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Forecasting the US Dollar-Korean Won Exchange Rate: A Factor-Augmented Model Approach

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Abstract

We propose factor-augmented out of sample forecasting models for the real exchange rate between Korea and the US. We estimate latent common factors by applying an array of data dimensionality reduction methods to a large panel of monthly frequency time series data. We augment benchmark forecasting models with common factor estimates to formulate out-of-sample forecasts of the real exchange rate. Major findings are as follows. First, our factor models outperform conventional forecasting models when combined with factors from the US macroeconomic predictors. Korean factor models perform overall poorly. Second, our factor models perform well at longer horizons when American real activity factors are employed, whereas American nominal/financial market factors help improve short-run prediction accuracy. Third, models with global PLS factors from UIP fundamentals overall perform well, while PPP and RIRP factors play a limited role in forecasting.

Keywords: Won/Dollar Real Exchange Rate; Principal Component Analysis; Partial Least Squares; LASSO; Out-of-Sample Forecast

JEL Classification: C38; C53; C55; F31; G17

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1 Introduction

This paper presents out-of-sample factor-augmented forecasting models for the real exchange rate between Korea and the US. We demonstrate that our models outperform commonly used benchmark models when factors are extracted from a large panel of macroeconomic time series data in the US. We report a very limited role of factors from Korean macroeconomic variables in predicting the real exchange rate at any forecast horizons.

In their seminal work, Meese and Rogoff (1983) demonstrate that the random walk (RW) model performs well in forecasting exchange rates in comparison with the models that are motivated by exchange rate determination theories. Cheung, Chinn, and Pascual (2005) add more recent evidence of such a disconnect between the exchange rate and economic fundamentals, showing that exchange rate models still do not consistently outperform the RW model in out-of-sample forecasting.[1] In a related work, Engel and West (2005) provide an interesting point that asset prices such as the exchange rate can show a near unit root process, though these prices are still consistent with asset pricing models, as the discount factor approaches one.[2]

A group of researchers, however, demonstrated that exchange rate models could outperform the RW model at longer horizons. For example, Mark (1995) used a regression model of multiple-period changes (long-differenced) in the nominal exchange rate on the deviation of the exchange rate from its fundamentals, then reported overall superior long-horizon predictability of fundamentals for the exchange rate. Chinn and Meese (1995) also report similar long-horizon evidence of greater predictability of exchange rate models relative to the RW. Using over two century-long annual frequency data, Lothian and Taylor (1996) report good out-of-sample predictability of fundamentals for the *real* exchange rate. Groen (2005) reports some long-horizon evidence of superior predictability of monetary fundamentals employing a vector autoregressive (VAR) model framework.[3] Also, Engel, Mark, and West (2008) show that out-of-sample predictability can be enhanced by focusing on panel estimation and long-horizon forecasts.

[1] However, Engel and Hamilton (1990) report some evidence that their nonlinear models outperform the RW. But their findings are still at odds with uncovered interest parity (UIP).

[2] Many researchers provide panel evidence of a close link between monetary models and exchange rate dynamics. See for example, Rapach and Wohar (2004), Groen (2000), and Mark and Sul (2001).

[3] They demonstrate that the monetary fundamentals-based *common* long-run model tends to outperform the RW model as well as the standard cointegrated vector autoregressive (VAR) model at 2 to 4 year horizons.

Another group of researchers started using Taylor Rule fundamentals in addition to conventional monetary fundamentals. See among others, Engel, Mark, and West (2008), Molodtsova, Nikolsko-Rzhevskyy, and Papell (2008), Molodtsova and Papell (2009), Molodtsova and Papell (2013), and Ince, Molodtsova, and Papell (2016). They show that models with Taylor Rule fundamentals tend to perform well in forecasting the exchange rate. See Rossi (2013) for a survey of research work that demonstrates the importance of Taylor Rule fundamentals in understanding exchange rate dynamics.[4]

We note that the pioneering work of Stock and Watson (2002) has initiated an influx of papers that utilize latent common factors in forecasting macroeconomic variables via principal components (PC) analysis. The current exchange rate literature is not an exception. A number of researchers began using large panels of time series data to better understand exchange rate dynamics. For instance, Engel, Mark, and West (2015) use cross-section information (PC factors) that are obtained from a panel of 17 bilateral exchange rates vis-à-vis the US dollar, then demonstrate that factor based forecasting models often outperform the RW model during the post-1999 sample period. They also report good forecasting performance of the dollar factor in combination with Purchasing Power Parity (PPP) factors. Chen, Jackson, Kim, and Resiandini (2014) used PC to extract latent common factors from 50 world commodity prices. Their first common factor turns out to be closely related with the dollar exchange rate, which is consistent with an observation that world commodities are priced in US dollars. They show that this first common factor yields superior out-of-sample predictive contents for the dollar exchange rate.

Greenaway-McGrevy, Mark, Sul, and Wu (2018) demonstrate that exchange rates are mainly driven by a dollar and an euro factor. They show that their dollar-euro factor model dominates the RW model in the out-of-sample prediction performance. Verdelhan (2018) uses portfolios of international currencies to extract the two risk factors (dollar factor and carry factor), which successfully explain exchange rate dynamics. Using a structural Bayesian vector autoregression (SBVAR), Ca' Zorzi, Kociecki, and Rubaszek (2015) demonstrate that real interest rate and PPP fundamentals are useful to forecast exchange rates at medium horizons, although it is still difficult to

[4] In a related work, Wang and Wu (2012) show the superior out-of-sample interval predictability of the Taylor Rule fundamentals at longer horizons. Also, many researchers report in-sample evidence that Taylor rul fundamentals help understanding exchange rate dynamics. See among others, Mark (2009), Molodtsova, Nikolsko-Rzhevskyy, and Papell (2008), Engel and West (2006), and Clarida and Waldman (2008).

beat the RW model in the short-run.

PC has been widely used in the current forecasting and empirical macroeconomics literature. However, it extracts latent common factors from predictors without considering the relationship between predictors and the target variable. As shown by Boivin and Ng (2006), its performance may be poor in forecasting the target variable if useful predictive contents are in a certain factors that are dominated by other factors. Recognizing this potential problem, we employ an alternative data dimensionality reduction method such as the partial least squares (PLS) method by Wold (1982). This method utilizes the covariance between the target and predictor variables to generate *target-specific* factors. See Kelly and Pruitt (2015) and Groen and Kapetanios (2016) for some comparisons between the PC and PLS approaches. Similar to Bai and Ng (2008) and Kelly and Pruitt (2015), we also use the Least Absolute Shrinkage and Selection Operator (LASSO) to select target-specific *groups* of the predictors among the full dataset to extract more relevant factors for the target.

In this paper, we suggest factor-augmented forecasting models for the KRW/USD real exchange rate. We employ PC and PLS as well as the LASSO in combination with PC and PLS to estimate common factors using large panels of 125 American and 192 Korean monthly frequency macroeconomic time series data from October 2000 to March 2019. Since most macroeconomic data are better approximated by a nonstationary integrated process (Nelson and Plosser, 1982), we extract common factors by applying these methods to first differenced predictors to consistently estimate factors (Bai and Ng, 2004). We also extract common factors from country-level global data using up to 43 country-level data for prices and interest rates, motivated by exchange rate determination theories such as Purchasing Power Parity (PPP), Uncovered Interest Parity (UIP), and Real Uncovered Interest Parity (RIRP). We then implement an array of out-of-sample forecasting exercises utilizing these factor estimates, and investigate what factors help improve the prediction accuracy for the exchange rate. We evaluate the out-of-sample predictability of our factor models via the ratio of the root mean squared prediction error (RRMSPE) criteria.

Our major findings are as follows. First, our factor-augmented forecasting models outperform the random walk (RW) and autoregressive $AR(1)$ type benchmark models only when they utilize American factors. Korean factor-augmented models overall got dominated by the AR model, although they still outperform the RW model when the forecast horizon is one-year or longer. It is well known that bilateral exchange rates relative to the US dollar tend to exhibit high cross-section correlations. That is, US common factors are likely to dominate dynamics of these exchange rates vis-à-vis dollars over idiosyncratic components in each country such as Korea. Combining Korean (idiosyncratic) factors with American factors slightly improves the forecasting performance but failed to observe sufficiently large improvement enough to outperform the AR model.

Second, our models tend to perform better in short horizons when they are combined with nominal/financial market factors in the US. On the other hand, American real activity factor models outperform both the RW and AR models at longer horizons. Put it differently, good short-run prediction performance of our models with the total factors seems to inherit the superior performance of our models with financial market factors, while superior long-run predictability seems to stem from information from real activity factors. These results are consistent with the work of Boivin and Ng (2006) in the sense that one may extract more useful information from subsets of predictors.

Third, forecasting models with UIP motivated PLS factors perform well when the US serves as the reference country. However, PPP and RIRP based factor models are dominated by the AR model whichever country serves as the reference. Overall, data-driven factor models seem to perform better than these propositon-based factor models.

The rest of the paper is organized as follows. Section 2 carefully describes how we estimate latent common factors via PC, PLS, and the LASSO for the real exchange rate when predictors obey an integrated process. Section 3 presents data descriptions and preliminary statistical analysis. We also report some in-sample fit analysis to investigate the source of latent common factors. In Section 4, we introduce our factor-augmented forecasting models and evaluation schemes. Then, we present and interpret our out-of-sample forecasting exercise results. We also report results with alternative identification approaches, proposition-based models and nonstationary forecasting models, in comparison with the data-driven factor forecasting models. Section 5 concludes.

2 Methods of Estimating Latent Common Factors

This section explains how we estimate latent common factors by applying Principal Component (PC), Partial Least Squares (PLS), and the Least Absolute Shrinkage and Selection Operator (LASSO) to a large panel of nonstationary predictors.

2.1 Principal Component Factors

Since the seminal work of Stock and Watson (2002), PC has been popularly used in the current forecasting literature. We begin with this approach to show how to estimate latent common factors when predictors are likely to be integratred $I(1)$ processes.

Consider a panel of N macroeconomic $T \times 1$ time series predictors/variables, $\mathbf{x} = [\mathbf{x}_1, \mathbf{x}_2, ..., \mathbf{x}_N]$, where $\mathbf{x}_i = [x_{i,1}, x_{i,2}, ..., x_{i,T}]'$, $i = 1, ..., N$. We assume that each predictor $\mathbf{x}_i$ has the following factor structure. Abstracting from deterministic terms,

$$x_{i,t} = \boldsymbol{\lambda}_i' \mathbf{f}_t^{PC} + \varepsilon_{i,t}, \tag{1}$$

where $\mathbf{f}_t = \left[f_{1,t}^{PC}, f_{2,t}^{PC}, \cdots, f_{R,t}^{PC}\right]'$ is an $R \times 1$ vector of *latent* time-varying common factors at time t and $\boldsymbol{\lambda}_i = [\lambda_{i,1}, \lambda_{i,2}, \cdots, \lambda_{i,R}]'$ denotes an $R \times 1$ vector of time-invariant idiosyncratic factor loading coefficients for $\mathbf{x}_i$. $\varepsilon_{i,t}$ is the idiosyncratic error term.

