat, leading to very similar coefficients. The Solow residual series $S = \exp(\alpha_0 + \epsilon)$ are compared. The present estimated coefficients relate directly to the per capita growth function. For capital and labor, $y = Y/L = AKL^{-1} = Ake^{\epsilon}$ where $k = K/L$ is estimated as $\ln y = \alpha_0 + \alpha \ln k + \epsilon$ leading to coefficients in (1) given CRS. The growth function adds energy per capita $n = E/L$ expanding to $y = Akne^{\epsilon}$ assuming CRS with $\alpha = 1 - \beta - \gamma$. The estimates of $\ln y = \alpha_0 + \alpha \ln k + \gamma \ln n + \epsilon$ and (2) constrained to CRS are identical. The physical production function (3) constrained for CRS leads to the same estimated coefficients as its implied growth function $y - 0.5 \ln k = \alpha_0 + \delta \ln n + \epsilon$. Means of the derived series $S = \exp(\alpha_0 + \epsilon)$ from (1)–(3) equal the estimated $A = \exp(\alpha_0)$ due to the white noise residuals. The mean and variance of the Solow residual series are lowest for the physical production function.

3 Estimated Production and Derived Cumulative Solow Residuals

Figure 1 shows the variables relative to means in (1)–(3) over the period 1949–2019. National income Y and the cost of fixed capital assets K from the Federal Reserve Economic Data (FRED) both steadily increase. The full-time equivalent labor force L from FRED grows at a slower rate. Total Btu energy input E from the Energy Information Agency (EIA) has the highest variation, increasing at a rising rate until the 1973 energy crisis before slowing to a steady positive trend. For reference, symbols of the variables are listed in Appendix 4.

* **Figure 1** *

Figure 2 shows plots of percentage changes that have means (std dev) of $\Delta \ln Y = 0.031$ (0.023), $\Delta \ln K = 0.030$ (0.012), $\Delta \ln L = 0.013$ (0.022), and $\Delta \ln E = 0.014$ (0.032). The average growth rates of output and capital are nearly equal with about twice the variation in output. The growth rate of labor is half as large with the variation of output. Energy input has the same growth rate as labor but the highest variation, accounting for output variation.

* **Figure 2** *

The per capita growth variables have means (std dev) of $\Delta \ln y = 0.018$ (0.019), $\Delta \ln k = 0.017$ (0.022), and $\Delta \ln e = 0.001$ (0.028). The $\Delta \ln y$ and $\Delta \ln k$ patterns are similar while $\Delta \ln e$ has higher variation, with a transition from positive to negative at the 1973 energy crisis. The scale of observed percentage changes varies over different time spans. Per capita income growth varies from 10% to 28% over the seven decades. The averages are higher before the energy crisis.

Table 1 reports unconstrained estimates of (1)-(3). Constant returns to scale CRS are not rejected as null hypotheses by Wald tests. Including energy input doubles explanatory power reducing both residual correlation in the Durbin-Watson DW statistic and heteroskedasticity in the autoregressive conditional heteroskedasticity ARCH and Bruesch-Pagan BP statistics.

* **Table 1** *

The capital coefficients 0.550 in (1) and 0.592 in (3) are in the neighborhood of the capital share but much lower and insignificant in (2) with dominating energy.

Labor productivity is reduced below its factor share by energy in (2) and (3). The average energy productivity in (2) and (3) of 0.434 is over five times its 8% factor share, suggesting effective monopsony power in the energy market.

A structural break for the 1973 energy crisis is significant in the estimate of (3) but not in (1) or (2). The constant term in (3) is insignificant prior to the break – complete TFP and no Solow residual due to the white noise residual. After the break, the mean of the derived Solow residual is lowest in (3). These unreported coefficients including the break are very similar to Table 1.

Table 1 presents diagnostic tests including the adjusted R-squared values for (2) and (3) above 70% compared to 48% for (1), reflecting the critical nature of energy. The Durbin-Watson (DW) statistics for (1) and (2) suggest no residual correlation, while (3) hints at positive autocorrelation. The ARCH tests for all three models suggest residuals are homoscedastic. The Breusch-Pagan (BP) test in (1) rejects the null hypothesis, indicating heteroskedasticity, although in (2) and (3) the null hypothesis is not rejected, again supporting including energy.

