Auburn University
Department of Economics
Working Paper Series

Superior Predictability of American Factors of the Dollar/Won Real Exchange Rate

Sarthak Behera*, Hyeongwoo Kim†, and Soohyon Kim‡

*Centre College; †Auburn University; ‡Chonnam National University

AUWP 2023-05

Superior Predictability of American Factors of the Dollar/Won Real Exchange Rate*

Sarthak Behera[†], Hyeongwoo Kim[‡], and Soohyon Kim[§]

June 2023

Abstract

This paper examines the asymmetric out-of-sample predictability of macroeconomic variables for the real exchange rate between the United States and Korea. While conventional models suggest that the bilateral real exchange rate is driven by the relative economic performance of the two countries, our research demonstrates the superior predictive power of our factor-augmented forecasting models only when factors are obtained from U.S. economic variables, whereas the inclusion of Korean factors fails to enhance predictability and behaves more like noise variables. Our models exhibit particularly strong performance at longer horizons when incorporating American real activity factors, while American nominal/financial market factors contribute to improved short-term prediction accuracy. We attribute the remarkable predictability of American factors to the significant cross-correlations observed among bilateral real exchange rates *vis-à-vis* the U.S. dollar, which suggests a limited influence of idiosyncratic factors specific to small countries. Moreover, we assess our factor-augmented forecasting models by incorporating proposition-based factors instead of macro factors. While macro factors generally exhibit superior performance, it is worth noting that the uncovered interest parity (UIP)-based global factors, with the dollar as the numéraire, consistently demonstrate strong overall performance. On the other hand, the purchasing power parity (PPP) and real uncovered interest parity (RIRP) factors have a limited role in forecasting the dollar/won real exchange rate.

Keywords: Dollar/Won Real Exchange Rate; Asymmetric Predictability; Principal Component Analysis; Partial Least Squares; LASSO; Out-of-Sample Forecast

JEL Classification: C38; C53; C55; F31; G17

---

*We thank seminar/conference participants at the Bank of Korea, Southern Economic Association Meetings, and the 7th International Workshop on Financial Markets and Nonlinear Dynamics for useful comments.

[†]Economics and Finance Program, Centre College, 319 Crounse Hall, Danville, KY 40422. Tel: +1-859-238-5235. Email: sarthak.behera@centre.edu.

[‡]Department of Economics, Auburn University, 138 Miller Hall, Auburn, AL 36849. Tel: +1-334-844-2928. Email: gmmkim@gmail.com.

[§]Chonnam National University, 77, Yongbong-ro, Buk-gu, Gwangju, 61186, Korea, Tel: +82-62-530-1540, Email: soohyon.kim@jnu.ac.kr.

1 Introduction

This paper investigates the asymmetric out-of-sample forecastability of macroeconomic predictors for the real exchange rate between the United States and Korea. Conventional real exchange rate models suggest that the bilateral real exchange rate is influenced by the relative economic performance of the two countries. However, our research demonstrates the superior predictive performance of latent factors obtained from a large panel of U.S. macroeconomic time series data, while Korean macroeconomic factors have a limited role, akin to noise variables, in forecasting the dollar/won real exchange rate. We interpret this remarkable predictability of American factors as a consequence of the significant cross-correlations observed between bilateral real exchange rates *vis-à-vis* the U.S. dollar. We further show that our factor forecasting models outperform commonly used benchmark models as well as theoretical proposition-based prediction models.

There is a substantial body of forecasting literature that extensively questions the effectiveness of exchange rate models in terms of their ability to predict exchange rates in out-of-sample scenarios. In their seminal work, Meese and Rogoff (1983) demonstrate that the random walk (RW) model performs well when it comes to forecasting exchange rates, surpassing models driven by exchange rate determination theories. Cheung, Chinn, and Pascual (2005) presented additional evidence highlighting the disconnection between exchange rates and economic fundamentals, revealing that exchange rate models do not consistently outperform the RW model in out-of-sample forecasting.[1] In a related study, Engel and West (2005) make an intriguing observation that asset prices, including the exchange rate, can exhibit a near unit root process, while remaining compatible with asset pricing models as the discount factor tends towards one.[2]

On the contrary, there exists a group of researchers who have demonstrated that exchange rate models can indeed outperform the RW model when considering longer time horizons. For instance, Mark (1995) employed a regression model that analyzed multi-period changes (long-differenced) in the nominal exchange rate based on deviations of the exchange rate from its fundamentals. The study reported overall superior predictability of fundamentals for the exchange rate at longer horizons. Similarly, Chinn and Meese (1995) also reported evidence supporting greater predictability of exchange rate models compared to the RW model when considering longer horizons. Using over two century-long annual frequency data, Lothian and Taylor (1996) report good out-of-sample predictability of fundamentals for the *real* exchange rate. Groen (2005) reported some evidence of superior predictability of monetary fundamentals at longer horizons using a vector autoregressive (VAR) model framework.[3] Additionally, Engel, Mark, and West (2008) showed that out-of-sample predictability can be enhanced by employing panel estimation techniques and focusing on long-horizon forecasts.

[1] However, Engel and Hamilton (1990) reported some evidence that nonlinear models can outperform the RW model. But their findings are still at odds with uncovered interest parity (UIP).

[2] Many researchers have provided panel evidence highlighting the close link between monetary models and exchange rate dynamics. See for example, Rapach and Wohar (2004), Groen (2000), and Mark and Sul (2001).

[3] It is worth noting that they demonstrated that the monetary fundamentals-based *common* long-run model tended to outperform both the RW model and the standard cointegrated VAR model at horizons ranging from 2 to 4 years.

Another group of researchers has started incorporating Taylor Rule fundamentals, along with conventional monetary fundamentals, into their exchange rate forecasting models. Notable studies in this area include Engel, Mark, and West (2008), Molodtsova, Nikolsko-Rzhevskyy, and Papell (2008), Molodtsova and Papell (2009), Molodtsova and Papell (2013), and Ince, Molodtsova, and Papell (2016). These studies emphasize the significance of Taylor Rule fundamentals in improving the accuracy of exchange rate forecasts. Additionally, Rossi (2013) provides a survey of related research that underscores the usefulness of Taylor Rules in understanding the dynamics of exchange rates.[4]

The pioneering work of Stock and Watson (2002) has initiated a wave of research utilizing latent common factors through principal components (PC) analysis for forecasting macroeconomic variables. This approach has also been applied in the field of exchange rate research. A number of researchers have employed large panels of time series data to gain deeper insights into exchange rate dynamics. For instance, Engel, Mark, and West (2015) utilized cross-section information, specifically PC factors that are obtained from a panel of 17 bilateral exchange rates relative to the US dollar. They demonstrated that forecasting models based on these factors often outperformed the RW model, particularly in the post-1999 sample period. The study also highlighted the strong forecasting performance of the dollar factor in combination with Purchasing Power Parity (PPP) factors. In a related study, Chen, Jackson, Kim, and Resiandini (2014) applied PC analysis to extract latent common factors from 50 world commodity prices. Interestingly, their first common factor was found to be closely linked to the dollar exchange rate, which aligns with the observation that world commodities are predominantly denominated in U.S. dollars. They show that this first common factor yields superior out-of-sample predictive contents for the dollar exchange rate.

Greenaway-McGrevy, Mark, Sul, and Wu (2018) demonstrate that exchange rates are primarily driven by a dollar factor and a euro factor. Their dollar-euro factor model outperformed the RW model in terms of out-of-sample prediction performance. Verdelhan (2018) utilized portfolios of international currencies to extract two risk factors, namely the dollar factor and the carry factor, which effectively explained exchange rate dynamics. Ca' Zorzi, Kociecki, and Rubaszek (2015) demonstrated that structural Bayesian vector autoregression (SBVAR) models with a Dornbusch prior tended to outperform the RW model at medium horizons, although beating the RW model in the short-run remained challenging.

While principal component (PC) analysis has been widely used in the forecasting and empirical macroeconomics literature, Boivin and Ng (2006) pointed out that its forecasting performance may be subpar if relevant predictive information is dominated by other factors within the analysis. This is because PC extracts latent common factors without considering the specific relationship between predictors and the target variable. To address this issue, we employ alternative data dimensionality reduction methods, such as partial least squares (PLS) proposed by Wold (1982). PLS utilizes the

[4] In a related work, Wang and Wu (2012) show the superior out-of-sample interval predictability of the Taylor Rule fundamentals at longer horizons. Also, many researchers report in-sample evidence that Taylor rule fundamentals help understand exchange rate dynamics. See among others, Mark (2009), Molodtsova, Nikolsko-Rzhevskyy, and Papell (2008), Engel and West (2006), Clarida and Waldman (2008), and Kim, Fujiwara, Hansen, and Ogaki (2015).

covariance between target and predictor variables to generate *target-specific* factors. For comparisons between PC and PLS approaches, Kelly and Pruitt (2015) and Groen and Kapetanios (2016) provide relevant studies. In line with Bai and Ng (2008) and Kelly and Pruitt (2015), the Least Absolute Shrinkage and Selection Operator (LASSO) technique is also used to select target-specific groups of predictors from the full dataset, extracting more relevant factors for the target variable.

In this paper, we propose factor-augmented forecasting models for the dollar/won real exchange rate. We utilize principal component (PC) analysis and partial least squares (PLS), as well as the LASSO in combination with PC and PLS, to estimate common factors. Our estimation is based on large panels of macroeconomic time series data consisting of 125 American and 192 Korean monthly frequency variables, covering the period from October 2000 to March 2019. Given that most macroeconomic data exhibit nonstationary integrated processes, see Nelson and Plosser (1982), we apply these methods to first-differenced predictors to consistently estimate the factors. See Bai and Ng (2004) for detailed explanations.

We also extract common factors from country-level global data using up to 43 country-level variables related to prices and interest rates. This approach is motivated by exchange rate determination theories such as Purchasing Power Parity (PPP), Uncovered Interest Parity (UIP), and Real Uncovered Interest Parity (RIRP). We conduct various out-of-sample forecasting exercises using these factor estimates and examine which factors contribute to improving the prediction accuracy of the exchange rate. We evaluate the out-of-sample predictability of our factor models using the ratio of the root mean squared prediction error (RRMSPE) criteria.

Our key findings can be summarized as follows. First, our factor-augmented forecasting models outperform the random walk (RW) and autoregressive AR(1) benchmark models only when American factors are incorporated. In contrast, the Korean factor-augmented models generally underperform the AR model. However, they still outperform the RW model when the forecast horizon is one year or longer. These findings deviate from conventional real exchange rate models, which suggest that the relative economic performance of the two countries drives the bilateral real exchange rate. We interpret these results in light of the observation that bilateral exchange rates relative to the U.S. dollar tend to exhibit highly positive cross-correlations and move in tandem. This suggests that American factors exert a dominant influence on the dynamics of these exchange rates *vis-à-vis* the dollar, overshadowing idiosyncratic components specific to each country, such as Korea.

Secondly, our findings indicate that our models tend to exhibit better performance at short horizons when combined with American nominal/financial market factors. Conversely, models incorporating American real activity factors outperform both the RW and AR models at longer horizons. These results align with the findings of Boivin and Ng (2006), who emphasized the importance of extracting more informative content from subsets of predictors.

Thirdly, we demonstrate that models incorporating factors motivated by Uncovered Interest Parity (UIP) perform well when the U.S. serves as the reference country. However, factor models based on Purchasing Power Parity (PPP) and Real Uncovered Interest Parity (RIRP) are outperformed by the AR model regardless of the reference country. Overall, the factor models driven by macroeconomic data tend to perform better than these proposition-based factor models.

The remainder of the paper is organized as follows. In Section 2, we provide a detailed description of how we estimate latent common factors using PC, PLS, and the LASSO when predictors follow an integrated process. Section 3 presents data descriptions and preliminary statistical analysis, including an examination of the in-sample fit to investigate the source of latent common factors. In Section 4, we introduce our factor-augmented forecasting models and evaluation schemes. We present and interpret the results of our out-of-sample forecasting exercises, and also compare the performance of our data-driven factor forecasting models with alternative identification approaches, proposition-based models, and nonstationary forecasting models. Finally, in Section 5, we provide concluding remarks.