Following Bai and Ng (2004), we estimate latent common factors by applying the PC method to first-differenced data. This is because, as shown by Nelson and Plosser (1982), most macroeconomic time series variables are better approximated by an integrated nonstationary stochastic process. Note that the PC estimator of $\mathbf{f}_t$ would be inconsistent if $\varepsilon_{i,t}$ is an integrated process. First differencing (1), we obtain the following factor structure.

$$\Delta x_{i,t} = \boldsymbol{\lambda}_i' \Delta \mathbf{f}_t^{PC} + \Delta \varepsilon_{i,t} \tag{2}$$

for $t = 2, \cdots, T$. We first normalize the data, $\Delta \tilde{\mathbf{x}} = [\Delta \tilde{\mathbf{x}}_1, \Delta \tilde{\mathbf{x}}_2, ..., \Delta \tilde{\mathbf{x}}_N]$, then apply PC to $\Delta \tilde{\mathbf{x}} \Delta \tilde{\mathbf{x}}'$ to obtain the factor estimates $\Delta \hat{\mathbf{f}}_t^{PC}$ along with their associated factor loading coefficients $\hat{\boldsymbol{\lambda}}_i$.[5] Naturally, estimates of the idiosyncratic component are obtained by taking the residual, $\Delta \hat{\varepsilon}_{i,t} = \Delta \tilde{x}_{i,t} - \hat{\boldsymbol{\lambda}}_i' \Delta \hat{\mathbf{f}}_t^{PC}$. The level variable estimates are recovered via cumulative summation as follows.

$$\hat{\varepsilon}_{i,t} = \sum_{s=2}^{t} \Delta \hat{\varepsilon}_{i,s}, \ \hat{\mathbf{f}}_t^{PC} = \sum_{s=2}^{t} \Delta \hat{\mathbf{f}}_s^{PC} \tag{3}$$

It should be noted that this procedure yields consistent factor estimates even when $\mathbf{x}$ includes some stationary $I(0)$ variables. For example, assume that $\mathbf{x}_j$, $j \in \{1, ..., N\}$ is $I(0)$. Differencing it once results in $\Delta \mathbf{x}_j$, which is still stationary, $I(-1)$. Therefore the PC estimator remains consistent. Alternatively, one may continue to difference

[5] This is because PC is not scale invariant. We demean and standardize each time series.

the variables until the null of nonstationarity hypothesis is rejected via a unit root test.[6] However, this may not be practically useful, when unit root tests provides contradicting statistical inferences as the test specification (i.e., number of lags, see) changes. See Cheung and Lai (1995) for related discussions.

2.2 Partial Least Squares Factors

We employ PLS for a scalar target variable q_t, which is somewhat neglected in the current literature. Unlike PC, the method of PLS generates target specific latent common factors, which is an attractive feature. As Boivin and Ng (2006) pointed out, PC factors might not be useful in forecasting the target when useful predictive contents are in a certain factor that may be dominated by other factors.

PLS is motivated by the following linear regression model. Abstracting from deterministic terms,

$$q_t = \Delta \mathbf{x}_t^{'} \boldsymbol{\beta} + e_t, \tag{4}$$

where $\Delta \mathbf{x}_t = [\Delta x_{1,t}, \Delta x_{2,t}, ..., \Delta x_{N,t}]'$ is an $N \times 1$ vector of predictor variables at time $t = 1, ..., T$, while $\boldsymbol{\beta}$ is an $N \times 1$ vector of coefficients. e_t is an error term. Again, we first-difference the predictor variables assuming that $\mathbf{x}_t$ is a vector of $I(1)$ variables..

PLS is especially useful for regression models that have many predictors, when N is large. To reduce the dimensionality, rewrite (4) as follows,

$$\begin{aligned} q_t &= \Delta \mathbf{x}_t^{'} \mathbf{w} \boldsymbol{\theta} + u_t \\ &= \Delta \mathbf{f}_t^{PLS'} \boldsymbol{\theta} + u_t \end{aligned} \tag{5}$$

where $\Delta \mathbf{f}_t^{pls} = \left[\Delta f_{1,t}^{PLS}, \Delta f_{2,t}^{PLS}, ..., \Delta f_{R,t}^{PLS} \right]^{'}$, $R < N$ is an $R \times 1$ vector of PLS factors. Note that $\Delta \mathbf{f}_t^{PLS}$ is a linear combination of *all* predictor variables, that is,

$$\Delta \mathbf{f}_t^{PLS} = \mathbf{w}^{'} \Delta \mathbf{x}_t, \tag{6}$$

where $\mathbf{w} = [\mathbf{w}_1, \mathbf{w}_2, ..., \mathbf{w}_R]$ is an $N \times R$ weighting matrix. That is, $\mathbf{w}_r = \left[w_{1,r}, w_{2,r}, ..., w_{N,r} \right]^{'}$, $r = 1, ..., R$, is an $N \times 1$ vector of weights on predictor variables for the r^{th} PLS factor, $\Delta f_{r,t}^{PLS}$. $\boldsymbol{\theta}$ is an $R \times 1$ vector of PLS regression coefficients. PLS regression minimizes the sum of squared residuals from the equation (5) for $\boldsymbol{\theta}$ instead of $\boldsymbol{\beta}$ in (4). It is

[6] This approach is used to construct the Fred-MD database.The Fred-MD is available at https://research.stlouisfed.org/econ/mccracken/fred-databases/.

important to note that we do not utilize $\boldsymbol{\theta}$ for our out-of-sample forecasting exercises in the present paper. To make it comparable to PC factors, we simply utilize PLS factors $\Delta \mathbf{f}_t^{PLS}$, then augment the benchmark forecasting model with estimated PLS factors $\Delta \hat{\mathbf{f}}_t^{PLS}$.

Among available PLS algorithms, see Andersson (2009) for a brief survey, we use the one proposed by Helland (1990) that is intuitively appealing. Helland's algorithm to estimate PLS factors for a scalar target variable q_t is as follows. First, $\Delta \hat{f}_{1,t}^{PLS}$ is pinned down by the linear combinations of the predictors in $\Delta \mathbf{x}_t$.

$$\Delta \hat{f}_{1,t}^{PLS} = \sum_{i=1}^{N} w_{i,1} \Delta x_{i,t}, \tag{7}$$

where the loading (weight) $w_{i,1}$ is given by $Cov(q_t, \Delta x_{i,t})$. Second, we regress q_t and $\Delta x_{i,t}$ on $\Delta \hat{f}_{1,t}^{PLS}$ then get residuals, $\tilde{q}_t$ and $\Delta \tilde{x}_{i,t}$, respectively, to remove the explained component by the first factor $\Delta \hat{f}_{1,t}^{PLS}$. Next, the second factor estimate $\Delta \hat{f}_{2,t}^{PLS}$ is obtained similarly as in (7) with $w_{i,2} = Cov(\tilde{q}_t, \Delta \tilde{x}_{i,t})$. We repeat until the R^{th} factor $\Delta \hat{f}_{R,t}^{PLS}$ is obtained.

2.3 Least Absolute Shrinkage and Selection Operator Factors

We employ a shrinkage and selection method for linear regression models, the LASSO, which is often used for sparse regression. Unlike ridge regression, the LASSO selects a subset ($\mathbf{x}^s$) of predictor variables from $\mathbf{x}$ by assigning 0 coefficient to the variables that are relatively less important in explaining the target variable. Putting it differently, we implement the *feature selection* task using the LASSO.

The LASSO puts a cap on the size of the estimated coefficients for the ordinary least squares (LS) driving the coefficient down to zero for some predictors. The LASSO solves the following constrained minimization problem using L_1-norm penalty on β.

$$\min_{\boldsymbol{\beta}} \left\{ \frac{1}{T} \sum_{t=1}^{T} (q_t - \Delta \mathbf{x}_t' \boldsymbol{\beta})^2 \right\}, \text{ s.t. } \sum_{j=1}^{N} |\beta_j| \le \tau \tag{8}$$

where $\Delta \mathbf{x}_t = [\Delta x_{1,t}, \Delta x_{2,t}, ..., \Delta x_{N,t}]'$ is an $N \times 1$ vector of predictor variables at time $t = 1, ..., T$, $\boldsymbol{\beta}$ is an $N \times 1$ vector of associated coefficients. As the value of tuning (penalty) parameter τ decreases, the LASSO returns a smaller subset of $\mathbf{x}$, setting more coefficients to zero.

Following Kelly and Pruitt (2015), we choose the value of τ to generate a certain number of predictors by applying the LASSO to $\Delta\mathbf{x}$. We then employ the PC or PLS approach to extract common factors, $\Delta\mathbf{f}_t^{PC/L}$ or $\Delta\mathbf{f}_t^{PLS/L}$, out of the predictor variables that are chosen by the LASSO regression. Similar to our PLS approach, we use the LASSO method only to obtain the subset of predictors that is closely related to the target.

3 In-Sample Analysis

3.1 Data Descriptions

We obtained a large panel of 126 American macroeconomic time series variables from the FRED-MD database. We also obtained 192 Korean macroeconomic time series data from the Bank of Korea. Korea has maintained a largely fixed exchange rate regime for the dollar-won exchange rate, then switched to a heavily managed floating exchange rate regime around 1980, and continue until the Asian Financial Crisis occurred in 1997, which forced Korea to adopt a market based exchange rate regime.

We focus on the free floating exchange rate regime in 2000's after the Korean economy fully recovered from the crisis. Observations are monthly and span from October 2000 to March 2019 to utilize reasonably many monthly predictors in Korea. We use the consumer price index (CPI) to transform the nominal KRW/USD exchange rate to the real exchange rate.

We categorized 192 Korean predictors into 13 groups. Groups #1 through #6 include real activity variables that include inventories and industrial productions, while groups #7 to #13 are nominal/financial market variables such as interest rates and prices. See Table 1 for more detailed information. Similarly, we categorized 126 American predictors into 9 groups of variables. Groups #1 through #4 represent the real activity variables, while groups #5 through #9 are considered as financial sector variable groups in the US. We log-transformed all quantity variables prior to estimations other than those expressed in percent (e.g., interest rates and unemployment rates).

Table 1 around here

3.2 Some Preliminary Analysis

3.2.1 Unit Root Tests

We first implement some specification tests for our analysis. Table 2 presents the augmented Dickey Fuller (ADF) test results for the real exchange rate (q_t) and the nominal exchange rate (s_t). The ADF test rejects the null of nonstationarity for q_t at the 5% significance level, while it fails to reject the null hypothesis for s_t at any conventional level. Note that these results are consistent with standard monetary models in international macroeconomics. For example, purchasing power parity (PPP) is consistent with stationary q_t and nonstationary s_t, because PPP implies a cointegrating relationship between s_t and the relative price for q_t in the long-run.

Next, we implement a panel unit root test for $\mathbf{x}_t^{Am}$ and $\mathbf{x}_t^{Kr}$, predictor variables in the US and in Korea, respectively, employing the Panel Analysis of Nonstationarity in Idiosyncratic and Common components (PANIC) analysis by Bai and Ng (2004). The PANIC procedure estimates common factors via PC as explained in the previous section, then test the null of nonstationarity for common factors via the ADF test with an intercept. It also implements a panel unit root test for de-factored idiosyncratic components of the data by the following statistic.

$$P_{\hat{e}} = \frac{-2\sum_{i=1}^{N} \ln p_{\hat{e}_i} - 2N}{2N^{1/2}},$$

where $p_{\hat{e}_i}$ denotes the p-value of the ADF statistic with no deterministic terms for de-factored $\Delta x_{i,t}$.[7]

Note that we also test the null hypothesis for the common factors of subsets of $\mathbf{x}_t$, that is, real and financial sector variables separately. This is because we are interested in the out-of-sample predictability of the common factors from these subsets of the data. In what follows, we show American real activity factors include more long-run predictive contents, whereas American financial market factors yield superior predictability in the short-run, which is consistent with the implications of Boivin and Ng (2006).

