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Abstract

We propose factor-based out of sample forecasting models for US dollar real exchange rates. We estimate latent common factors employing an array of data dimensionality reduction approaches that include the Principal Component Analysis, Partial Least Squares, and the LASSO for a large panel of 125 monthly frequency US macroeconomic time series data. We augment two benchmark models, a stationary autoregressive model and the random walk model, with estimated common factors to formulate out-of-sample forecasts of the real exchange rate. Empirical findings demonstrate that our factor augmented models outperform the benchmark models at longer horizons when factors are extracted from real activity variables excluding financial sector variables. Factors obtained from financial market variables overall play a limited role in forecasting. Our data-driven models tend to perform better than models with international factors that are motivated by exchange rate determination theories.

Keywords: US Dollar Real Exchange Rate; Principal Component Analysis; Partial Least Squares; LASSO; Out-of-Sample Forecast

JEL Classification: C38; C53; C55; F31; G17

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1 Introduction

This paper presents factor augmented out-of-sample forecasting models for dollar real exchange rates in a data rich environment during the post-Bretton Woods system era. We demonstrate that our models outperform the benchmark models that are commonly used in the current literature at longer horizons, when factors are extracted from real activity variables excluding financial market predictors.

In their seminal paper, Meese and Rogoff (1983) demonstrate that the random walk (RW) model performs well in predicting the nominal exchange rate in comparison with models that utilize economic fundamental variables, which are motivated by exchange rate determination theories. Engel and Hamilton (1990), however, report the better predictability of their nonlinear models over the RW, although their findings are still at odds with uncovered interest parity (UIP). Cheung, Chinn, and Pascual (2005) update Meese and Rogoff's work, showing that exchange rate models still do not consistently outperform the RW model in out-of-sample forecasting at any forecast horizons. In relation to such a disconnect between the exchange rate and its macroeconomic fundamentals, Engel and West (2005) provide an interesting point that asset prices such as the exchange rate can behave similar to a RW process as the discount factor approaches one, even when those prices are still consistent with models of asset pricing.[1]

On the other hand, a number of studies reported evidence of greater predictability of exchange rate models over the RW model at longer horizons. Using regression models of multiple-period changes (long differencing) in the exchange rate on the deviation of the exchange rate from its fundamentals, Mark (1995) reports overall superior long-horizon predictability of fundamentals for the exchange rate. Chinn and Meese (1995) also show that exchange rate models can beat the RW at long horizons. Lothian and Taylor (1996) report good out-of-sample predictability of fundamentals utilizing simple stationary models of the real exchange rate using over two century-long annual frequency data. Groen (2005) reports that the monetary fundamentals-based *common* long-run model outperforms the RW model as well as standard cointegrated

[1] Many researchers provide panel evidence of monetary models that connect to exchange rate dynamics. See for example, Rapach and Wohar (2004), Groen (2000), and Mark and Sul (2001). Also, Mark (2009), Molodtsova, Nikolsko-Rzhevskyy, and Papell (2008), Engel and West (2006), and Clarida and Waldman (2008) explored empirical performance of the models based on Taylor-rule fundamentals.

vector autoregressive (VAR) model-based forecasts at 2 to 4 year horizons. Engel, Mark, and West (2008) also show that out-of-sample predictability can be enhanced by focusing on panel estimation and long-horizon forecasts.

There are quite a few researchers who show that Taylor Rule fundamentals provide useful predictive contents for the exchange rate. See among others, Engel, Mark, and West (2008), Molodtsova, Nikolsko-Rzhevskyy, and Papell (2008), Molodtsova and Papell (2009), Molodtsova and Papell (2013), and Ince, Molodtsova, and Papell (2016). In a related work, Wang and Wu (2012) show the superior out-of-sample interval predictability of the Taylor Rule fundamentals at longer horizons. See also Rossi (2013) for a survey of researches that demonstrates the importance of Taylor Rule fundamentals in understanding exchange rate dynamics.

More recently, a group of researchers started using a large panel of time series data for understanding exchange rate dynamics. Since the pioneering work of Stock and Watson (2002), there has been an influx of papers that use factor analysis to perform predictions of macroeconomic variables, utilizing latent common factors via principal components (PC) analysis. For instance, Engel, Mark, and West (2015) use cross-section information from PC factors that are obtained from a panel of 17 bilateral exchange rates vis-à-vis the US dollar, and show that factor based forecasting models often outperform the random walk model during the post-1999 sample period. They also show that the dollar factor combined with Purchasing Power Parity (PPP) factors perform well. Chen, Jackson, Kim, and Resiandini (2014) use PC to extract latent common factors from 50 commodity prices, motivated by the fact that world commodities are priced in dollars. Their first common factor turns out to be closely related with the dollar exchange rate. They show that the first common factor yields superior out-of-sample predictive contents for the dollar exchange rate.

Verdelhan (2018) proposes to use the two risk factors (dollar factor and carry factor) that are constructed from portfolios of international currencies for better understanding exchange rate dynamics. Greenaway-McGrevy, Mark, Sul, and Wu (2018) demonstrate that exchange rates are mainly driven by a dollar and an euro factor but not the carry factor. They show that the out-of-sample prediction performance from their dollar-euro factor model dominates the RW (no change) model. Ca' Zorzi, Kociecki, and Rubaszek (2015) use a structural Bayesian vector autoregression (SBVAR) and demonstrate that forecasting on the basis of real interest rate and convergence to PPP holds good in the medium horizons, although it is still difficult to beat the

RW model in the short-run.

Note that PC obtains common factors solely from a group of predictor variables. Boivin and Ng (2006), however, pointed out that the performance of the PC method may be poor in forecasting the target variable if useful predictive contents are in a certain factor that may be dominated by other factors. In order to address this issue, we employ the partial least squares (PLS) method which is proposed by Wold (1982). This method utilizes the covariance between the target and predictor variables to generate target-specific factors. See Kelly and Pruitt (2015) and Groen and Kapetanios (2016) for some comparisons between the PC and PLS approaches. Similar to Bai and Ng (2008) and Kelly and Pruitt (2015), we also use the Least Absolute Shrinkage and Selection Operator (LASSO) to select a target-specific *group* of variables among the full dataset to extract more relevant factors for the target.

This paper first estimates latent common factors for the dollar real exchange rate from a large panel U.S. macroeconomic time series data during the post-Bretton Woods system, then augment both the stationary and non-stationary benchmark forecasting models with the factor estimates. We employ an array of data dimensionality reduction approaches including aforementioned methods of PC and PLS as well as the LASSO in combination with PC and PLS. As shown by Nelson and Plosser (1982), most macroeconomic data are better approximated with a non-stationary stochastic process. Recognizing this, we apply these methods to first differenced predictors in order to obtain factor estimates consistently. See Bai and Ng (2004) for detailed explanations. In addition, we extract common factors from country-level data motivated by exchange rate determination theories including Purchasing Power Parity (PPP), Uncovered Interest Parity (UIP), and Real Uncovered Interest Parity (RIRP). We evaluate the out-of-sample predictability of our factor augmented forecasting models using the ratio of the root mean squared prediction error (RRMSPE) criteria.

Our major findings are as follows. First, stationary factor models perform better than the random walk based factor models, although conventional unit root tests tend to be in favor of non-stationarity of the real exchange rate.

Second, forecasting models augmented with real activity factors tend to outperform the models that utilize factors from all predictor variables at longer horizons. This is in line with the work of Boivin and Ng (2006) who demonstrate that more data are not necessarily useful. On the other hand, factors from financial variables provide overall very limited predictive contents.

Third, labor market variables are identified as the main source of superior out-of-sample predictive contents among real activity variables. Excluding labor variables, real factor models lose their predictability substantially. On the other hand, price variables seem to cause the poor performance of financial factors. Factors from non-price financial predictors substantially improve the prediction performance.

Fourth, the models with UIP factors perform well in the short-run horizons, while other models with PPP and RIRP factors overall perform poorly in comparison with our data-driven factor augmented forecasting models.

The rest of the paper is organized as follows. Section 2 describes our factor augmented forecasting models for the real exchange rate. We also describe our factor estimation strategies and the evaluation method via a fixed-size rolling window scheme. Section 3 presents data descriptions and some preliminary statistical analysis. Then we report our major empirical findings of the paper including the in-sample analysis and the evaluations of the out-of-sample (OOS) predictability of our factor-augmented forecasting models. In Section 4, we introduce alternative identification approaches based on proposition-based models, then report the OOS performance of the models in comparison with the data-driven factor forecasting models. Section 5 concludes.

2 The Empirical Model

2.1 Factor Models for Foreign Exchange Rates

We utilize an array of data dimensionality reduction approaches to extract latent common factors from a large panel of macroeconomic time series data that might contain useful predictive contents of the real exchange rate. In addition to the Principal Component (PC) method, we employ Partial Least Squares (PLS) that generates target specific common factors. We also utilize the Least Absolute Shrinkage and Selection Operator (LASSO) in combination with PC and PLS.

2.1.1 Principal Component Factors

We begin with PC that has been popularly employed in the current literature since the seminal work of Stock and Watson (2002).

Consider a panel of N macroeconomic $T \times 1$ time series predictors/variables, $\mathbf{x} = [\mathbf{x}_1, \mathbf{x}_2, ..., \mathbf{x}_N]$, where $\mathbf{x}_i = [x_{i,1}, x_{i,2}, ..., x_{i,T}]'$, $i = 1, ..., N$. Abstracting from deterministic terms, each predictor $\mathbf{x}_i$ is assumed to have the following factor structure,

$$x_{i,t} = \boldsymbol{\lambda}_i' \mathbf{f}_t^{PC} + \varepsilon_{i,t}, \tag{1}$$

where $\mathbf{f}_t = \left[f_{1,t}^{PC}, f_{2,t}^{PC}, \cdots, f_{R,t}^{PC} \right]'$ is an $R \times 1$ vector of *latent* time-varying common factors at time t. $\boldsymbol{\lambda}_i = [\lambda_{i,1}, \lambda_{i,2}, \cdots, \lambda_{i,R}]'$ denotes an $R \times 1$ vector of time-invariant but idiosyncratic factor loading coefficients for $\mathbf{x}_i$. $\varepsilon_{i,t}$ is the idiosyncratic error term.

As Nelson and Plosser (1982) demonstrated, most macroeconomic time series variables are better approximated by an integrated non-stationary stochastic process. Following Bai and Ng (2004), we estimate $\mathbf{f}_t$ and $\boldsymbol{\lambda}_i$ via the PC method for the first-differenced data, because the PC estimator of $\mathbf{f}_t$ would be inconsistent if $\varepsilon_{i,t}$ is an integrated process. For this, rewrite (1) as follows.

$$\Delta x_{i,t} = \boldsymbol{\lambda}_i' \Delta \mathbf{f}_t^{PC} + \Delta \varepsilon_{i,t} \tag{2}$$

for $t = 2, \cdots, T$. Since PC is not scale invariant, we first normalize the data, $\Delta\tilde{\mathbf{x}} = [\Delta\tilde{\mathbf{x}}_1, \Delta\tilde{\mathbf{x}}_2, ..., \Delta\tilde{\mathbf{x}}_N]$, then apply PC to $\Delta\tilde{\mathbf{x}}\Delta\tilde{\mathbf{x}}'$ to obtain the factor estimates $\Delta\hat{\mathbf{f}}_t^{PC}$ along with their associated factor loading coefficients $\hat{\boldsymbol{\lambda}}_i$. Estimates for the idiosyncratic components are naturally given by residuals $\Delta\hat{\varepsilon}_{i,t} = \Delta\tilde{x}_{i,t} - \hat{\boldsymbol{\lambda}}_i' \Delta\hat{\mathbf{f}}_t^{PC}$. Level variables are recovered via cumulative summation as follows.

$$\hat{\varepsilon}_{i,t} = \sum_{s=2}^{t} \Delta\hat{\varepsilon}_{i,s}, \;\; \hat{\mathbf{f}}_t^{PC} = \sum_{s=2}^{t} \Delta\hat{\mathbf{f}}_s^{PC} \tag{3}$$

Note that we obtain consistent factor estimates even when $\mathbf{x}$ includes some stationary variables. Assume that one of $\mathbf{x}$ is stationary, say, $\mathbf{x}_j$, $j \in \{1, ..., N\}$ is $I(0)$. Since $\Delta\mathbf{x}_j$ is still stationary, $I(-1)$, the PC estimator remains consistent. Alternatively, one may continue to difference the variables until the null of nonstationarity hypothesis is rejected via unit root test as is done to construct the Fred-MD.[2] However, this may not be practically useful, even though it is statistically more rigorous, when unit root tests provides contradicting statistical inferences as the test specification (i.e., number of lags, see) changes. See Cheung and Lai (1995) for related

[2] The Fred-MD is available at https://research.stlouisfed.org/econ/mccracken/fred-databases/.

discussions.

2.1.2 Partial Least Squares Factors

PLS for a scalar target variable q_t is motivated by the following linear regression model. Abstracting from deterministic terms,

$$q_t = \Delta\mathbf{x}_t^{'}\boldsymbol{\beta} + e_t, \tag{4}$$

where $\Delta\mathbf{x}_t = [\Delta x_{1,t}, \Delta x_{2,t}, ..., \Delta x_{N,t}]'$ is an $N \times 1$ vector of predictor variables at time $t = 1, ..., T$, while $\boldsymbol{\beta}$ is an $N \times 1$ vector of associated coefficients. e_t is an error term. Note that we again first-difference the predictor variables with an assumption that $\mathbf{x}_t$ is a vector of $I(1)$ variables..

PLS is especially useful for regression models that have many predictors, that is, when N is large. Rewrite (4) as follows,

$$\begin{aligned} q_t &= \Delta\mathbf{x}_t^{'}\mathbf{w}\boldsymbol{\theta} + u_t \\ &= \Delta\mathbf{f}_t^{PLS'}\boldsymbol{\theta} + u_t \end{aligned} \tag{5}$$

where $\Delta\mathbf{f}_t^{pls} = \left[\Delta f_{1,t}^{PLS}, \Delta f_{2,t}^{PLS}, ..., \Delta f_{R,t}^{PLS}\right]'$, $R < N$ is an $R \times 1$ vector of PLS factors. Note that the PLS factor is a linear combination of *all* predictor variables,

$$\Delta\mathbf{f}_t^{PLS} = \mathbf{w}^{'}\Delta\mathbf{x}_t, \tag{6}$$

where $\mathbf{w} = [\mathbf{w}_1, \mathbf{w}_2, ..., \mathbf{w}_R]$ is an $N \times R$ weighting matrix. That is, $\mathbf{w}_r = [w_{1,r}, w_{2,r}, ..., w_{N,r}]^{'}$, $r = 1, ..., R$, is an $N \times 1$ vector of weights on predictor variables for the r^{th} PLS factor, $\Delta f_{r,t}^{PLS}$. $\boldsymbol{\theta}$ is an $R \times 1$ vector of PLS regression coefficients. PLS regression minimizes the sum of squared residuals from the equation (5) for $\boldsymbol{\theta}$ instead of $\boldsymbol{\beta}$ in (4).

It should be noted that we do not utilize $\boldsymbol{\theta}$ for our out-of-sample forecasting exercises in the present paper. Instead, we augment the benchmark forecasting model with estimated PLS factors $\Delta\hat{\mathbf{f}}_t^{PLS}$ to make our models to be comparable with the PC-based factors introduced in the previous section.

Among available PLS algorithms, see Andersson (2009) for a brief survey, we use the one proposed by Helland (1990) that is more appealing intuitively to forecast a scalar target variable q_t as follows. First, $\Delta\hat{f}_{1,t}^{PLS}$ is pinned down by the linear combinations of the predictors in $\Delta \mathbf{x}_t$.

$$\Delta \hat{f}_{1,t}^{PLS} = \sum_{i=1}^{N} w_{i,1} \Delta x_{i,t}, \tag{7}$$

where the loading (weight) $w_{i,1}$ is given by $Cov(q_t, \Delta x_{i,t})$. Next, we regress q_t and $\Delta x_{i,t}$ on $\Delta \hat{f}_{1,t}^{PLS}$ then get residuals, $\tilde{q}_t$ and $\Delta \tilde{x}_{i,t}$, respectively, to remove the explained component by the first factor $\Delta \hat{f}_{1,t}^{PLS}$. The second factor estimate $\Delta \hat{f}_{2,t}^{PLS}$ is then obtained similarly as in (7) with $w_{i,2} = Cov(\tilde{q}_t, \Delta \tilde{x}_{i,t})$. We repeat until the R^{th} factor $\Delta \hat{f}_{R,t}^{PLS}$ is obtained.

