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Multilateral Comparative Advantage

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This paper shows relative price competition based on comparative advantage can lead to diversified production and complex trade for more than two countries and two goods . The present paper develops the trade patterns for three and four countries and goods based on trade between the extreme countries in each relative price ranking with the directions of trade for middle countries depending on the terms of trade. The possible trade patterns include diversified exports, nontraded goods, nontrading countries, two-way trade in the same good, and separate trade groups.

Key terms: comparative advantage; trade patterns

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Multilateral Comparative Advantage

The example of comparative advantage for England and Portugal trading cloth and wine in Ricardo (1817) leads to specialized production and trade in the constant cost model. Bastable (1903), Viner (1937), and Graham (1948) note the familiar comparison of bilateral relative inputs does not identify comparative advantage for three countries and three goods. McKenzie (1954) and Jones (1961) identify the efficient good for any number of countries in the minimum cross product of the global unit input matrix. The further unlikely low price relative to every other good in every other country is necessary to ensure specialized production, a condition that can be called global comparative advantage (GCA).

The present paper develops the types of trade patterns that can arise based on multilateral comparative advantage (MCA) for three and four countries trading the same number of goods. Trade between each pair of countries in each relative price ranking is considered starting with the extreme countries trading those two goods. The terms of trade determine the directions of trade for the middle countries in each ranking. For three countries and three goods, the possible trade patterns include multiple exports, nontraded goods, nontrading countries, and perhaps the most surprising two-way trade. Countries would then export and import the same good in exchange for different goods. With four countries and four goods, separate groups of trading countries are also possible. These realistic trade patterns are based purely on constant cost MCA.

Adding Germany and grains to the example of Ricardo, England might diversify exporting grains along with cloth to Portugal. Germany could be left producing its own wine. Portugal and Germany might not trade with each other. Portugal could export wine to England for cloth meanwhile importing

wine from Germany in exchange for grains. Adding America and minerals to the example, America and Portugal might trade the four goods completely separated from the trading countries Germany and England.

Section 1 reviews comparative advantage for two countries or two goods. Section 2 and 3 develop the conditions of MCA in the 3x3 and 4x4 models. Section 4 examines the unlikely condition of GCA necessary for specialization. Sections 5 and 6 derive the range of possible trade patterns in the 3x3 and 4x4 models. Section 7 projects to MCA and trade among many countries and goods, followed by the Conclusion.

1. Bilateral comparative advantage

The 2x2 constant cost model begins with the global input matrix with a_{hk} as the unit input of labor for good h in country k,

$$\begin{pmatrix} a_{1A} & a_{1B} \\ a_{2A} & a_{2B} \end{pmatrix}. \tag{1}$$

Assume efficient goods are along the main diagonal leading to $a_{1A}a_{2B} < a_{1B}a_{2A}$. Define relative inputs as $A_{12} \equiv a_{1A}/a_{2A}$ and $B_{12} \equiv a_{1B}/a_{2B}$ reducing the efficiency condition to,

$$A_{12} < B_{12}. \tag{2}$$

The potential output of good h in country k is $x_{hk} = F_k/a_{hk}$ where F_k is the factor endowment. Based on (2) country A can produce relatively more good 1,

$$x_{1A}/x_{2A} > x_{1B}/x_{2B}. \tag{3}$$

Specialization maximizes global output as $x_{1A}x_{2B} > x_{1B}x_{2A}$ assuming country sizes F_k are consistent as developed in Thompson (2018). The gains from trade G_k expressed in terms of the input for consumption bundle c_{ik} is $G_k = \Sigma_i a_{ik}(c_{ik} - x_{ik}) = \Sigma_i a_{ik}c_{ik} - F_k$.

Competition implies factor price w_k and price of good h in country k are tied together, $p_{hk} = w_k a_{hk}$ implying $p_{ik}^A/p_{hk}^A = a_{ik}/a_{hk}$ where the superscript^A denotes autarky price. Specialization requires the terms of trade $tt_{12} = p_1/p_2$ must be flanked by relative prices according to $A_{12} < tt_{12} < B_{12}$. The necessary conditions for trade $p_{1A} < p_{1B}$ and $p_{2B} < p_{2A}$ imply the limits on the relative factor price,

$$a_{2B}/a_{2A} < w_A/w_B < a_{1B}/a_{1A}. \quad (4)$$

Improved technology in export (import) production raises the upper (lower) limit on the relative factor price of a country.