2 Methods of Estimating Latent Common Factors

This section explains how we estimate latent common factors by applying Principal Component (PC), Partial Least Squares (PLS), and the Least Absolute Shrinkage and Selection Operator (LASSO) to a large panel of *nonstationary* predictors.

2.1 Principal Component Factors

Since the seminal work of Stock and Watson (2002), PC has been popularly used in the current forecasting literature. We begin with this approach to show how to estimate latent common factors when predictors obey an integrated $I(1)$ process, largely based on the work of Bai and Ng (2004, 2010).

Consider a panel of N macroeconomic $T \times 1$ time series predictors/variables, $\mathbf{x} = [\mathbf{x}_1, \mathbf{x}_2, ..., \mathbf{x}_N]$, where $\mathbf{x}_i = [x_{i,1}, x_{i,2}, ..., x_{i,T}]'$, $i = 1, ..., N$. We assume that each predictor $\mathbf{x}_i$ has the following factor structure. Abstracting from deterministic terms,

$$x_{i,t} = \boldsymbol{\lambda}_i' \mathbf{f}_t^{PC} + \varepsilon_{i,t}, \tag{1}$$

where $\mathbf{f}_t = \left[f_{1,t}^{PC}, f_{2,t}^{PC}, \cdots, f_{R,t}^{PC} \right]'$ is an $R \times 1$ vector of *latent* time-varying common factors at time t and $\boldsymbol{\lambda}_i = [\lambda_{i,1}, \lambda_{i,2}, \cdots, \lambda_{i,R}]'$ denotes an $R \times 1$ vector of time-invariant idiosyncratic factor loading coefficients for $\mathbf{x}_i$. $\varepsilon_{i,t}$ is the idiosyncratic error term.

Following Bai and Ng (2004, 2010), we estimate latent common factors by applying the PC method to first-differenced data. This is because, as shown by Nelson and Plosser (1982), most macroeconomic time series variables are better approximated by an integrated nonstationary stochastic process. Note that the PC estimator of $\mathbf{f}_t$ would be inconsistent if $\varepsilon_{i,t}$ is an integrated process. Differencing both sides of (1), we obtain the following.

$$\Delta x_{i,t} = \boldsymbol{\lambda}_i' \Delta \mathbf{f}_t^{PC} + \Delta \varepsilon_{i,t} \tag{2}$$

for $t = 2, \cdots, T$. We first normalize the data, $\Delta\tilde{\mathbf{x}} = [\Delta\tilde{\mathbf{x}}_1, \Delta\tilde{\mathbf{x}}_2, ..., \Delta\tilde{\mathbf{x}}_N]$, then apply PC to $\Delta\tilde{\mathbf{x}}\Delta\tilde{\mathbf{x}}'$ to obtain the factor estimates $\Delta\hat{\mathbf{f}}_t^{PC}$ along with their associated factor loading coefficients $\hat{\boldsymbol{\lambda}}_i$.[5] Naturally, estimates of the idiosyncratic component are obtained by taking the residual, $\Delta\hat{\varepsilon}_{i,t} = \Delta\tilde{x}_{i,t} - \hat{\boldsymbol{\lambda}}_i'\Delta\hat{\mathbf{f}}_t^{PC}$. The level variable estimates are then recovered via cumulative summation as follows.

$$\hat{\varepsilon}_{i,t} = \sum_{s=2}^{t} \Delta\hat{\varepsilon}_{i,s}, \ \hat{\mathbf{f}}_t^{PC} = \sum_{s=2}^{t} \Delta\hat{\mathbf{f}}_s^{PC} \tag{3}$$

It should be noted that this procedure yields consistent factor estimates even when $\mathbf{x}$ includes some stationary $I(0)$ variables. For example, assume that $\mathbf{x}_j$, $j \in \{1, ..., N\}$ is $I(0)$. Differencing it once results in $\Delta\mathbf{x}_j$, which is still a stationary $I(-1)$ process. Therefore the PC estimator remains consistent. Alternatively, one may continue to difference the variables until the null of nonstationarity hypothesis is rejected via a unit root test.[6] However, this may not be practically useful when unit root tests provides contradicting statistical inferences under alternative test specifications (i.e., number of lags, number of observations) in finite samples. See Cheung and Lai (1995) for related discussions.

2.2 Partial Least Squares Factors

We employ PLS for a scalar target variable q_t, which is a somewhat overlooked estimator in the current literature. Unlike PC, the method of PLS generates *target specific* latent common factors, which is an attractive feature. As Boivin and Ng (2006) pointed out, PC factors might not be useful in forecasting the target variable when useful predictive contents are in a certain factor that may be dominated by other factors.

PLS is motivated by the following linear regression model. Abstracting from deterministic terms,

$$q_t = \Delta\mathbf{x}_t'\boldsymbol{\beta} + e_t, \tag{4}$$

where $\Delta\mathbf{x}_t = [\Delta x_{1,t}, \Delta x_{2,t}, ..., \Delta x_{N,t}]'$ is an $N \times 1$ vector of predictor variables at time $t = 1, ..., T$, while $\boldsymbol{\beta}$ is an $N \times 1$ vector of coefficients. e_t is an error term. Again, we first-difference the predictor variables assuming that $\mathbf{x}_t$ is a vector of $I(1)$ variables..

PLS is especially useful for regression models that have many predictors, that is, when N is large.[7] To reduce the dimensionality, rewrite (4) as follows,

$$\begin{aligned} q_t &= \Delta\mathbf{x}_t'\mathbf{w}\boldsymbol{\theta} + u_t \\ &= \Delta\mathbf{f}_t^{PLS'}\boldsymbol{\theta} + u_t \end{aligned} \tag{5}$$

where $\Delta\mathbf{f}_t^{PLS} = \left[\Delta f_{1,t}^{PLS}, \Delta f_{2,t}^{PLS}, ..., \Delta f_{R,t}^{PLS}\right]'$, $R < N$ is an $R \times 1$ vector of PLS factors.

[5] This is because PC is not scale invariant. We demean and standardize each time series.

[6] This approach is used to construct the Fred-MD database. The Fred-MD is available at https://research.stlouisfed.org/econ/mccracken/fred-databases/.

[7] For example, the least squares estimator is not even feasible when N is greater than T.

Note that $\Delta \mathbf{f}_t^{PLS}$ is a linear combination of *all* predictor variables, similar to ridge regression, that is,

$$\Delta \mathbf{f}_t^{PLS} = \mathbf{w}' \Delta \mathbf{x}_t, \tag{6}$$

where $\mathbf{w} = [\mathbf{w}_1, \mathbf{w}_2, ..., \mathbf{w}_R]$ is an $N \times R$ weighting matrix. That is, $\mathbf{w}_r = [w_{1,r}, w_{2,r}, ..., w_{N,r}]'$, $r = 1, ..., R$, is an $N \times 1$ vector of weights on predictor variables for the r^{th} PLS factor, $\Delta f_{r,t}^{PLS}$. $\boldsymbol{\theta}$ is an $R \times 1$ vector of PLS regression coefficients. PLS regression minimizes the sum of squared residuals from the equation (5) for $\boldsymbol{\theta}$ instead of $\boldsymbol{\beta}$ in (4).

It should be noted that we do not utilize $\boldsymbol{\theta}$ for our out-of-sample forecasting exercises in the present paper. To make it comparable to PC factors, we utilize PLS factors $\Delta \mathbf{f}_t^{PLS}$, then augment the benchmark forecasting model with estimated PLS factors $\Delta \hat{\mathbf{f}}_t^{PLS}$.

Among available PLS algorithms, see Andersson (2009) for a brief survey, we use the one proposed by Helland (1990) that is intuitively appealing. Helland's algorithm to estimate PLS factors for a scalar target variable q_t is as follows.

First, $\Delta \hat{f}_{1,t}^{PLS}$ is pinned down by the linear combinations of the predictors in $\Delta \mathbf{x}_t$.

$$\Delta \hat{f}_{1,t}^{PLS} = \sum_{i=1}^{N} w_{i,1} \Delta x_{i,t}, \tag{7}$$

where the loading (weight) $w_{i,1}$ is given by $Cov(q_t, \Delta x_{i,t})$. Second, we regress q_t and $\Delta x_{i,t}$ on $\Delta \hat{f}_{1,t}^{PLS}$ then get residuals, $\tilde{q}_t$ and $\Delta \tilde{x}_{i,t}$, respectively, to remove the explained component by the first factor $\Delta \hat{f}_{1,t}^{PLS}$. Next, the second factor estimate $\Delta \hat{f}_{2,t}^{PLS}$ is obtained similarly as in (7) with $w_{i,2} = Cov(\tilde{q}_t, \Delta \tilde{x}_{i,t})$. We repeat until the R^{th} factor $\Delta \hat{f}_{R,t}^{PLS}$ is obtained. Note that this algorithm generates mutually orthogonal factors.

2.3 Least Absolute Shrinkage and Selection Operator Factors

We employ a shrinkage and selection method for linear regression models, the LASSO, which is often used for sparse regression. Unlike ridge regression, the LASSO selects a subset ($\mathbf{x}^s$) of predictor variables from $\mathbf{x}$ by assigning 0 coefficient to the variables that are relatively less important in explaining the target variable. Putting it differently, we implement the *feature selection* task using the LASSO.

The LASSO puts a cap on the size of the estimated coefficients for the ordinary least squares (LS) driving the coefficient down to zero for some predictors. The LASSO solves the following constrained minimization problem using L_1-norm penalty on β.

$$\min_{\boldsymbol{\beta}} \left\{ \frac{1}{T} \sum_{t=1}^{T} (q_t - \Delta \mathbf{x}_t' \boldsymbol{\beta})^2 \right\}, \text{ s.t. } \sum_{j=1}^{N} |\beta_j| \leq \tau \tag{8}$$

where $\Delta \mathbf{x}_t = [\Delta x_{1,t}, \Delta x_{2,t}, ..., \Delta x_{N,t}]'$ is an $N \times 1$ vector of predictor variables at time $t = 1, ..., T$, $\boldsymbol{\beta}$ is an $N \times 1$ vector of associated coefficients. As the value of tuning (penalty) parameter τ decreases, the LASSO returns a smaller subset of $\mathbf{x}$, setting more coefficients to zero.

Following Kelly and Pruitt (2015), we choose the value of τ to generate a certain number of predictors by applying the LASSO to $\Delta\mathbf{x}$. We then employ the PC and PLS approaches to extract common factors, $\Delta\mathbf{f}_t^{PC/L}$ or $\Delta\mathbf{f}_t^{PLS/L}$, out of the predictor variables that are chosen by the LASSO regression. The variables selected from the regression were based on the entire period and the tuning parameter was selected accordingly. Similar to our PLS approach, we use the LASSO method only to obtain the subset of predictors that are closely related to the target. Having obtained them, we use a rolling window approach to extract the common factors using PC and PLS methods described above.

3 In-Sample Analysis

3.1 Data Descriptions

We employ two sets of large panel macroeconomic data in the U.S. and Korea to assess and compare the predictability of these dataset for the real dollar/won exchange rate. For this purpose, we obtained a panel of 126 American macroeconomic time series variables from the FRED-MD database. We also obtained 192 Korean macroeconomic time series data from the Bank of Korea. Korea has maintained a largely fixed exchange rate regime for the dollar-won exchange rate until around 1980, then switched to a heavily managed floating exchange rate regime. Korea continued such dirty float or managed float until they were forced to adopt a market based exchange rate regime after they got hit by the Asian Financial Crisis in 1997.