2.1.3 Least Absolute Shrinkage and Selection Operator Factors

We employ a shrinkage and selection method for linear regression models, the LASSO, which is popularly used for sparse regression. Unlike ridge regression, the LASSO selects a subset ($\mathbf{x}^s$) of predictor variables from $\mathbf{x}$ by dropping the variables that are relatively less important in explaining the target variable. Putting it differently, we implement the *feature selection* task using the LASSO.

The LASSO puts a cap on the size of the estimated coefficients for the ordinary least squares (LS), and thereby drives the coefficient down to zero for some variables. Put it differently, it solves the following minimization problem of the sum of squared errors (as in the LS) subject to the constraint on the L_1-norm value of β.

$$\min_{\boldsymbol{\beta}} \left\{ \frac{1}{T} \sum_{t=1}^{T} (q_t - \Delta \mathbf{x}_t' \boldsymbol{\beta})^2 \right\}, \text{ s.t. } \sum_{j=1}^{N} |\beta_j| \leq \tau \tag{8}$$

where $\Delta \mathbf{x}_t = [\Delta x_{1,t}, \Delta x_{2,t}, ..., \Delta x_{N,t}]'$ is an $N \times 1$ vector of predictor variables at time $t = 1, ..., T$, $\boldsymbol{\beta}$ is an $N \times 1$ vector of associated coefficients. We again use the first-differenced predictor variables, assuming that $\mathbf{x}_t$ is a vector of $I(1)$ variables. The tuning parameter τ controls the overall strength of the penalty. As the value of τ decreases, the LASSO returns a smaller subset of $\mathbf{x}$, setting more coefficients to zero.

In what follows, we choose the value of τ that result in a certain number of predictors by applying the LASSO to $\Delta \mathbf{x}$. We then employ the PC or PLS approach to extract the common factors, $\Delta \mathbf{f}_t^{PC/L}$ or $\Delta \mathbf{f}_t^{PLS/L}$, out of the predictor variables that are chosen by the LASSO regression. Note that we use the LASSO approach only to obtain the subset of predictors that is closely related to the target. See Kelly and Pruitt (2015) for similar approaches.

2.2 Factor Augmented Forecasting Models

As we will demonstrate in the next section, real exchange rates tend to exhibit highly persistent dynamics that is virtually indistinguishable from the unit root process. Recognizing this observational equivalence issue, we augment both the stationary and non-stationary benchmark models by adding factor estimates to improve the out-of-sample predictability.

Our first benchmark model is motivated by the following *non-stationary* random walk process.

$$q_{t+1}^{BM_{RW}} = q_t + \eta_{t+1}, \tag{9}$$

where η_{t+1} is a white noise process. The j-period ahead forecast from this benchmark RW model is,

$$\widehat{q}_{t+j|t}^{BM_{RW}} = q_t \tag{10}$$

Abstracting from deterministic terms, the factor augmented random walk model is the following.

$$q_{t+j}^{F_{RW}} = q_t + \boldsymbol{\gamma}_j' \boldsymbol{\Delta} \hat{\mathbf{f}}_t + e_{t+j}, \; j = 1, 2, .., k, \tag{11}$$

where $\Delta \hat{\mathbf{f}}_t$ is a vector of factor estimates that are obtained via one of the methods explained in the previous sections. e_{t+j} in (11) is a partial sum of the white noise process η_t, that is, $e_{t+j} = \sum_{s=1}^{j} \eta_{t+s}$. Note that (11) nests the random walk (RW) process (9) when $\boldsymbol{\gamma}_j = \mathbf{0}$.[3]

Since the coefficient on q_t is fixed, we cannot use the unrestricted LS for (11). To resolve this problem, we first regress the long-differenced target variable $q_{t+j} - q_t$ on $\Delta \hat{\mathbf{f}}_t$ and obtain the consistent estimate $\hat{\boldsymbol{\gamma}}_j$.[4] Adding q_t back to the fitted value yields the following j-period ahead forecast for q_{t+j},

$$\widehat{q}_{t+j|t}^{F_{RW}} = q_t + \hat{\boldsymbol{\gamma}}_j' \Delta \hat{\mathbf{f}}_t \tag{12}$$

[3] Note that this specification is inconsistent with our earlier specification described in (4) that requires stationarity of the target variable q_t. Practically speaking, the random walk type models often perform well in forecasting persistent variables such as the real exchange rate. Furthermore, it is often difficult to distinguish highly persistent or near unit root variables from stationary variables (observational equivalence), leading us to the two mutually exclusive stochastic processes described in (11) and (13).

[4] That is, we assume that $q_{t+j} - q_t$ is stationary.

Our second factor forecast model is motivated by the following *stationary* AR(1)-type stochastic process.

$$q_{t+j}^{BM_{AR}} = \alpha_j q_t + u_{t+j},\ j = 1, 2, .., k, \tag{13}$$

where α_j is less than one in absolute value for stationarity. Note that we regress q_{t+j} *directly* on the current value q_t. Alternatively, one may employ a *recursive* forecasting approach with an AR(1) model, $q_{t+1} = \alpha q_t + \varepsilon_{t+1}$, which implies $\alpha_j = \hat{\alpha}^j$ under this approach. Under the direct forecasting approach (13), the j-period ahead forecast is,

$$\widehat{q}_{t+j|t}^{BM_{AR}} = \hat{\alpha}_j q_t, \tag{14}$$

where $\hat{\alpha}_j$ is the LS estimate of α_j.

Abstracting from deterministic terms, our factor augmented stationary AR(1)-type model is the following.

$$q_{t+j}^{F_{AR}} = \alpha_j q_t + \boldsymbol{\beta}_j^{'} \boldsymbol{\Delta}\hat{\mathbf{f}}_t + u_{t+j},\ j = 1, 2, .., k \tag{15}$$

Note that we again employ a *direct* forecasting approach by regressing the j-period ahead target variable (q_{t+j}) directly on the current period target variable (q_t) and the estimated factors ($\Delta\hat{\mathbf{f}}_t$). Note that (15) is an exact AR(1) process for $j = 1$ extended by the factor covariates $\Delta\hat{\mathbf{f}}_t$. We obtain the following j-period ahead forecast for the target variable,

$$\widehat{q}_{t+j|t}^{F_{AR}} = \hat{\alpha}_j q_t + \hat{\boldsymbol{\beta}}_j^{'} \Delta\hat{\mathbf{f}}_t, \tag{16}$$

where $\hat{\alpha}_j$ and $\hat{\boldsymbol{\beta}}_j$ are the least squares coefficient estimates. (15) nests the stationary benchmark model (13) when $\boldsymbol{\Delta}\hat{\mathbf{f}}_t$ does not contain any useful predictive contents for q_{t+j}, that is, $\boldsymbol{\beta}_j = 0$.

2.3 Evaluation Methods

We evaluate the out-of-sample predictability of our models using a fixed-size rolling window scheme as follows.[5]

We begin with estimating the first set of factors $\left\{\Delta\hat{\mathbf{f}}_t\right\}_{t=1}^{T_0}$ using the data di-

[5]Rolling window schemes tend to perform better than the recursive method in the presence of structural breaks. However, results with recursive approaches were qualitatively similar.

mensionality reduction methods for the initial $T_0 < T$ observations, $\{q_t, \Delta x_{i,t}\}_{t=1}^{T_0}$, $i = 1, 2, ..., N$. Then, we formulate the first forecast $\widehat{q}_{t+j|t}$ as explained in the previous section. Next, we add one observation but drop one earliest observation for the second round forecasting. That is, we re-estimate $\left\{\Delta\hat{\mathbf{f}}_t\right\}_{t=2}^{T_0+1}$ from $\{q_t, \Delta x_{i,t}\}_{t=2}^{T_0+1}$, $i = 1, 2, ..., N$, maintaining the same number of observations (T_0), which is used to formulate the second round forecast, q_{T_0+j+1}. We repeat until we forecast the last observation, q_T.

To evaluate the out-of-sample prediction accuracy of our factor augmented models, we use the ratio of the root mean square prediction error $(RRMSPE)$ defined as follows,

$$RRMSPE(j) = \frac{\sqrt{\frac{1}{T-T_0-j}\sum_{t=T_0+j}^{T}\left(\varepsilon_{t+j|t}^{BM}\right)^2}}{\sqrt{\frac{1}{T-T_0-j}\sum_{t=T_0+j}^{T}\left(\varepsilon_{t+j|t}^{F}\right)^2}}, \tag{17}$$

where

$$\varepsilon_{t+j|t}^{BM} = q_{t+j} - \widehat{q}_{t+j|t}^{BM},\ \varepsilon_{t+j|t}^{F} = q_{t+j} - \widehat{q}_{t+j|t}^{F} \tag{18}$$

Note that our factor models outperform the benchmark models when $RRMSPE$ is greater than 1.[6]

3 Empirical Findings

3.1 Data Descriptions

We use the real trade weighted dollar indices (major currency index, TWEXMPA; broad currency index, TWEXBPA), obtained from the Federal Reserve Economic Data (FRED). Observations are monthly frequency from January 1973 to December 2018, which correspond to the floating exchange rate regime followed by the collapse of the Bretton Woods system. As can be seen in Figure 1, these indices exhibit quite persistent dynamics, showing multiple long swings. For example, both real exchange rates began rising from around 1978 until they reached a peak in 1985. Then, the G5 nations signed the Plaza Accord, agreeing to depreciate US dollars

[6] Alternatively, one may employ the ratio of the root mean absolute prediction error ($RRMAPE$). That is, the loss function is defined with the absolute value instead of the squared value. $RRMAPE$ tends to perform more reliably in the presence of outliers. Results are overall qualitatively similar.

against the Japanese yen and the German Deutsche mark, which resulted in a 10-year long downward trend.

Figure 1 around here

We obtained 125 macroeconomic time series variables from the FRED-MD database for the same sample period. We log-transformed all quantity variables prior to estimations other than those in percent (e.g., interest rates and unemployment rates). These 125 variables are categorized into 9 groups. See Table 1. Group #1 includes 16 output and income variables, while 31 labor market variables belong to group #2. Groups #3 and #4 include various housing and manufacturers' consumption related variables. Group #5 has money and credit variables, while groups #6 and #7 include interest rates and price level variables, respectively. Groups #8 and #9 have the stock market and exchange rate variables, respectively. Note that groups #1 through #4 represent the real activity variables, while groups #5 through #9 are considered as financial sector variable groups in the US.

Table 1 around here

3.2 Some Preliminary Analysis

We perform an array of preliminary analysis including univariate and panel unit root tests (Table 2) and the persistence parameter estimations (Table 3) via a median unbiased estimator.

3.2.1 Unit Root Tests

We first employ the conventional univariate Augmented Dickey Fuller (ADF) test with an intercept for the two real exchange rates, the real major currency dollar index (q_t^M) and the broad dollar index (q_t^B). As can be seen in Table 2, the test fails to reject the null of nonstationarity, implying that these variables obey either a highly persistent stationary process or a unit root process that are not distinguishable with each other due to observational equivalence.

We also implemented a panel unit root test for $\mathbf{x}_t$, the Panel Analysis of Nonstationarity in Idiosyncratic and Common components (PANIC) analysis by Bai and Ng (2004), which tests the nonstationarity null hypothesis for common factors as well as de-factored idiosyncratic components of the data. Note that we also test the null hypothesis for the common factors from subsets of $\mathbf{x}_t$, real and financial sector variables separately, since we are particularly interested in evaluating the out-of-sample predictability of the common factors from these subsets of the data motivated by Boivin and Ng (2006). It should be noted that PLS factors from q_t^M are different from those from q_t^B, because PLS generates target specific factors, which is not the case for PC factors that are estimated solely from $\mathbf{x}_t$.

The PANIC fails to reject the null of nonstationarity for all common factor estimates at any conventional significance levels. Its panel test rejects the null hypothesis that states all variables are $I(1)$ processes for 6 out of 9 cases.[7] Note that nonstationarity of the common factor eventually dominates stationarity of de-factored series, confirming the nonstationarity of $\mathbf{x}_t$. Putting it differently, given the strong evidence of nonstationarity in common factors, the test results provide strong evidence in favor of nonstationarity in the predictor variables $\mathbf{x}_t$, which is consistent with Nelson and Plosser (1982).

Table 2 around here

3.2.2 Persistence of the Real Exchange Rate

As we discussed earlier, it is virtually impossible to distinguish highly persistent stationary processes from a unit root process (observational equivalence). To check this possibility in a deeper level, we obtained the median unbiased estimates of the persistence parameter of the real exchange rates employing the grid bootstrap procedure by Hansen (1999).

For this purpose, consider the following $AR(1)$ process for the real exchange rate q_t.[8]

$$q_{t+1} = \alpha q_t + \varepsilon_{t+1} \tag{19}$$

[7] Its alternative hypothesis is that there is at least one stationary variable.

[8] If q_t is of higher order $AR(p), p > 1$, process, we can obtain the approximately median unbiased estimator for α in the presence of nuisance parameters.

Define the following grid-t statistic at each fine grid point α over an interval $[\hat{\alpha} \pm 6 \times se(\hat{\alpha})]$ where $\hat{\alpha}$ is the least squares estimate of α and $se(\hat{\alpha})$ is its standard error.

$$t_T(\alpha) = \frac{\hat{\alpha} - \alpha}{se(\hat{\alpha})} \tag{20}$$

We implement $10,000$ nonparametric bootstrap simulations at 100 fine grid points, then obtain the (p quantile) grid-t bootstrap quantile functions, $Q^*_{T,p}(\alpha)$.[9] The median unbiased estimator is defined as,

$$\alpha_{MUE} = \alpha \in R, \text{ s.t. } t_T(\alpha) = Q^*_{n,50\%}(\alpha), \tag{21}$$

while the 95% grid-t confidence interval is defined by the following.

$$\left[\alpha \in R : Q^*_{T,2.5\%}(\alpha) \leq t_T(\alpha) \leq Q^*_{T,97.5\%}(\alpha)\right], \tag{22}$$

Results are reported in Table 3 and confirmed high degree persistence of the real exchange rates. Half-life point estimates were 5.730 and 6.612 years for the major and the broad real exchange rate index, respectively.[10] 95% confidence bands extended to positive infinity. These findings provide justifications that we employ both the stationary and non-stationary benchmark models.

Table 3 around here

3.3 Factor Model In-Sample Analysis

This section describes in-sample properties of the factor estimates we discussed in Section 2. The bottom panel of Figure 1 reports three first common factor estimates via PC, PLS for the major real exchange rate (q_t^M), and PLS for the broad currency real exchange rate (q_t^B). We present level factors that are visually more tractable, that is, $\hat{f}_{1,t} = \sum_{s=2}^{t} \Delta \hat{f}_{1,s}$. Note that PLS yields target-specific factors, so two PLS

[9] Each function is evaluated at each grid point α, not at the point estimate. If they are evaluated at the point estimate, the quantile functions correspond to the bootstrap-t quantile functions. See Efron and Tibshirani (1994).

[10] Half-life (HL) estimates are obtained by $\ln(0.5)/\ln(\hat{\alpha}_{MUE})$. HL estimates are annualized by muliplying it by 12 since the data is monthly frequency.

factors for each real exchange rate are presented in addition to the PC factor that is extracted only from the predictor variables.

As the Figure 1 shows, PLS factors demonstrate a strong degree co-movement with each other, reflecting similar dynamics of q_t^M and q_t^B. Although fluctuating somewhat differently, PLS factors also show strong correlations with the PC factor. See Table 4. For instance, $\Delta f_{1,t}^{PLS}(q_t^M)$ and $\Delta f_{1,t}^{PLS}(q_t^B)$ have 0.376 and 0.418 correlations with $\Delta f_{1,t}^{PC}$, respectively. The second common factors also exhibit 0.393 and 0.298 correlations, respectively. These results imply that real exchange rates share substantial common driving forces with macroeconomic predictors.