The PANIC test fails to reject the null of nonstationarity for all common factor estimates at the 5% significance level with an exception of the second financial factor in the US. Its panel unit root test rejects the null hypothesis that states all variables

[7] $P_{\hat{e}}$ statistic has an asymptotic standard normal distribution. The panel test utilizes the p-value of the ADF statistics with no deterministic terms, because de-factored variables are mean-zero residuals.

are $I(1)$ processes for all cases.[8] However, nonstationary common factors eventually dominate stationary dynamics of de-factored idiosyncratic components. Hence, test results in Table 2 provide strong evidence in favor of nonstationarity in the predictor variables $\mathbf{x}_t$, which is consistent with Nelson and Plosser (1982).

Table 2 around here

3.2.2 Persistence of the Real Exchange Rate

Exchange rates often exhibit very persistent dynamics, which is often indistinguishable from a unit root process due to the so-called observational equivalence problem.[9] To investigate this possibility, we employ the grid bootstrap procedure by Hansen (1999) to obtain median unbiased estimates for the persistence parameter of the real exchange rates. For this purpose, consider the following $AR(1)$ process for q_t.[10]

$$q_{t+1} = \alpha + \beta q_t + \varepsilon_{t+1} \tag{9}$$

Define the following grid-t statistics at each fine grid point $\beta \in [\beta_{\min}, \beta_{\max}]$.

$$t_T(\beta) = \frac{\hat{\beta} - \beta}{se(\hat{\beta})} \tag{10}$$

We implement $10,000$ nonparametric bootstrap simulations at 100 fine grid points over $[\hat{\beta} \pm 6 \times se(\hat{\beta})]$, where $\hat{\beta}$ is the (biased) least squares estimate of β and $se(\hat{\beta})$ is its standard error, totaling 1 million bootstrap simulations, which generates the p^{th} grid-t bootstrap quantile functions, $Q^*_{T,p}(\beta)$.[11] The median unbiased estimator ($\hat{\beta}_{MUE}$) is then defined as,

$$\beta \in R : t_T(\beta) = Q^*_{T,50\%}(\beta), \tag{11}$$

[8] The alternative hypothesis is that there is at least one stationary variable.

[9] For example, it is virtually impossible to distinguish a near unit root process, say the persistence parameter equals 0.999, from the unit root process via unit root tests.

[10] If q_t is of higher order $AR(p)$, $p > 1$, process, we can obtain the approximately median unbiased estimator for β in the presence of nuisance parameters.

[11] Each function is evaluated at each grid point β, not at the point estimate. If they are evaluated at the point estimate, the quantile functions correspond to the bootstrap-t quantile functions. See Efron and Tibshirani (1994).

while the 95% grid-t confidence interval can be obtained by the following.

$$\left[\beta \in R : Q^*_{T,2.5\%}(\beta) \leq t_T(\beta) \leq Q^*_{T,97.5\%}(\beta)\right], \tag{12}$$

We report point estimates $\hat{\beta}_{MUE}$ as well as their corresponding 95% confidence bands in Table 3. We also report annualized half-life point estimates, $\left(\ln(0.5)/\ln(\hat{\beta}_{MUE})\right)\times 12$, and their bootstrap confidence bands. Results confirm a high degree of persistence in both the real and nominal exchange rates. Half-life point estimates were 2.227 and 1.979 years for q_t and s_t, respectively. Even though the half-life of s_t is slightly shorter, we obtained more compact confidence band of q_t in comparison with that of s_t, which is consistent with the ADF test results in Table 2.[12]

Table 3 around here

3.3 Factor Model In-Sample Analysis

This section describes some in-sample properties of the factor estimates we discussed in previous section. Figure 1 presents in-sample fit analysis for the KRW/USD real exchange rate. Three figures in top row report cumulative R^2 statistics of PC and PLS factors obtained from all predictors, real activity predictors, and financial sector predictors in the US, respectively from left to right. Three figures in bottom row provide cumulative R^2 values of Korean factors. Some interesting findings are as follows.

First, the PLS factors (dashed lines) provide a notably better in-sample fit in comparison with the performance of PC factors (solid lines). This is because PLS utilizes the covariance between the target and the predictor variables, while PC factors are extracted from the predictor variables only. It is also interesting to see that the cumulative R^2 statistics of PLS factors overall exhibit a positive slope at a decreasing rate as the number of factors increases, whereas additional contributions of PC factors show no such patterns. This is mainly due to the fact that our PLS algorithm sequentially estimates orthogonalized common factors after removing explanatory power of previously estimated factors. The PC method extracts common

[12] Greater ADF test statistic in absolute value of q_t than that of s_t is due to smaller standard error of q_t than that of s_t.

factors without considering the target variable, hence the contribution of additional PC factors does not necessarily decrease.

Second, American factors greatly outperform Korean factors. Cumulative R^2 values of American PLS factors reach well above 60%, while Korean PLS factors cumulatively explain less than 40% of variations in the real exchange rate. Note that Korean PC factors yield virtually no explanatory power. These findings imply that Korean macroeconomic variables might not play an important role in explaining real exchange rate dynamics, while American predictors contain substantial predictive contents for it. We relate such findings with a strong degree of cross-section correlations of many bilateral exchange rates relative to the US dollars, implying that American factors better explain dynamics of exchange rates vis-à-vis US dollar than idiosyncratic factors in local countries such as Korea.

Third, PLS factors from American real activity groups and financial variable groups explain variations in the KRW/USD real exchange rate as well as those from entire predictors. On the other hand, the contribution of PLS Korean factors mostly stem from that of PLS Korean financial sector factors.

Figure 1 around here

Next, we investigate the source of the common factor estimates, employing the marginal R^2 analysis. That is, we regress each predictor onto the common factor and record what proportion of the variation in each predictor can be explained by the common factor. Results are reported in Figures 2 to 4 for the first common factor from the entire predictors, real activity variables, and nominal/financial market variables, respectively.

As can be seen in Figure 2, the marginal R^2 statistics of the first American PC factor (solid lines) are very similar to those of the first PLS American factor (bar graphs). On the other hand, the marginal R^2 statistics of the first Korean PC and PLS factors are very different. More specifically, the marginal R^2 statistics of the Korean PLS factor are negligibly low in comparison with those of the PC factor. Since PC factors are obtained only from the predictors with no reference on the target variable, the marginal R^2 values of the PC factor are expected to be high. However, since PLS factors are estimated using the covariance of the target variable and the predictor, low R^2 statistics of the Korean PLS factor imply that Korean predictors are largely disconnected from the KRW/USD real exchange rate. This is consistent with cumulative R^2 statistics in Figure 1.

Note also that PLS American factors are more closely connected with groups #1 (industrial production) and #2 (labor market) than other groups. Put it differently, the first PLS American factor seems to be strongly driven by these real activity variables rather than financial market variables and other real activity variables.

Figure 2 around here

We investigate the source of the common factors in a more disaggregated level, looking at the marginal R^2 statistics of the real and financial market factors. Figure 3 reports the R^2 statistics of the first American real activity factor. Again, we notice the PLS and PC factors explain the variations in real activity variables similarly well. We also note that the American real activity factor is mainly driven by industrial production (Group #1) and labor market (Group #2) variables. Again, we note that the PLS Korean real activity factor explains negligible variations in Korean real activity variables, while the first PC factor exhibits reasonably high R^2 statistics. This again confirms our previous findings. Similar results were observed from the marginal R^2 analysis for the first financial market PLS and PC factors in Figure 4. The American PLS and PC nominal/financial market factors seem to be driven mostly by CPIs and PPIs in the US.

Figures 3 and 4 around here

4 Out-of-Sample Prediction Performance

4.1 Factor-Augmented Forecasting Models

This section reports our out-of-sample forecast exercise results using factor-augmented forecasting models for the KRW/USD real exchange rate. Based on the ADF test results in Table 2, we employ the following *stationary* $AR(1)$-type stochastic process for the real exchange rate. Abstracting from an intercept,

$$q_{t+j} = \alpha_j q_t + u_{t+j}, \ j = 1, 2, .., k, \tag{13}$$

where α_j is less than one in absolute value for stationarity. Note that we regress the j-period ahead target variable (q_{t+j}) *directly* on the current period target variable (q_t) instead of using a *recursive* forecasting approach with an AR(1) model, $q_{t+1} = \alpha q_t + \varepsilon_{t+1}$, which implies $\alpha_j = \alpha^j$ under this approach. With this specification, the j-period ahead forecast is,

$$\widehat{q}^{AR}_{t+j|t} = \hat{\alpha}_j q_t, \tag{14}$$

where $\hat{\alpha}_j$ is the least squares (LS) estimate of α_j.

We augment (13) by adding factor estimates. That is, our factor augmented stationary $AR(1)$-type forecasting model is the following.

$$q_{t+j} = \alpha_j q_t + \boldsymbol{\beta}_j^{'} \boldsymbol{\Delta}\hat{\mathbf{f}}_t + u_{t+j},\ j = 1, 2, .., k \tag{15}$$

We again employ a *direct* forecasting approach by regressing q_{t+j} directly on q_t and the estimated factors $(\Delta\hat{\mathbf{f}}_t)$. Note that (15) coincides with an exact $AR(1)$ process when $j = 1$, but extended by the factor covariates $\Delta\hat{\mathbf{f}}_t$. We obtain the following j-period ahead forecast for the target variable,

$$\widehat{q}^{F_{AR}}_{t+j|t} = \hat{\alpha}_j q_t + \hat{\boldsymbol{\beta}}_j^{'} \Delta\hat{\mathbf{f}}_t, \tag{16}$$

where $\hat{\alpha}_j$ and $\hat{\boldsymbol{\beta}}_j$ are the LS coefficient estimates. Note also that (15) nests the stationary benchmark model (13) when $\boldsymbol{\Delta}\hat{\mathbf{f}}_t$ does not contain any useful predictive contents for q_{t+j}, that is, $\boldsymbol{\beta}_j = 0$.