For any number of countries, relative prices determine comparative advantage. Consider the 2x3 model adding country C,

$$\begin{pmatrix} a_{1A} & a_{1B} & a_{1C} \\ a_{2A} & a_{2B} & a_{2C} \end{pmatrix}. \quad (5)$$

If $A_{12} < B_{12} < C_{12}$ then A has thes lowest price of 1, and C the lowest price of 2. The export pattern is A1-C2 with middle country B exporting 1 to C or 2 to A depending on tt_{12}.

For any number of goods between two countries, relative inputs also determine comparative advantage. In the 3x2 model,

$$\begin{pmatrix} a_{1A} & a_{1B} \\ a_{2A} & a_{2B} \\ a_{3A} & a_{3B} \end{pmatrix}, \quad (6)$$

assume $A_{12} < B_{12}$ and $A_{23} < B_{23}$ implying $A_{13} < B_{13}$. Country A has a lower price of 1 relative to both other goods, as does B for 3. While A has a lower price of 2 relative to 3, B has a lower price of 2 relative to 1. The result is A1-B3 trade with one specialized country importing two goods or both countries diversified producing 2.

2. Multilateral comparative advantage in the 3x3 model

The 3x3 input matrix,

$$\begin{pmatrix} a_{1A} & a_{1B} & a_{1C} \\ a_{2A} & a_{2B} & a_{2C} \\ a_{3A} & a_{3B} & a_{3C} \end{pmatrix}, \tag{7}$$

has the six cross products,

$$\begin{aligned} &\text{(a)}\ a_{1A}a_{2B}a_{3C} && \text{(b)}\ a_{2A}a_{3B}a_{1C} \\ &\text{(c)}\ a_{3A}a_{1B}a_{2C} && \text{(d)}\ a_{3A}a_{2B}a_{1C} \\ &\text{(e)}\ a_{1A}a_{3B}a_{2C} && \text{(f)}\ a_{2A}a_{1B}a_{3C}. \end{aligned} \tag{8}$$

Assume the minimum (8a) along the main diagonal in the McKenzie-Jones efficiency A1-B2-C3. The three inequalities in the negative direction (a) < (f, e, d) reduce to the familiar bilateral inequalities,

$$\begin{aligned} &\text{(a)}\ A_{12} < B_{12} \\ &\text{(b)}\ A_{13} < C_{13} \\ &\text{(c)}\ B_{23} < C_{23}. \end{aligned} \tag{9}$$

Country A has a lower price of 1 relative to 2 and B in (a) and relative to 3 and C in (b). Similar conditions hold for B-2 in (a) and (c), and for C-3 in (b) and (c). Each country has a lower price of its efficient good relative to the efficient good in the two other countries.

The two inequalities in (8) in the positive direction (a) < (b, c) reduce to each country competing with the other two countries,

$$\begin{aligned} &\text{(a)}\ A_{12} < C_{13}/B_{23} = C_{13}B_{32} \equiv CB_{12} \\ &\text{(b)}\ A_{13} < B_{12}C_{23} \equiv BC_{13}. \end{aligned} \tag{10}$$

Jones (1961) points out these conditions compare the price of the efficient good in each country to the price in the other two trading countries. The products of two relative inputs in (10) are a relative price for those two countries trading among themselves. The term CB_{12} in (a) is the implied price of good 1

relative to 2 with C and B trading in $C_{13}B_{32} = (a_{1C}/a_{3C})(a_{3B}/a_{2B})$. Country A must be able to beat CB_{12} to be able to export 1 to B for 2. If $A_{12} > CB_{12}$ country C would diversify exporting 1 along with 3 to B. Similarly (b) is necessary for A to export 1 to C. MCA is stated by (9) and (10).

The conditions in (10) can be stated for each country competing with the other two. In (a) country B faces $B_{23} < C_{13}/A_{12} = A_{21}C_{13} \equiv AC_{23}$ related to the price of 2 relative to 3 as A exports 2 and 1 to C for 3. Similar conditions can be stated for A and C based on (a) and for each country in (b). The bilateral inequalities in (9) imply at least one of the inequalities in (10).