We focus on the free floating exchange rate regime in 2000's after the Korean economy fully recovered from the crisis. Observations are monthly and span from October 2000 to March 2019 to utilize reasonably many monthly predictors in Korea. We use the consumer price index (CPI) to transform the nominal KRW/USD exchange rate to the real exchange rate.

We categorized 126 American predictors into 9 groups. Groups #1 through #4 include real activity variables that include industrial productions and labor market variables, while groups #5 to #9 are nominal/financial market variables such as interest rates and prices in the U.S. Similarly, we categorized 192 Korean predictors into 13 groups. Groups #1 through #6 include real activity variables that include inventories and industrial productions, while groups #7 to #13 are nominal/financial market variables such as interest rates and monetary aggregates. See Table 1 for more detailed information. We log-transformed all quantity variables prior to estimations other than those expressed in percent (e.g., interest rates and unemployment rates).

Table 1 around here

3.2 Unit Root Tests

We first implement some specification tests for our analysis. Table 2 presents the augmented Dickey Fuller (ADF) test results for the log real exchange rate ($q_t = s_t + p_t^{US} - p_t^{KR}$) and the log nominal exchange rate (s_t). The ADF test rejects the null of nonstationarity for q_t at the 5% significance level, while it fails to reject the null hypothesis for s_t at any conventional level. Note that these results are consistent with standard monetary models in international macroeconomics. Recall, for example, that purchasing power parity (PPP) is consistent with stationary q_t and nonstationary s_t, because PPP implies a cointegrating relationship [1, 1] between s_t and the relative price ($relp_t = p_t^{US} - p_t^{KR}$) for the real exchange rate q_t in the long-run.

Next, we implement a panel unit root test for predictor variables in the US ($\mathbf{x}_t^{US}$) and in Korea ($\mathbf{x}_t^{KR}$) via the Panel Analysis of Nonstationarity in Idiosyncratic and Common components (PANIC) analysis by Bai and Ng (2004, 2010). The PANIC procedure estimates common factors ($f_{r,t}^{PC}, r = 1, 2, .., R$) via PC as explained in the previous section, then test the null of nonstationarity for common factors via the ADF test with an intercept. It also implements a panel unit root test for de-factored idiosyncratic components of the data by the following statistic.

$$P_{\hat{e}} = \frac{-2\sum_{i=1}^{N} \ln p_{\hat{e}_i} - 2N}{2N^{1/2}},$$

where $p_{\hat{e}_i}$ denotes the p-value of the ADF statistic with no deterministic terms for de-factored $\Delta x_{i,t}$.[8]

Note that we also test the null hypothesis for the common factors of subsets of $\mathbf{x}_t$, that is, real and financial sector variables separately. This is because we are interested in the out-of-sample predictability of the common factors from these subsets of the data. In what follows, we show American real activity factors ($f_{r,t}^{PC,R}, r = 1, 2, .., R$) include more long-run predictive contents, whereas American financial market factors ($f_{r,t}^{PC,F}, r = 1, 2, .., R$) yield superior predictability in the short-run, which is consistent with the implications of Boivin and Ng (2006).

The PANIC test fails to reject the null of nonstationarity for all common factor estimates at the 5% significance level with an exception of the second financial factor in the US. Its panel unit root test rejects the null hypothesis that states all variables are $I(1)$ processes for all cases.[9] However, nonstationary common factors eventually dominate stationary dynamics of de-factored idiosyncratic components.[10] Hence, test results in Table 2 provide strong evidence in favor of nonstationarity in the predictor variables $\mathbf{x}_t$, which is consistent with Nelson and Plosser (1982).

Table 2 around here

[8] $P_{\hat{e}}$ statistic has an asymptotic standard normal distribution. The panel test utilizes the p-value of the ADF statistics with no deterministic terms, because defactored variables are mean-zero residuals.

[9] The alternative hypothesis is that there is at least one stationary variable.

[10] See Kim and Kim (2018) for a simulation study that shows the dominance of stationary components over non-stationary components in small samples.

3.3 Factor Model In-Sample Analysis

This section describes some in-sample properties of the factor estimates we discussed in previous section. The in-sample factors are estimated based on the entire available data, and therefore these factors will be different that of the *out-of-sample* factors obtained in the next section. Figure 1 presents in-sample fit analysis for the KRW/USD real exchange rate. Three figures in top row report cumulative R^2 statistics of PC and PLS factors obtained from all predictors, real activity predictors, and financial sector predictors in the US, respectively from left to right. Three figures in bottom row provide cumulative R^2 values of Korean factors similarly. Some interesting findings are as follows.

First, the PLS factors (dashed lines) provide a notably better in-sample fit in comparison with the performance of PC factors (solid lines). This is because PLS utilizes the covariance between the target and the predictor variables, while PC factors are extracted from the predictor variables only. It is also interesting to see that the cumulative R^2 statistics of PLS factors overall exhibit a positive slope at a decreasing rate as the number of factors increases, whereas additional contributions of PC factors show no such patterns. This is mainly due to the fact that our PLS algorithm sequentially estimates orthogonalized common factors after removing explanatory power of previously estimated factors. The PC method extracts common factors without considering the target variable, hence the additional contribution of PC factors does not necessarily decrease.

Second, American factors greatly outperform Korean factors. Cumulative R^2 values of American PLS factors reach well above 60%, while Korean PLS factors cumulatively explain less than 40% of variations in the real exchange rate. Note that Korean PC factors yield virtually no explanatory power. These findings imply that Korean macroeconomic variables might not play an important role in explaining the Won/Dollar real exchange rate dynamics, while American predictors contain substantial predictive contents for it.

Figure 1 around here

We relate such findings with a strong degree of cross-correlations of many bilateral exchange rates relative to the US dollars, implying that American factors better explain dynamics of exchange rates *vis-à-vis* the U.S. dollar than idiosyncratic factors in local countries such as Korea. To see this, we implement a formal test by Pesaran (2021) for the cross-section dependence in 36 bilateral real exchange rates against the U.S. dollar including 16 euro-zone countries.[11]

[11] We obtained all nominal exchange rates and CPIs from the IFS for the sample period from September 2000 to December 2018. 16 eurozone countries that are included are Austria, Belgium, Finland, France, Germany, Greece, Ireland, Italy, Latvia, Lithuania, Luxembourg, Netherlands, Portugal, Slovakia, Slovenia, and Spain. We were able to retrieve 20 non-eurozone countries including Brazil, Canada, Chile, China, Colombia, Czech Republic, Denmark, Hungary, India, Indonesia, Israel, Japan, Korea, Mexico, Poland, Russia, Singapore, Switzerland, Sweden, and the UK. We obtained the Singapore CPI from the Department of Statistics of Singapore.

Consider the following test statistic.

$$CD = \sqrt{\frac{2T}{N(N-1)}} \left(\sum_{i=1}^{N-1} \sum_{j=i+1}^{N} \hat{\rho}_{i,j} \right) \longrightarrow^d N(0,1)$$

where $\hat{\rho}_{i,j}$ is the pair-wise correlation coefficients from the residuals of the ADF regressions for each real exchange rate.[12] The CD statistic was 176.748 ($pv = 0.000$) implying strong empirical evidence of cross-section dependence at any conventional significance level.

The heat map in Figure 2 clearly demonstrates such strong cross-correlations in the real exchange rate relative to the U.S. dollar. With some exceptions such as China that often employs managed float, most real exchange rates exhibit highly correlated contemporaneous relations. The average $\hat{\rho}_{i,Korea}$ was 0.446, while the average $\hat{\rho}_{i,j}$ of all countries was 0.481.

We obtained similar results even when we exclude all euro-zone countries. The CD statistic was 59.019 ($pv = 0.000$) and the average $\hat{\rho}_{i,j}$ was 0.293. Such strong cross-correlations of many bilateral real exchange rates imply a dominant role of the reference country, that is, the U.S. in determining the dynamics of these bilateral exchange rates.

Figure 2 around here

Third, PLS factors from American real activity groups and financial variable groups explain variations in the KRW/USD real exchange rate similarly well. On the other hand, the contribution of PLS Korean factors mostly stem from that of PLS Korean financial sector factors. PLS real Korean factors explain less than 10% of variations jointly even when 12 factors are utilized.

Next, we investigate the source of these common factor estimates via the marginal R^2 analysis. That is, we regress each predictor onto the common factor and record what proportion of the variation can be explained by the common factor. Results are reported in Figures 3 to 5 for the first common factor from the entire predictors, real activity variables, and nominal/financial market variables, respectively.

As can be seen in Figure 3, we note that the marginal R^2 statistics of the first American PC factor (solid lines) are very similar to those of the first PLS American factor (bar graphs). On the other hand, the marginal R^2 statistics of the first Korean PC factor is quite different from the PLS factor. More specifically, the marginal R^2 statistics of the Korean PLS factor are negligibly low in comparison with those of the PC factor. Since PC factors are obtained only from the predictors with no reference on the target variable, the marginal R^2 values of the PC factor are expected to be high. However, since PLS factors are estimated using the covariance of the target variable and the predictor, low R^2 statistics of the Korean PLS factor imply that Korean predictors are

[12] We implemented the ADF regression for each real exchange rate relative to the U.S. dollar via the general-to-specific rule with maximum 6 lags, then calculated pair-wise correlation coefficients using the ADF regression residuals of 35 real exchange rates.

largely disconnected from the KRW/USD real exchange rate. This is consistent with negligibly low cumulative R^2 statistics in Figure 1.

Note also that PLS American factors are more closely connected with groups #1 (industrial production) and #2 (labor market) than other groups. Put it differently, the first PLS American factor seems to be strongly driven by these real activity variables rather than financial market variables and other real activity variables. We also point out that these two groups include key variables that influence the Fed's decision making process of U.S. monetary policy stance under its dual mandate. That is, they are potentially useful predictors of financial market conditions.

Figure 3 around here

We investigate the source of the common factors in a more disaggregated level, looking at the marginal R^2 statistics of the real and financial market factors. Figure 4 reports the R^2 statistics of the first American real activity factor. Again, the PLS and PC factors turn out to explain the variations in real activity variables similarly well. We also note that the American real activity factor is mainly driven by industrial production (Group #1) and labor market (Group #2) variables. However, the PLS Korean real activity factor explains negligible variations in Korean real activity variables, while the first PC factor exhibits reasonably high R^2 statistics. This again confirms our previous findings. Similar results were observed from the marginal R^2 analysis for the first financial market PLS and PC factors in Figure 5. The American PLS and PC nominal/financial market factors seem to be driven mostly by CPIs and PPIs in the US.

Figures 4 and 5 around here

4 Out-of-Sample Prediction Performance

4.1 Factor-Augmented Forecasting Models

This section reports our out-of-sample forecast exercise results using factor-augmented forecasting models for the KRW/USD real exchange rate. Based on the ADF test results in Table 2, we employ the following *stationary* $AR(1)$-type stochastic process for the real exchange rate q_t. Abstracting from an intercept,

$$q_{t+j} = \alpha_j q_t + u_{t+j}, \ j = 1, 2, .., k, \tag{9}$$

where α_j is less than one in absolute value for stationarity. Note that we regress the j-period ahead target variable (q_{t+j}) *directly* on the current period target variable (q_t) instead of using a *recursive* forecasting approach with an AR(1) model, $q_{t+1} = \alpha q_t + \varepsilon_{t+1}$, which implies $\alpha_j = \alpha^j$ under that approach. With this specification, the j-period ahead forecast is,

$$\widehat{q}^{AR}_{t+j|t} = \hat{\alpha}_j q_t, \tag{10}$$

where $\hat{\alpha}_j$ is the least squares (LS) estimate of α_j in (9).