We evaluate the out-of-sample predictability of our factor-augmented forecasting model $\widehat{q}^{F_{AR}}_{t+j|t}$ using a fixed-size rolling window scheme as follows.[13] We use the initial $T_0 < T$ observations, $\{q_t, \Delta x_{i,t}\}_{t=1}^{T_0}$, $i = 1, 2, ..., N$ to estimate the first set of factors $\left\{\Delta\hat{\mathbf{f}}_t\right\}_{t=1}^{T_0}$ using one of our data dimensionality reduction methods. We formulate the first forecast $\widehat{q}^{F_{AR}}_{T_0+j|T_0}$, then calculate and keep the forecast error $(\varepsilon^{F_{AR}}_{T_0+j|T_0})$. Then, we add one next observation $(t = T_0+1)$ but drop one earliest observation $(t = 2)$ for the second round forecasting. That is, we re-estimate $\left\{\Delta\hat{\mathbf{f}}_t\right\}_{t=2}^{T_0+1}$ using $\{q_t, \Delta x_{i,t}\}_{t=2}^{T_0+1}$, $i = 1, 2, ..., N$, maintaining the same number of observations (T_0) in order to formulate the second round forecast, $\hat{q}^{F_{AR}}_{T_0+j+1|T_0+1}$. We repeat until we forecast the last observation, q_T. We implement the same procedure for the benchmark forecast $\widehat{q}^{AR}_{t+j|t}$ by (14) in

[13] Rolling window schemes tend to perform better than the recursive method in the presence of structural breaks. However, results with recursive approaches were qualitatively similar.

addition to the no-change Random Walk (RW) benchmark $\widehat{q}_{t+j|t}^{RW} = q_t$.[14]

We employ the ratio of the root mean square prediction error ($RRMSPE$) to evaluate the out-of-sample prediction accuracy of our factor augmented models. That is,

$$RRMSPE(j) = \frac{\sqrt{\frac{1}{T-j-T_0+1}\sum_{t=T_0}^{T-j}\left(\varepsilon_{t+j|t}^{BM}\right)^2}}{\sqrt{\frac{1}{T-j-T_0+1}\sum_{t=T_0}^{T-j}\left(\varepsilon_{t+j|t}^{F_{AR}}\right)^2}}, \tag{17}$$

where

$$\varepsilon_{t+j|t}^{BM} = q_{t+j} - \widehat{q}_{t+j|t}^{BM},\ \varepsilon_{t+j|t}^{F} = q_{t+j} - \widehat{q}_{t+j|t}^{F_{AR}},\ BM = AR, RW \tag{18}$$

Note that our factor models outperform the benchmark models when $RRMSPE$ is greater than 1.[15]

4.2 Prediction Accuracy Evaluations for the Real Exchange Rate

We implement out-of-sample forecast exercises using a fixed-size (50% split point) rolling window method with up to 4 (k) latent factor estimates.[16] We obtained latent common factors via the PLS, PC, and LASSO methods for large panels of macroeconomic data in the US and in Korea.

Table 4 reports the $RRMSPE$ statistics of our forecasting model $\widehat{q}_{t+j|t}^{F_{AR}}$ relative to $\widehat{q}_{t+j|t}^{AR}$ and $\widehat{q}_{t+j|t}^{RW}$. Recall that our models outperform the RW model when the $RRMSPE$ is greater than one. The numbers in bold denote $\widehat{q}_{t+j|t}^{F_{AR}}$ outperforms $\widehat{q}_{t+j|t}^{RW}$, while the superscript $*$ means that $\widehat{q}_{t+j|t}^{F_{AR}}$ outperforms both $\widehat{q}_{t+j|t}^{RW}$ and $\widehat{q}_{t+j|t}^{AR}$.[17] Our major findings are as follows.

First, American predictors yield superior predictive contents for the KRW/USD real exchange rate, while Korean factor models perform relatively poorly. That is, $\widehat{q}_{t+j|t}^{F_{AR}}$ frequently outperform both the RW and AR models when augmented by American factors. The models with Korean factors overall got dominated by the AR model although still outperform the RW model when the forecast horizon is one-year or

[14] Consider a random walk model, $q_{t+1}^{BM_{RW}} = q_t + \eta_{t+1}$,where η_{t+1} is a white noise process. Therefore, j-period ahead forecast from this benchmark RW model is simply q_t.

[15] Alternatively, one may employ the ratio of the root mean absolute prediction error ($RRMAPE$). Results are overall qualitatively similar.

[16] We obtained qualitatively similar results with a 70% sample split point.

[17] That is, the AR benchmark model performs better thant the RW model.

longer. Recall that these empirical findings are consistent with our in-sample fit analysis shown in the previous section. When we combine American factors with Korean factors, the performance slightly improves, that is, the $RRMSPE$ increases a little, but failed to generate sufficiently big improvement enough to outperform the AR model.

Second, we observe that our American factor models tend to perform better at short horizons when combined with nominal/financial market factors, while real activity factors improve the predictability at longer horizons. That is, the good prediction performance of our models with the total factors, $\Delta\hat{\mathbf{f}}_t^{PLS}$ or $\Delta\hat{\mathbf{f}}_t^{PC}$, at 1-period horizon seem to inherit the superior performance of our models with financial market factors, $\Delta\hat{\mathbf{f}}_t^{PLS,F}$ or $\Delta\hat{\mathbf{f}}_t^{PC,F}$. Similarly, superior long-run predictability seems to stem from predictive contents of real activity factors, $\Delta\hat{\mathbf{f}}_t^{PLS,R}$ or $\Delta\hat{\mathbf{f}}_t^{PC,R}$. These results imply that factors obtained from subsets may provide more useful information than factors from the entire predictor variables, which is consistent with Boivin and Ng (2006).

Table 4 around here

We also employ the LASSO to select the subsets of the predictor variables that are useful to explain the target variable. The idea behind this is to estimate the factors using fewer but more informative predictor variables as discussed by Bai and Ng (2008). Following Kelly and Pruitt (2015), we adjust the tuning parameter τ in (8) to choose a group of 30 predictors from each panel of macroeconomic variables in the US and in Korea, while 20 predictors were chosen from each of the real activity and the financial market variable groups. Then, we employed PLS and PC to estimate up to 4 common factors to augment the benchmark AR model.

As we can see in Table 5, results are qualitatively similar to previous ones. Financial factor augmented forecasting models tend to perform better for the 1-period ahead forecasts, while real factors provide superior predictive contents for the real exchange rate at longer horizons. Korean factor-augmented models perform overall poorly relative to the AR model, although they still outperform the RW model when the forecast horizon is 1-year or longer.

Table 5 around here

4.3 Prediction Accuracy Evaluations for Exchange Rate Returns

This section employs an alternative specification for the exchange rate. That is, we evaluate the prediction accuracy of our factor models for exchange rate returns, motivated by an assumption that exchange rates obey a nonstationary stochastic process. Recall that this specification can be justified not only empirically (see Table 2) but also theoretically by the monetary models of the *nominal* exchange rate. Assuming an integrated process for s_t, we consider the following $AR(1)$-type model for the exchange rate return (Δs_t). Abstracting from an intercept,

$$\Delta s_{t+j} = \alpha_j \Delta s_t + u_{t+j},\ j = 1, 2, .., k, \tag{19}$$

where α_j is less than one in absolute value for stationarity.[18] Note that we regress the j-period ahead target variable (Δs_{t+j}) *directly* on the current period exchange rate return (Δs_t). Then, the j-period ahead forecast is,

$$\Delta \hat{s}^{AR}_{t+j} = \hat{\alpha}_j \Delta s_t \tag{20}$$

The corresponding factor augmented forecasting model is the following.

$$\Delta s_{t+j} = \alpha_j \Delta s_t + \boldsymbol{\beta}^{'}_j \boldsymbol{\Delta} \hat{\mathbf{f}}_t + u_{t+j},\ j = 1, 2, .., k \tag{21}$$

which augment (19) by adding factor estimates ($\Delta \hat{\mathbf{f}}_t$). The j-period ahead forecast for the exchange rate return is,

$$\Delta \widehat{s}^{F_{AR}}_{t+j|t} = \hat{\alpha}_j \Delta s_t + \hat{\boldsymbol{\beta}}^{'}_j \Delta \hat{\mathbf{f}}_t, \tag{22}$$

where $\hat{\alpha}_j$ and $\hat{\boldsymbol{\beta}}_j$ are the LS coefficient estimates.

Recall that (13) or (15) are motivated by a *long-run* cointegrating relationship between the nominal exchange rate (s_t) and the relative price. On the other hand, (19) or (22) describes a *short-run* stochastic process of the nominal exchange rate return. In Table 6, therefore, we report the $RRMSPE$ statistics of the 1-period ahead out-of-sample forecasts of our factor models relative to the RW and AR models.

One interesting finding is that PLS American factor augmented models tend to

[18] That is, we assume that there are two eigenvalues for the level exchange rate, 1 and α_1.

perform better than PC factor models. The PLS models outperform both the RW and AR models in 9 out of 12 cases, whereas the PC models outperform both benchmark models only for 4 out of 12 cases. Korean factor models again perform poorly even relative to the RW model.

Table 6 around here

Even though the ADF test in Table 2 provides empirical evidence in favor of the stationarity of the real KRW/USD exchange rate, the persistence of the real exchange rate is similar to the nominal exchange rate as can be seen in Table 3. So we also experiment our forecasting exercises with the real exchange rate return (Δq_t). Table 7 reports qualitatively similar results. PLS American factors again yield superior 1-period ahead forecast performance for Δq_t. PLS models outperform both the benchmark models for 8 out of 12 cases, while PC models perform better than those models only 4 out of 12.

Table 7 around here

4.4 Model Predictability with Proposition Based Global Factors

This section implements out-of-sample forecast exercises using models that utilize global factors that are motivated by exchange rate determination theories: Purchasing Power Parity (PPP); Uncovered Interest Parity (UIP); Real Interest Rate Parity (RIRP).

4.4.1 Purchasing Power Parity: Relative Price Factors

We first consider PPP to motivate the strategy to identify common factors. When PPP holds, there exists a cointegrating vector [1, 1] for the log nominal (bilateral) exchange rate s_t (foreign currency price of 1 US dollar) and the log relative price $(p_t - p_t^*)$, $relp_t$, where p_t and p_t^* are the log prices in the US and in the foreign country, respectively. That is, under PPP, the real exchange rate, $q_t = s_t + relp_t$, is stationary, while the nominal exchange rate and the relative price are nonstationary $I(1)$ processes. Note that our unit root test results in Table 2 are consistent with PPP.

We obtained the Consumer Price Index (CPI) of 43 countries from the IFS database including 18 euro-zone countries. Assuming that the US is the home/reference country, we constructed 42 relative prices $(p_t - p_t^*)$, then estimated the first common factor from these relative prices after taking the first difference $(\Delta p_t - \Delta p_t^*)$ or $(\pi_t - \pi_t^*)$, since the relative price is an integrated process. We also extracted the common factor from the 42 relative prices with Korea as the reference country.[19]

We report j-period ahead out-of-sample predictability exercise results (50% split point) using one factor models when each of the US and Korea serves as the reference country in Table 8. Results imply overall poor performance of PPP motivated factor models irrespective of the choice of the reference country.

Our factor models outperform the RW model when the forecast horizon is 1-year or longer, which is consistent with PPP that is a long-run proposition. However, our PPP factor models rarely beat the AR benchmark model nor our data-driven macroeconomic factor models presented in the previous section. It is not surprising to find similar performance of the Korean reference factor model as the American factor model, since they include fundamentally similar information of CPIs in 43 countries.

Table 8 around here

4.4.2 Uncovered Interest Parity: Interest Rate Spread

The second proposition we adopt is Uncovered Interest Parity (UIP). Abstracting from risk premium, UIP states the following.

$$\Delta s_{t+1} = i_t^* - i_t + \varepsilon_{t+1}, \tag{23}$$

[19]For the group of developed countries, we include 11 euro-zone countries (Austria, Belgium, Finland, France, Germany, Greece, Italy, Luxembourg, Netherlands, Portugal, Spain) and 8 non-euro-zone countries (Canada, Denmark, Japan, Singapore, Switzerland, Sweden, United Kingdom, United States). All data are obtained from the IFS database with an exception of Singapore. We obtained the Singapore CPI from the Department of Statistics of Singapore. In addition to the group of 19 developed countries, we added 7 the rest of euro-zone countries (Cyprus, Estonia, Ireland, Latvia, Lithuania, Slovakia, Slovenia) except Malta, and 17 non-euro-zone countries (Brazil, China, Chile, Colombia, Czech Republic, Hong Kong, Hungary, India, Indonesia, Israel, Korea, Malaysia, Mexico, Poland, Romania, Russia, Saudi Arabia).

where Δs_{t+1} is the nominal exchange rate return, that is, appreciation (depreciation) rate of the home (foreign) currency, while i_t and i_t^* are nominal short-run interest rates in the home and foreign countries, respectively. $\varepsilon_{t+1} = \Delta s_{t+1} - E_t \Delta s_{t+1}$ is the mean-zero ($E_t \varepsilon_{t+1} = 0$) rational expectation error term.