As an example of apparent comparative advantage, consider the example

$$\begin{pmatrix} 1 & 2.1 & 1.9 \\ 0.5 & 1 & 1.1 \\ 0.6 & 1 & 1 \end{pmatrix}. \qquad (11)$$

While the bilateral inequalities in (9) hold, neither condition in (10) holds. The minimum cross product $a_{1C}a_{2A}a_{3B} = 0.95$ indicates the McKenzie-Jones efficiency. Rename the countries according to A $\rightarrow$ B, B $\rightarrow$ C, and C $\rightarrow$ A leading to MCA with $A_{12} = 1.7 < B_{12} = 2$, $A_{13} = 1.9 < C_{13} = 2.1$, and $B_{23} = .83 < C_{23} = 1$ in (9) and $A_{12} < CB_{12} = 2.5$ and $A_{13} < BC_{13} = 2$ in (10).

Efficient specialization maximizes global output $x_{1A} + x_{2B} + x_{3C}$ based on the relative output rankings,

$$\begin{aligned} &\text{(a)}\ x_{1A}/x_{2A} > x_{1B}/x_{2B} \\ &\text{(b)}\ x_{1A}/x_{3A} > x_{1C}/x_{3C} \\ &\text{(c)}\ x_{2B}/x_{3B} > x_{2C}/x_{3C}. \end{aligned} \qquad (12)$$

Country A can produce more of 1 relative to 2 than can B in (a) and more of 1 relative to 3 than can C in (b). Similar conditions hold for B-2 in (a) and (b), and for C-3 in (b) and (c).

The three limits on relative factor prices are,

(a) $a_{2B}/a_{2A} < w_A/w_B < a_{1B}/a_{1A}$

(b) $a_{3C}/a_{3A} < w_A/w_C < a_{1C}/a_{1A}$ (13)

(c) $a_{3C}/a_{3B} < w_B/w_C < a_{2C}/a_{2B}$.

Improved efficiency in export (import) production would raise the two upper (lower) limits on the relative factor price of a country. Analogous to the 2xn and rx2 models, MCA can be extended adding middle countries in the 3xn model and middle goods in the rx3 model.

4. MCA in the 4x4 model

Efficiency along the main diagonal of the 4x4 input matrix,

$$\begin{pmatrix} a_{1A} & a_{1B} & a_{1C} & a_{1D} \\ a_{2A} & a_{2B} & a_{2C} & a_{2D} \\ a_{3A} & a_{3B} & a_{3C} & a_{3D} \\ a_{4A} & a_{4B} & a_{4C} & a_{4D} \end{pmatrix}, \quad (14)$$

in A1-B2-C3-D4 implies $a_M \equiv a_{1A}a_{2B}a_{3C}a_{4D}$ is less than the other seven cross products. Two inequalities in the negative direction $a_M < a_{1C}a_{2B}a_{3A}a_{4D}$ and $a_M < a_{1A}a_{2D}a_{3C}a_{4B}$ each contain two of the four coefficients in a_M leading to bilateral relative price inequalities of efficient goods,

(a) $A_{13} < C_{13}$ (15)

(b) $B_{24} < D_{24}$.

Bilateral inequalities become irrelevant with five or more countries and goods. The inequality $a_M < a_{3A}a_{4B}a_{1C}a_{2D}$ in the positive direction contains the coefficients in (15) leading to the redundant condition $A_{13}B_{24} < D_{24}C_{13}$.

The two other inequalities in the negative direction $a_M < a_{1B}a_{2A}a_{3D}a_{4C}$ and $a_M < a_{1D}a_{2C}a_{3B}a_{4A}$ lead to conditions involving efficient goods,

(a) $A_{12}C_{34} < B_{12}D_{34}$ (16)

(b) $A_{14}B_{23} < D_{14}C_{23}$.

This condition (a) is weaker than the two underlying inequalities $A_{12} < B_{12}$ and $C_{34} < D_{34}$. Similarly (b) implies either $A_{14} < D_{14}$ or $B_{23} < C_{23}$ but not necessarily both.