Table 2 2 reports estimates of (1)–(3) constrained to CRS have lower explanatory power, especially for (1), again supporting including energy. Capital productivity falls by 18% in (1) but rises 25% and becomes significant adding energy in (2). Labor productivity is consistent in (1) and (2) but falls 43% in (3) due to capital-energy interaction. Including energy input at least doubles explanatory power, reducing residual correlation and heteroskedasticity, consistent with Table 1. Labor coefficients are well below their observed factor shares, indicating monopoly power in the labor market. The adjusted R-squared value for (3) is somewhat lower than in Table 1. The negative R-squared value for (1) again supports including energy.

* **Table 2** *

The present estimates extend the results in Copeland and Thompson (2022) to another decade of data, confirming insight on the critical nature of capital-energy interaction. The high energy productivity is consistent with optimal depletion of the energy resource in Thompson (2012).

Figure 3 shows plots of the estimated Solow residuals $S = \exp(\alpha_0 + \epsilon)$ from Table 1. The physical Solow residual is consistently closest to zero. Table 3 3 summarizes the descriptive statistics for the series with means 1.0068 for KL, the highest 1.0149 for KLE, and the lowest 1.0048 for physical production representing the term A or TFP. Including energy reduces the variance by about half and favors a white noise Solow residual.

* **Figure 3** *

* **Table 3** *

Figure 4 compares this cumulative effect of the Solow residual accumulating according to $C_t = S_t S_{t-1}$ in (1) and (3). Over the sample, the cumulative KL effect grows 55% compared to 37% for physical production and 185% for the KLE effect. The cumulative physical effect is nearly constant before the 1973 energy crisis. After 1973, the growth rate for physical production is higher at 36% than for KL at 29%. The 0.018 variance of physical production is about half that of KL at 0.034.

* **Figure 4** *

In Figure 4, the cumulative residual Solow C derived for discrete time periods as in the literature varies considerably. Table 4 4 compares this cumulative Solow C across decades for the capital-labor and physical specifications. Including energy input typically but not always lowers the Solow C. The wide range illustrates arbitrary time, making general lessons a challenge.

* **Table 4** *

4 Conclusion

In the literature on economic growth, Solow's neoclassical model stands out as the dominant paradigm even though several alternative models including endogenous growth, Schumpeterian creative destruction, and semi-endogenous have been developed over the last few decades. In the Solow model, output is a function of capital and labor with all other variables influencing output relegated to the residual known as total factor productivity TFP.

The present paper suggests that energy should be included as an input along with capital and labor in the Solow model. This suggestion rests on our finding that the inclusion of energy strengthens empirical performance doubling the explanatory power of the estimating equation and reducing autocorrelation and heteroscedasticity. Moreover, quite crucially, including energy as a separate input reduces the mean and variance of TFP generating a more reliable estimate.

Scholars of economic growth theory including Gordon (2016) and Aghion and Howitt (2009) calculate TFP as a residual by assigning a value of 0.7 for the labor share and 0.3 for the capital share of output. Our empirical findings suggest these values should be modified to accommodate for an energy share of 0.07. By doing so, analysts would obtain a more realistic look into the Solow residual.

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Tables, Figures, & Appendix

Tables

Table 1. Estimated production

No Constraints	KL (1)	KLE (2)	Physical (3)
α_0	0.007 (0.005)	0.015*** (0.004)	0.005* (0.003)
K	0.550*** (0.167)	0.182 (0.118)	—
L	0.563*** (0.091)	0.298*** (0.067)	—
E	—	0.470*** (0.050)	—
KL	—	—	0.194*** (0.067)
KE	—	—	0.398*** (0.050)
RTS Wald	1.11 (0.480)	0.95 (0.641)	1.18 (0.057)
AdjR^2	0.484	0.771	0.729
DW	2.15	1.98	1.73
Arch(1)	0.66	0.48	0.73
Breusch-Pagan	0.05	0.59	0.21