Motivated by (23), we obtained 18 international short-term interest rates from the FRED and the OECD database to construct nominal interest rate spreads by subtracting US interest rate (i_t) from the national interest rate (i_t^*).[20] We estimate the first common factor via PLS and PC from the balanced panel of 17 interest rate spreads relative to American interest rate. We took the first difference of the spreads to make sure we estimate factors consistently.[21] Similarly, we estimated the first common factor from 17 interest rate spreads relative to Korean interest rate.

Table 9 reports the $RRMSPE$ statistics of the j-period ahead out-of-sample forecasts for the nominal exchange rate return (Δs_t) using UIP motivated factors. American PLS factor forecasting models perform overall well relative to both benchmark models. It is interesting that our short-run proposition based forecasting models perform fairly well both in the short-run and in the long-run. However, Korean factor models overall performed poorly.

Table 9 around here

4.4.3 Real Interest Rate Parity: Real Interest Rate Spread

The last proposition we employ is Real Interest Rate Parity (RIRP), which combines PPP with UIP. Taking the first difference to the PPP equation ($q_t = s_t + p_t - p_t^*$) at time $t+1$,

$$\Delta q_{t+1} = \Delta s_{t+1} + \pi_{t+1} - \pi_{t+1}^* \tag{24}$$

Combining (23) and (24), we obtain the following expression for RIRP.

$$\Delta q_{t+1} = r_t^* - r_t + \varepsilon_{t+1}, \tag{25}$$

[20] For the group of developed countries, we include 11 euro-zone countries (Austria, Belgium, Finland, France, Germany, Greece, Italy, Luxembourg, Netherlands, Portugal, Spain) and 7 non-euro-zone countries (Canada, Denmark, Japan, Switzerland, Sweden, United Kingdom, United States). All data are obtained from the OECD database and the FRED.

[21] The PANIC test provides strong evidence of nonstationarity for the interest rate spreads.

where $r_t = i_t - \pi_{t+1}$ and $r_t^* = i_t^* - \pi_{t+1}^*$ are the *ex post* real interest rates in the home and foreign country, respectively.

Using international CPIs and short-run interest rates we used, we estimated the first common factors by applying PLS and PC to a panel of 17 real interest rate spreads $(r_t^* - r_t)$ without taking differences. This is because we obtained very strong evidence in favor of stationarity for real interest rate spreads.[22] We report results in Table 10. Our factor augmented forecasting models outperformed the RW model, but performed poorly relative to the AR benchmark model.

Table 10 around here

5 Concluding Remarks

In this paper, we propose parsimonious factor-augmented forecasting models for the KRW/USD real exchange rate in a data rich environment. We apply an array of data dimensionality reduction methods to large panels of 125 American and 192 Korean monthly frequency macroeconomic time series data from October 2000 to March 2019. In addition to Principal Component (PC) analysis that has been frequently used in the current literature, we employ the Partial Least Squares (PLS) approach and the Least Absolute Shrinkage and Selection Operator (LASSO) combined with PC and PLS to extract target-specific common factors for the KRW/USD exchange rate.

We augment benchmark forecasting models with estimated common factors to formulate out-of-sample forecasts. Then, using the ratio of the root mean squared prediction error ($RRMSPE$) criteria, we evaluate the predictive accuracy of our proposed models for the real exchange rate relative to the random walk (RW) and the stationary autoregressive (AR) models.

Our forecasting models outperform both the RW and the AR benchmark models only when we utilize latent common factors from American predictors. In particular, our models that utilize real activity factors perform well at longer horizons, while nominal/financial market factors help improve the prediction performance at short horizons. The superior performance with factors from subset of predictors is in line with the work of Boivin and Ng (2006) who demonstrated the importance of relevant

[22] The PANIC test results are available upon request.

common factors for the target variable. Models with Korean factors overall perform poorly relative to the AR model, while they still outperform the RW model when the forecast horizon is 1-year or longer.

We also implement forecasting exercises using global factors that are motivated by exchange rate determination theories such as Purchasing Power Parity (PPP), Uncovered Interest Parity (UIP), and Real Uncovered Interest Parity (RIRP). Forecasting models with UIP common factors turn out to perform fairly well when the US serves as the reference country, while models with either PPP or RIRP factors perform overall poorly whichever is used for the reference country.

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Table 1. Macroeconomic Data Descriptions

American Data

Group ID	Data ID	Data Description
#1	1-16	Industrial Production Indices
#2	17-47	Labor Market Variables
#3	48-57	Housing Inventories
#4	58-65	Manufacturers' Consumption/ New Orders
#5	66-79	Monetary Aggregates
#6	80-96	Domestic Interest Rates
#7	97-116	Producer/Consumer Prices
#8	117-121	Stock Indices
#9	122-126	Exchange Rates

Korean Data

Group ID	Data ID	Data Description
#1	1-27	New Orders
#2	28-34	Inventory
#3	35-52	Housing
#4	53-74	Retails/Manufacturing
#5	75-87	Labor
#6	88-98	Industrial Production
#7	99-102	Business Condition
#8	103-114	Stock Indices
#9	115-127	Interest Rates
#10	128-145	Exports/Imports Prices
#11	146-163	Prices
#12	164-180	Money
#13	181-192	Exchange Rates

Note: We obtained the American data from the FRED-MD website. The Korean Data was obtained from the Bank of Korea.

Table 2. Unit Root Test Results

	ADF Test			
	q_t	$-2.966^{\dagger}$ (0.038)	s_t	-1.953 (0.308)
	PANIC Test			
	$\mathbf{x}_t^{Am}$		$\mathbf{x}_t^{Kr}$	
All variables	$f_{1,t}^{PC}$	-2.519 (0.100)	$f_{1,t}^{PC}$	1.186 (0.998)
	$f_{2,t}^{PC}$	-1.270 (0.641)	$f_{2,t}^{PC}$	-2.085 (0.237)
	$P_{\hat{e}}$	$11.341^{\ddagger}$ (0.000)	$P_{\hat{e}}$	$16.667^{\ddagger}$ (0.000)
Real Variables	$f_{1,t}^{PC,R}$	-2.443 (0.124)	$f_{1,t}^{PC,R}$	-0.634 (0.868)
	$f_{2,t}^{PC,R}$	-2.393 (0.133)	$f_{2,t}^{PC,R}$	-2.057 (0.254)
	$P_{\hat{e}}$	$5.763^{\ddagger}$ (0.000)	$P_{\hat{e}}$	$15.552^{\ddagger}$ (0.000)
Financial Variables	$f_{1,t}^{PC,F}$	-1.888 (0.326)	$f_{1,t}^{PC,F}$	1.250 (0.999)
	$f_{2,t}^{PC,F}$	$-3.437^{\ddagger}$ (0.009)	$f_{2,t}^{PC,F}$	-1.544 (0.512)
	$P_{\hat{e}}$	$6.425^{\ddagger}$ (0.000)	$P_{\hat{e}}$	$8.711^{\ddagger}$ (0.000)

Note: q_t and s_t are the CPI-based real bilateral KRW/USD exchange rate and the nominal bilateral KRW/USD exchnage rate, respectively. PLS produces target specific factors for q_t and s_t separately, while PC yields the same common factors independent on the target variable. Real variables are from group #1 through #4 for American factors and groups #1 through #7 for Korean factors, while financial variables include group #5 through #9 for American factors and groups #8 through #13 for Korean factors. The augmented Dickey-Fuller (*ADF*) test reports the *ADF* t-statitics when an intercept is included. P-values are in parenthesis. For the PANIC test results, we report the *ADF* t-statistics with an intercept for each common factor estimate. $P_{\hat{e}}$ denotes the panel test statistics from the de-factored idiosyncratic components. ‡ and † denote a rejection of the null hypothesis at the 1% and 5% level, respectively.

Table 3. Median Unbiased Estimates of the Persistence Parameter

	β	C.I	HL	C.I
q_t	0.974	$[0.938, 1.006]$	2.227	$[0.905, \infty)$
s_t	0.971	$[0.932, 1.006]$	1.979	$[0.822, \infty)$

Note: q_t and s_t are the CPI-based real bilateral KRW/USD exchange rate and the nominal bilateral KRW/USD exchnage rate, respectively. β denotes the persistent parameter from an autoregressive process of degree 1, AR(1), specificiation of each real exchange rate. We corrected the median bias following Hansen's (1999) grid bootstrap technique. We employed 100 fine evenly spaced grid points on the interval $[\hat{\beta} \pm 6 \times se(\hat{\beta})]$, where $\hat{\beta}$ is the least squares estimate of β and se is its standard error. 10,000 nonparametric bootstrap simulations were done at each grid point to construct quantile function estimates. HL denotes the implied half-life point estimate in years. C.I denotes the 95% median unbiased confidence band.

Table 4. Out-of-Sample Predictability for the Real Exchange Rate

				American Factors			
j	#Factors	$\Delta\mathbf{f}_t^{PLS}$	$\Delta\mathbf{f}_t^{PLS,R}$	$\Delta\mathbf{f}_t^{PLS,F}$	$\Delta\mathbf{f}_t^{PC}$	$\Delta\mathbf{f}_t^{PC,R}$	$\Delta\mathbf{f}_t^{PC,F}$
1	1	**1.0115***	**1.0077**	**1.0074**	**1.0251***	**1.0088***	**1.0144***
	2	**1.0355***	0.9971	**1.0198***	**1.0148***	**1.0023**	**1.0155***
	3	**1.0266***	0.9941	**1.0223***	**1.0142***	0.9680	**1.0223***
	4	0.9985	0.9882	**1.0388***	**1.0262***	0.9575	**1.0528***
12	1	**1.3440***	**1.3611***	**1.2499***	**1.3194***	**1.3375***	**1.2380**
	2	**1.3389***	**1.3469***	**1.0965**	**1.2573***	**1.3375***	**1.2352**
	3	**1.2421***	**1.2246**	**1.0243**	**1.2572***	**1.3308***	**1.1217**
	4	**1.2625***	**1.2046**	**1.0128**	**1.1701**	**1.2850***	**1.0164**
36	1	**1.4269***	**1.4466***	**1.2710**	**1.3742***	**1.4271***	**1.2409**
	2	**1.2736**	**1.3948***	**1.2316**	**1.3142**	**1.3980***	**1.2585**
	3	**1.2232**	**1.4229***	**1.2715**	**1.3225**	**1.3882***	**1.2300**
	4	**1.2029**	**1.4147***	**1.2980**	**1.2430**	**1.3593**	**1.2719**