The final two cross products in (14) lead to prices in each country relative to the other three countries trading among themselves. The two inequalities $a_M < a_{2A}a_{3B}a_{4C}a_{1D}$ and $a_M < a_{4A}a_{1B}a_{2C}a_{3D}$ in the positive direction imply,

$$\text{(a)}\ \ A_{12} < D_{14}C_{43}B_{32} = (a_{1D}/a_{4D})(a_{4C}/a_{3C})(a_{3B}/a_{2B}) \equiv DCB_{1432} \qquad (17)$$

$$\text{(b)}\ \ A_{14} < B_{12}C_{23}D_{34} = (a_{1B}/a_{2B})(a_{2C}/a_{3C})(a_{3D}/a_{4D}) \equiv BCD_{1234}.$$

The price of 1 relative to 2 in A is compared to the other three countries trading among themselves in (a). Country A must beat the offer for good 1 that B has trading with C and D. Trade among those three countries leads to the relative price DCB_{1432} that country A must faces to export 1 to B for 2. Similarly in (b) country A must beat BCD_{1234} to export 1 to D.

The conditions in (17) can be stated for each country competing with the other three. For instance (a) can be stated $B_{23} < A_{21}D_{14}C_{43} \equiv ADC_{2143}$ related to the price of good 2 that B faces when C trades with A and D. For C and D conditions analogous to (17a) are $C_{34} < B_{32}A_{21}D_{14} \equiv BAD_{3214}$ and $D_{41} < C_{43}B_{32}A_{21} \equiv CBA_{4321}$.

The conditions for applications or tests of MCA are (15)-(16)-(17). Two out of the total of six familiar bilateral inequalities are part of MCA. For five or more countries and goods, all bilateral relative price comparisons are irrelevant to MCA.

5. Global comparative advantage

A low price of each efficient good relative to every other good in every other country ensures specialized production expanding (9) in the 3x3 model to include relative prices of inefficient goods in the three terms A_{23}-B_{13}-C_{12},

(a) $A_{12} < C_{12} < B_{12}$

(b) $A_{13} < B_{13} < C_{13}$ (18)

(c) $B_{23} < A_{23} < C_{23}$.

For example, the relative price C_{12} in (a) includes the relative price of the two inefficient goods in country C. In (a) and (b) country A has lower prices of good 1 relative to the other goods in the other countries. The same holds for B-2 in (a) and (c), and for C-3 in (b) and (c).

GCA is a stronger condition than MCA in (9)-(10). As an example GCA does not hold in the reordered example (11) where $C_{12} > B_{12}$, $B_{13} > A_{13}$, and $A_{23} > C_{23}$. For arbitrary input matrices GCA is highly unlikely. The 27 possible rankings of the nine relative prices in (18) imply a probability of less than 4% in the 3x3 model for arbitrary input matrices.

GCA is necessary for specialization but not sufficient as middle countries in the three rankings could also trade. Country C in (18a) could either export 1 to B or 2 to A depending on those terms of trade. The same holds for B in (18b) and A in (18a). Even assuming GCA there are six potential trade patterns involving diversified production.

In the 4x4 model GCA starts with (15) and adds the two middle countries in the two relative price rankings. The inefficient goods added to (15a) are B_{13} and D_{13}, and to (15b) A_{24} and C_{24}. Similar rankings for the other four pairs of goods 1-2, 1-4, 2-3, and 3-4 enter GCA. The point is that diversified production characterizes the constant cost model for more than two countries and two goods.

5. Trade patterns in the 3x3 model

The present approach is to consider every instance of bilateral trade starting with the two extreme countries in a relative price ranking and considering the possible directions of trade for middle countries depending on the terms of trade. Consider the two positions for the terms of trade tt_{jk} between goods j and k,

$$A_{jk} < tt_{jk}^{L} < B_{jk} < tt_{jk}^{H} < C_{jk}. \quad (19)$$

Country A will export j to C in exchange for k. At the low terms of trade tt_{jk}^{L} middle country B exports k to A, while at the high tt_{jk}^{H} country B exports j to C.

Deardorff (1984) notes that not every instance of bilateral trade is possible given competitive pricing. Two countries m and n cannot export the same two goods as competitive pricing implying $p_j = w_m a_{jm} = w_n a_{jn}$ would imply w_m/w_n is equal to to two arbitrary input ratios. Two corollary restrictions can be stated:

(i) Countries cannot export different pairs of goods

(ii) If one country exports every good, the others must specialize.