Table 2. Constant Returns to Scale

Constant Returns to Scale	KL (1)	KLE (2)	Physical (3)
α_0	0.011***	0.013***	0.009***
	(0.002)	(0.002)	(0.001)
K	0.451***	0.228***	—
	(0.089)	(0.064)	
L	0.549***	0.306***	—
	(0.089)	(0.064)	
E	—	0.466***	—
		(0.049)	
KL	—	—	0.110***
			(0.050)
KE	—	—	0.390***
			(0.050)
AdjR^2	-0.209	0.710	0.500
DW	2.16	1.98	1.69
Arch(1)	0.84	0.48	0.50
Breusch-Pagan	0.05	0.59	0.021

Table 3. Derived Solow residual series

	KL (1)	KLE (2)	Physical (3)
Mean	1.0068	1.0149	1.0048
Std Dev	0.016	0.011	0.012
Variance	0.00026	0.00011	0.00014
Kurtosis	0.668	0.404	0.220
Skewness	-0.096	-0.110	-0.011
Range	0.087	0.057	0.059

Table 4. Decade-long Solow residuals

	KL model (1)	Physical (3)
1950s	5.1%	-0.2%
1960s	9.9%	-1.4%
1970s	-0.6%	1.7%
1980s	6.1%	7.6%
1990s	12.7%	10.7%
2000s	6.6%	4.8%
2010s	1.0%	4.0%