Table 4 around here

In Figure 2, we report the cumulative R^2 statistics of PC and PLS factors for up to 12 factors. By construction, the PLS factors provide a better in-sample fit than the PC factors as they utilize the covariance between the target and the predictor variables, while PC factors are extracted from the variance-covariance structure of predictors only. Unlike PC factors, the cumulative R^2 statistics of PLS factors overall exhibit a positive slope at a decreasing rate. That is, the first common factor explains more than the second one, which adds more explanatory power than the third one, and so forth. This is because our PLS algorithm sequentially estimates orthogonalized common factors using residuals of the target and predictors as explained earlier in Section 2. Since the PC method uses only the predictor variables without considering the target variable, additional R^2 values do not necessarily decrease. For both the exchange rates, the cumulative R^2 value with up to 12 PLS factors is about 0.42 (using all the variables), whereas that with PC factors is about 0.10 for both. The cumulative R^2 obtained from the PC factors extracted from financial variables is marginally better at about 0.13. In a nutshell, the PLS method yields superior in-sample performance in comparison with the PC method.

Figure 2 around here

To investigate the source of the common factor estimates, we employ the marginal R^2 analysis using the predictors and the common factor estimate. That is, we regress each predictor onto the common factor to see what proportion of the variation of each predictor can be explained by the common factor. Results are reported in Figures 3 to 5 for the first common factors.[11]

Note that the marginal R^2 statistics of the PC factor (line) are the same given the set of predictor variables since PC doesn't yield target specific factors. As can be seen in Figure 3, the first PC factor is overall closely related with many predictors, especially with Groups #1, 2, 5, and 7. The first PLS factors of q_t^M (top panel) and q_t^B (bottom panel) also exhibit similar patterns. On the other hand, the PC factor from the real activity variables seems to be driven mainly by the first two groups, #1 and #2 variables. See Figure 4. PLS factors, however, tend to be more closely related with labor market variables. The financial factor estimates reported in Figure 5 highlight the important role of producer/consumer prices (Group #7), although monetary aggregates (Group #5) also exhibit overall high R^2 values.

Figures 3 to 5 around here

3.3.1 Quantile Regression Analysis

We also employ quantile regression analysis with the first two common factor estimates. That is, we investigate the relationship between j-period ahead real exchange rates and the common factors using the conditional distribution function at different quantiles to supplement the least squares (LS) approach, which is based on the conditional mean function. The τ^{th} quantile regression coefficient estimator $\hat{\beta}_{\tau,j}$ for s_{t+j} is defined as follows.

$$\hat{\beta}_{\tau,j} = \underset{\beta}{\operatorname{argmin}} \frac{1}{T} \sum_{t=1}^{T} \rho_\tau \left(q_{t+j} - \beta \Delta \hat{f}_k \right), \ k = 1, 2 \tag{23}$$

where

$$\rho(u) = \begin{cases} -(1-\tau)u, & u < 0 \\ \tau u, & u \geqq 0 \end{cases}$$

Figure 6 reports $\hat{\beta}_{\tau,0}$ estimates of the PLS common factors for the contemporaneous major (q_t^B, first two rows) and the broad (q_t^B, last two rows) real exchange rates at

[11] See Appendix for the analysis with the second common factors.

the 5%, 25%, 50% (median), 75%, and 95% percentiles in addition to their associated standard errors that are obtained from 5,000 nonparametric bootstrap. Also, 90% LS nonparametric confidence intervals appear as shaded areas. Quantile regression estimates overall confirm the robustness of our LS approaches because most 90% confidence bands of the quantile and LS estimates overlap each other. $\hat{\beta}_{\tau,j}$ estimates of the PLS common factors for the 1-, 2-, and 3-year ahead real exchange rates are reported in Appendix. They exhibit similar patterns, concluding our LS-based analysis are overall robust to potential outliers.

Figure 6 around here

3.4 Out-of-Sample Prediction Performance

We implement out-of-sample forecast exercises using a fixed-size (50% split point) rolling window method using up to 4 (k) latent factor estimates.[12] We utilize PC, PLS and the LASSO for all 125 monthly frequency time series variables as well as the following two sub-groups: 65 real activity predictors (groups #1 through #4) and 60 financial sector variables (groups #5 through #9). In what follows, we demonstrate that our forecasting models outperform the benchmark models at longer horizons when combined with real activity factors, excluding financial factors, which is consistent with Boivin and Ng (2006).

We report the $RRMSPE$ statistics (17) with the random walk benchmark model for an array of factor augmented forecasting models in Tables 5 (major currencies real exchange rate, q_t^M) and 6 (broad currencies real exchange rate, q_t^B). Recall that our factor models perform better than the benchmark model when the $RRMSPE$ is greater than one.

As we can see in the top panels for the non-stationary model forecasts ($\widehat{q}_{t+j|t}^{F_{RW}}$), estimated common factors perform better than the benchmark RW model only in limited cases. That is, for q_t^M, the factor model forecast $\widehat{q}_{t+j|t}^{F_{RW}}$ was overall better than $\widehat{q}_{t+j|t}^{B_{RW}}$ when real activity PLS factors ($\Delta\mathbf{f}_t^{PLS,R}$) were used for 1 and 2 year-ahead forecasts. The PLS total factor ($\Delta\mathbf{f}_t^{PLS}$) models perform similarly, implying

[12]We obtained qualitatively similar results with a 70% sample split point.

that their predictive contents are mostly from $\Delta \mathbf{f}_t^{PLS,R}$. For q_t^B, non-stationary factor models overall perform poorly relative to the RW benchmark model.

It should be noted that most of our factor augmented stationary models ($\widehat{q}_{t+j|t}^{F_{AR}}$) outperform the RW model at 1, 2, 3-year forecast horizons (j). More importantly, our models perform the best when they are augmented with real activity factors, $\Delta \mathbf{f}_t^{PLS,R}$ and $\Delta \mathbf{f}_t^{PC,R}$, outperforming both RW and AR benchmark models (superscript *), in most cases for q_t^M when the forecast horizon is 1 year or longer. We obtained qualitatively similar results for q_t^B. Stationary factor augmented models with q_t^B outperform the RW model in the longer run, but they perform better than the AR model less frequently compared with the cases for q_t^M.

Tables 5 and 6 around here

We highlight the important role of real factors in out-of-sample predictions in comparison with financial market factors in Figures 7 and 8. For q_t^M, $\widehat{q}_{t+j|t}^{F_{AR}}$ formulated using $\Delta \mathbf{f}_t^{PLS,R}$ or $\Delta \mathbf{f}_t^{PC,R}$ mostly outperform both the AR and RW benchmark models when the forecast horizon is 1 year or longer. However, the $RRMSPE$ of stationary models ($\widehat{q}_{t+j|t}^{F_{AR}}$) augmented by financial factors, $\Delta \mathbf{f}_t^{PLS,F}$ or $\Delta \mathbf{f}_t^{PC,F}$, is less than the one of the AR model in all cases, although they often outperform the RW model. We observe similar findings for q_t^B, which implies greater predictive contents of real factors in comparison with those of financial factors, although our factor models outperform the benchmark AR model less frequently. These findings clearly demonstrate that more data are not always better, which is consistent with the work of Boivin and Ng (2006). Putting it differently, real activity variables/factors contain important longer run predictive contents for the real exchange rate, whereas financial variables/factors are likely to add noise instead of predictive contents in the longer run.

Figures 7 and 8 around here

We also employ the LASSO to obtain the subsets of the predictor variables that are useful to explain the target variable. The idea behind this is to estimate the factors using fewer but more informative predictor variables. See Bai and Ng (2008). Following Kelly and Pruitt (2015), we adjust the tuning parameter τ in (8) to select a group of 30 predictors from all 125 time series variables, while 20 predictors were chosen from each of the real activity and the financial variables groups. Then, we employed PLS and PC to estimate up to 4 common factors that are used to augment the benchmark RW and AR models. We report results in Tables 7 (q_t^M) and 8 (q_t^B).

We obtain qualitatively similar results as those from pure PLS or PC models. LASSO-based *non-stationary* models in combination with PLS or PC perform better than previous results in Tables 5 and 6, although overall performance is weak in comparison with the benchmark models. On the other hand, LASSO/PLS and LASSO/PC *stationary* models again outperform the benchmark RW and AR models when factors were estimated from real activity variables. That is, $\Delta\mathbf{f}_t^{PLS/L,R}$ and $\Delta\mathbf{f}_t^{PC/L,R}$ seem to contain useful predictive contents of the real exchange rate at longer forecast horizons. Again, $\widehat{q}_{t+j|t}^{F_{AR}}$ with financial factors, $\Delta\mathbf{f}_t^{PLS/L,F}$ and $\Delta\mathbf{f}_t^{PC/L,F}$, perform overall poorly in comparison with the benchmark models. Qualitatively similar but a little weaker out-of-sample predictability of our factor models was observed for q_t^B.

Our exercises imply stronger out-of-sample predictability of our factor models for q_t^M relative to that of q_t^B. We believe this is due to the fact we estimate common factors using a large panel of US macroeconomic data. q_t^M is constructed using mostly developed countries' currencies such as the euro, the British pound, and the Japanese yen, whereas q_t^B uses exchange rates and their associated trade weights of many countries including developing countries such as China, India, and Thailand. If underlying driving forces of the American economy are more closely connected with those of developed economies rather than developing countries, factor estimates obtained from US economic data would contain greater predictive contents for q_t^M, because more information on developing economies would be needed to better forecast q_t^B.

Tables 7 and 8 around here

Our forecasting exercises in this section clearly demonstrate that real activity factors contain useful long-run predictive contents for the real exchange rate. Among real activity predictors (Groups 1 to 4, 65 variables), Figure 4 shows, via marginal R^2 analysis, that PLS/PC real factors are closely related with labor market predictors

(Group 2). Motivated by this finding, we further investigate the source of the predictive contents for the real exchange rate by utilizing labor market related factors that are estimated from 30 labor market variables. Results are reported in Table 9. Since non-stationary models perform relatively poorly, we report results only from stationary models.

We further provide visual evidence of such findings. Figure 7 reports the $RRMSPE$ statistics of real factor ($\Delta\mathbf{f}_t^{PC,R}$, $\Delta\mathbf{f}_t^{PLS,R}$) augmented models for q_t^M in comparison with those of financial factor ($\Delta\mathbf{f}_t^{PC,F}$, $\Delta\mathbf{f}_t^{PLS,F}$) augmented ones. For $j \geq 12$, the former mostly outperforms both the AR and RW models, whereas the latter is overall dominated by the AR model, although they are still doing fairly well relative to the RW model. Figure 8 provides qualitatively similar results for q_t^B, although a little weaker performance of real factor models.

It is interesting to see stationary models ($\widehat{q}_{t+j|t}^{F_{AR}}$) with labor factors outperform (first two columns of Table 9) the RW model for both exchange rates when the forecast horizon is over 1 year. $\widehat{q}_{t+j|t}^{F_{AR}}$ perform better than the AR model especially for most 1-year ahead or longer forecasts for q_t^M. For q_t^B, labor factors that are estimated via PC perform better than PLS factor augmented models.

To see whether labor variables are the main source of predictive contents among all real activity variables, we implement similar exercises using real variables excluding labor variables, and report the results in the next two columns. We notice that out-of-sample prediction performances are getting worse when factors from non-labor real variables are used. We observe that the one-period ahead predictability somewhat improves but $\widehat{q}_{t+j|t}^{F_{AR}}$ outperforms only the RW model but was dominated by the AR model for all cases of $j \geq 12$. The last two columns report the cases when all predictors but labor market variables are used, which result in poor out-of-sample predictability. All together, these findings imply that the labor market contains substantial predictive contents for the real exchange rate.

Table 9 around here

We also present visual evidence of these findings. As we can see in Figure 9 for the major currency real exchange rate, labor factor augmented models outperform the forecasting models (PLSAR-L, PCAR-L) with non-labor real activity factors

(PLSAR-NL, PCAR-NL). We obtain qualitatively similar but a little weaker performance of the former relative to the latter for the broad currency real exchange rate. See Figure 10.

Figures 9 and 10 around here

As seen in previous sections, our factor models perform poorly if augmented by financial market factors. Marginal R^2 analysis in Figure 5 shows that the first financial common factor is closely related with consumer/producer prices. This motivates us to investigate whether price factors behave as noise in out-of-sample forecasts for the real exchange rate. For this purpose, we estimate the factors only from price variables and implement similar forecasting exercises with those factors. Results are shown in Table 10.

Price factor augmented models (first two columns) perform quite poorly even relative to the RW model, while financial factors excluding price variables outperform both the RW and AR models frequently especially when factors were estimated via PLS. The last two columns show overall prediction performances can improve when price variables are excluded.

Table 10 around here

Figures 11 and 12 clearly demonstrate these interesting findings. Note that the $RRMSPE$ statistics stays mostly below 1, indicating that price factor models are dominated by the RW model, when price factors were used. Excluding prices, however, financial factor augmented models dramatically improve the forecasting performances when $j \geq 12$, indicating that poor performances of financial factor models are mostly due to price variables.

Figures 11 and 12 around here

4 Alternative Identifications

4.1 Proposition Based Global Factors

This section implements out-of-sample forecast exercises, utilizing global factors that are motivated by existing propositions in the current open economy macroeconomics literature. Via PC and PLS for international data, we extract common factors from the predictors based on Purchasing Power Parity (PPP) and Uncovered Interest Parity (UIP) as well as Real Interest Rate Parity (RIRP), then implement out-of-sample prediction exercises with these parsimoniously estimated factors.

4.1.1 Purchasing Power Parity: Relative Price Factors

The log real exchange rate is defined as follows.

$$q_t = s_t + relp_t, \tag{24}$$

where s_t denotes the log nominal (bilateral) exchange rate as the foreign currency price of 1 US dollar, $relp_t$ is the log relative price $(p_t - p_t^*)$, and p_t and p_t^* are the log prices in the US and in the foreign country, respectively. Under PPP, q_t obeys a stationary $I(0)$ stochastic process, which is consistent with our stationary models, while s_t and the relative price $(p_t - p_t^*)$ obey a non-stationary $I(1)$ process. That is, s_t and $(p_t - p_t^*)$ are cointegrated with a known cointegrating vector $[1, \ -1]$.

We obtained the Consumer Price Index (CPI) of 43 countries from the IFS database including 18 euro-zone countries. Assuming the US as the home country, we constructed 42 relative prices $(p_t - p_t^*)$ with different sample periods. We estimated the first common factor from the following two groups of relative prices after taking the first difference $(\Delta p_t - \Delta p_t^*)$ or $(\pi_t - \pi_t^*)$, that is, inflation differences, since the relative price is an integrated process $I(1)$.

$\Delta f_t^{P,D}$ denotes the common factor obtained by applying PLS and PC to a panel of 18 developed countries' relative prices from January 1973 to December 2018.[13] We do not use the nominal exchange rate s_t as a predictor variable, because 11 out of

[13] For the group of developed countries, we include 11 euro-zone countries (Austria, Belgium, Finland, France, Germany, Greece, Italy, Luxembourg, Netherlands, Portugal, Spain) and 8 non-euro-zone countries (Canada, Denmark, Japan, Singapore, Switzerland, Sweden, United Kingdom, United States). All data are obtained from the IFS database with an exception of Singapore. We obtained the Singapore CPI from the Department of Statistics of Singapore.

19 developed countries are euro-zone countries, resulting in only 9 nominal exchange rates that makes it difficult to apply factor analysis. $\Delta f_t^{P,F}$ is the common factor from all 42 prices relative to the US from the same sample period. Since the data is not a balanced panel, we obtained the single common factor by taking the cross-section average (CSA).[14]

We report j-period ahead out-of-sample predictability exercise results using a single factor for each model with a 50% split point in Table 11. Again, non-stationary models ($\widehat{q}_{t+j|t}^{F_{RW}}$) mostly perform worse than the RW model, while stationary models ($\widehat{q}_{t+j|t}^{F_{AR}}$) outperform the RW benchmark model in most cases for both real exchange rates, q_t^M and q_t^B, whereas . However, the factor models perform better than the benchmark AR model only for 3 and 1 out of 12 cases for q_t^M and q_t^B, respectively. That is, our parsimonious factor models with PPP-based factors perform overall poorly in comparison with the models with real activity factors, although they still perform better than the RW model.