We augment (9) by adding factor estimates. That is, our factor augmented stationary $AR(1)$-type forecasting model is the following.

$$q_{t+j} = \alpha_j q_t + \boldsymbol{\beta}_j^{'} \boldsymbol{\Delta}\hat{\mathbf{f}}_t + u_{t+j},\ j = 1, 2, .., k \tag{11}$$

We again employ a *direct* forecasting approach by regressing q_{t+j} directly on q_t and the estimated factors ($\boldsymbol{\Delta}\hat{\mathbf{f}}_t$). Note that (11) coincides with an exact $AR(1)$ process when $j = 1$, but extended by the factor covariates $\boldsymbol{\Delta}\hat{\mathbf{f}}_t$. Note also that (11) nests the stationary benchmark model (9) when $\boldsymbol{\Delta}\hat{\mathbf{f}}_t$ does not contain any useful predictive contents for q_{t+j}, that is, $\boldsymbol{\beta}_j = 0$. We obtain the following j-period ahead forecast for q_{t+j},

$$\widehat{q}^{F_{AR}}_{t+j|t} = \hat{\alpha}_j q_t + \hat{\boldsymbol{\beta}}_j^{'} \Delta\hat{\mathbf{f}}_t, \tag{12}$$

where $\hat{\alpha}_j$ and $\hat{\boldsymbol{\beta}}_j$ are the LS coefficient estimates from (11).

We evaluate the out-of-sample predictability of our factor-augmented forecasting model $\widehat{q}^{F_{AR}}_{t+j|t}$ using a fixed-size rolling window scheme as follows.[13] We use initial $T_0 < T$ observations, $\{q_t, \Delta x_{i,t}\}_{t=1}^{T_0}$, $i = 1, 2, ..., N$ to estimate the first set of factors $\left\{\Delta\hat{\mathbf{f}}_t\right\}_{t=1}^{T_0}$ using one of our data dimensionality reduction methods. We formulate the first forecast $\widehat{q}^{F_{AR}}_{T_0+j|T_0}$ by (12), then calculate and keep the forecast error ($\varepsilon^{F_{AR}}_{T_0+j|T_0}$). Next, we add one observation ($t = T_0+1$) but drop one earliest observation ($t = 2$) for the second round forecasting. That is, we re-estimate $\left\{\Delta\hat{\mathbf{f}}_t\right\}_{t=2}^{T_0+1}$ using $\{q_t, \Delta x_{i,t}\}_{t=2}^{T_0+1}$, $i = 1, 2, ..., N$, maintaining the same number of observations (T_0) in order to formulate the second round forecast, $\hat{q}^{F_{AR}}_{T_0+j+1|T_0+1}$, and its resulting forecast error $\varepsilon^{F_{AR}}_{T_0+j+1|T_0+1}$. We repeat until we forecast the last observation, q_T. We implement the same procedure for the benchmark forecast $\widehat{q}^{AR}_{t+j|t}$ by (10) in addition to the no-change Random Walk (RW) benchmark $\widehat{q}^{RW}_{t+j|t} = q_t$.[14]

We employ the ratio of the root mean square prediction error ($RRMSPE$) to evaluate the j-period ahead out-of-sample prediction accuracy of our factor augmented models relative to the benchmark. That is,

$$RRMSPE(j) = \frac{\sqrt{\frac{1}{T-j-T_0+1}\sum_{t=T_0}^{T-j}\left(\varepsilon^{BM}_{t+j|t}\right)^2}}{\sqrt{\frac{1}{T-j-T_0+1}\sum_{t=T_0}^{T-j}\left(\varepsilon^{F_{AR}}_{t+j|t}\right)^2}}, \tag{13}$$

[13] Rolling window schemes tend to perform better than the recursive method in the presence of structural breaks. However, results with recursive approaches were qualitatively similar.

[14] Consider a random walk model, $q^{BM_{RW}}_{t+1} = q_t + \eta_{t+1}$,where η_{t+1} is a white noise process. Therefore, j-period ahead forecast from this benchmark RW model is simply q_t.

where

$$\varepsilon^{BM}_{t+j|t} = q_{t+j} - \widehat{q}^{BM}_{t+j|t},\ \varepsilon^{F}_{t+j|t} = q_{t+j} - \widehat{q}^{F_{AR}}_{t+j|t},\ BM = AR, RW \tag{14}$$

Note that $RRMSPE(j) > 1$ indicates that our factor models outperform the benchmark models.[15]

4.2 Prediction Accuracy Evaluations for the Real Exchange Rate

We implement out-of-sample forecast exercises using a fixed-size (50% split point, that is, $T_0 = T/2$) rolling window method with up to 3 (k) latent factor estimates.[16] Latent common factors are acquired via the PLS, PC, and LASSO methods for large panels of macroeconomic data of the U.S. and Korea.

Table 3 reports the $RRMSPE$ statistics of our forecasting model $\widehat{q}^{F_{AR}}_{t+j|t}$ relative to $\widehat{q}^{RW}_{t+j|t}$. Recall that our models outperform the RW benchmark when the $RRMSPE$ is greater than one. We also obtained the $RRMSPE$ statistics of $\widehat{q}^{F_{AR}}_{t+j|t}$ relative to $\widehat{q}^{AR}_{t+j|t}$ (not reported to save space), and the superscript $*$ indicates that $\widehat{q}^{F_{AR}}_{t+j|t}$ outperforms $\widehat{q}^{AR}_{t+j|t}$. In fact, since the AR benchmark performs better than the RW model, it implies that $\widehat{q}^{F_{AR}}_{t+j|t}$ outperforms both the benchmarks $\widehat{q}^{RW}_{t+j|t}$ and $\widehat{q}^{AR}_{t+j|t}$. Our major findings are as follows.

First, the American predictors yield superior predictive contents for the KRW/USD real exchange rate, while the Korean factor models perform relatively poorly. That is, our factor models outperform both the RW and AR models only when the American factors are employed. The models with the Korean factors overall got dominated by the AR model although they still outperform the RW model when the forecast horizon is one-year or longer. Recall that these empirical findings are consistent with our in-sample fit analysis shown in the previous section. Interestingly, when we combine the American factors with equal numbers of the Korean factors, the performance tends to deteriorate. That is, the $RRMSPE$ often decreases in comparison with those of the American factor augmented models implying that the Korean factors may add noise in our combined models.

Second, we observe that our American factor models tend to perform better at short horizons when combined with nominal/financial market factors, while real activity factors improve the predictability at longer horizons. That is, the good prediction performance of our models with the total factors, $\Delta\hat{\mathbf{f}}^{PLS}_t$ or $\Delta\hat{\mathbf{f}}^{PC}_t$, at 1-period horizon seem to inherit the superior performance of our models with financial market factors, $\Delta\hat{\mathbf{f}}^{PLS,F}_t$ or $\Delta\hat{\mathbf{f}}^{PC,F}_t$, while superior long-run predictability seems to stem from predictive contents of real activity factors, $\Delta\hat{\mathbf{f}}^{PLS,R}_t$ or $\Delta\hat{\mathbf{f}}^{PC,R}_t$. These results imply that factors obtained from subsets may provide more useful information than factors from the entire predictor variables, which is consistent with Boivin and Ng (2006).

Table 3 around here

[15] Alternatively, one may employ the ratio of the root mean absolute prediction error ($RRMAPE$). Results are overall qualitatively similar.

[16] We obtained qualitatively similar results with a 70% sample split point.

We also employ the LASSO approach to select the subsets of the predictor variables that are useful to explain the target variable. The idea behind that is to estimate the factors using fewer but more informative predictor variables as discussed by Bai and Ng (2008). Following Kelly and Pruitt (2015), we adjust the tuning parameter τ in (8) to choose a group of 30 predictors from each panel of macroeconomic variables in the US and in Korea, while 20 predictors were chosen from each of the real activity and the financial market variable groups. Then, we employed PLS and PC to estimate up to 3 common factors to augment the benchmark AR model.

As we can see in Table 4, results are qualitatively similar to previous ones. The financial factor augmented forecasting models tend to perform better for the 1-period ahead forecasts, while real factors provide superior predictive contents for the real exchange rate at longer horizons. The Korean factor-augmented models perform overall poorly relative to the AR model, although they still outperform the RW model when the forecast horizon is 1-year or longer.

Table 4 around here

4.3 Model Predictability with Proposition Based Global Factors

This section implements out-of-sample forecast exercises using the models that are augmented by global factors, motivated by three exchange rate determination theories: Purchasing Power Parity (PPP); Uncovered Interest Parity (UIP); Real Interest Rate Parity (RIRP).

4.3.1 Purchasing Power Parity: Relative Price Factors

We first employ PPP to identify common global factors. Under PPP, the real exchange rate q_t ($= s_t + relp_t$) is stationary, while the log nominal exchange rate s_t and the log relative price $relp_t$ ($= p_t - p_t^*$) are nonstationary $I(1)$ processes. Put it differently, there exists a cointegrating vector $[1,\ 1]$ for s_t (foreign currency price of 1 USD) and $relp_t$, where p_t and p_t^* are the log prices in the U.S. and in the foreign country, respectively. Recall that our unit root test results in Table 2 are consistent with PPP.

We obtained the Consumer Price Index (CPI) of 43 countries from the IFS database including 18 euro-zone countries. Assuming that the U.S. is the home/reference country, we constructed 42 relative prices ($p_t - p_t^*$), then estimated the first common factor from these relative prices after taking the first difference ($\Delta p_t - \Delta p_t^*$), or ($\pi_t - \pi_t^*$), since the relative price is an integrated process. Also, we extracted the common factors from the 42 relative prices with Korea as the reference country.[17,18]

We report j-period ahead out-of-sample predictability exercise results (50% split point) for the Won-Dollar real exchange rate using one factor models when the U.S. or Korea serves as the reference country in Table 5. Results imply overall poor performance of PPP motivated factor models irrespective of the choice of the numéraire currency.

Our factor models overall outperform the RW model when the forecast horizon is 1-year or longer, which is consistent with the view that PPP is a long-run proposition. However, our PPP factor models rarely beat the AR benchmark model nor our data-driven macroeconomic factor models presented in the previous section. It is not surprising to find similar performance of the Korean reference factor model as the American factor model, since they include fundamentally similar information of CPIs in 43 countries.

Table 5 around here

4.3.2 Uncovered Interest Parity: Interest Rate Spread

The second proposition we adopt is Uncovered Interest Parity (UIP). Abstracting from risk premium, UIP states the following.

$$\Delta s_{t+1} = i_t^* - i_t + \varepsilon_{t+1}, \tag{15}$$

where Δs_{t+1} is the nominal exchange rate return, that is, appreciation (depreciation) rate of the home (foreign) currency, while i_t and i_t^* are nominal short-run interest rates in the home and foreign countries, respectively. $\varepsilon_{t+1} = \Delta s_{t+1} - E_t \Delta s_{t+1}$ is the mean-zero ($E_t \varepsilon_{t+1} = 0$) rational expectation error term.

Motivated by (15), we obtained 18 international short-term interest rates from the FRED and the OECD database to construct nominal interest rate spreads by subtracting US interest rate (i_t) from the national interest rate (i_t^*).[19] We estimate the first common factor via PLS and PC from the balanced panel of 17 interest rate spreads relative to American interest rate. We took the first difference of the spreads to make sure we estimate factors consistently.[20] Similarly, we estimated the first common factor from 17 interest rate spreads relative to Korean interest rate.

[17] For the group of developed countries, we include 11 euro-zone countries (Austria, Belgium, Finland, France, Germany, Greece, Italy, Luxembourg, Netherlands, Portugal, Spain) and 8 non-euro-zone countries (Canada, Denmark, Japan, Singapore, Switzerland, Sweden, United Kingdom, United States). All data are obtained from the IFS database with an exception of Singapore. We obtained the Singapore CPI from the Department of Statistics of Singapore. In addition to the group of 19 developed countries, we added 7 the rest of euro-zone countries (Cyprus, Estonia, Ireland, Latvia, Lithuania, Slovakia, Slovenia) except Malta, and 17 non-euro-zone countries (Brazil, China, Chile, Colombia, Czech Republic, Hong Kong, Hungary, India, Indonesia, Israel, Korea, Malaysia, Mexico, Poland, Romania, Russia, Saudi Arabia).