				Korean Factors			
j	#Factors	$\Delta\mathbf{f}_t^{PLS}$	$\Delta\mathbf{f}_t^{PLS,R}$	$\Delta\mathbf{f}_t^{PLS,F}$	$\Delta\mathbf{f}_t^{PC}$	$\Delta\mathbf{f}_t^{PC,R}$	$\Delta\mathbf{f}_t^{PC,F}$
1	1	0.9006	**1.0029**	0.9382	0.6836	0.8443	0.9646
	2	0.6329	0.9441	0.9450	0.4931	0.8644	0.2871
	3	0.4880	0.8965	0.5315	0.2348	0.7819	0.1707
	4	0.4316	0.8660	0.6469	0.2275	0.6987	0.1362
12	1	**1.2607***	**1.2162**	**1.2698***	**1.2381**	**1.2422***	**1.2074**
	2	**1.2431***	**1.1635**	**1.2362**	**1.2067**	**1.2433***	**1.1877**
	3	**1.2380**	**1.0719**	**1.2288**	**1.2159**	**1.2298**	**1.2485***
	4	**1.2203**	**1.0664**	**1.2545***	**1.2289**	**1.2084**	**1.2318**
36	1	**1.3737***	**1.3787***	**1.3883***	**1.3752***	**1.3652**	**1.3797***
	2	**1.3498**	**1.2832**	**1.3850***	**1.3843***	**1.3425**	**1.3809***
	3	**1.1916**	**1.1056**	**1.3377**	**1.3847***	**1.3435**	**1.3687**
	4	**1.1716**	**1.0641**	**1.3359**	**1.3367**	**1.3177**	**1.3611**

				American and Korean Factors			
j	#Factors	$\Delta\mathbf{f}_t^{PLS}$	$\Delta\mathbf{f}_t^{PLS,R}$	$\Delta\mathbf{f}_t^{PLS,F}$	$\Delta\mathbf{f}_t^{PC}$	$\Delta\mathbf{f}_t^{PC,R}$	$\Delta\mathbf{f}_t^{PC,F}$
1	2	0.9469	**1.0082***	0.9542	0.8242	0.8925	0.9781
	4	0.9441	0.9419	0.8800	0.4030	0.8900	0.2400
	6	0.5604	0.8730	0.5753	0.2230	0.7975	0.1871
	8	0.5408	0.8029	0.9025	0.2096	0.7784	0.1803
12	2	**1.3503***	**1.3203***	**1.2704***	**1.3054***	**1.3351***	**1.1799**
	4	**1.3055***	**1.2448***	**1.0548**	**1.1873**	**1.3342***	**1.1753**
	6	**1.1377**	**1.0888**	0.9167	**1.1621**	**1.3277***	**1.1377**
	8	**1.1442**	**1.0450**	0.9340	**1.1567**	**1.2331**	0.9798
36	2	**1.4278***	**1.4534***	**1.2882**	**1.3655**	**1.4190***	**1.2216**
	4	**1.2430**	**1.3358**	**1.2300**	**1.2845**	**1.3804***	**1.2352**
	6	**1.1017**	**1.1954**	**1.2246**	**1.2733**	**1.3578**	**1.2321**
	8	**1.0833**	**1.1244**	**1.2869**	**1.1854**	**1.3005**	**1.2677**

Note: We report the *RRMSPE* statistics employing a rolling window scheme with a 50% sample split point. *RRMSPE* denotes the ratio of the root mean squared prediction errors, which is the mean squared prediction error (*RMSPE*) from the benchmark random walk (RW) model divided by the *RMSPE* from each competing model with k factors. Superscript R and F mean that factors were estimated from real and financial variables, respectively. *RRMSPE* statistics in bold denote that the competing model outperforms the benchmark RW model. $*$ denotes that the competing model outperforms the benchmark AR model as well as the RW model.

Table 5. Out-of-Sample Predictability with LASSO for the Real Exchange Rate

		American Factors					
j	#Factors	$\Delta\mathbf{f}_t^{PLS}$	$\Delta\mathbf{f}_t^{PLS,R}$	$\Delta\mathbf{f}_t^{PLS,F}$	$\Delta\mathbf{f}_t^{PC}$	$\Delta\mathbf{f}_t^{PC,R}$	$\Delta\mathbf{f}_t^{PC,F}$
1	1	**1.0162***	**1.0218***	**1.0188***	**1.0392***	**1.0184***	**1.0329***
	2	**1.0388***	0.9849	**1.0261***	**1.0451***	**1.0160**	**1.0520***
	3	**1.0387***	0.9807	**1.0379***	**1.0419***	**1.0278***	**1.0474***
	4	**1.0502***	0.9933	**1.0236***	**1.0456***	**1.0145***	**1.0411***
12	1	**1.2145**	**1.3792***	**1.1614**	**1.2515***	**1.3354***	**1.2413**
	2	**1.1729**	**1.3456***	**1.1068**	**1.2305**	**1.3549***	**1.1742**
	3	**1.1734**	**1.3395***	**1.1350**	**1.0718**	**1.3060***	**1.1282**
	4	**1.1316**	**1.2497***	**1.1100**	**1.0864**	**1.2917***	**1.1534**
36	1	**1.2970**	**1.5208***	**1.2753**	**1.3555**	**1.4750***	**1.3515**
	2	**1.3264**	**1.4849***	**1.3718**	**1.2840**	**1.4654***	**1.3337**
	3	**1.2664**	**1.4729***	**1.3596**	**1.3029**	**1.4816***	**1.3415**
	4	**1.2589**	**1.4242***	**1.2998**	**1.2794**	**1.4763***	**1.3809***
		Korean Factors					
j	#Factors	$\Delta\mathbf{f}_t^{PLS}$	$\Delta\mathbf{f}_t^{PLS,R}$	$\Delta\mathbf{f}_t^{PLS,F}$	$\Delta\mathbf{f}_t^{PC}$	$\Delta\mathbf{f}_t^{PC,R}$	$\Delta\mathbf{f}_t^{PC,F}$
1	1	0.9735	**1.0022**	0.9773	0.5423	0.9743	0.5820
	2	0.4596	0.9929	0.6262	0.5492	0.9646	0.3462
	3	0.5694	0.9760	0.6352	0.3133	0.9632	0.2832
	4	0.4563	0.9744	0.6065	0.2911	0.9286	0.2604
12	1	**1.2271**	**1.2144**	**1.2199**	**1.1532**	**1.2434***	**1.1571**
	2	**1.1769**	**1.2247**	**1.1618**	**1.1656**	**1.2418***	**1.1594**
	3	**1.1812**	**1.2154**	**1.1611**	**1.1720**	**1.2274**	**1.1600**
	4	**1.1718**	**1.2148**	**1.1536**	**1.1695**	**1.1975**	**1.1220**
36	1	**1.3667**	**1.2937**	**1.3728**	**1.3919***	**1.3654**	**1.3920***
	2	**1.3624**	**1.3071**	**1.3783***	**1.3936***	**1.3648**	**1.3919***
	3	**1.3794***	**1.2798**	**1.2943**	**1.4058***	**1.3316**	**1.3155**
	4	**1.3342**	**1.2690**	**1.2935**	**1.2495**	**1.3285**	**1.2894**
		American and Korean Factors					
j	#Factors	$\Delta\mathbf{f}_t^{PLS}$	$\Delta\mathbf{f}_t^{PLS,R}$	$\Delta\mathbf{f}_t^{PLS,F}$	$\Delta\mathbf{f}_t^{PC}$	$\Delta\mathbf{f}_t^{PC,R}$	$\Delta\mathbf{f}_t^{PC,F}$
1	2	0.9954	**1.0169***	0.9854	0.5882	0.9919	0.6359
	4	0.5416	0.9679	0.7121	0.6120	0.9696	0.4283
	6	0.6234	0.9530	0.6667	0.3999	0.9695	0.3120
	8	0.5272	0.9706	0.6231	0.3538	0.9193	0.2736
12	2	**1.2325**	**1.3289***	**1.1751**	**1.1768**	**1.3298***	**1.1787**
	4	**1.0980**	**1.3127***	**1.0330**	**1.1387**	**1.3483***	**1.0954**
	6	**1.0936**	**1.2745***	**1.0212**	**1.0291**	**1.2780***	**1.0795**
	8	**1.0107**	**1.1975**	0.9970	**1.0501**	**1.2380**	**1.0466**
36	2	**1.2664**	**1.4650***	**1.2462**	**1.3389**	**1.4520***	**1.3508**
	4	**1.2798**	**1.4418***	**1.3483**	**1.2810**	**1.4358***	**1.3306**
	6	**1.2135**	**1.3949***	**1.2337**	**1.2589**	**1.4167***	**1.3051**
	8	**1.2523**	**1.3107**	**1.1827**	**1.1572**	**1.4132***	**1.2956**

Note: We report the *RRMSPE* statistics employing a rolling window scheme with a 50% sample split point. *RRMSPE* denotes the ratio of the root mean squared prediction errors, which is the mean squared prediction error (*RMSPE*) from the benchmark random walk (RW) model divided by the *RMSPE* from each competing model with k factors. Superscript R and F mean that factors were estimated from real and financial variables, respectively. *RRMSPE* statistics in bold denote that the competing model outperforms the benchmark RW model. $*$ denotes that the competing model outperforms the benchmark AR model as well as the RW model.

Table 6. 1-period ahead Predictability for the Nominal Exchange Rate Return

			American Factors			
#Factors	$\Delta\mathbf{f}_t^{PLS}$	$\Delta\mathbf{f}_t^{PLS,R}$	$\Delta\mathbf{f}_t^{PLS,F}$	$\Delta\mathbf{f}_t^{PC}$	$\Delta\mathbf{f}_t^{PC,R}$	$\Delta\mathbf{f}_t^{PC,F}$
1	**1.0371***	**1.0318***	**1.0309***	**1.0281***	**1.0225**	**1.0249**
2	**1.0455***	**1.0252**	**1.0576***	**1.0154**	**1.0282***	**1.0232**
3	**1.0703***	**1.0153**	**1.0685***	**1.0158**	0.9877	**1.0233**
4	**1.0552***	0.9987	**1.0680***	**1.0312***	0.9771	**1.0367***

			Korean Factors			
#Factors	$\Delta\mathbf{f}_t^{PLS}$	$\Delta\mathbf{f}_t^{PLS,R}$	$\Delta\mathbf{f}_t^{PLS,F}$	$\Delta\mathbf{f}_t^{PC}$	$\Delta\mathbf{f}_t^{PC,R}$	$\Delta\mathbf{f}_t^{PC,F}$
1	0.9191	0.9839	**1.0030**	0.5361	0.7685	0.9923
2	0.7552	**1.0042**	0.5207	0.5829	0.9124	0.6725
3	0.5044	0.9567	0.9014	0.2566	0.8513	0.2561
4	0.4619	0.7603	0.6702	0.2698	0.6652	0.2733

			American and Korean Factors			
#Factors	$\Delta\mathbf{f}_t^{PLS}$	$\Delta\mathbf{f}_t^{PLS,R}$	$\Delta\mathbf{f}_t^{PLS,F}$	$\Delta\mathbf{f}_t^{PC}$	$\Delta\mathbf{f}_t^{PC,R}$	$\Delta\mathbf{f}_t^{PC,F}$
2	0.9072	0.9890	**1.0132**	0.5436	0.8024	0.9831
4	0.7558	0.9954	0.5552	0.4477	0.9440	0.5707
6	0.4049	0.9517	0.6317	0.2581	0.8699	0.2555
8	0.3370	0.7699	0.4400	0.2403	0.7215	0.3216

Note: We report the *RRMSPE* statistics employing a rolling window scheme with a 50% sample split point. *RRMSPE* denotes the ratio of the root mean squared prediction errors, which is the mean squared prediction error (*RMSPE*) from the benchmark random walk (RW) model divided by the *RMSPE* from each competing model with k factors. Superscript R and F mean that factors were estimated from real and financial variables, respectively. *RRMSPE* statistics in bold denote that the competing model outperforms the benchmark RW model. $*$ denotes that the competing model outperforms the benchmark AR model as well as the RW model.