Table 11 around here

4.1.2 Uncovered Interest Parity: Interest Rate Spread

We also utilize factors extracted from up to 17 international short-run interest rate spread relative to the US interest rate, motivated by the two propositions: Uncovered Interest Parity (UIP) and Real Interest Rate Parity (RIRP).[15] Abstracting from risk premium, UIP states the following.

$$\Delta s_{t+1} = i_t^* - i_t + \varepsilon_{t+1}, \tag{25}$$

where Δs_{t+1} is the appreciation (depreciation) rate of the home (foreign) currency, while i_t and i_t^* are nominal short-run interest rates in the home and foreign countries,

[14] In addition to the group of 19 developed countries, we added 7 the rest of euro-zone countries (Cyprus, Estonia, Ireland, Latvia, Lithuania, Slovakia, Slovenia) except Malta, and 17 non-euro-zone countries (Brazil, China, Chile, Colombia, Czech Republic, Hong Kong, Hungary, India, Indonesia, Israel, Korea, Malaysia, Mexico, Poland, Romania, Russia, Saudi Arabia).

[15] For the group of developed countries, we include 11 euro-zone countries (Austria, Belgium, Finland, France, Germany, Greece, Italy, Luxembourg, Netherlands, Portugal, Spain) and 7 non-euro-zone countries (Canada, Denmark, Japan, Switzerland, Sweden, United Kingdom, United States). All data are obtained from the OECD database and the FRED.

respectively. $\varepsilon_{t+1} = \Delta s_{t+1} - E_t \Delta s_{t+1}$ is the mean-zero rational expectation error term, that is, $E_t \varepsilon_{t+1} = 0$.

We obtained the short-term interest rate data from the FRED and the OECD database for the same group of countries in the previous section for PPP. We constructed the nominal interest rate spread by subtracting the US interest rate (i_t) from the national interest rate (i_t^*). Ex post real interest rates were obtained by subtracting one-period ahead CPI-based inflation rate from the nominal interest rate, then were used to produce real interest rate spreads, $r_t^* - r_t$.

Table 12 reports j-period ahead out-of-sample predictability exercise results using a single factor that is motivated by UIP. We estimate $\Delta f_t^{U,F}$ by taking the cross-section average of the unbalanced panel of all 17 countries' nominal interest rate spreads relative to the US, $\Delta(i_t^* - i_t)$, from January 1973 to December 2018, while $\Delta f_t^{U,S}$ was estimated via PLS and PC for the balanced panel of 16 interest rate spreads of developed countries relative to the US from August 1985 to December 2018.

Note that we use first differenced interest rate spreads, which may be inconsistent with (25) that implies stationarity of the interest rate spread. As a preliminary test, we implemented the PANIC test for panels of $(i_t^* - i_t)$ data, which provided strong evidence of nonstationarity. The p values of the first two PLS common factors of the $i_t^* - i_t$ for q_t^M were 0.165 and 0.197, respectively. The p values of the first two PLS common factors for q_t^B were 0.062 and 0.294, respectively.[16] That is, the PANIC test fails to reject the null of nonstationarity at the 5% significance level for all cases. The p values of the first two PC common factors were 0.071 and 0.021, respectively, implying the nonstationarity of the panel was caused by the integrated first common factor. Therefore, the PANIC test overall provide empirical evidence in favor of nonstationarity. Based on these observations, we implemented out-of-sample forecasting exercises with factors from $\Delta(i_t^* - i_t)$ to make sure of the consistency of our factor estimates.

Our stationary models ($\widehat{q}_{t+j|t}^{F_{AR}}$) outperform again the RW benchmark model in all cases for both q_t^M and q_t^B, whereas non-stationary models ($\widehat{q}_{t+j|t}^{F_{RW}}$) overall perform worse than the RW model. The factor models perform better than the benchmark AR model only for 7 out of 12 and 8 out of 12 cases for q_t^M and q_t^B, respectively. That is, our models with UIP-based factors perform overall poorly in comparison with the models with real activity factors at longer horizons, although they perform better

[16] We don't consider the possibility that the two common factors are cointegrated.

than the RW model. Interestingly, UIP models perform better than real activity models in addition to both benchmark models at 1-period horizon for 10 out of 12 cases.

Table 12 around here

4.1.3 Real Interest Rate Parity: Real Interest Rate Spread

Assuming PPP holds, take the first difference to (24) at time $t+1$,

$$\Delta q_{t+1} = \Delta s_{t+1} + \pi_{t+1} - \pi^*_{t+1} \tag{26}$$

Combining (25) and (26), we obtain the following expression for Real Interest Rate Parity (RIRP).

$$\Delta q_{t+1} = r^*_t - r_t + \varepsilon_{t+1}, \tag{27}$$

where $r_t = i_t - \pi_{t+1}$ and $r^*_t = i^*_t - \pi^*_{t+1}$ are the *ex post* real interest rates in the home and foreign country, respectively.

Table 13 reports j-period ahead out-of-sample predictability exercise results using a single factor, motivated by RIRP. We extract $\Delta f^{R,F}_t$ by taking the cross-section average of the unbalanced panel of all 17 countries' real interest rate spreads relative to the US, $(r^*_t - r_t)$, from February 1973 to December 2018, while $\Delta f^{R,S}_t$ was estimated via applying PLS and PC to the balance panel of real interest rate spreads of 16 developed countries relative to the US from August 1985 to December 2018.

Note that, unlike the UIP factors, we estimate the common factors from $(r^*_t - r_t)$ without taking differences. This is because we obtained very strong evidence in favor of stationarity for real interest rate spreads. The PANIC rejects the null of nonstationarity for the first two PLS/PC common factors of the $r^*_t - r_t$ for both real exchange rates at the 1% significance level. And the panel test for de-factored idiosyncratic components provided strong evidence of stationarity at any conventional significance level.

Our factor augmented forecasting models perform poorly even compared with the RW benchmark model when combined with RIRP factors. Furthermore, the AR benchmark model outperforms all RIRP-based factor models.

Table 13 around here

5 Concluding Remarks

In this paper, we propose parsimonious factor-augmented forecasting models for the dollar real exchange rate in a data rich environment. We apply an array of data dimensionality reduction methods to a large panel of 125 monthly frequency macroeconomic variables from January 1973 to December 2018 to extract latent common factors. In addition to the Principal Component (PC) analysis, we employ the Partial Least Squares (PLS) approach that are largely overlooked in the current literature. We also employ the Least Absolute Shrinkage and Selection Operator (LASSO) combined with PC and PLS. We augment the random walk (RW) benchmark and a stationary autoregressive (AR) type benchmark model with estimated common factors to examine if these factors provide additional predictive contents for the real exchange rate.

We implemented an array of out-of-sample prediction exercises using a fixed-size rolling window scheme for 1-month to 3-year forecast horizons, then evaluated our proposed factor-augmented forecasting models relative to the two benchmark models via the ratio of the root mean squared prediction error ($RRMSPE$). Our stationary forecasting models outperform the RW benchmark consistently when the forecast horizon is 1-year or longer. In particular, our models that utilize real activity factors overall outperform both the RW and the AR benchmark when the forecast horizon is 1-year or longer. Factors obtained from financial/nominal predictor variables fail to contribute to out-of-sample predictability. These findings are in line with the work of Boivin and Ng (2006) who demonstrated the importance of relevant common factors for the target variable.

We also implement forecasting exercises using the factors that are motivated by exchange rate determination theories. Using up to 43 country-level data for prices and interest rates, we extract common factors based on Purchasing Power Parity (PPP), Uncovered Interest Parity (UIP), and Real Uncovered Interest Parity (RIRP) for out-of-sample forecasting exercises. Forecasting models with UIP common factors turn out to perform well in the short-run, while the models with either PPP or RIRP factors perform overall poorly. That is, these proposition-based factors fail to yield greater predictive contents than the data-driven real factors at longer horizons.

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Table 1. Macroeconomic Data Descriptions

Group ID	Data ID	Data Description
#1	1-16	Industrial Production Indices
#2	17-47	Labor Market Variables
#3	48-57	Housing Inventories
#4	58-65	Manufacturers' Consumption/ New Orders
#5	66-79	Monetary Aggregates
#6	80-96	Domestic Interest Rates
#7	97-116	Producer/Consumer Prices
#8	117-121	Stock Indices
#9	122-125	Bilateral Exchange Rates

Note: We obtained all data from the FRED-MD website, https://research.stlouisfed.org/econ/mccracken/fred-databases/.

Table 2. Unit Root Test Results

			ADF Test			
	q_t^M	-2.131 (0.232)	q_t^B	-2.027 (0.275)		
			PANIC Test			
	q_t^M		q_t^B		$\mathbf{x}_t$	
All variables	$f_{1,t}^{PLS}$	-1.435 (0.569)	$f_{1,t}^{PLS}$	-1.200 (0.674)	$f_{1,t}^{PC}$	-1.274 (0.641)
	$f_{2,t}^{PLS}$	-0.779 (0.827)	$f_{2,t}^{PLS}$	-0.873 (0.795)	$f_{2,t}^{PC}$	-0.626 (0.868)
	$P_{\hat{e}}$	$3.605^{\ddagger}$ (0.000)	$P_{\hat{e}}$	$2.062^{\dagger}$ (0.020)	$P_{\hat{e}}$	$5.982^{\ddagger}$ (0.000)
Real Variables	$f_{1,t}^{PLS,R}$	-1.275 (0.641)	$f_{1,t}^{PLS,R}$	-1.591 (0.488)	$f_{1,t}^{PC,R}$	-1.938 (0.302)
	$f_{2,t}^{PLS,R}$	-1.746 (0.399)	$f_{2,t}^{PLS,R}$	-1.171 (0.690)	$f_{2,t}^{PC,R}$	-1.098 (0.722)
	$P_{\hat{e}}$	$4.809^{\ddagger}$ (0.000)	$P_{\hat{e}}$	$6.911^{\ddagger}$ (0.000)	$P_{\hat{e}}$	1.094 (0.137)
Financial Variables	$f_{1,t}^{PLS,F}$	-1.617 (0.472)	$f_{1,t}^{PLS,F}$	-0.975 (0.763)	$f_{1,t}^{PC,F}$	-0.524 (0.892)
	$f_{2,t}^{PLS,F}$	-0.740 (0.835)	$f_{2,t}^{PLS,F}$	-0.804 (0.819)	$f_{2,t}^{PC,F}$	-1.747 (0.399)
	$P_{\hat{e}}$	-0.056 (0.522)	$P_{\hat{e}}$	0.048 (0.481)	$P_{\hat{e}}$	$2.324^{\ddagger}$ (0.010)

Note: q_t^M and q_t^B are the real trade weighted US dollar index with major and broad currencies, respectively. PLS produces target specific factors for q_t^M and q_t^Bseparately, while PC yields the same common factors independent on the target variable. Real variables are from group #1 through #4, while financial variables include group #5 through #9. The augmented Dickey-Fuller (ADF) test reports the ADF t-statitics when an intercept is included. P-values are in parenthesis. For the PANIC test results, we report the ADF t-statistics with an intercept for each common factor estimate. $P_{\hat{e}}$ denotes the panel test statistics from the de-factored idiosyncratic components. The ADF test fails to reject the null of nonstationarity even at the 10% significance level for all cases. The panel test rejects the null of nonstationarity at the 5% for 6 out of 9 cases. ‡ and † denote a rejection of the null hypothesis at the 1% and 5% level, respectively. Note, however, that the panel results do not provide any evidence of stationarity for $\mathbf{x}_t$ given strong evidence of nonstationarity of common factors.

Table 3. Median Unbiased Estimates of the Persistence Parameter

	β	C.I	HL	C.I
q_t^M	0.990	$[0.977, 1.002]$	5.730	$[2.444, \infty)$
q_t^B	0.991	$[0.980, 1.002]$	6.612	$[2.789, \infty)$

Note: q_t^M and q_t^B are the real trade weighted US dollar index with major currencies and broad range of currencies, respectively. β denotes the persistent parameter from an autoregressive process of degree 1, AR(1), specificiation of each real exchange rate. We corrected the median bias following Hansen's (1999) grid bootstrap technique. We employed 100 fine evenly spaced grid points on the interval $[\hat{\beta} \pm 6 \times se(\hat{\beta})]$, where $\hat{\beta}$ is the least squares estimate of β and se is its standard error. 10,000 nonparametric bootstrap simulations were done at each grid point to construct quantile function estimates. HL denotes the implied half-life point estimate in years. C.I denotes the 95% median unbiased confidence band.

Table 4. Correlation Matrix of Factor Estimates

	Major Currencies (q_t^M)					
	$\Delta f_{1,t}^{PLS}$	$\Delta f_{1,t}^{PLS,R}$	$\Delta f_{1,t}^{PLS,F}$	$\Delta f_{1,t}^{PC}$	$\Delta f_{1,t}^{PC,R}$	$\Delta f_{1,t}^{PC,F}$
$\Delta f_{1,t}^{PLS}$	1.0000					
$\Delta f_{1,t}^{PLS,R}$	0.5929	1.0000				
$\Delta f_{1,t}^{PLS,F}$	0.9464	0.3010	1.0000			
$\Delta f_{1,t}^{PC}$	0.3761	0.1094	0.4015	1.0000		
$\Delta f_{1,t}^{PC,R}$	0.1470	0.0826	0.1409	0.9284	1.0000	
$\Delta f_{1,t}^{PC,F}$	0.6368	0.0230	0.7448	0.4742	0.1680	1.0000
	$\Delta f_{2,t}^{PLS}$	$\Delta f_{2,t}^{PLS,R}$	$\Delta f_{2,t}^{PLS,F}$	$\Delta f_{2,t}^{PC}$	$\Delta f_{2,t}^{PC,R}$	$\Delta f_{2,t}^{PC,F}$
$\Delta f_{2,t}^{PLS}$	1.0000					
$\Delta f_{2,t}^{PLS,R}$	0.4223	1.0000				
$\Delta f_{2,t}^{PLS,F}$	0.8684	0.1106	1.0000			
$\Delta f_{2,t}^{PC}$	0.3927	0.3830	0.5590	1.0000		
$\Delta f_{2,t}^{PC,R}$	0.2477	0.0433	0.2188	0.3358	1.0000	
$\Delta f_{2,t}^{PC,F}$	0.1704	0.2749	0.0195	0.2948	0.0769	1.0000
	Broad Currencies (q_t^B)					
	$\Delta f_{1,t}^{PLS}$	$\Delta f_{1,t}^{PLS,R}$	$\Delta f_{1,t}^{PLS,F}$	$\Delta f_{1,t}^{PC}$	$\Delta f_{1,t}^{PC,R}$	$\Delta f_{1,t}^{PC,F}$
$\Delta f_{1,t}^{PLS}$	1.0000					
$\Delta f_{1,t}^{PLS,R}$	0.5989	1.0000				
$\Delta f_{1,t}^{PLS,F}$	0.9595	0.3491	1.0000			
$\Delta f_{1,t}^{PC}$	0.4179	0.1273	0.4442	1.0000		
$\Delta f_{1,t}^{PC,R}$	0.1306	0.0155	0.1474	0.9284	1.0000	
$\Delta f_{1,t}^{PC,F}$	0.8068	0.2280	0.8639	0.4742	0.1680	1.0000
	$\Delta f_{2,t}^{PLS}$	$\Delta f_{2,t}^{PLS,R}$	$\Delta f_{2,t}^{PLS,F}$	$\Delta f_{2,t}^{PC}$	$\Delta f_{2,t}^{PC,R}$	$\Delta f_{2,t}^{PC,F}$
$\Delta f_{2,t}^{PLS}$	1.0000					
$\Delta f_{2,t}^{PLS,R}$	0.4085	1.0000				
$\Delta f_{2,t}^{PLS,F}$	0.8667	0.2614	1.0000			
$\Delta f_{2,t}^{PC}$	0.2978	0.1337	0.4286	1.0000		
$\Delta f_{2,t}^{PC,R}$	0.1707	0.8223	0.1620	0.3358	1.0000	
$\Delta f_{2,t}^{PC,F}$	0.0577	0.0852	0.1022	0.2948	0.0769	1.0000

Note: We report the absolute value of the correlation coefficient statistics of the two factor estimates, because the factor loading and latent factors are jointly estimated, thus the sign of the factor estimates is not relevant.