[18] We focus on the relative price factor because of lack of available nominal exchange rates in the presence of euro-zone countries.

[19] For the group of developed countries, we include 11 euro-zone countries (Austria, Belgium, Finland, France, Germany, Greece, Italy, Luxembourg, Netherlands, Portugal, Spain) and 7 non-euro-zone countries (Canada, Denmark, Japan, Switzerland, Sweden, United Kingdom, United States). All data are obtained from the OECD database and the FRED.

[20] The PANIC test provides strong evidence of nonstationarity for the interest rate spreads.

We also employ a similar factor model as (9) but for the nominal exchange rate return Δs_t. Since s_t is an integrated process, we employ the following $AR(1)$-type model for Δs_t. Abstracting from an intercept,

$$\Delta s_{t+j} = \alpha_j \Delta s_t + u_{t+j},\ j = 1, 2, .., k, \tag{16}$$

where α_j is less than one in absolute value for stationarity.[21] Note that we regress the j-period ahead target variable (Δs_{t+j}) *directly* on the current period exchange rate return (Δs_t). Then, the j-period ahead forecast is,

$$\Delta \hat{s}^{AR}_{t+j} = \hat{\alpha}_j \Delta s_t, \tag{17}$$

while the corresponding factor augmented forecasting model is given as follows.

$$\Delta s_{t+j} = \alpha_j \Delta s_t + \boldsymbol{\beta}_j^{'} \boldsymbol{\Delta} \hat{\mathbf{f}}_t + u_{t+j},\ j = 1, 2, .., k \tag{18}$$

which augment (16) by adding factor estimates ($\Delta \hat{\mathbf{f}}_t$). The j-period ahead forecast for the exchange rate return is,

$$\Delta \widehat{s}^{F_{AR}}_{t+j|t} = \hat{\alpha}_j \Delta s_t + \hat{\boldsymbol{\beta}}_j^{'} \Delta \hat{\mathbf{f}}_t, \tag{19}$$

where $\hat{\alpha}_j$ and $\hat{\boldsymbol{\beta}}_j$ are the LS coefficient estimates.

The first 2 columns of Table 6 reports the $RRMSPE$ statistics of the 1-period ahead out-of-sample forecasts for the nominal exchange rate return (Δs_t) using the UIP motivated factors. The American UIP-PLS factor forecasting models perform well relative to both benchmark models. The American macroeconomic data driven PLS models perform similarly well. The Korean UIP factor models overall performed poorly except when we included two UIP factors. However, they perform better than the Korean macroeconomic factor augmented models, highlighting marginal but potentially important contributions of the UIP global factors.

Table 6 around here

4.3.3 Real Interest Rate Parity: Real Interest Rate Spread

The last proposition we employ is Real Interest Rate Parity (RIRP), which combines PPP with UIP. Taking the first difference to the PPP equation ($q_t = s_t + p_t - p_t^*$) at time $t+1$,

$$\Delta q_{t+1} = \Delta s_{t+1} + \pi_{t+1} - \pi^*_{t+1} \tag{20}$$

Combining (15) and (20), we obtain the following expression for RIRP.

$$\Delta q_{t+1} = r_t^* - r_t + \varepsilon_{t+1}, \tag{21}$$

[21] That is, we assume that there are two eigenvalues for the level exchange rate, 1 and α_1.

where $r_t = i_t - \pi_{t+1}$ and $r_t^* = i_t^* - \pi_{t+1}^*$ are the *ex post* real interest rates in the home and foreign country, respectively. Note that this specification is inconsistent with the ADF test results in Table 2 that provides strong empirical evidence in favor of the stationarity of the real KRW/USD exchange rate, although it shows highly persistent dynamics similar to that of the nominal exchange rate.

Using international CPIs and short-run interest rates we used, we estimated the first common factors by applying PLS and PC to a panel of 17 real interest rate spreads $(r_t^* - r_t)$ without taking differences. This is because we obtained very strong evidence in favor of stationarity for real interest rate spreads.[22]

Similar to the previous section, we estimate the common factors from the macroeconomic data for Δq_t to compare the results with the RIRP based models. Results are reported in Table 7.

$RRMSPE$ statistics in the first two columns demonstrate the RIRP global common factors are not useful whichever country serves as the reference country. However, our macroeconomic data driven PLS factor models outperform both benchmark models when the American factors are employed. Again, the Korean factor models perform quite poorly relative to both benchmark models.

Table 7 around here

5 Concluding Remarks

In this paper, our objective is to propose parsimonious factor-augmented forecasting models for the dollar/won real exchange rate within a data-rich environment. We leverage various data dimensionality reduction techniques on extensive panels of macroeconomic time series data, consisting of 125 American and 192 Korean monthly frequency variables, covering the period from October 2000 to March 2019. In addition to the widely used Principal Component (PC) analysis in the literature, we introduce the Partial Least Squares (PLS) approach and combine the Least Absolute Shrinkage and Selection Operator (LASSO) with PC and PLS to extract target-specific common factors specifically for the dollar/won real exchange rate.

We enhance benchmark forecasting models by incorporating the estimated common factors and generate out-of-sample forecasts. To assess the prediction accuracy of our proposed models for the real exchange rate compared to the random walk (RW) and stationary autoregressive (AR) models, we employ the ratio of the root mean squared prediction error ($RRMSPE$) criteria. By employing these methodological approaches, we aim to provide more accurate and parsimonious forecasting models for the dollar/won real exchange rate, improving upon traditional benchmark models.

Our proposed forecasting models consistently demonstrate superior performance compared to both the random walk (RW) and autoregressive (AR) benchmark models, but this is only achieved

[22]The PANIC test results are available upon request.

when utilizing latent common factors derived from the American predictors. Specifically, models incorporating American real activity factors exhibit strong performance at longer horizons, while American nominal/financial market factors enhance prediction accuracy at shorter horizons. These findings align with the research of Boivin and Ng (2006), who emphasized the importance of identifying relevant common factors for the target variable. In contrast, models incorporating Korean factors generally underperform compared to the AR model, although they still outperform the RW model for forecast horizons of 1 year or longer.

In addition, we conduct forecasting exercises using global common factors motivated by exchange rate determination theories such as Purchasing Power Parity (PPP), Uncovered Interest Parity (UIP), and Real Uncovered Interest Parity (RIRP). The models incorporating UIP common factors perform reasonably well when the US is considered as the reference country. However, models incorporating PPP or RIRP factors exhibit subpar performance regardless of the reference country. Overall, our findings highlight the importance of selecting relevant factors and considering the specific economic context when constructing forecasting models for the dollar/won real exchange rate.

References

Andersson, M. (2009): "A Comparison of Nine PLS1 Algorithms," *Journal of Chemometrics*, 23, 518–529.

Bai, J., and S. Ng (2002): "Determining the Number of Factors in Approximate Factor Models," *Econometrica*, 70(1), 191–221.

——— (2004): "A PANIC Attack on Unit Roots and Cointegration," *Econometrica*, 72(4), 1127–1177.

——— (2008): "Forecasting Economic Time Series Using Targeted Predictors," *Journal of Econometrics*, 146(2), 304–317.

——— (2010): "Panel Unit Root Tests With Cross-Section Dependence: A Further Investigation," *Econometric Theory*, 26(4), 1088–1114.

Boivin, J., and S. Ng (2006): "Are More Data Always Better for Factor Analysis?," *Journal of Econometrics*, 132(1), 169–194.

Ca' Zorzi, M., A. Kociecki, and M. Rubaszek (2015): "Bayesian Forecasting of Real Exchange Rates with a Dornbusch Prior," *Economic Modelling*, 46(C), 53–60.

Chen, S.-L., J. Jackson, H. Kim, and P. Resiandini (2014): "What Drives Commodity Prices?," *American Journal of Agricultural Economics*, 96(5), 1455–1468.

Cheung, Y.-W., M. Chinn, and A. G. Pascual (2005): "Empirical Exchange Rate Models of The Nineties: Are Any Fit to Survive?," *Journal of International Money and Finance*, 24(7), 1150–1175.

Cheung, Y.-W., and K. S. Lai (1995): "Lag Order and Critical Values of the Augmented Dickey-Fuller Test," *Journal of Business & Economic Statistics*, 13(3), 277–80.

Chinn, M., and R. Meese (1995): "Banking on Currency Forecasts: How Predictable is Change in Money?," *Journal of International Economics*, 38(1-2), 161–178.

Clarida, R. H., and D. Waldman (2008): "Is Bad News About Inflation Good News for the Exchange Rate? And, If So, Can That Tell Us Anything about the Conduct of Monetary Policy?," in *Asset Prices and Monetary Policy*, NBER Chapters, pp. 371–396. National Bureau of Economic Research, Inc.

Engel, C., and J. Hamilton (1990): "Long Swings in the Dollar: Are They in the Data and Do Markets Know It?," *American Economic Review*, 80(4), 689–713.

Engel, C., N. C. Mark, and K. D. West (2008): "Exchange Rate Models Are Not As Bad As You Think," in *NBER Macroeconomics Annual 2007, Volume 22*, NBER Chapters, pp. 381–441. National Bureau of Economic Research, Inc.

——— (2015): "Factor Model Forecasts of Exchange Rates," *Econometric Reviews*, 34(1-2), 32–55.

Engel, C., and K. D. West (2005): "Exchange Rates and Fundamentals," *Journal of Political Economy*, 113(3), 485–517.

——— (2006): "Taylor Rules and the Deutschmark: Dollar Real Exchange Rate," *Journal of Money, Credit and Banking*, 38(5), 1175–1194.

Greenaway-McGrevy, R., N. C. Mark, D. Sul, and J.-L. Wu (2018): "Identifying Exchange Rate Common Factors," *International Economic Review*, 59(4), 2193–2218.

Groen, J. (2005): "Exchange Rate Predictability and Monetary Fundamentals in a Small Multi-country Panel," *Journal of Money, Credit and Banking*, 37(3), 495–516.

Groen, J., and G. Kapetanios (2016): "Revisiting Useful Approaches to Data-Rich Macroeconomic Forecasting," *Computational Statistics and Data Analysis*, 100, 221–239.

Groen, J. J. J. (2000): "The monetary exchange rate model as a long-run phenomenon," *Journal of International Economics*, 52(2), 299–319.

Helland, I. S. (1990): "Partial Least Squares Regression and Statistical Models," *Scandinavian Journal of Statistics*, 17, 97–114.

Ince, O., T. Molodtsova, and D. H. Papell (2016): "Taylor rule deviations and out-of-sample exchange rate predictability," *Journal of International Money and Finance*, 69(C), 22–44.

Kelly, B., and S. Pruitt (2015): "The Three-Pass Regression Filter: A New Approach to Forecasting Using Many Predictors," *Journal of Econometrics*, 186(2), 294–316.

Kim, H., I. Fujiwara, B. E. Hansen, and M. Ogaki (2015): "Purchasing Power Parity and the Taylor Rule," *Journal of Applied Econometrics*, 30(6), 874–903.

Kim, H., and J. Kim (2018): "London calling: Nonlinear mean reversion across national stock markets," *The North American Journal of Economics and Finance*, 44(C), 265–277.

Lothian, J. R., and M. P. Taylor (1996): "Real Exchange Rate Behavior: The Recent Float from the Perspective of the Past Two Centuries," *Journal of Political Economy*, 104(3), 488–509.

Mark, N. (1995): "Exchange Rates and Fundamentals: Evidence on Long-Horizon Predictability," *American Economic Review*, 85(1), 201–18.

Mark, N., and D. Sul (2001): "Nominal Exchange Rates and Monetary Fundamentals: Evidence From a Small Post-Bretton Woods Panel," *Journal of International Economics*, 53(1), 29–52.

Mark, N. C. (2009): "Changing Monetary Policy Rules, Learning, and Real Exchange Rate Dynamics," *Journal of Money, Credit and Banking*, 41(6), 1047–1070.