In the example of global comparative advantage GCA in (18) each country exports its efficient good in the country-export pairs A1-B2-C3. The result would be specialized production disregarding trade by middle countries. Relaxing GCA replace (18c) with

$$(c)\ B_{23} < C_{23} < A_{23}, \quad (20)$$

as in the example (11) after the reordering. Country A would diversify to export 3 in (20) and 1 in (18a) to B in exchange for 2. Country A also exports 1 to C in (18b) for 3 implying two-way trade in 3 implying $p_3 = a_{3A}w_A = a_{3C}w_C$ and the relative wage $w_A/w_C = a_{3C}/a_{3A}$. Country B could specialize in 2 as it imports 1 and 3 from A leaving C to produce nontraded good 2. Middle country C in (20) given $tt_{23} < C_{23}$ would export 3 to B for 2. Both B and C would specialize as B imports 3 from both C and A. Country A would diversify producing 1 and 3.

Figure 1 illustrates this diversified production for country A on its production frontier abc. Specialized country A would produce at point $a = F_A/a_{1A}$ on the corner of abc. Consumption frontier

dgh is determined by the terms of trade tt_{12} and tt_{13} from the point of diversified production d where $x_1 = e < a$, $x_2 = 0$, and $x_3 = f$.

* Figure 1 *

The terms of trade tt_{12} improve for A with diversification as p_1 rises due to its decreased supply and p_2 falls due to increased supply. The terms of trade tt_{13} also improve for A due to the higher p_1. The consumption frontier dgh extends beyond the production frontier abc leading to diversification if utility is higher on dgh than on abc. Starting with GCA in (18) there are six possible trade patterns that result from moving a middle country to an extreme position.

Figure 2 pictures middle country trade based on the GCA in (18). Each country exports its efficient good to the other two with exports indicated close to the country of origin. Assuming the terms of trade $tt_{12} < C_{12}$ in (18a) middle C exports 2 to A implying the relative factor price $w_B/w_C = a_{2C}/a_{2B}$. If $B_{13} < tt_{13}$ in (18b) middle B could export 1 to C for 3 implying $w_A/w_B = a_{1B}/a_{1A}$ and $w_A/w_C = a_{1B}a_{2B}/a_{1A}a_{2C}$. Country A could not then trade as a middle country in (18c) under competitive pricing. There are 18 similar possible trade patterns with two diversified middle countries.

* Figure 2 *

Returning to (20) assume $C_{23} < tt_{23}$ leading middle C to export 2 to A for 3. Given (18b) there is two-way trade in 3 between A and C. Both B and C would export 2 implying $w_B/w_C = a_{2C}/a_{2A}$ and $w_A/w_B = a_{2B}a_{3C}/a_{2C}a_{3A}$ given the two-way trade in 3. Countries B and C would not trade. Country B must specialize in 2 relying on imports of 1 and 3 from A.

Another type of trade pattern occurs when the same country is in the middle of every ranking. Maintaining (18a) and (20) assume,

$$\text{(b) } A_{13} < C_{13} < B_{13}. \tag{21}$$

Countries A and B trade their efficient goods 1 and 2 with two-way trade in 3 implying $p_3 = w_A a_{3A} = w_B a_{3B}$ and $w_A/w_B = a_{3B}/a_{3A}$. Country C could specialize in 2 exporting 2 to A for 1 in (18a) and 2 to A for 3 in (20). Country C could produce the single 3 to B for 2 in (20) and produce its own nontraded good 1. There are six trade patterns with C exporting a single good. Combinations of middle country trade are feasible in two or three of the rankings.

6. Trade patterns in the 4x4 model

The 4x4 model (14) adds the possibility of separate trade groups. The unlikely condition of global comparative advantage GCA in (15)-(17) would result in specialization assuming no middle country trade. Separate trade groups arise if $C_{12} < A_{12} < B_{12} < D_{12}$ denoted by 12-CABD along with 13-ACDB, 14-CABD, 23-DBAC, 24-BCDA, and 34-BCDA with extreme country trade entirely within groups A-B and C-D. Country A exports 1 and 4 in exchange for 2 and 3 from B. In group C-D country D exports 2 and 4 in exchange for 1 and 3 from C. Each country produces a different pair of goods. Every good is produced in two nontrading countries.