Table 5. j-Period ahead Out-of-Sample Predictability: Major Currencies

j	#Factors	$\Delta\mathbf{f}_t^{PLS}$	$\Delta\mathbf{f}_t^{PLS,R}$	$\Delta\mathbf{f}_t^{PLS,F}$	$\Delta\mathbf{f}_t^{PC}$	$\Delta\mathbf{f}_t^{PC,R}$	$\Delta\mathbf{f}_t^{PC,F}$
				Non-Stationary Models ($\widehat{q}_{t+j\|t}^{F_{RW}}$)			
1	1	0.9934	0.9943	0.9972	0.9928	0.9961	0.9885
	2	0.9887	0.9900	0.9917	0.9872	0.9975	0.9851
	3	0.9817	0.9933	0.9917	0.9814	0.9973	0.9839
	4	0.9788	0.9919	0.9886	0.9843	0.9931	0.9716
12	1	**1.0149**	**1.0085**	0.9939	0.9850	0.9846	0.9752
	2	**1.0183**	**1.0122**	0.9854	0.9726	0.9998	0.9715
	3	0.9948	**1.0211**	0.9720	0.9741	**1.0088**	0.9477
	4	**1.0070**	**1.0256**	0.9769	0.9658	0.9948	0.9455
24	1	**1.0290**	**1.0257**	0.9913	0.9534	0.9642	0.9504
	2	**1.0175**	**1.0244**	0.9820	0.9435	0.9689	0.9440
	3	0.9976	**1.0302**	0.9494	0.9463	0.9518	0.9222
	4	**1.0095**	0.9993	0.9470	0.9427	0.9328	0.9192
36	1	**1.0187**	**1.0109**	0.9837	0.9220	0.9236	0.9154
	2	0.9572	0.9727	0.9575	0.8830	0.9242	0.9054
	3	0.9482	0.9588	0.9041	0.8847	0.8894	0.8824
	4	0.9723	0.9257	0.9127	0.8722	0.8777	0.8824
				Stationary Models ($\widehat{q}_{t+j\|t}^{F_{AR}}$)			
j	#Factors	$\Delta\mathbf{f}_t^{PLS}$	$\Delta\mathbf{f}_t^{PLS,R}$	$\Delta\mathbf{f}_t^{PLS,F}$	$\Delta\mathbf{f}_t^{PC}$	$\Delta\mathbf{f}_t^{PC,R}$	$\Delta\mathbf{f}_t^{PC,F}$
1	1	0.9867	0.9927	0.9932	0.9915	0.9971	0.9865
	2	0.9797	0.9851	0.9896	0.9832	0.9977	0.9827
	3	0.9797	0.9918	0.9880	0.9760	0.9969	0.9811
	4	0.9748	0.9914	0.9869	0.9798	0.9931	0.9710
12	1	**1.0388**	**1.0557***	**1.0249**	**1.0442***	**1.0448***	**1.0188**
	2	**1.0318**	**1.0435***	**1.0231**	**1.0195**	**1.0642***	**1.0118**
	3	**1.0253**	**1.0607***	**1.0035**	**1.0187**	**1.0783***	0.9992
	4	**1.0169**	**1.0560***	0.9946	**1.0081**	**1.0622***	0.9820
24	1	**1.0911**	**1.1545***	**1.0612**	**1.1020***	**1.1185***	**1.0394**
	2	**1.0512**	**1.1267***	**1.0330**	**1.0703**	**1.1463***	**1.0252**
	3	**1.0415**	**1.1431***	0.9949	**1.0688**	**1.1513***	**1.0124**
	4	**1.0301**	**1.0914**	0.9778	**1.0471**	**1.1188***	0.9767
36	1	**1.1507**	**1.2432***	**1.1159**	**1.1785***	**1.1872***	**1.0510**
	2	**1.0297**	**1.1856***	**1.0345**	**1.0791**	**1.2125***	**1.0298**
	3	**1.0291**	**1.1912***	0.9740	**1.0791**	**1.1998***	**1.0179**
	4	**1.0253**	**1.1197**	0.9723	**1.0380**	**1.1734***	0.9869

Note: We report the *RRMSPE* statistics employing a rolling window scheme with a 50% sample split point. *RRMSPE* denotes the ratio of the root mean squared prediction errors, which is the mean squared prediction error (*RMSPE*) from the benchmark random walk (RW) model divided by the *RMSPE* from each competing model with k factors. *RRMSPE* statistics in bold denote that the competing model outperforms the benchmark RW model. $*$indicates that the factor model outperforms both the AR and RW benchmark models.

Table 6. j-Period ahead Out-of-Sample Predictability: Broad Currencies

		Non-Stationary Models ($\widehat{q}^{F_{RW}}_{t+j\|t}$)					
j	#Factors	$\Delta\mathbf{f}_t^{PLS}$	$\Delta\mathbf{f}_t^{PLS,R}$	$\Delta\mathbf{f}_t^{PLS,F}$	$\Delta\mathbf{f}_t^{PC}$	$\Delta\mathbf{f}_t^{PC,R}$	$\Delta\mathbf{f}_t^{PC,F}$
1	1	0.9917	0.9956	0.9945	0.9944	0.9968	0.9895
	2	0.9935	0.9887	0.9930	0.9866	0.9979	0.9882
	3	0.9832	0.9925	0.9901	0.9805	0.9975	0.9856
	4	0.9803	0.9875	0.9776	0.9891	0.9950	0.9744
12	1	0.9804	0.9823	0.9800	0.9816	0.9847	0.9664
	2	0.9934	0.9881	0.9693	0.9575	**1.0016**	0.9588
	3	0.9794	**1.0015**	0.9709	0.9566	**1.0115**	0.9367
	4	0.9892	0.9969	0.9641	0.9473	0.9972	0.9308
24	1	0.9725	0.9922	0.9556	0.9756	0.9863	0.9380
	2	0.9825	0.9972	0.9405	0.9499	**1.0049**	0.9274
	3	0.9779	**1.0020**	0.9459	0.9512	**1.0048**	0.9053
	4	0.9732	0.9670	0.9380	0.9374	0.9735	0.8900
36	1	0.9266	0.9576	0.9104	0.9498	0.9485	0.8891
	2	0.9116	0.9320	0.8925	0.8833	0.9660	0.8735
	3	0.9143	0.9253	0.8945	0.8845	0.9464	0.8486
	4	0.9206	0.8963	0.8868	0.8627	0.9213	0.8408
		Stationary Models ($\widehat{q}^{F_{AR}}_{t+j\|t}$)					
j	#Factors	$\Delta\mathbf{f}_t^{PLS}$	$\Delta\mathbf{f}_t^{PLS,R}$	$\Delta\mathbf{f}_t^{PLS,F}$	$\Delta\mathbf{f}_t^{PC}$	$\Delta\mathbf{f}_t^{PC,R}$	$\Delta\mathbf{f}_t^{PC,F}$
1	1	0.9887	0.9957	0.9926	0.9950	0.9987	0.9888
	2	0.9867	0.9859	0.9926	0.9852	0.9986	0.9871
	3	0.9833	0.9933	0.9876	0.9780	0.9974	0.9847
	4	0.9773	0.9913	0.9776	0.9856	0.9954	0.9749
12	1	**1.0052**	**1.0193**	**1.0229**	**1.0342**	**1.0416**	**1.0151**
	2	**1.0226**	**1.0157**	**1.0277**	**1.0033**	**1.0498**	**1.0060**
	3	**1.0240**	**1.0424**	**1.0134**	**1.0007**	**1.0572***	0.9978
	4	**1.0140**	**1.0330**	0.9953	0.9916	**1.0431**	0.9795
24	1	**1.0145**	**1.0888**	**1.0367**	**1.0862**	**1.1154***	**1.0250**
	2	**1.0409**	**1.0855**	**1.0316**	**1.0442**	**1.1355***	**1.0101**
	3	**1.0403**	**1.1140***	**1.0164**	**1.0397**	**1.1501***	0.9946
	4	**1.0082**	**1.0604**	0.9844	**1.0146**	**1.1119***	0.9458
36	1	**1.0146**	**1.1430***	**1.0283**	**1.1461***	**1.1720***	**1.0057**
	2	**1.0153**	**1.1518***	0.9962	**1.0398**	**1.2033***	0.9852
	3	**1.0148**	**1.1834***	0.9817	**1.0387**	**1.2170***	0.9731
	4	0.9886	**1.1154**	0.9540	0.9936	**1.1837***	0.9249

Note: We report the *RRMSPE* statistics employing a rolling window scheme with a 50% sample split point. *RRMSPE* denotes the ratio of the root mean squared prediction errors, which is the mean squared prediction error (*RMSPE*) from the benchmark random walk (RW) model divided by the *RMSPE* from each competing model with k factors. *RRMSPE* statistics in bold denote that the competing model outperforms the benchmark RW model. ∗indicates that the factor model outperforms both the AR and RW benchmark models.

Table 7. j**-Period ahead Out-of-Sample Predictability with LASSO: Major Currencies**

		Non-Stationary Models ($\widehat{q}^{F_{RW}}_{t+j\|t}$)					
j	#Factors	$\Delta\mathbf{f}_t^{PLS/L}$	$\Delta\mathbf{f}_t^{PLS/L,R}$	$\Delta\mathbf{f}_t^{PLS/L,F}$	$\Delta\mathbf{f}_t^{PC/L}$	$\Delta\mathbf{f}_t^{PC/L,R}$	$\Delta\mathbf{f}_t^{PC/L,F}$
1	1	0.9959	0.9931	0.9989	0.9903	0.9964	0.9923
	2	0.9901	0.9927	0.9937	0.9920	0.9972	0.9878
	3	0.9916	0.9920	0.9863	0.9980	0.9977	0.9747
	4	0.9819	0.9909	0.9804	0.9812	0.9958	0.9695
12	1	0.9954	**1.0147**	0.9843	0.9999	0.9911	0.9786
	2	0.9875	**1.0228**	0.9781	0.9681	**1.0039**	0.9581
	3	**1.0214**	**1.0093**	0.9836	0.9621	0.9861	0.9364
	4	**1.0040**	**1.0100**	0.9811	0.9598	0.9857	0.9450
24	1	**1.0040**	**1.0351**	0.9891	0.9850	0.9321	0.9542
	2	0.9891	**1.0034**	0.9627	0.8961	0.9350	0.9290
	3	**1.0874**	0.9886	0.9858	0.8886	0.9168	0.8785
	4	**1.0443**	0.9869	0.9631	0.9037	0.9164	0.8963
36	1	0.9920	**1.0010**	0.9982	0.9512	0.8713	0.9152
	2	0.9475	0.9162	0.9724	0.8234	0.8737	0.8888
	3	**1.0264**	0.9050	0.9706	0.8259	0.8664	0.8247
	4	0.9802	0.8982	0.9277	0.8374	0.8628	0.8462
		Stationary Models ($\widehat{q}^{F_{AR}}_{t+j\|t}$)					
j	#Factors	$\Delta\mathbf{f}_t^{PLS/L}$	$\Delta\mathbf{f}_t^{PLS/L,R}$	$\Delta\mathbf{f}_t^{PLS/L,F}$	$\Delta\mathbf{f}_t^{PC/L}$	$\Delta\mathbf{f}_t^{PC/L,R}$	$\Delta\mathbf{f}_t^{PC/L,F}$
1	1	0.9924	0.9914	0.9928	0.9844	0.9966	0.9835
	2	0.9920	0.9945	0.9899	0.9859	**1.0001**	0.9769
	3	0.9922	0.9923	0.9854	0.9897	0.9983	0.9765
	4	0.9845	0.9907	0.9835	0.9750	0.9971	0.9656
12	1	**1.0305**	**1.0522***	**1.0204**	**1.0357**	**1.0626***	**1.0025**
	2	**1.0404**	**1.0691***	**1.0250**	**1.0236**	**1.0696***	0.9519
	3	**1.0574***	**1.0470***	**1.0303**	**1.0193**	**1.0471***	0.9546
	4	**1.0438***	**1.0468***	**1.0221**	**1.0164**	**1.0466***	0.9592
24	1	**1.0801**	**1.1527***	**1.0465**	**1.0933***	**1.1246***	0.9968
	2	**1.0535**	**1.1342***	**1.0097**	**1.0606**	**1.1261***	0.9139
	3	**1.1207***	**1.1023***	**1.0257**	**1.0320**	**1.1004***	0.9071
	4	**1.0858**	**1.0971***	0.9846	**1.0273**	**1.0993***	0.9159
36	1	**1.1384**	**1.2239***	**1.0993**	**1.1358**	**1.1752***	0.9958
	2	**1.0502**	**1.1825***	**1.0371**	**1.0697**	**1.1748***	0.9080
	3	**1.1067**	**1.1516**	**1.0411**	**1.0634**	**1.1794***	0.8924
	4	**1.0870**	**1.1365**	0.9845	**1.0604**	**1.1760***	0.9025

Note: We report the *RRMSPE* statistics employing a rolling window scheme with a 50% sample split point. *RRMSPE* denotes the ratio of the root mean squared prediction errors, which is the mean squared prediction error (*RMSPE*) from the benchmark random walk (RW) model divided by the *RMSPE* from each competing model with k factors. *RRMSPE* statistics in bold denote that the competing model outperforms the benchmark RW model. ∗indicates that the factor model outperforms both the AR and RW benchmark models.

Table 8. j-Period ahead Out-of-Sample Predictability with LASSO: Broad Currencies

		Non-Stationary Models ($\widehat{q}_{t+j\|t}^{F_{RW}}$)					
j	#Factors	$\Delta\mathbf{f}_t^{PLS/L}$	$\Delta\mathbf{f}_t^{PLS/L,R}$	$\Delta\mathbf{f}_t^{PLS/L,F}$	$\Delta\mathbf{f}_t^{PC/L}$	$\Delta\mathbf{f}_t^{PC/L,R}$	$\Delta\mathbf{f}_t^{PC/L,F}$
1	1	0.9976	0.9951	0.9991	0.9946	0.9968	0.9919
	2	0.9917	0.9925	0.9970	0.9959	0.9978	0.9894
	3	0.9798	0.9971	0.9879	0.9888	**1.0011**	0.9898
	4	0.9712	0.9988	0.9823	0.9869	0.9953	0.9922
12	1	0.9972	0.9846	0.9935	**1.0013**	0.9960	0.9750
	2	**1.0056**	**1.0029**	0.9864	0.9892	0.9917	0.9570
	3	**1.0085**	0.9929	**1.0247**	0.9587	0.9753	0.9308
	4	0.9851	0.9865	0.9986	0.9510	0.9674	0.9377
24	1	**1.0176**	0.9965	0.9934	0.9964	0.9811	0.9524
	2	**1.0229**	**1.0037**	0.9714	0.9667	0.9535	0.9058
	3	**1.0499**	0.9927	**1.0264**	0.8953	0.9413	0.8580
	4	**1.0126**	**1.0074**	0.9339	0.8814	0.9340	0.8531
36	1	**1.0001**	0.9538	0.9733	0.9744	0.9179	0.8947
	2	0.9709	0.9188	0.9224	0.8875	0.8870	0.8432
	3	**1.0260**	0.9208	0.9864	0.7873	0.8783	0.7969
	4	0.9774	0.9396	0.8414	0.7693	0.8782	0.7662
		Stationary Models ($\widehat{q}_{t+j\|t}^{F_{AR}}$)					
j	#Factors	$\Delta\mathbf{f}_t^{PLS/L}$	$\Delta\mathbf{f}_t^{PLS/L,R}$	$\Delta\mathbf{f}_t^{PLS/L,F}$	$\Delta\mathbf{f}_t^{PC/L}$	$\Delta\mathbf{f}_t^{PC/L,R}$	$\Delta\mathbf{f}_t^{PC/L,F}$
1	1	0.9959	0.9962	0.9967	0.9899	0.9973	0.9882
	2	0.9924	0.9948	0.9956	0.9895	**1.0003**	0.9818
	3	0.9815	0.9986	0.9843	0.9788	**1.0040***	0.9837
	4	0.9731	0.9993	0.9779	0.9761	0.9984	0.9896
12	1	**1.0227**	**1.0191**	**1.0331**	**1.0306**	**1.0491**	**1.0136**
	2	**1.0587***	**1.0429**	**1.0310**	**1.0307**	**1.0451**	0.9802
	3	**1.0549***	**1.0288**	**1.0486**	0.9673	**1.0224**	0.9528
	4	**1.0288**	**1.0214**	**1.0190**	0.9560	**1.0079**	0.9658
24	1	**1.0439**	**1.0912**	**1.0591**	**1.0736**	**1.1315***	**1.0175**
	2	**1.0843**	**1.1251***	**1.0416**	**1.0814**	**1.1064***	0.9276
	3	**1.1014***	**1.0953**	**1.0640**	0.9264	**1.0898**	0.8871
	4	**1.0583**	**1.1052***	0.9698	0.8908	**1.0743**	0.8836
36	1	**1.0425**	**1.1438***	**1.0591**	**1.1108**	**1.1905***	0.9653
	2	**1.0643**	**1.2083***	0.9818	**1.0920**	**1.1733***	0.8723
	3	**1.1173**	**1.1800***	**1.0209**	0.8772	**1.1858***	0.8483
	4	**1.0752**	**1.1981***	0.8955	0.8501	**1.1704***	0.8079

Note: We report the *RRMSPE* statistics employing a rolling window scheme with a 50% sample split point. *RRMSPE* denotes the ratio of the root mean squared prediction errors, which is the mean squared prediction error (*RMSPE*) from the benchmark random walk (RW) model divided by the *RMSPE* from each competing model with k factors. *RRMSPE* statistics in bold denote that the competing model outperforms the benchmark RW model. $*$ indicates that the factor model outperforms both the AR and RW benchmark models.