Meese, R. A., and K. Rogoff (1983): "Empirical Exchange Rate Models of The Seventies: Do They Fit Out of Sample?," *Journal of International Economics*, 14(1), 3 – 24.

Molodtsova, T., A. Nikolsko-Rzhevskyy, and D. Papell (2008): "Taylor Rules With Real-Time Data: A Tale of Two Countries And One Exchange Rate," *Journal of Monetary Economics*, 55(Supplement 1), S63–S79.

Molodtsova, T., and D. Papell (2009): "Out-of-Sample Exchange Rate Predictability With Taylor Rule Fundamentals," *Journal of International Economics*, 77(2), 167 – 180.

Molodtsova, T., and D. H. Papell (2013): "Taylor Rule Exchange Rate Forecasting during the Financial Crisis," *NBER International Seminar on Macroeconomics*, 9(1), 55–97.

Nelson, C. R., and C. I. Plosser (1982): "Trends and Random Walks in Macroeconmic Time Series : Some Evidence And Implications," *Journal of Monetary Economics*, 10(2), 139–162.

Pesaran, M. H. (2021): "General Diagnostic Tests for Cross-Sectional Dependence in Panels," *Empirical Economics*, 60, 13–50.

Rapach, D. E., and M. Wohar (2004): "Testing the Monetary Model of Exchange Rate Determination: A Closer Look at Panels," *Journal of International Money and Finance*, 23(6), 867–895.

Rossi, B. (2013): "Exchange Rate Predictability," *Journal of Economic Literature*, 51(4), 1063–1119.

Stock, J. H., and M. W. Watson (2002): "Macroeconomic Forecasting using Diffusion Indexes," *Journal of Business and Economic Statistics*, 20(2), 147–162.

Verdelhan, A. (2018): "The Share of Systematic Variation in Bilateral Exchange Rates," *Journal of Finance*, 73(1), 375–418.

Wang, J., and J. J. Wu (2012): "The Taylor Rule and Forecast Intervals for Exchange Rates," *Journal of Money, Credit and Banking*, 44(1), 103–144.

Wold, H. (1982): *Soft Modelling: The Basic Design and Some Extensions*, vol. 1 of *Systems under indirect observation, Part II.* North-Holland, Amsterdam.

Table 1. Macroeconomic Data Descriptions

American Data

Group ID	Data ID	Data Description
#1	1-16	Industrial Production Indices
#2	17-47	Labor Market Variables
#3	48-57	Housing Inventories
#4	58-65	Manufacturers' Consumption/ New Orders
#5	66-79	Monetary Aggregates
#6	80-96	Domestic Interest Rates
#7	97-116	Producer/Consumer Prices
#8	117-121	Stock Indices
#9	122-126	Exchange Rates

Korean Data

Group ID	Data ID	Data Description
#1	1-27	New Orders
#2	28-34	Inventory
#3	35-52	Housing
#4	53-74	Retails/Manufacturing
#5	75-87	Labor
#6	88-98	Industrial Production
#7	99-102	Business Condition
#8	103-114	Stock Indices
#9	115-127	Interest Rates
#10	128-145	Exports/Imports Prices
#11	146-163	Prices
#12	164-180	Money
#13	181-192	Exchange Rates

Note: We obtained the American data from the FRED-MD website. The Korean Data was obtained from the Bank of Korea.

Table 2. Unit Root Test Results

ADF Test

q_t	$-2.966^{\dagger}$ (0.038)	s_t	-1.953 (0.308)

PANIC Test

$\mathbf{x}_t^{US}$		$\mathbf{x}_t^{KR}$	
$f_{1,t}^{PC}$	-2.519 (0.100)	$f_{1,t}^{PC}$	1.186 (0.998)
$f_{2,t}^{PC}$	-1.270 (0.641)	$f_{2,t}^{PC}$	-2.085 (0.237)
$P_{\hat{e}}$	$11.341^{\ddagger}$ (0.000)	$P_{\hat{e}}$	$16.667^{\ddagger}$ (0.000)
$f_{1,t}^{PC,R}$	-2.443 (0.124)	$f_{1,t}^{PC,R}$	-0.634 (0.868)
$f_{2,t}^{PC,R}$	-2.393 (0.133)	$f_{2,t}^{PC,R}$	-2.057 (0.254)
$P_{\hat{e}}$	$5.763^{\ddagger}$ (0.000)	$P_{\hat{e}}$	$15.552^{\ddagger}$ (0.000)
$f_{1,t}^{PC,F}$	-1.888 (0.326)	$f_{1,t}^{PC,F}$	1.250 (0.999)
$f_{2,t}^{PC,F}$	$-3.437^{\ddagger}$ (0.009)	$f_{2,t}^{PC,F}$	-1.544 (0.512)
$P_{\hat{e}}$	$6.425^{\ddagger}$ (0.000)	$P_{\hat{e}}$	$8.711^{\ddagger}$ (0.000)

Note: q_t and s_t are the CPI-based real and the nominal bilateral Won/Dollar exchange rate, respectively. PLS estimates target specific factors for q_t and s_t separately, while PC yields common factors independent on the target variable. Real variables are from group #1 through #4 for American factors and groups #1 through #7 for Korean factors, while financial variables include group #5 through #9 for American factors and groups #8 through #13 for Korean factors. The augmented Dickey-Fuller (ADF) test reports the ADF t-statistics when an intercept is included. P-values are in parenthesis. For the PANIC test results, we report the ADF t-statistics with an intercept for each common factor estimate. $P_{\hat{e}}$ denotes the panel test statistics from the de-factored idiosyncratic components. ‡ and † denote a rejection of the null hypothesis at the 1% and 5% level, respectively.

Table 3. Out-of-Sample Predictability for the Real Exchange Rate

j	#Factors	$\Delta \mathbf{f}_t^{PLS}$	$\Delta \mathbf{f}_t^{PLS,R}$	$\Delta \mathbf{f}_t^{PLS,F}$	$\Delta \mathbf{f}_t^{PC}$	$\Delta \mathbf{f}_t^{PC,R}$	$\Delta \mathbf{f}_t^{PC,F}$
				American Factors			
1	1	1.0115*	1.0077	1.0074	1.0251*	1.0088*	1.0144*
	2	1.0355*	0.9971	1.0198*	1.0148*	1.0023	1.0155*
	3	1.0266*	0.9941	1.0223*	1.0142*	0.9680	1.0223*
12	1	1.3440*	1.3611*	1.2499*	1.3194*	1.3375*	1.2380
	2	1.3389*	1.3469*	1.0965	1.2573*	1.3375*	1.2352
	3	1.2421*	1.2246	1.0243	1.2572*	1.3308*	1.1217
36	1	1.4269*	1.4466*	1.2710	1.3742*	1.4271*	1.2409
	2	1.2736	1.3948*	1.2316	1.3142	1.3980*	1.2585
	3	1.2232	1.4229*	1.2715	1.3225	1.3882*	1.2300
				Korean Factors			
j	#Factors	$\Delta \mathbf{f}_t^{PLS}$	$\Delta \mathbf{f}_t^{PLS,R}$	$\Delta \mathbf{f}_t^{PLS,F}$	$\Delta \mathbf{f}_t^{PC}$	$\Delta \mathbf{f}_t^{PC,R}$	$\Delta \mathbf{f}_t^{PC,F}$
1	1	0.9006	1.0029	0.9382	0.6836	0.8443	0.9646
	2	0.6329	0.9441	0.9450	0.4931	0.8644	0.2871
	3	0.4880	0.8965	0.5315	0.2348	0.7819	0.1707
12	1	1.2607*	1.2162	1.2698*	1.2381	1.2422*	1.2074
	2	1.2431*	1.1635	1.2362	1.2067	1.2433*	1.1877
	3	1.2380	1.0719	1.2288	1.2159	1.2298	1.2485*
36	1	1.3737*	1.3787*	1.3883*	1.3752*	1.3652	1.3797*
	2	1.3498	1.2832	1.3850*	1.3843*	1.3425	1.3809*
	3	1.1916	1.1056	1.3377	1.3847*	1.3435	1.3687
				American and Korean Factors			
j	#Factors	$\Delta \mathbf{f}_t^{PLS}$	$\Delta \mathbf{f}_t^{PLS,R}$	$\Delta \mathbf{f}_t^{PLS,F}$	$\Delta \mathbf{f}_t^{PC}$	$\Delta \mathbf{f}_t^{PC,R}$	$\Delta \mathbf{f}_t^{PC,F}$
1	2	0.9469	1.0082*	0.9542	0.8242	0.8925	0.9781
	4	0.9441	0.9419	0.8800	0.4030	0.8900	0.2400
	6	0.5604	0.8730	0.5753	0.2230	0.7975	0.1871
12	2	1.3503*	1.3203*	1.2704*	1.3054*	1.3351*	1.1799
	4	1.3055*	1.2448*	1.0548	1.1873	1.3342*	1.1753
	6	1.1377	1.0888	0.9167	1.1621	1.3277*	1.1377
36	2	1.4278*	1.4534*	1.2882	1.3655	1.4190*	1.2216
	4	1.2430	1.3358	1.2300	1.2845	1.3804*	1.2352
	6	1.1017	1.1954	1.2246	1.2733	1.3578	1.2321

Note: We report the *RRMSPE* statistics employing a rolling window scheme with a 50% sample split point. *RRMSPE* denotes the mean squared prediction error (*RMSPE*) from the benchmark random walk (RW) model divided by the *RMSPE* from each competing model with k factors. Superscript R and F mean that factors were estimated from real and financial variables, respectively. *RRMSPE* statistics that are greater than 1 indicate that the competing model outperforms the benchmark RW model. $*$ denotes the cases that the competing model outperforms the benchmark AR model as well as the RW model.

Table 4. Out-of-Sample Predictability with LASSO for the Real Exchange Rate

				American Factors			
j	#Factors	$\Delta\mathbf{f}_t^{PLS}$	$\Delta\mathbf{f}_t^{PLS,R}$	$\Delta\mathbf{f}_t^{PLS,F}$	$\Delta\mathbf{f}_t^{PC}$	$\Delta\mathbf{f}_t^{PC,R}$	$\Delta\mathbf{f}_t^{PC,F}$
1	1	1.0162*	1.0218*	1.0188*	1.0392*	1.0184*	1.0329*
	2	1.0388*	0.9849	1.0261*	1.0451*	1.0160	1.0520*
	3	1.0387*	0.9807	1.0379*	1.0419*	1.0278*	1.0474*
12	1	1.2145	1.3792*	1.1614	1.2515*	1.3354*	1.2413
	2	1.1729	1.3456*	1.1068	1.2305	1.3549*	1.1742
	3	1.1734	1.3395*	1.1350	1.0718	1.3060*	1.1282
36	1	1.2970	1.5208*	1.2753	1.3555	1.4750*	1.3515
	2	1.3264	1.4849*	1.3718	1.2840	1.4654*	1.3337
	3	1.2664	1.4729*	1.3596	1.3029	1.4816*	1.3415

				Korean Factors			
j	#Factors	$\Delta\mathbf{f}_t^{PLS}$	$\Delta\mathbf{f}_t^{PLS,R}$	$\Delta\mathbf{f}_t^{PLS,F}$	$\Delta\mathbf{f}_t^{PC}$	$\Delta\mathbf{f}_t^{PC,R}$	$\Delta\mathbf{f}_t^{PC,F}$
1	1	0.9735	1.0022	0.9773	0.5423	0.9743	0.5820
	2	0.4596	0.9929	0.6262	0.5492	0.9646	0.3462
	3	0.5694	0.9760	0.6352	0.3133	0.9632	0.2832
12	1	1.2271	1.2144	1.2199	1.1532	1.2434*	1.1571
	2	1.1769	1.2247	1.1618	1.1656	1.2418*	1.1594
	3	1.1812	1.2154	1.1611	1.1720	1.2274	1.1600
36	1	1.3667	1.2937	1.3728	1.3919*	1.3654	1.3920*
	2	1.3624	1.3071	1.3783*	1.3936*	1.3648	1.3919*
	3	1.3794*	1.2798	1.2943	1.4058*	1.3316	1.3155