Introducing middle country trade complicates trade patterns. If $A_{12} < tt_{12} < D_{12}$ in 12-CABD then A would export 1 to D for 2. If $B_{12} < tt_{12} < D_{12}$ then B would join A exporting 1 to D for 2. If $C_{12} < tt_{12} < A_{12}$ both would export 2 to C for imports of 1. Another type of pattern would be A and B trading 1 and 2 at their own unique terms of trade $A_{12} < tt_{12}^{AB} < B_{12}$ leaving the trade groups intact. Middle country trade in the other six rankings leads to a wide range of potential trade patterns.

7. MCA and trade for many countries and goods

General properties of constant cost production and trade for many countries and goods are developed in Chipman (1965), Jones and Neary (1986), and Thompson (2001). With five or more

countries and goods, familiar bilateral relative price inequalities in (15) are replaced by weaker conditions involving their products as in (16).

The conditions of MCA also relate to each country competing with the rest of the trading world. In the 5x5 model, there are five weak inequalities of efficient goods and four inequalities relating efficient goods to the rest of the trading world. For instance $A_{12} < E_{15}D_{54}C_{43}B_{32}$ would have to hold for A to export 1 to B in exchange for 2 as otherwise B would export 2 through C and D in exchange for 1 from E.

In a 10x10 model, MCA amounts to ten weak conditions relating products of efficient goods and nine conditions for competing with the rest of the trading world. In the data of Eaton and Kortum (2012) with 31 countries and goods, there are 31 weak product and 30 inequalities related to the rest of the world. These conditions define MCA. The potential trade patterns depending on the various terms of trade are highly diverse and seemingly realistic.

9. Conclusion

The present paper points out that comparative advantage in the constant cost trade model should be associated with diversified production and complex trade patterns for three or more countries trading the same number of goods. Specialized production is a special case in the model with two countries and two goods. Relative price competition among three or more countries trading the same number of goods leads to multiple exports, nontraded goods, nontrading countries, two-way trade, and separate trade groups. In trade theory, these observed trade patterns are motivated by increasing costs, product differentiation, transport costs, border effects, skilled labor, natural resources, and utility functions.

Regarding applications or tests, most countries export most products at practical levels of aggregation contradicting the assumption of competitive pricing. The parametric link between output and average cost in Thompson (2003) relaxes competitive pricing without assumptions on production. Estimation of the terms of trade would simplify the predicted middle country trade in the relative price rankings. The level of aggregation would affect theoretical predictions.

Other constant cost frameworks can be related to the present complex trade patterns. Ruffin (2002, 2013) and Maneschi (2004) treat the coefficients of Ricardo as total labor inputs leading to results consistent with various assumptions about production. The missing link model of Ruffin (1988) includes a separate factor of production for each good leading to properties similar to the factor proportions model. Including capital input in the fixed factor proportions model introduces the influence of factor abundance and intensity as developed in Thompson (2010). Trade patterns based on MCA can be developed in these theoretical frameworks.

Simulations of the global trade equilibrium including country sizes and utility maximization would solve for the theoretical terms of trade along with the levels of production and trade. The offer curves of each country could in principle be simulated. While the theory is straightforward, simulations involve multi-objective optimization presenting a computational challenge.

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Figure 1. Consumption possibility frontier with diversified exports

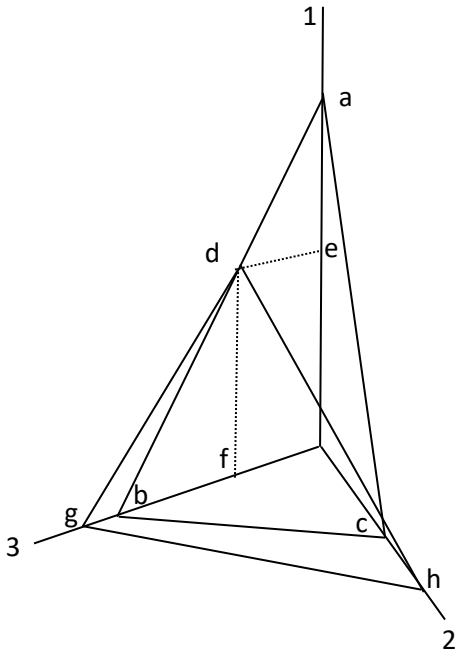

Figure 2. GCA with middle country exports for B and C