Table 9. j-Period ahead Out-of-Sample Predictability: Labor Market Factors

		Major Currencies (q_t^M): *Stationary Models* ($\widehat{q}_{t+j\|t}^{F_{AR}}$)					
		Labor Variables		*Real excluding Labor*		*Total excluding Labor*	
j	#Factors	$\Delta\mathbf{f}_t^{PLS}$	$\Delta\mathbf{f}_t^{PC}$	$\Delta\mathbf{f}_t^{PLS}$	$\Delta\mathbf{f}_t^{PC}$	$\Delta\mathbf{f}_t^{PLS}$	$\Delta\mathbf{f}_t^{PC}$
1	1	0.9947	0.9987	0.9998	**1.0011***	0.9923	0.9845
	2	0.9918	0.9960	**1.0095***	0.9940	0.9865	0.9805
	3	0.9932	0.9867	**1.0015***	0.9945	0.9829	0.9799
	4	0.9934	0.9865	0.9946	0.9911	0.9794	0.9724
12	1	**1.0603***	**1.0633***	**1.0334**	**1.0350**	**1.0249**	**1.0212**
	2	**1.0652***	**1.0663***	**1.0349**	**1.0364**	**1.0243**	**1.0118**
	3	**1.0527***	**1.0476***	**1.0413**	**1.0348**	**1.0132**	**1.0096**
	4	**1.0515***	**1.0487***	**1.0413**	**1.0292**	**1.0006**	0.9948
24	1	**1.1723***	**1.1352***	**1.0823**	**1.0917**	**1.0604**	**1.0511**
	2	**1.1309***	**1.1468***	**1.0813**	**1.0890**	**1.0331**	**1.0369**
	3	**1.0998***	**1.0998***	**1.0882**	**1.0843**	**1.0204**	**1.0353**
	4	**1.1076***	**1.0933***	**1.0881**	**1.0795**	**1.0018**	**1.0089**
36	1	**1.2685***	**1.1925***	**1.1656**	**1.1588**	**1.1182**	**1.0873**
	2	**1.1934***	**1.1925***	**1.1602**	**1.1596**	**1.0395**	**1.0426**
	3	**1.1540**	**1.1470**	**1.1538**	**1.1551**	**1.0453**	**1.0455**
	4	**1.1585**	**1.1242**	**1.1550**	**1.1556**	**1.0064**	**1.0148**
		Broad Currencies (q_t^B): *Stationary Models* ($\widehat{q}_{t+j\|t}^{F_{AR}}$)					
		Labor Variables		*Real excluding Labor*		*Total excluding Labor*	
j	#Factors	$\Delta\mathbf{f}_t^{PLS}$	$\Delta\mathbf{f}_t^{PC}$	$\Delta\mathbf{f}_t^{PLS}$	$\Delta\mathbf{f}_t^{PC}$	$\Delta\mathbf{f}_t^{PLS}$	$\Delta\mathbf{f}_t^{PC}$
1	1	0.9952	0.9994	**1.0115***	**1.0038***	0.9932	0.9895
	2	0.9891	0.9965	**1.0095***	0.9971	0.9901	0.9817
	3	0.9905	0.9874	**1.0033***	0.9986	0.9857	0.9840
	4	0.9902	0.9840	0.9984	0.9964	0.9839	0.9811
12	1	**1.0213**	**1.0518***	**1.0375**	**1.0431**	**1.0209**	**1.0178**
	2	**1.0365**	**1.0539***	**1.0407**	**1.0417**	**1.0272**	**1.0040**
	3	**1.0429**	**1.0443**	**1.0390**	**1.0397**	**1.0158**	**1.0022**
	4	**1.0418**	**1.0436**	**1.0362**	**1.0268**	0.9862	0.9900
24	1	**1.0956**	**1.1332***	**1.0782**	**1.0968**	**1.0290**	**1.0348**
	2	**1.0977**	**1.1426***	**1.0880**	**1.0933**	**1.0274**	**1.0218**
	3	**1.0965**	**1.1070***	**1.0936**	**1.0879**	**1.0247**	**1.0185**
	4	**1.0955**	**1.0989**	**1.0867**	**1.0768**	0.9754	0.9887
36	1	**1.1585***	**1.1993***	**1.1269**	**1.1339**	**1.0257**	**1.0387**
	2	**1.1851***	**1.1992***	**1.1323**	**1.1350**	0.9939	**1.0034**
	3	**1.1724***	**1.1623***	**1.1378**	**1.1297**	0.9921	**1.0034**
	4	**1.1788***	**1.1391**	**1.1365**	**1.1301**	0.9465	0.9706

Note: We report the *RRMSPE* statistics employing a rolling window scheme with a 50% sample split point. *RRMSPE* denotes the ratio of the root mean squared prediction errors, which is the mean squared prediction error (*RMSPE*) from the benchmark random walk (RW) model divided by the *RMSPE* from each competing model with k factors. *RRMSPE* statistics in bold denote that the competing model outperforms the benchmark RW model. $*$ indicates that the factor model outperforms both the AR and RW benchmark models.

Table 10. j-Period ahead Out-of-Sample Predictability: Price Factors

		Major Currencies (q_t^M): *Stationary Models* ($\widehat{q}_{t+j\|t}^{F_{AR}}$)					
		Price Variables		*Financial excl. Prices*		*Total excluding Prices*	
j	#Factors	$\Delta\mathbf{f}_t^{PLS}$	$\Delta\mathbf{f}_t^{PC}$	$\Delta\mathbf{f}_t^{PLS}$	$\Delta\mathbf{f}_t^{PC}$	$\Delta\mathbf{f}_t^{PLS}$	$\Delta\mathbf{f}_t^{PC}$
1	1	0.9926	0.9918	0.9997	0.9915	0.9912	0.9946
	2	0.9864	0.9886	0.9913	0.9886	0.9817	0.9827
	3	0.9884	0.9863	0.9863	0.9756	0.9754	0.9747
	4	0.9816	0.9895	0.9746	0.9623	0.9712	0.9717
12	1	**1.0145**	**1.0018**	**1.0405**	**1.0419***	**1.0434***	**1.0463***
	2	0.9713	0.9574	**1.0403**	**1.0410**	**1.0439***	**1.0484***
	3	0.9481	0.9511	**1.0471***	**1.0364**	**1.0386**	**1.0432***
	4	0.9452	0.9397	**1.0378**	0.9989	**1.0322**	**1.0463***
24	1	**1.0484**	**1.0096**	**1.0954***	**1.0905**	**1.1294***	**1.1210***
	2	0.9283	0.9081	**1.0910**	**1.0907**	**1.1144***	**1.1193***
	3	0.8964	0.8866	**1.1113***	**1.0922***	**1.1041***	**1.1226***
	4	0.8888	0.8797	**1.1110***	**1.0625**	**1.1000***	**1.0894**
36	1	**1.0834**	**1.0035**	**1.1689***	**1.1659**	**1.2277***	**1.2053***
	2	0.8808	0.8750	**1.1662**	**1.1783***	**1.1813***	**1.1846***
	3	0.8564	0.8416	**1.1772***	**1.1814***	**1.1394**	**1.1904***
	4	0.8493	0.8332	**1.1919***	**1.1364**	**1.1388**	**1.1587**
		Broad Currencies (q_t^B): *Stationary Models* ($\widehat{q}_{t+j\|t}^{F_{AR}}$)					
		Price Variables		*Financial excl. Prices*		*Total excluding Prices*	
j	#Factors	$\Delta\mathbf{f}_t^{PLS}$	$\Delta\mathbf{f}_t^{PC}$	$\Delta\mathbf{f}_t^{PLS}$	$\Delta\mathbf{f}_t^{PC}$	$\Delta\mathbf{f}_t^{PLS}$	$\Delta\mathbf{f}_t^{PC}$
1	1	0.9941	0.9939	**1.0009**	0.9956	0.9952	0.9968
	2	0.9928	0.9912	0.9941	0.9919	0.9898	0.9861
	3	0.9940	0.9901	0.9937	0.9802	0.9829	0.9805
	4	0.9820	**1.0004**	0.9804	0.9746	0.9852	0.9800
12	1	0.9998	**1.0019**	**1.0565***	**1.0505**	**1.0347**	**1.0418**
	2	0.9829	0.9484	**1.0528***	**1.0482**	**1.0438**	**1.0413**
	3	0.9644	0.9465	**1.0617***	**1.0436**	**1.0426**	**1.0350**
	4	0.9518	0.9371	**1.0552***	**1.0041**	**1.0366**	**1.0382**
24	1	0.9918	0.9994	**1.1047***	**1.0977**	**1.0932**	**1.1138***
	2	0.9332	0.8805	**1.0969**	**1.0915**	**1.1120***	**1.1090***
	3	0.8809	0.8699	**1.1145***	**1.0870**	**1.1074***	**1.1112***
	4	0.8712	0.8650	**1.0950**	**1.0495**	**1.0916**	**1.0883**
36	1	0.9636	0.9661	**1.1390**	**1.1362**	**1.1508***	**1.1860***
	2	0.8617	0.8079	**1.1347**	**1.1461***	**1.1873***	**1.1684***
	3	0.8075	0.7823	**1.1480***	**1.1499***	**1.1543***	**1.1762***
	4	0.7978	0.7766	**1.1237**	**1.0858**	**1.1384**	**1.1658***

Note: We report the *RRMSPE* statistics employing a rolling window scheme with a 50% sample split point. *RRMSPE* denotes the ratio of the root mean squared prediction errors, which is the mean squared prediction error (*RMSPE*) from the benchmark random walk (RW) model divided by the *RMSPE* from each competing model with k factors. *RRMSPE* statistics in bold denote that the competing model outperforms the benchmark RW model. $*$ indicates that the factor model outperforms both the AR and RW benchmark models.

Table 11. j**-Period ahead Out-of-Sample Predictability: PPP Based Models**

		Major Currencies (q_t^M)			
		Non-Stationary Models ($\widehat{q}_{t+j\|t}^{F_{RW}}$)		*Stationary Models* ($\widehat{q}_{t+j\|t}^{F_{AR}}$)	
	j	CSA		CSA	
$\Delta f_t^{P,F}$	1	0.9993		**1.0018***	
	12	0.9788		**1.0421***	
	24	0.9693		**1.0901**	
	36	0.9590		**1.1620**	
	j	PLS	PC	PLS	PC
$\Delta f_t^{P,D}$	1	0.9928	0.9973	0.9881	0.9978
	12	0.9660	0.9899	0.9850	**1.0457***
	24	0.9539	0.9688	**1.0270**	**1.0875**
	36	0.9201	0.9392	**1.1312**	**1.1636**
		Broad Currencies (q_t^B)			
		Non-Stationary Models ($\widehat{q}_{t+j\|t}^{F_{RW}}$)		*Stationary Models* ($\widehat{q}_{t+j\|t}^{F_{AR}}$)	
	j	CSA		CSA	
$\Delta f_t^{P,F}$	1	**1.0017**		**1.0042***	
	12	0.9857		**1.0496**	
	24	0.9755		**1.0958**	
	36	0.9596		**1.1317**	
		PLS	PC	PLS	PC
$\Delta f_t^{P,D}$	1	0.9972	0.9986	0.9939	0.9992
	12	0.9845	0.9921	**1.0006**	**1.0480**
	24	0.9676	0.9773	**1.0219**	**1.0904**
	36	0.9260	0.9434	**1.0750**	**1.1350**

Note: $\Delta f_t^{P,F}$ is the cross-section average of broad range panel of relative prices with respect to the US, starting from 1973M1 to 2018 M12. $\Delta f_t^{P,D}$ is the first common factor from a panel of relative prices of developed countries with respect to the US, starting from 1973 M1 to 2018 M12. We report the *RRMSPE* statistics employing a rolling window scheme and a 50% sample split. *RRMSPE* denotes the ratio of the root mean squared prediction errors, which is the mean squared prediction error (*RMSPE*) from the benchmark random walk (RW) model divided by the *RMSPE* from each competing model. *RRMSPE* statistics in bold denote that the competing model outperforms the benchmark RW model. $*$ indicates that the factor model outperforms both the AR and RW benchmark models.

Table 12. j-Period ahead Out-of-Sample Predictability: UIP Based Models

		Major Currencies (q_t^M)			
		Non-Stationary Models ($\widehat{q}_{t+j\|t}^{F_{RW}}$)		*Stationary Models* ($\widehat{q}_{t+j\|t}^{F_{AR}}$)	
	j	CSA		CSA	
$\Delta f_t^{U,F}$	1	**1.0033**		**1.0046***	
	12	0.9784		**1.0406**	
	24	0.9695		**1.0920***	
	36	0.9619		**1.1716***	
	j	PLS	PC	PLS	PC
$\Delta f_t^{U,S}$	1	**1.0093***	**1.0115***	**1.0105***	**1.0138***
	12	0.9639	0.9648	**1.0024**	**1.0065**
	24	0.9041	0.9079	0.9241	0.9330
	36	0.8162	0.8249	0.8935	0.8989
		Broad Currencies (q_t^B)			
		Non-Stationary Models ($\widehat{q}_{t+j\|t}^{F_{RW}}$)		*Stationary Models* ($\widehat{q}_{t+j\|t}^{F_{AR}}$)	
	j	CSA		CSA	
$\Delta f_t^{U,F}$	1	**1.0050**		**1.0063***	
	12	0.9858		**1.0517***	
	24	0.9766		**1.1008**	
	36	0.9649		**1.1425***	
	j	PLS	PC	PLS	PC
$\Delta f_t^{U,S}$	1	**1.0139***	**1.0163***	**1.0138***	**1.0171***
	12	0.9495	0.9520	0.9885	0.9926
	24	0.8869	0.8899	0.9003	0.9098
	36	0.8050	0.8080	0.8114	0.8200