Figures

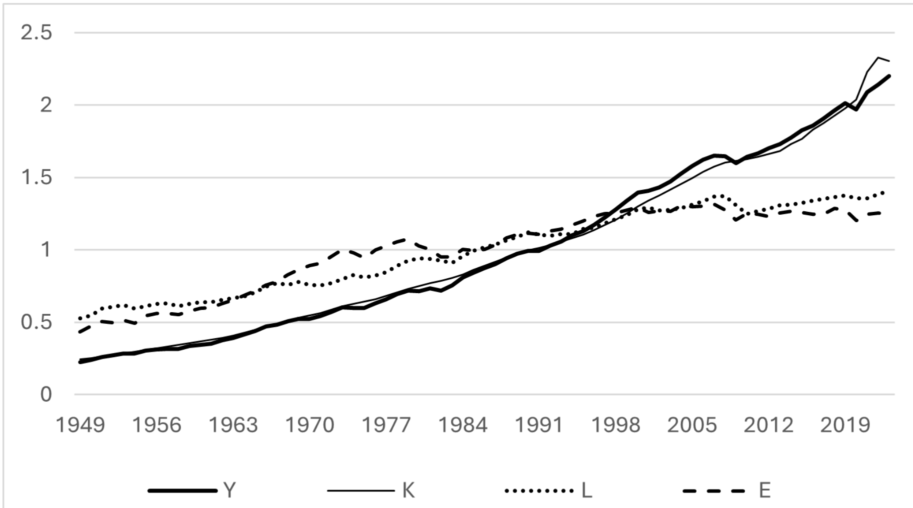

Figure 1. Trends in series

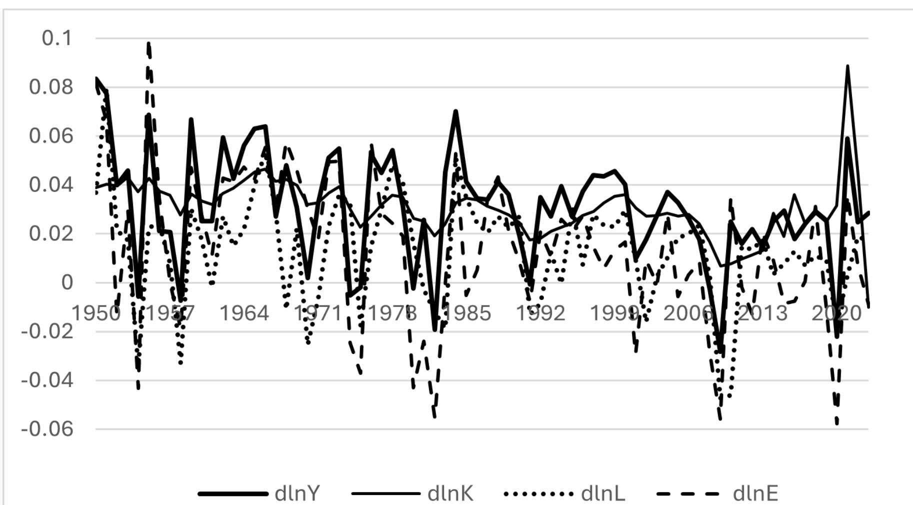

Figure 2. Percentage changes

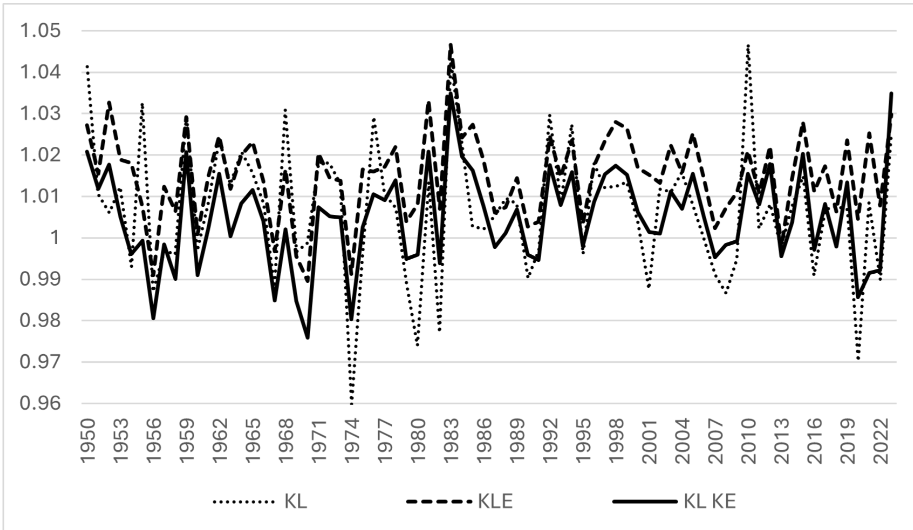

Figure 3. Solow residual series

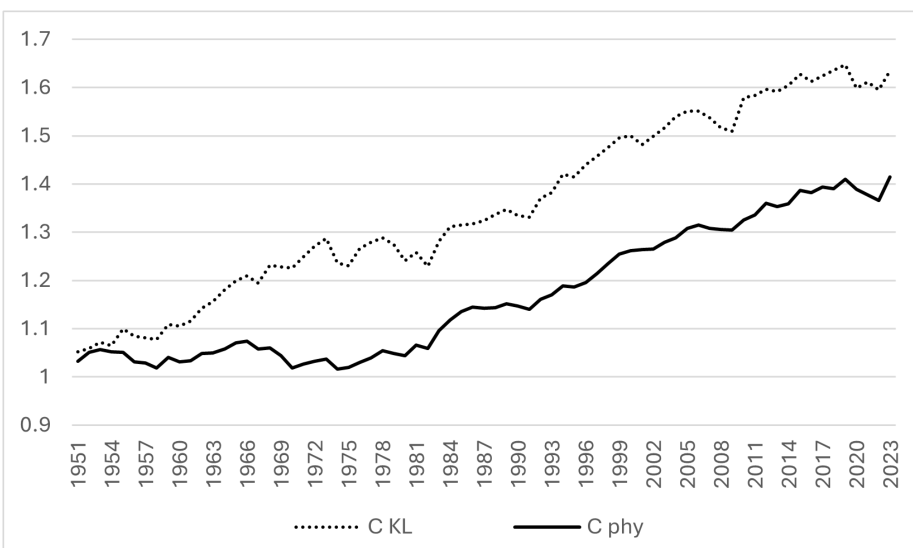

Figure 4. Cumulative Solow residuals

Appendix A – Variable Definitions & Data Sources

Variable	Definition & Source
Y	National Income, Federal Reserve Economic Data (FRED)
K	Fixed Capital Assets (Cost), Federal Reserve Economic Data (FRED)
L	Labor Force (Full-time Equivalent), Federal Reserve Economic Data (FRED)
E	Energy Input (Total Btu), Energy Information Agency (EIA)
KL	Interaction variable, K and L
KE	Interaction variable, K and E