Table 7. 1-period ahead Predictability for the Real Exchange Rate Return

			American Factors			
#Factors	$\Delta\mathbf{f}_t^{PLS}$	$\Delta\mathbf{f}_t^{PLS,R}$	$\Delta\mathbf{f}_t^{PLS,F}$	$\Delta\mathbf{f}_t^{PC}$	$\Delta\mathbf{f}_t^{PC,R}$	$\Delta\mathbf{f}_t^{PC,F}$
1	**1.0290***	**1.0260***	**1.0201***	**1.0198***	**1.0151**	**1.0150**
2	**1.0358***	**1.0215***	**1.0404***	**1.0061**	**1.0276***	**1.0123**
3	**1.0598***	**1.0086**	**1.0568***	**1.0061**	0.9895	**1.0132**
4	**1.0440***	**1.0004**	**1.0554***	**1.0294***	0.9750	**1.0249***

			Korean Factors			
#Factors	$\Delta\mathbf{f}_t^{PLS}$	$\Delta\mathbf{f}_t^{PLS,R}$	$\Delta\mathbf{f}_t^{PLS,F}$	$\Delta\mathbf{f}_t^{PC}$	$\Delta\mathbf{f}_t^{PC,R}$	$\Delta\mathbf{f}_t^{PC,F}$
1	0.8899	0.9832	0.9923	0.6676	0.8165	0.8348
2	0.8123	**1.0050**	0.6365	0.4295	0.8601	0.5331
3	0.3470	0.9574	0.7707	0.2443	0.7875	0.2736
4	0.4283	0.7916	0.6183	0.2624	0.6692	0.2924

			American and Korean Factors			
#Factors	$\Delta\mathbf{f}_t^{PLS}$	$\Delta\mathbf{f}_t^{PLS,R}$	$\Delta\mathbf{f}_t^{PLS,F}$	$\Delta\mathbf{f}_t^{PC}$	$\Delta\mathbf{f}_t^{PC,R}$	$\Delta\mathbf{f}_t^{PC,F}$
2	0.8903	0.9908	0.9958	0.6806	0.8459	0.8232
4	0.8484	**1.0052**	0.6932	0.3501	0.8963	0.4958
6	0.3470	0.9491	0.5696	0.2462	0.8307	0.2670
8	0.2975	0.7909	0.3985	0.2195	0.7457	0.2962

Note: We report the *RRMSPE* statistics employing a rolling window scheme with a 50% sample split point. *RRMSPE* denotes the ratio of the root mean squared prediction errors, which is the mean squared prediction error (*RMSPE*) from the benchmark random walk (RW) model divided by the *RMSPE* from each competing model with k factors. Superscript R and F mean that factors were estimated from real and financial variables, respectively. *RRMSPE* statistics in bold denote that the competing model outperforms the benchmark RW model. $*$ denotes that the competing model outperforms the benchmark AR model as well as the RW model.

Table 8. PPP Based Models for the Real Exchange Rate

		American Reference		*Korean Reference*	
j	#Factors	$\Delta\mathbf{f}_t^{PLS}$	$\Delta\mathbf{f}_t^{PC}$	$\Delta\mathbf{f}_t^{PLS}$	$\Delta\mathbf{f}_t^{PC}$
1	1	**1.0061**	**1.0057**	0.9678	0.9670
	2	0.9905	0.9866	0.9639	0.9710
	3	0.9831	0.9697	0.9566	0.9636
12	1	**1.2381***	**1.2372**	**1.2422***	**1.2441***
	2	**1.1760**	**1.2265**	**1.2108**	**1.2153**
	3	**1.1868**	**1.1836**	**1.0970**	**1.1617**
24	1	**1.4852**	**1.4836**	**1.5278***	**1.5213**
	2	**1.4995**	**1.4423**	**1.4262**	**1.4693**
	3	**1.4894**	**1.4652**	**1.2865**	**1.3131**
36	1	**1.3415**	**1.3364**	**1.4172***	**1.4151***
	2	**1.3406**	**1.3401**	**1.3710**	**1.2233**
	3	**1.3167**	**1.3140**	**1.0971**	**1.2321**

Note: We report the *RRMSPE* statistics employing a rolling window scheme with a 50% sample split point. *RRMSPE* denotes the ratio of the root mean squared prediction errors, which is the mean squared prediction error (*RMSPE*) from the benchmark random walk (RW) model divided by the *RMSPE* from each competing model with k factors. *RRMSPE* statistics in bold denote that the competing model outperforms the benchmark RW model. $*$ denotes that the competing model outperforms the benchmark AR model as well as the RW model.

Table 9. UIP Based Models for the Nominal Returns

		American Factors		*Korean Factors*	
j	#Factors	$\Delta \mathbf{f}_t^{PLS}$	$\Delta \mathbf{f}_t^{PC}$	$\Delta \mathbf{f}_t^{PLS}$	$\Delta \mathbf{f}_t^{PC}$
1	1	**1.0357***	**1.0317***	**1.0161**	**1.0121**
	2	**1.0529***	**1.0258**	**1.0334***	**1.0264***
	3	**1.0261***	**1.0237**	**1.0166**	**1.0250**
12	1	0.9983	0.9980	0.9990	0.9984
	2	0.9969	0.9988	0.9941	0.9943
	3	0.9973	0.9930	0.9938	0.9947
24	1	**1.0028***	**1.0026***	0.9975	0.9974
	2	**1.0019***	0.9994	0.9968	0.9994
	3	0.9871	**1.0040***	0.9788	**1.0005***
36	1	**1.0257***	**1.0244***	0.9967	0.9959
	2	**1.0194***	**1.0281***	0.9872	0.9944
	3	**1.0157***	**1.0231***	0.9781	**1.0313***

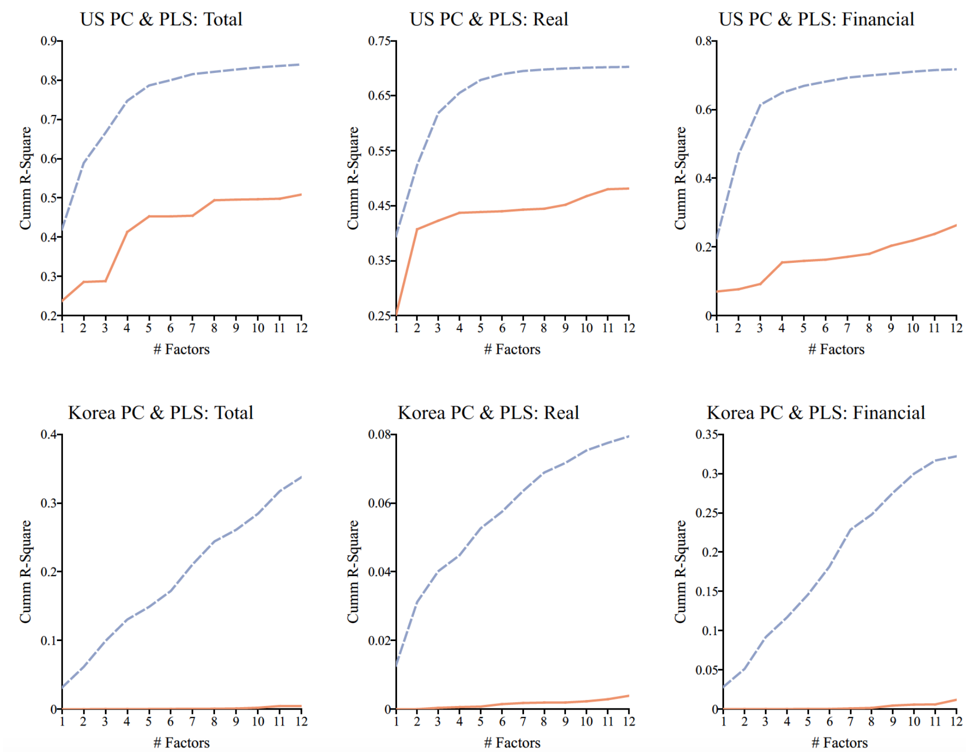
Note: We report the *RRMSPE* statistics employing a rolling window scheme with a 50% sample split point. *RRMSPE* denotes the ratio of the root mean squared prediction errors, which is the mean squared prediction error (*RMSPE*) from the benchmark random walk (RW) model divided by the *RMSPE* from each competing model with k factors. *RRMSPE* statistics in bold denote that the competing model outperforms the benchmark RW model. $*$ denotes that the competing model outperforms the benchmark AR model as well as the RW model.

Table 10. RIRP Based Models for the Real Exchange Rate

j	#Factors	*American Factors* $\Delta\mathbf{f}_t^{PLS}$	$\Delta\mathbf{f}_t^{PC}$	*Korean Factors* $\Delta\mathbf{f}_t^{PLS}$	$\Delta\mathbf{f}_t^{PC}$
1	1	**1.0057**	**1.0066**	0.9833	0.9792
	2	**1.0042**	0.9882	0.9742	0.9836
	3	0.9924	0.9754	0.9697	0.9631
12	1	**1.2000**	**1.2311***	**1.2370***	**1.2505***
	2	**1.1999**	**1.2290***	**1.2233**	**1.2444***
	3	**1.1701**	**1.2450***	**1.1503**	**1.2495***
24	1	**1.5022**	**1.4930**	**1.5172***	**1.5287***
	2	**1.4630**	**1.4182**	**1.5192***	**1.5539***
	3	**1.4098**	**1.5244***	**1.4452**	**1.3247**
36	1	**1.4415***	**1.3576**	**1.3535**	**1.3523**
	2	**1.3619**	**1.3440**	**1.3451**	**1.1982**
	3	**1.2887**	**1.3696**	**1.1703**	**1.0686**

Note: We report the *RRMSPE* statistics employing a rolling window scheme with a 50% sample split point. *RRMSPE* denotes the ratio of the root mean squared prediction errors, which is the mean squared prediction error (*RMSPE*) from the benchmark random walk (RW) model divided by the *RMSPE* from each competing model with k factors. *RRMSPE* statistics in bold denote that the competing model outperforms the benchmark RW model. $*$ denotes that the competing model outperforms the benchmark AR model as well as the RW model.