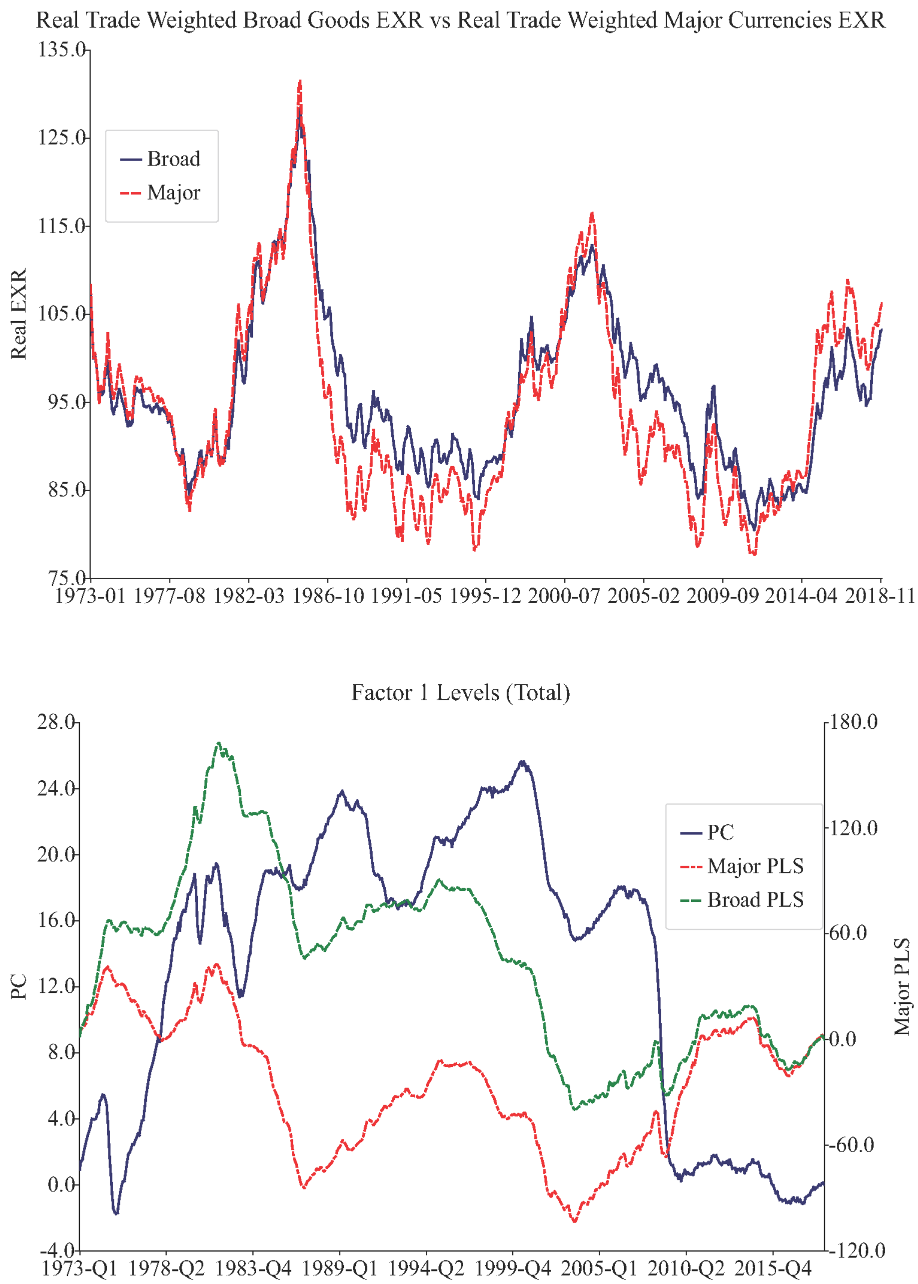
Note: $\Delta f_t^{U,F}$ is the cross-section average of broad range panel of interest rate spreads with respect to the US, starting from 1973M1 to 2018 M12. $\Delta f_t^{U,S}$ denotes the first common factor from broad range panel of interest rate spreads with respect to the US starting from 1985 M8 to 2018 M12. Sample split point is 50%, but results were similar with a 70% split point specification. *RRMSPE* denotes the ratio of the root mean squared prediction errors, which is the mean squared prediction error (*RMSPE*) from the benchmark random walk (RW) model divided by the *RMSPE* from each competing model. *RRMSPE* statistics in bold denote that the competing model outperforms the benchmark RW model. $*$ indicates that the factor model outperforms both the AR and RW benchmark models.

Table 13. j-Period ahead Out-of-Sample Predictability: RIRP Based Models

		Major Currencies (q_t^M)			
		Non-Stationary Models ($\widehat{q}_{t+j\|t}^{F_{RW}}$)		*Stationary Models* ($\widehat{q}_{t+j\|t}^{F_{AR}}$)	
	j	CSA		CSA	
$\Delta f_t^{R,F}$	1	0.9988		0.9978	
	12	0.9860		**1.0268**	
	24	0.9765		**1.0890**	
	36	0.9584		**1.1639**	
	j	PLS	PC	PLS	PC
$\Delta f_t^{R,S}$	1	0.9922	0.9917	0.9949	0.9953
	12	0.9599	0.9510	0.9723	0.9739
	24	0.8939	0.8885	0.8444	0.8618
	36	0.8249	0.8221	0.7685	0.7895
		Broad Currencies (q_t^B)			
		Non-Stationary Models ($\widehat{q}_{t+j\|t}^{F_{RW}}$)		*Stationary Models* ($\widehat{q}_{t+j\|t}^{F_{AR}}$)	
	j	CSA		CSA	
$\Delta f_t^{R,F}$	1	0.9999		0.9972	
	12	0.9921		**1.0122**	
	24	0.9926		**1.0557**	
	36	0.9799		**1.0716**	
	j	PLS	PC	PLS	PC
$\Delta f_t^{R,S}$	1	0.9874	0.9871	0.9864	0.9873
	12	0.9366	0.9231	0.9414	0.9453
	24	0.8595	0.8553	0.8286	0.8472
	36	0.7770	0.7770	0.7228	0.7503

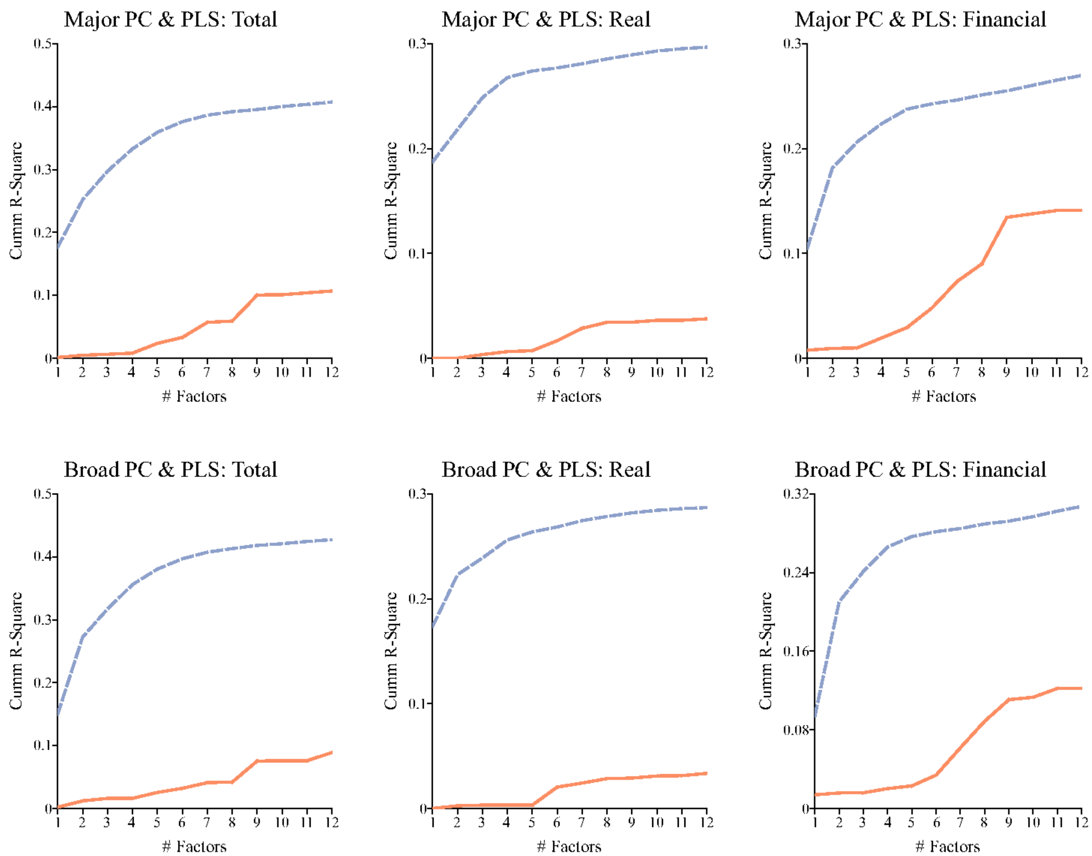
Note: $\Delta f_t^{R,F}$ is the cross-section average of broad range panel of interest rate spreads with respect to the US, starting from 1973M2 to 2018 M12. $\Delta f_t^{R,S}$ denotes the first common factor from broad range panel of interest rate spreads with respect to the US starting from 1985 M8 to 2018 M12. Sample split point is 50%, but results were similar with a 70% split point specification. *RRMSPE* denotes the ratio of the root mean squared prediction errors, which is the mean squared prediction error (*RMSPE*) from the benchmark random walk (RW) model divided by the *RMSPE* from each competing model. *RRMSPE* statistics in bold denote that the competing model outperforms the benchmark RW model. $*$ indicates that the factor model outperforms both the AR and RW benchmark models.

Figure 1. Real Exchange Rates and Common Factor Estimates



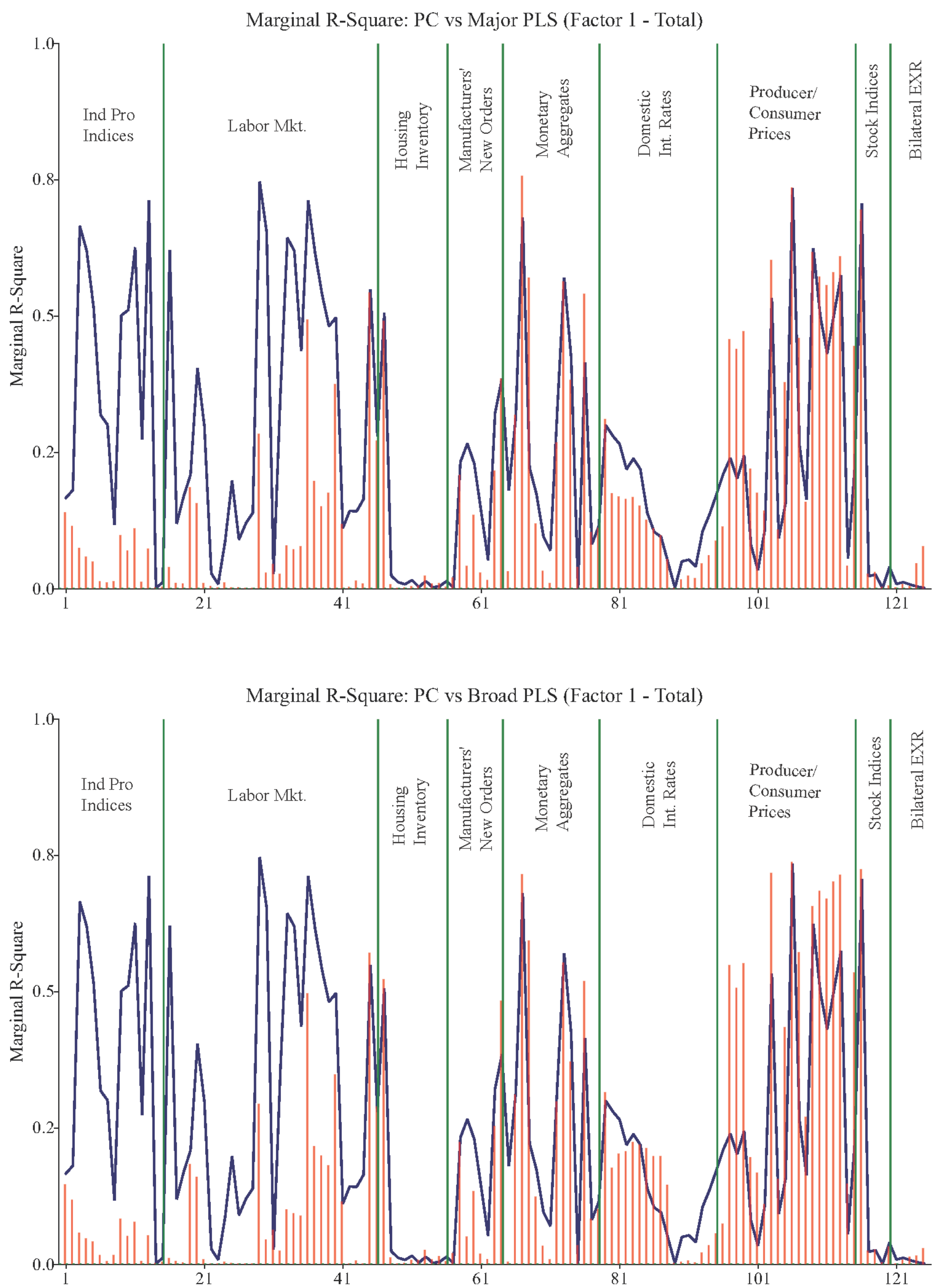
Note: We obtained the US dollar real exchange rates from the FRED. We report the first level common factor estimates that were recovered by cumulatively summation of differenced factors. Partial Least Squares (PLS) factors are target specific, thus we separately obtained factors for the major and broad currencies exchange rates.

Figure 2. Cumulative R^2 Values of PC (solid) and PLS Factors



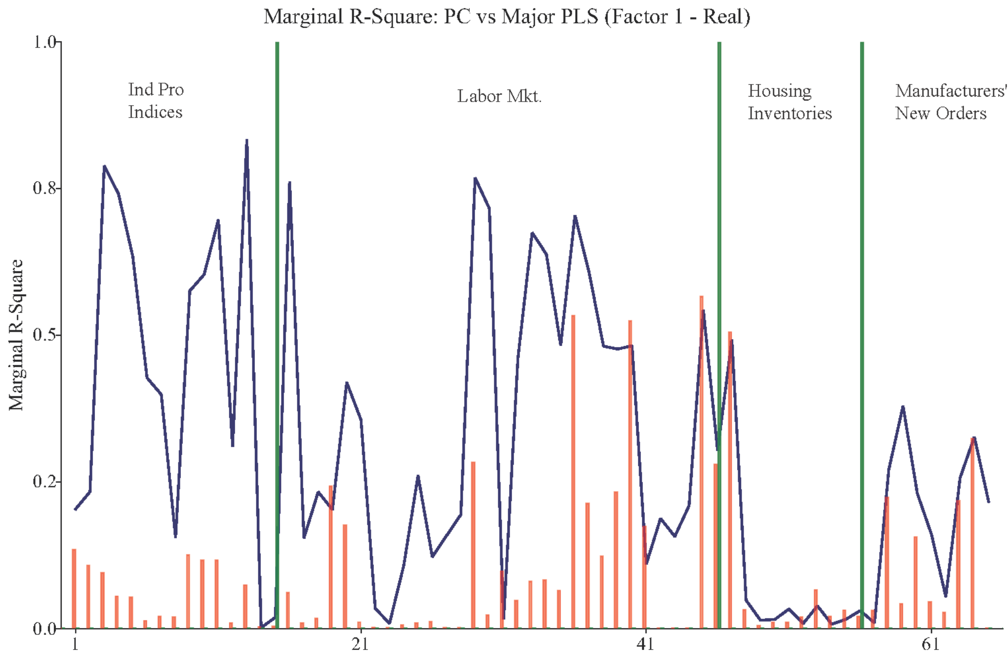
Note: We regress the real exchange rate on each factor and obtain the R^2 statistics. Since we use orthogonalized factors, we report the cumulative R^2 values.

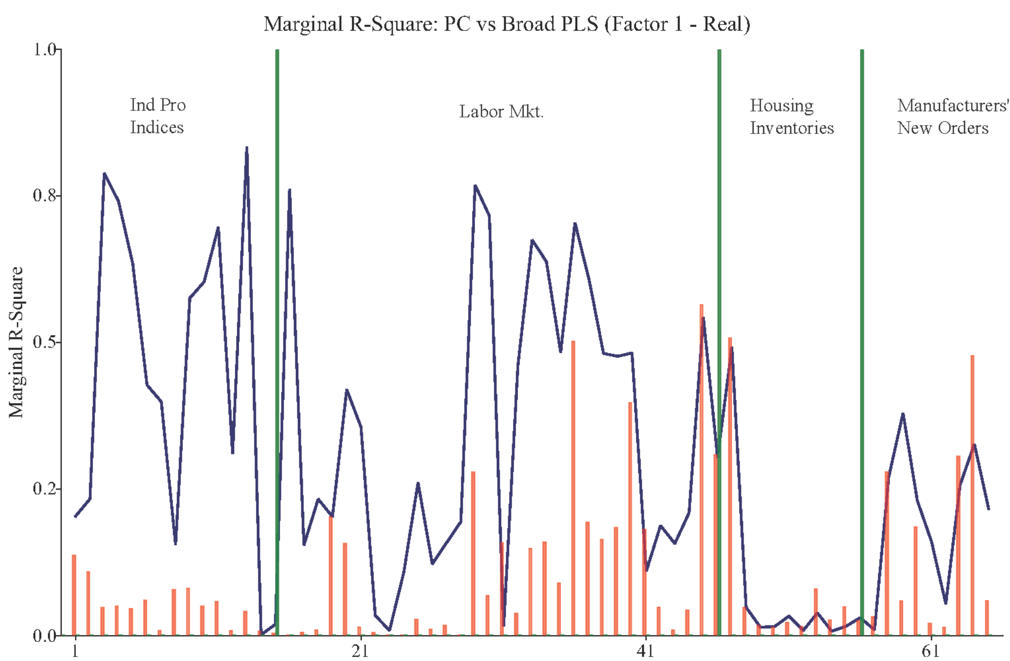
Figure 3. Marginal R^2 Analysis for PC (line) and PLS Factors (bar)



Note: We report the R^2 values that were obtained by regressing each predictor on the common factor estimate. That is, the horizontal axis is the predictor IDs.