				American and Korean Factors			
j	#Factors	$\Delta\mathbf{f}_t^{PLS}$	$\Delta\mathbf{f}_t^{PLS,R}$	$\Delta\mathbf{f}_t^{PLS,F}$	$\Delta\mathbf{f}_t^{PC}$	$\Delta\mathbf{f}_t^{PC,R}$	$\Delta\mathbf{f}_t^{PC,F}$
1	2	0.9954	1.0169*	0.9854	0.5882	0.9919	0.6359
	4	0.5416	0.9679	0.7121	0.6120	0.9696	0.4283
	6	0.6234	0.9530	0.6667	0.3999	0.9695	0.3120
12	2	1.2325	1.3289*	1.1751	1.1768	1.3298*	1.1787
	4	1.0980	1.3127*	1.0330	1.1387	1.3483*	1.0954
	6	1.0936	1.2745*	1.0212	1.0291	1.2780*	1.0795
36	2	1.2664	1.4650*	1.2462	1.3389	1.4520*	1.3508
	4	1.2798	1.4418*	1.3483	1.2810	1.4358*	1.3306
	6	1.2135	1.3949*	1.2337	1.2589	1.4167*	1.3051

Note: We report the *RRMSPE* statistics employing a rolling window scheme with a 50% sample split point. *RRMSPE* denotes the mean squared prediction error (*RMSPE*) from the benchmark random walk (RW) model divided by the *RMSPE* from each competing model with k factors. Superscript R and F mean that factors were estimated from real and financial variables, respectively. *RRMSPE* statistics that are greater than 1 indicate that the competing model outperforms the benchmark RW model. $*$ denotes the cases that the competing model outperforms the benchmark AR model as well as the RW model.

Table 5. PPP Based Models for the Real Exchange Rate

		American Reference		*Korean Reference*	
j	#Factors	$\Delta\mathbf{f}_t^{PPP,PLS}$	$\Delta\mathbf{f}_t^{PPP,PC}$	$\Delta\mathbf{f}_t^{PPP,PLS}$	$\Delta\mathbf{f}_t^{PPP,PC}$
1	1	1.0061	1.0057	0.9678	0.9670
	2	0.9905	0.9866	0.9639	0.9710
	3	0.9831	0.9697	0.9566	0.9636
12	1	1.2381*	1.2372	1.2422*	1.2441*
	2	1.1760	1.2265	1.2108	1.2153
	3	1.1868	1.1836	1.0970	1.1617
36	1	1.3415	1.3364	1.4172*	1.4151*
	2	1.3406	1.3401	1.3710	1.2233
	3	1.3167	1.3140	1.0971	1.2321

Note: We report the *RRMSPE* statistics employing a rolling window scheme with a 50% sample split point. *RRMSPE* denotes the mean squared prediction error (*RMSPE*) from the benchmark random walk (RW) model divided by the *RMSPE* from each competing model with k factors. *RRMSPE* statistics that are greater than 1 indicate that the competing model outperforms the benchmark RW model. $*$denotes the cases that the competing model outperforms the benchmark AR model as well as the RW model. $\Delta f_t^{PPP,PLS(PC)}$ denotes PLS (or PC) factors based on PPP.

Table 6. UIP Based Models for the Nominal Exchange Rate Returns

		American Factors		
#Factors	$\Delta\mathbf{f}_t^{UIP,PLS}$	$\Delta\mathbf{f}_t^{UIP,PC}$	$\Delta\mathbf{f}_t^{PLS}$	$\Delta\mathbf{f}_t^{PC}$
1	1.0357*	1.0317*	1.0371*	1.0281*
2	1.0529*	1.0258	1.0455*	1.0154
3	1.0261*	1.0237	1.0703*	1.0158

		Korean Factors		
#Factors	$\Delta\mathbf{f}_t^{UIP,PLS}$	$\Delta\mathbf{f}_t^{UIP,PC}$	$\Delta\mathbf{f}_t^{PLS}$	$\Delta\mathbf{f}_t^{PC}$
1	1.0161	1.0121	0.9191	0.5361
2	1.0334*	1.0264*	0.7552	0.5829
3	1.0166	1.0250	0.5044	0.2566

Note: We report the *RRMSPE* statistics for the one-period ahead forecast employing a rolling window scheme with a 50% sample split point. *RRMSPE* denotes the mean squared prediction error (*RMSPE*) from the benchmark random walk (RW) model divided by the *RMSPE* from each competing model with k factors. *RRMSPE* statistics that are greater than 1 indicate that the competing model outperforms the benchmark RW model. $*$ denotes the cases that the competing model outperforms the benchmark AR model as well as the RW model. $\Delta f_t^{UIP,PLS(PC)}$ denotes PLS (or PC) factors based on the UIP condition, whereas $\Delta f_t^{PLS(PC)}$ denotes PLS (or PC) factors based on data-driven macroeconomic factors for the nominal exchange rate return.

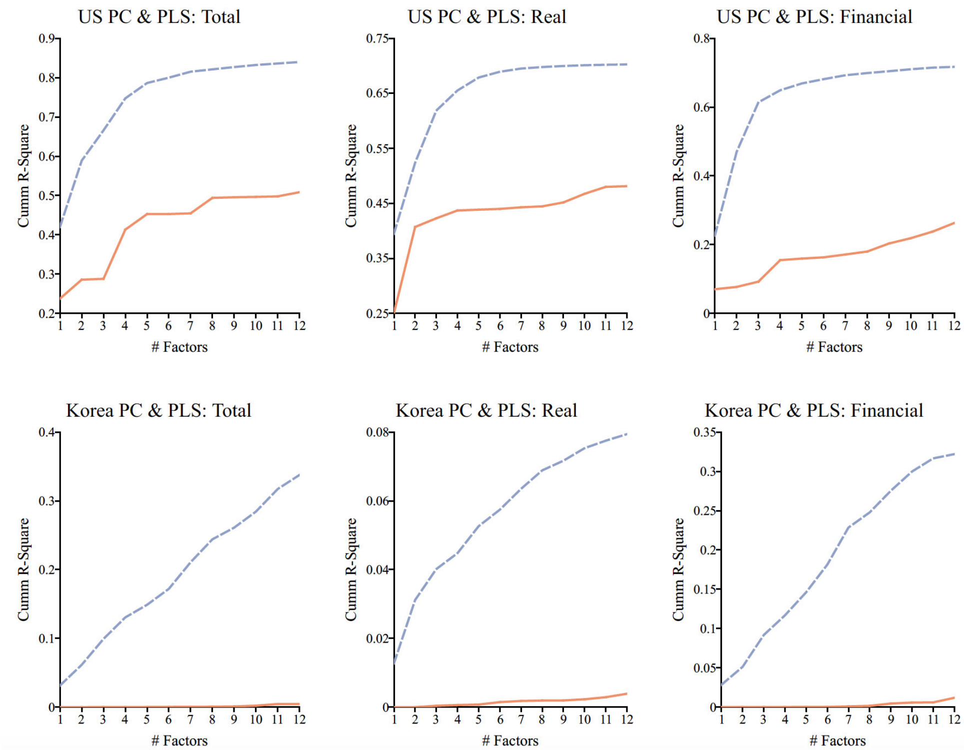
Table 7. RIRP Based Models for the Real Exchange Rate Return

	American Factors			
#Factors	$\Delta\mathbf{f}_t^{RIRP,PLS}$	$\Delta\mathbf{f}_t^{RIRP,PC}$	$\Delta\mathbf{f}_t^{PLS}$	$\Delta\mathbf{f}_t^{PC}$
1	1.0057	1.0066	1.0290*	1.0198*
2	1.0042	0.9882	1.0358*	1.0061
3	0.9924	0.9754	1.0598*	1.0061

	Korean Factors			
#Factors	$\Delta\mathbf{f}_t^{RIRP,PLS}$	$\Delta\mathbf{f}_t^{RIRP,PC}$	$\Delta\mathbf{f}_t^{PLS}$	$\Delta\mathbf{f}_t^{PC}$
1	0.9833	0.9792	0.9191	0.6676
2	0.9742	0.9836	0.7552	0.4295
3	0.9697	0.9631	0.5044	0.2443

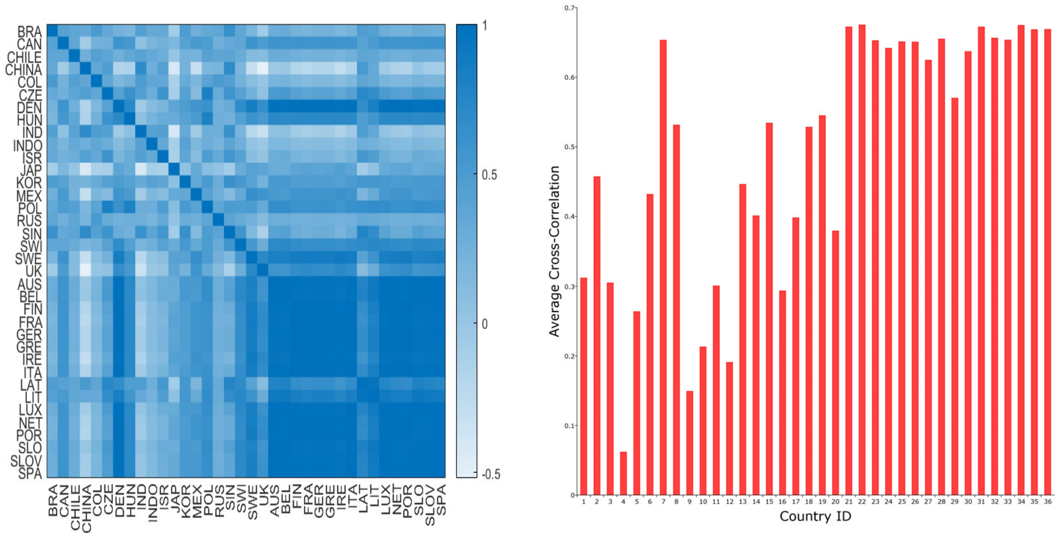
Note: We report the *RRMSPE* statistics for the one-period ahead forecast employing a rolling window scheme with a 50% sample split point. *RRMSPE* denotes the mean squared prediction error (*RMSPE*) from the benchmark random walk (RW) model divided by the *RMSPE* from each competing model with k factors. *RRMSPE* statistics that are greater than 1 indicate that the competing model outperforms the benchmark RW model. $*$ denotes the cases that the competing model outperforms the benchmark AR model as well as the RW model. $\Delta f_t^{RIRP,PLS(PC)}$ denotes PLS (or PC) factors based on RIRP, whereas $\Delta f_t^{PLS(PC)}$denotes PLS (or PC) factors based on data-driven macroeconomic factors for the real exchange rate return.

Figure 1. Cumulative R^2 Analysis



Note: We regress the real exchange rate on each factor and obtain the R^2 statistics. Since we use orthogonalized factors, we report the cumulative R^2 statistics. Dotted lines are for PLS factors and solid lines are for PC factors.