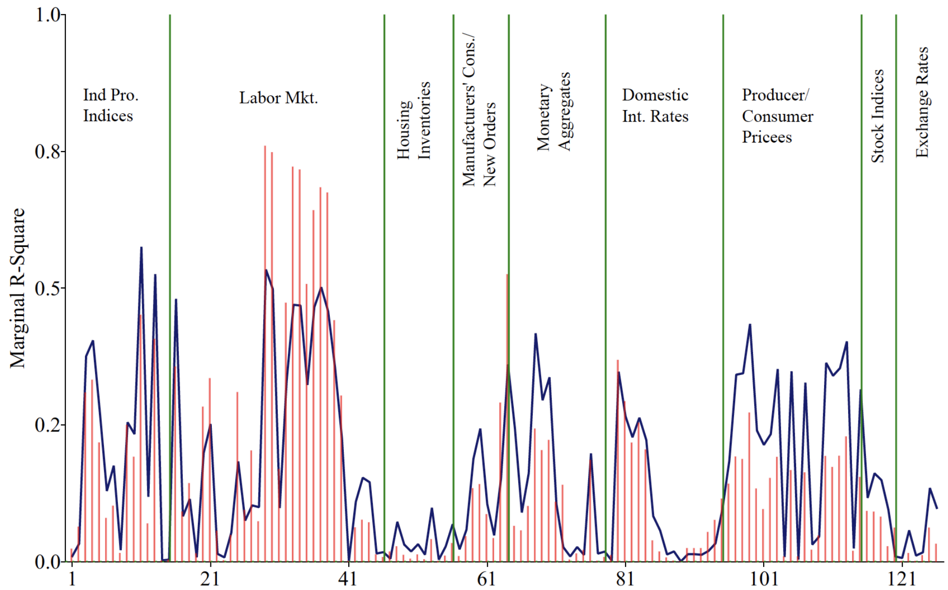
Figure 1. Cumulative R^2 Analysis



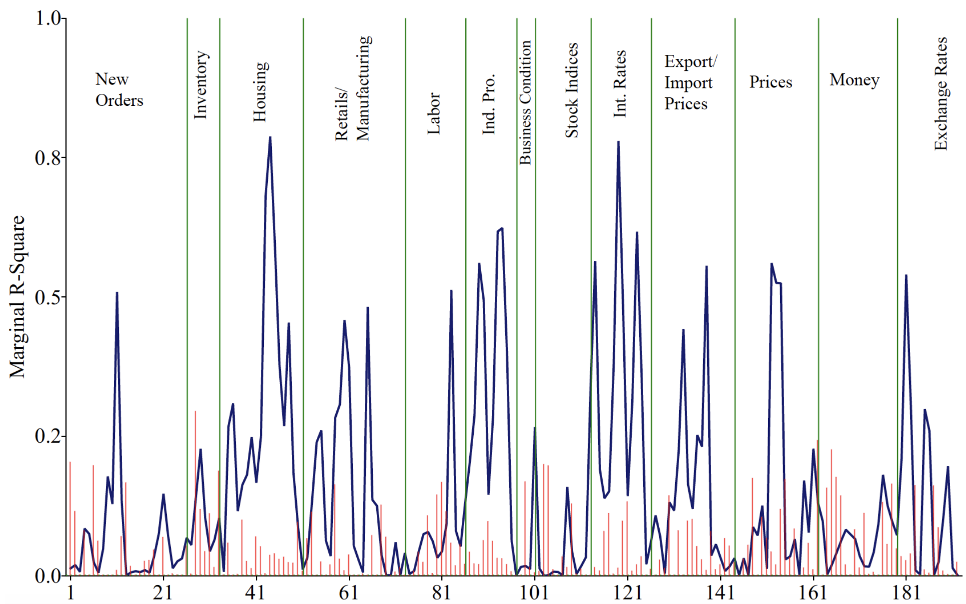
Note: We regress the real exchange rate on each factor and obtain the R^2 statistics. Since we use orthogonalized factors, we report the cumulative R^2 statistics.Dotted lines are for PLS factors and solid lines are for PC factors.

Figure 2. Marginal R^2 Analysis: Total Factors

American Factors



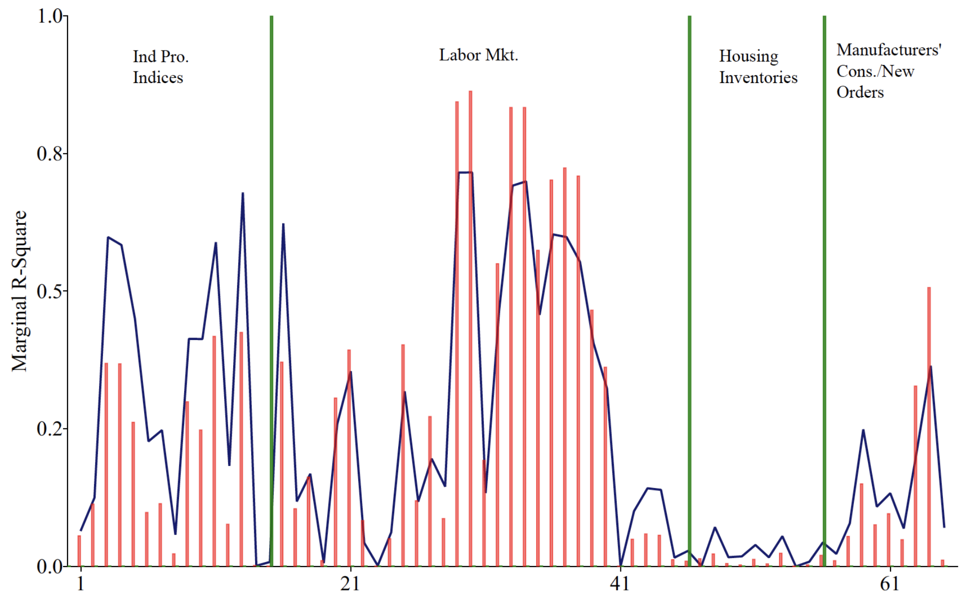
Korean Factors



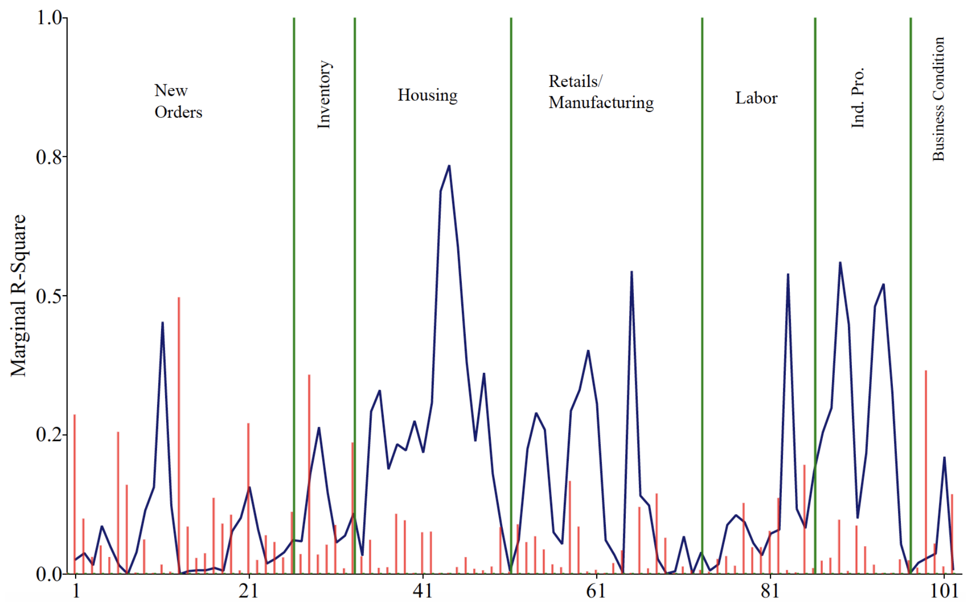
Note: We report the R^2 statistics that were obtained by regressing each predictor on the common factor estimate. That is, the horizontal axis is the predictor IDs. Solid lines are for PC factors, while bar graphs are for PLS factors.

Figure 3. Marginal R^2 Analysis: Real Activity Factors

American Factors



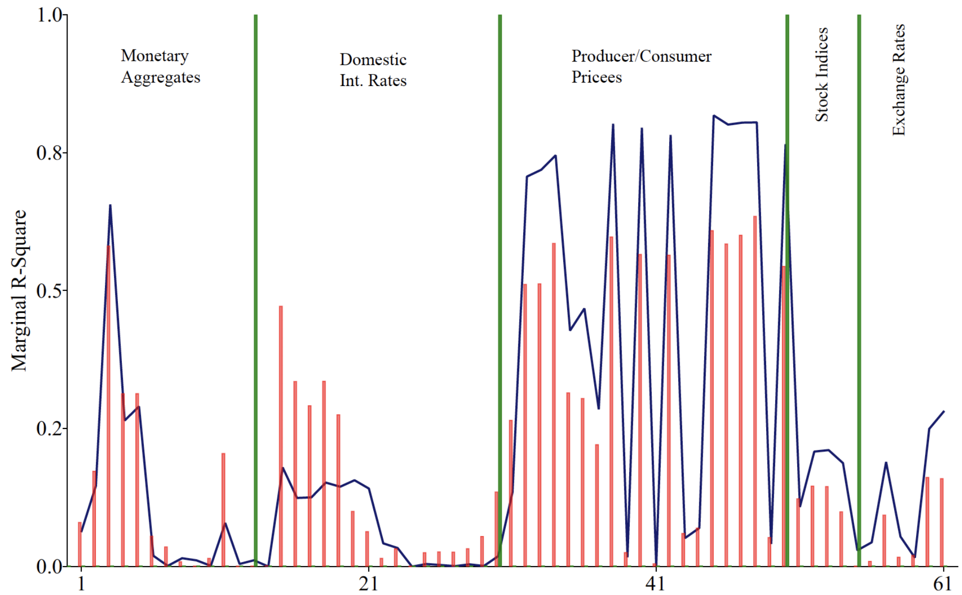
Korean Factors



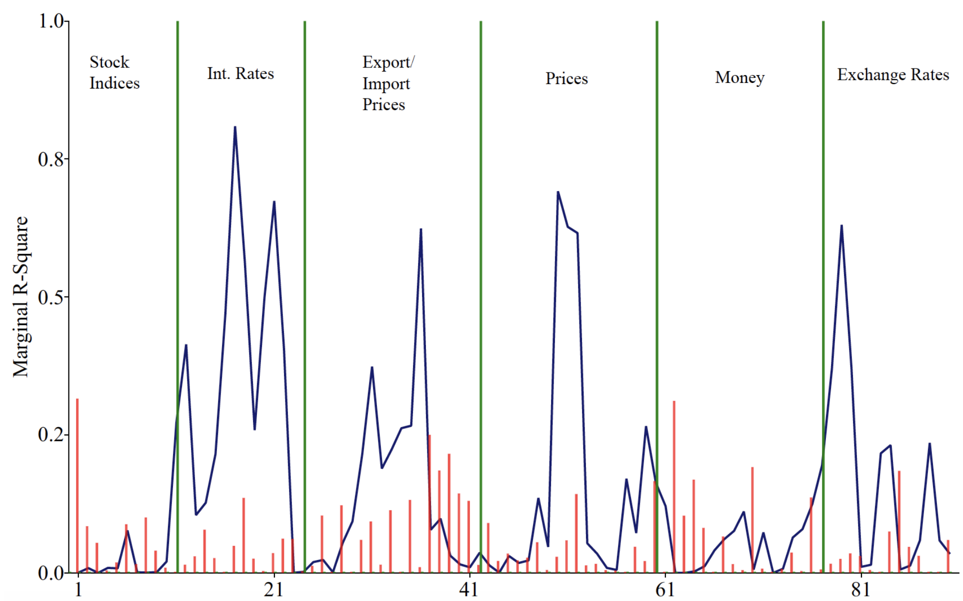
Note: We report the R^2 statistics that were obtained by regressing each predictor on the common factor estimate. That is, the horizontal axis is the predictor IDs. Solid lines are for PC factors, while bar graphs are for PLS factors.

Figure 4. Marginal R^2 Analysis: Nominal/Financial Factors

American Factors



Korean Factors



Note: We report the R^2 statistics that were obtained by regressing each predictor on the common factor estimate. That is, the horizontal axis is the predictor IDs. Solid lines are for PC factors, while bar graphs are for PLS factors.