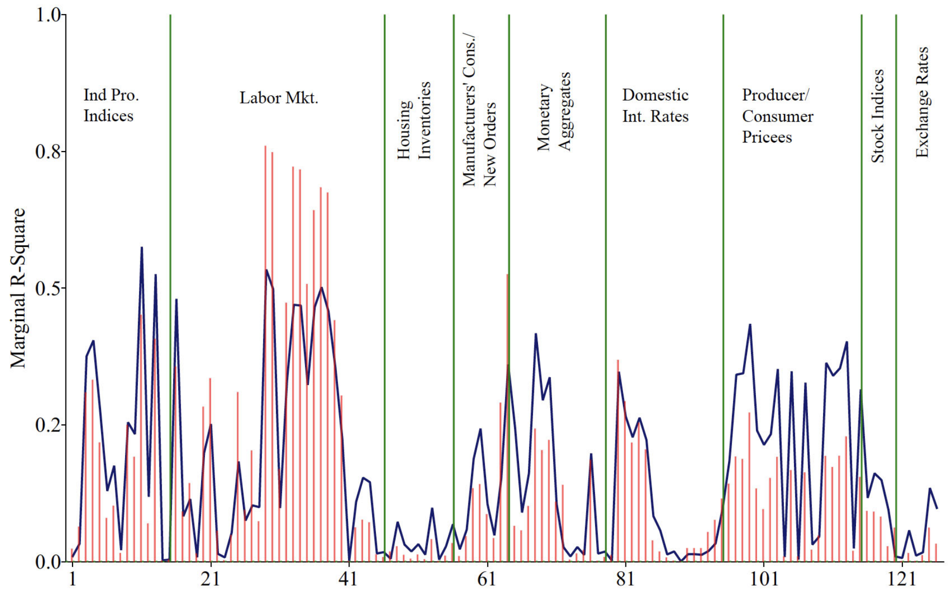
Figure 2. Cross-Section Dependence of the Real Exchange Rates realtive to the Dollar



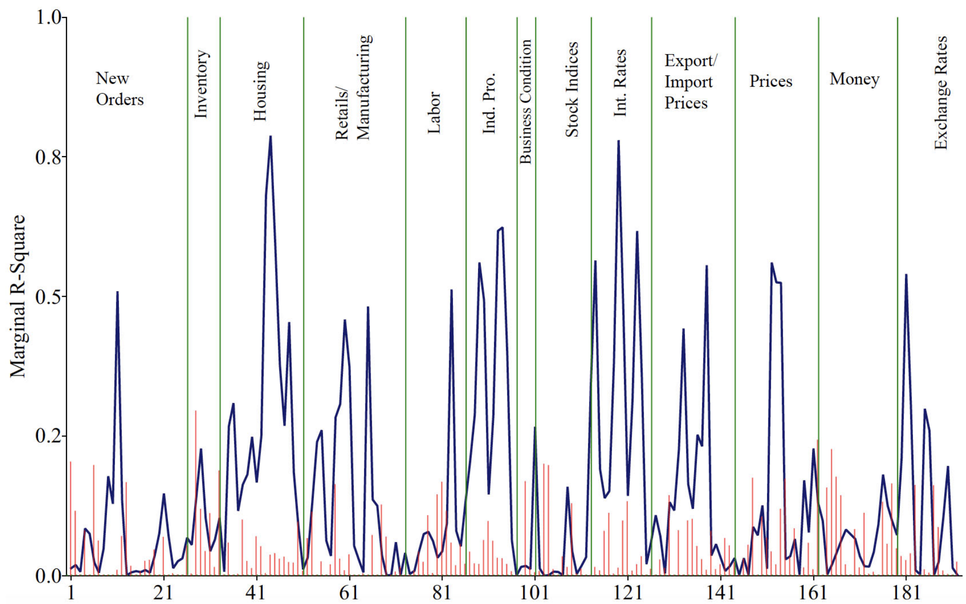
Note: The heat map on the left shows the cross-correlations of the ADF regression residuals of 36 real exchange rate relative to the U.S. dollar. We report the average cross-correlations of each real exchange rate in the figure on the right. The country ID is of the same order as in the heat map on the left. For example, Korea's ID 13.

Figure 3. Marginal R^2 Analysis: Total Factors

American Factor #1



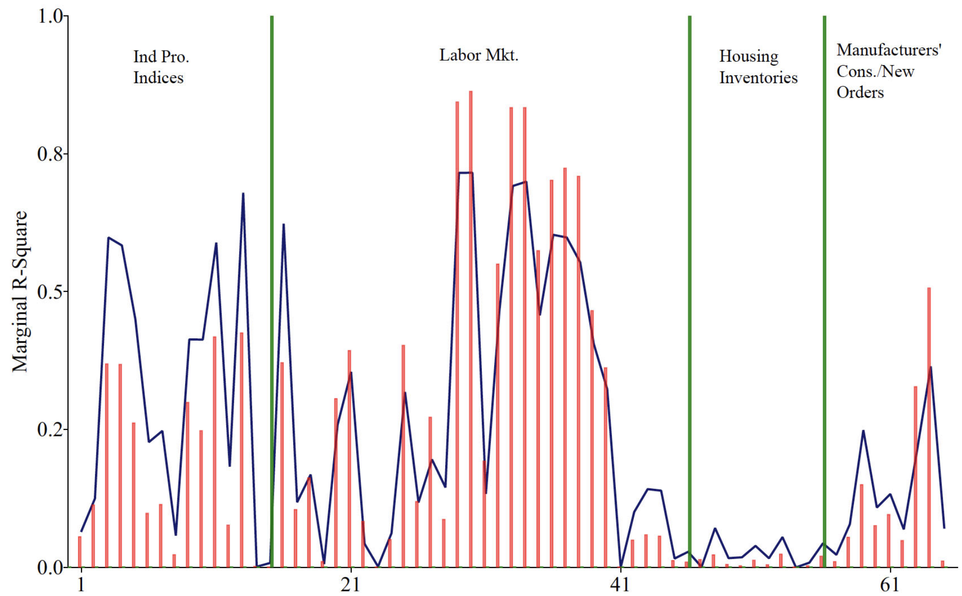
Korean Factor #1



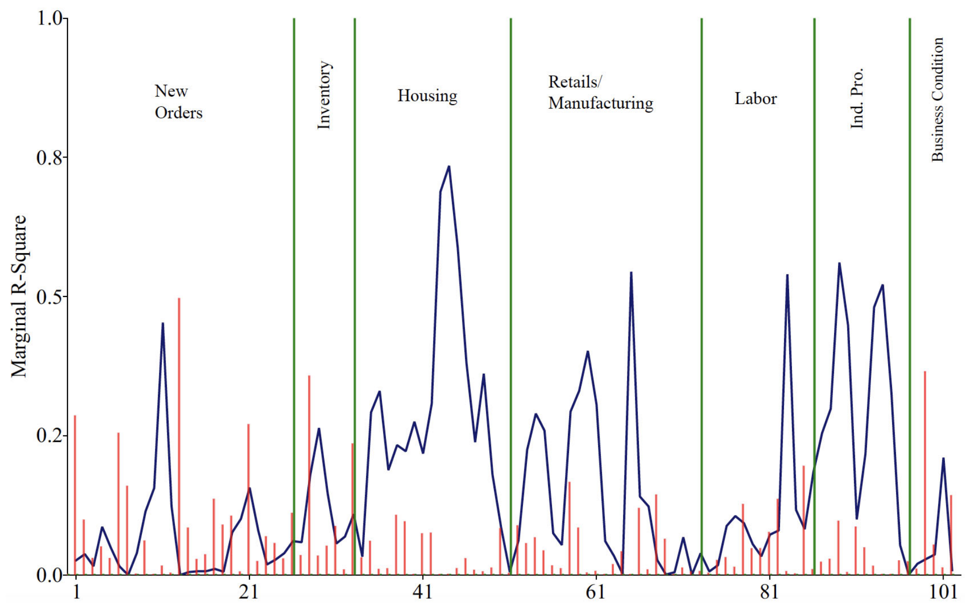
Note: We report the R^2 statistics that were obtained by regressing each predictor on the first common factor estimate. That is, the horizontal axis is the predictor IDs. Solid lines are for the PC factor, while bar graphs are for the PLS factor.

Figure 4. Marginal $\mathbf{R}^2$ Analysis: Real Activity Factors

American Factor #1



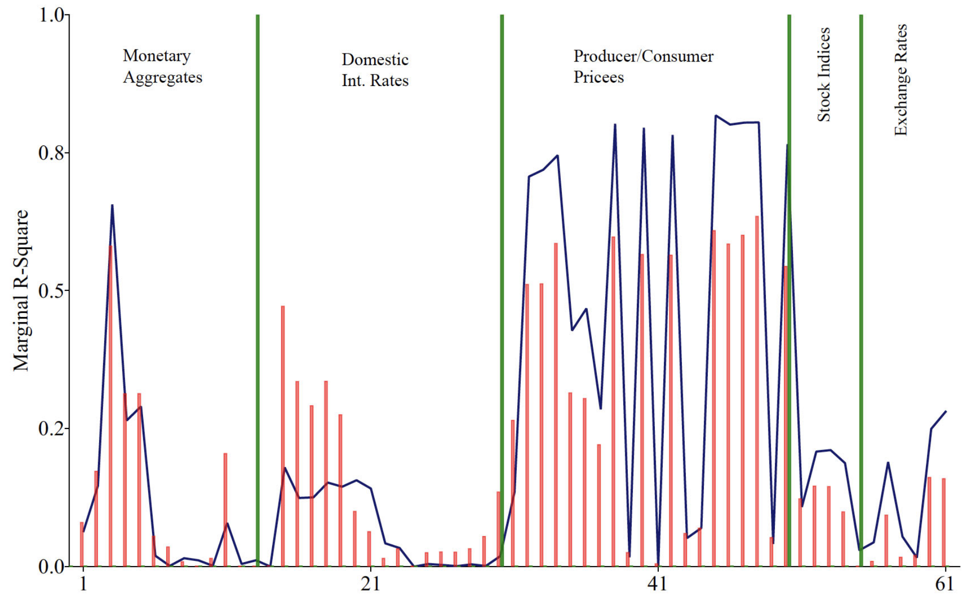
Korean Factor #1



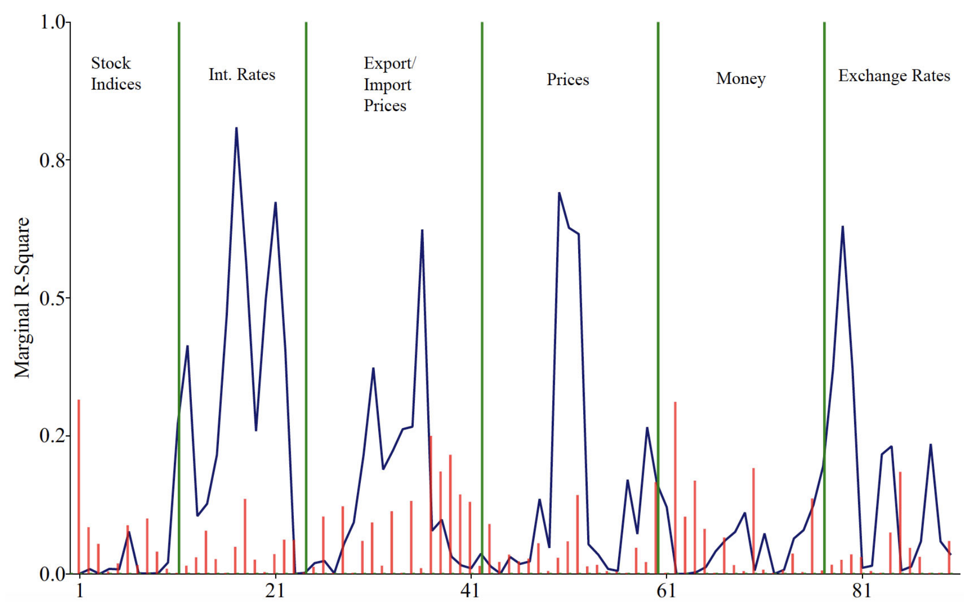
Note: We report the R^2 statistics that were obtained by regressing each predictor on the first common factor estimate. That is, the horizontal axis is the predictor IDs. Solid lines are for the PC factor, while bar graphs are for the PLS factor.

Figure 5. Marginal R^2 Analysis: Nominal/Financial Factors

American Factor #1



Korean Factor #1



Note: We report the R^2 statistics that were obtained by regressing each predictor on the first common factor estimate. That is, the horizontal axis is the predictor IDs. Solid lines are for the PC factor, while bar graphs are for the PLS factor.