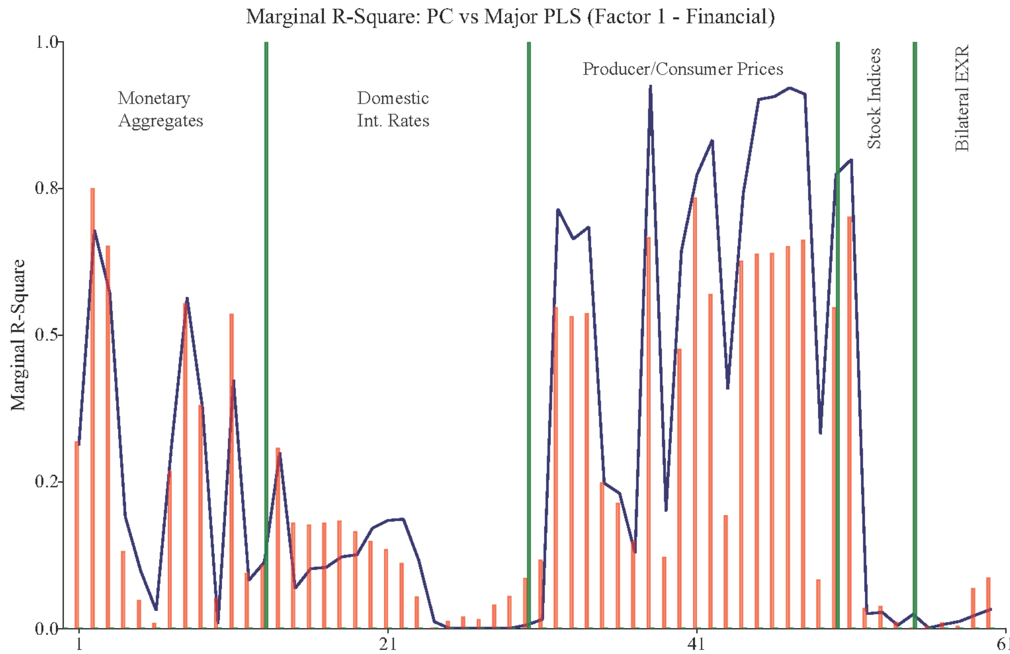
Figure 4. Marginal R^2 Analysis for PC (line) and PLS Factors (bar)

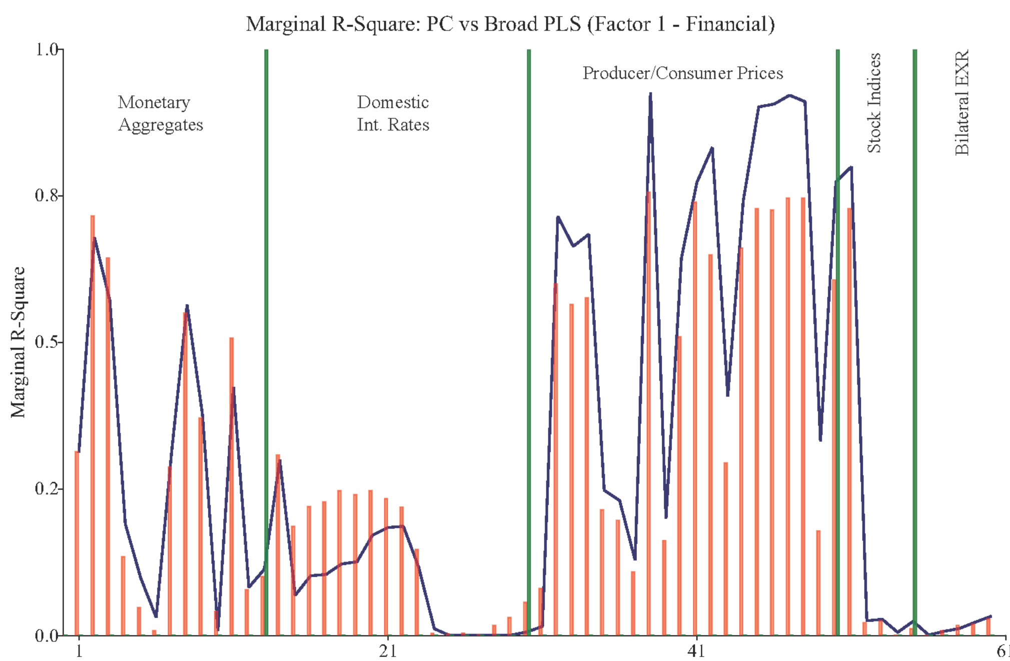




Note: We report the R^2 values that were obtained by regressing each predictor on the common factor estimate. That is, the horizontal axis is the predictor IDs.

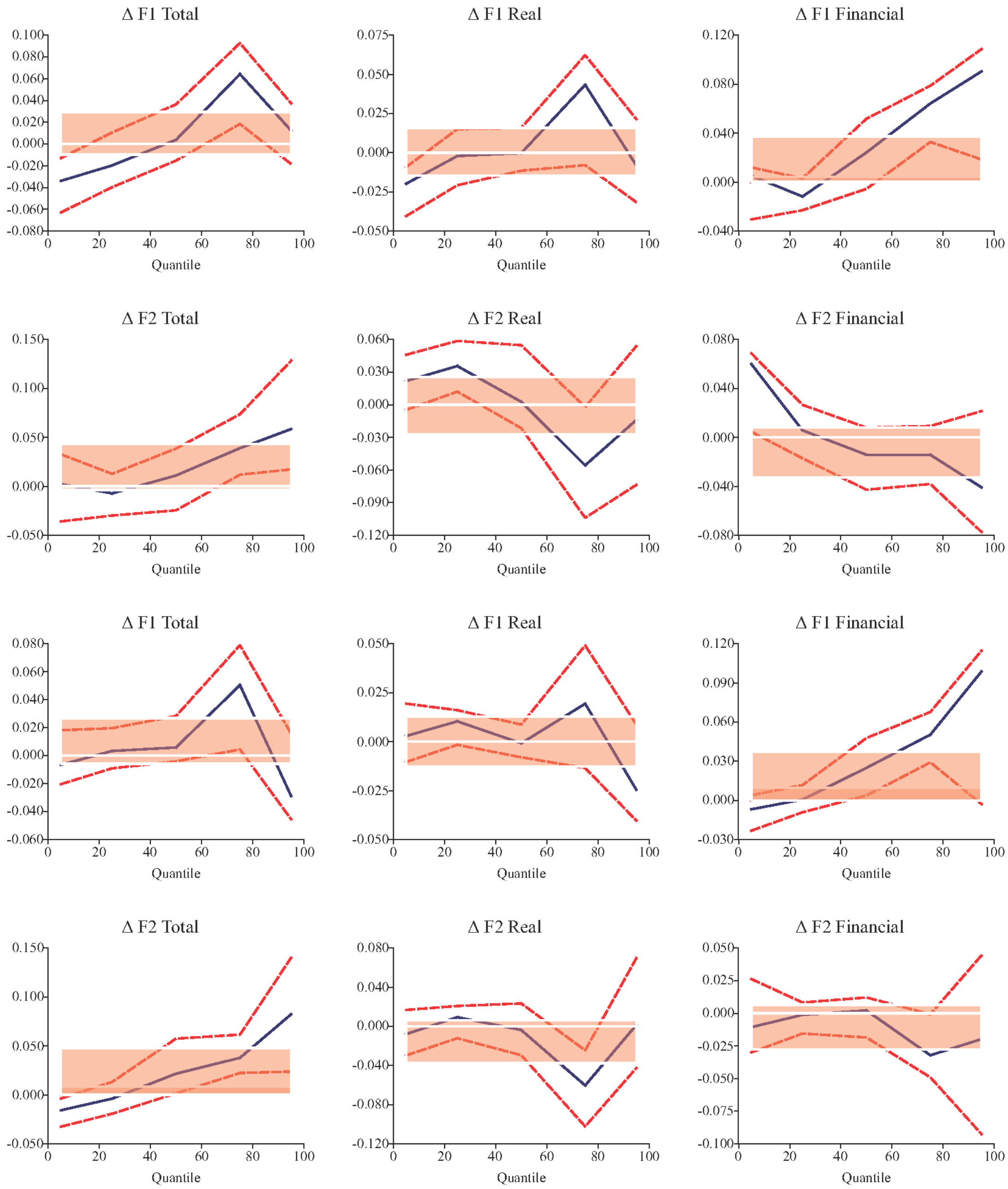
Figure 5. Marginal R^2 Analysis for PC (line) and PLS Factors (bar)





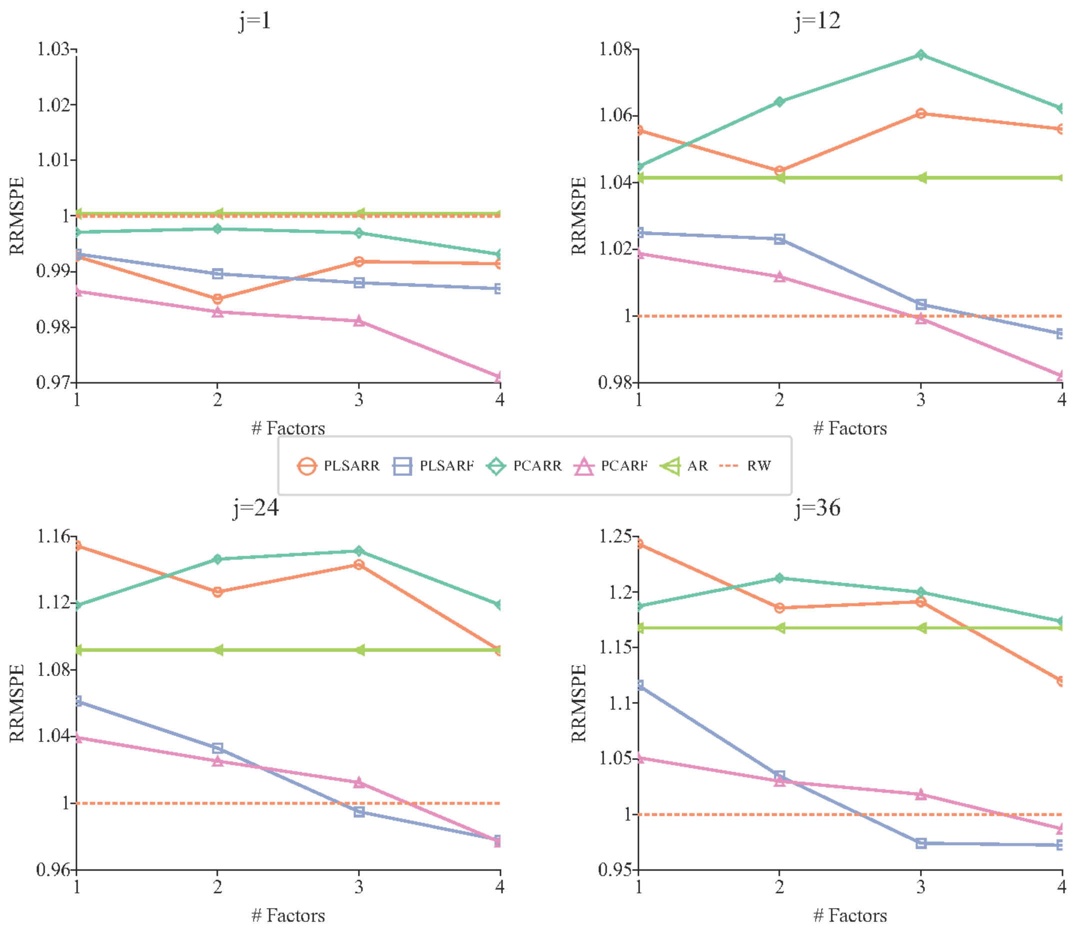
Note: We report the R^2 values that were obtained by regressing each predictor on the common factor estimate. That is, the horizontal axis is the predictor IDs.

Figure 6. Quantile Regression Results - Contemporaneous



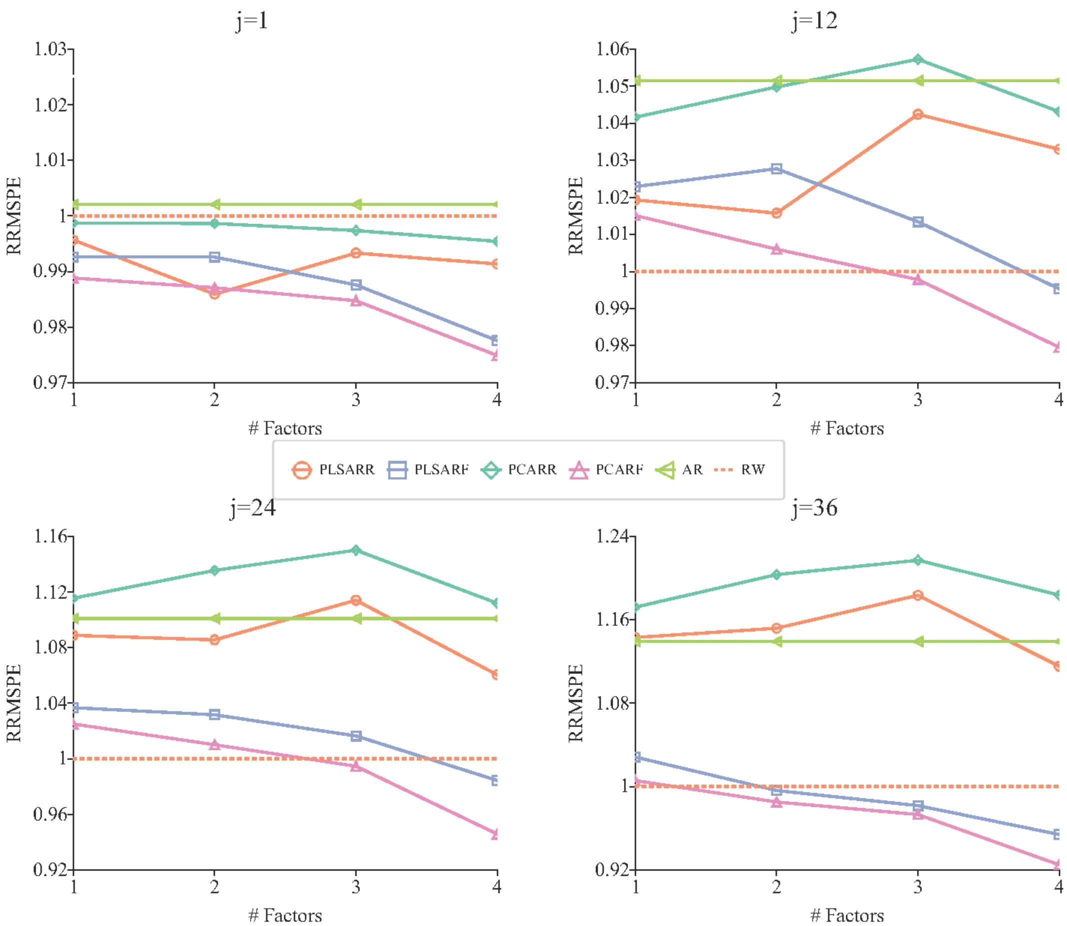
Note: We report the quantile regression coefficient estimate of the first PLS common factor with the 90% confidence band that are obtained via nonparametric bootstrap. We chose 5, 25, 50, 75, and 95 percentiles. The shaded areas indicate the 90% confidence bands of the ordinary least squares coefficient estimate. The first two rows are for the major real exchange rate, while the last two rows are for the broad real exchange rate.

Figure 7. j-Period ahead Out-of-Sample Predictability: Major Currencies (q_t^M)



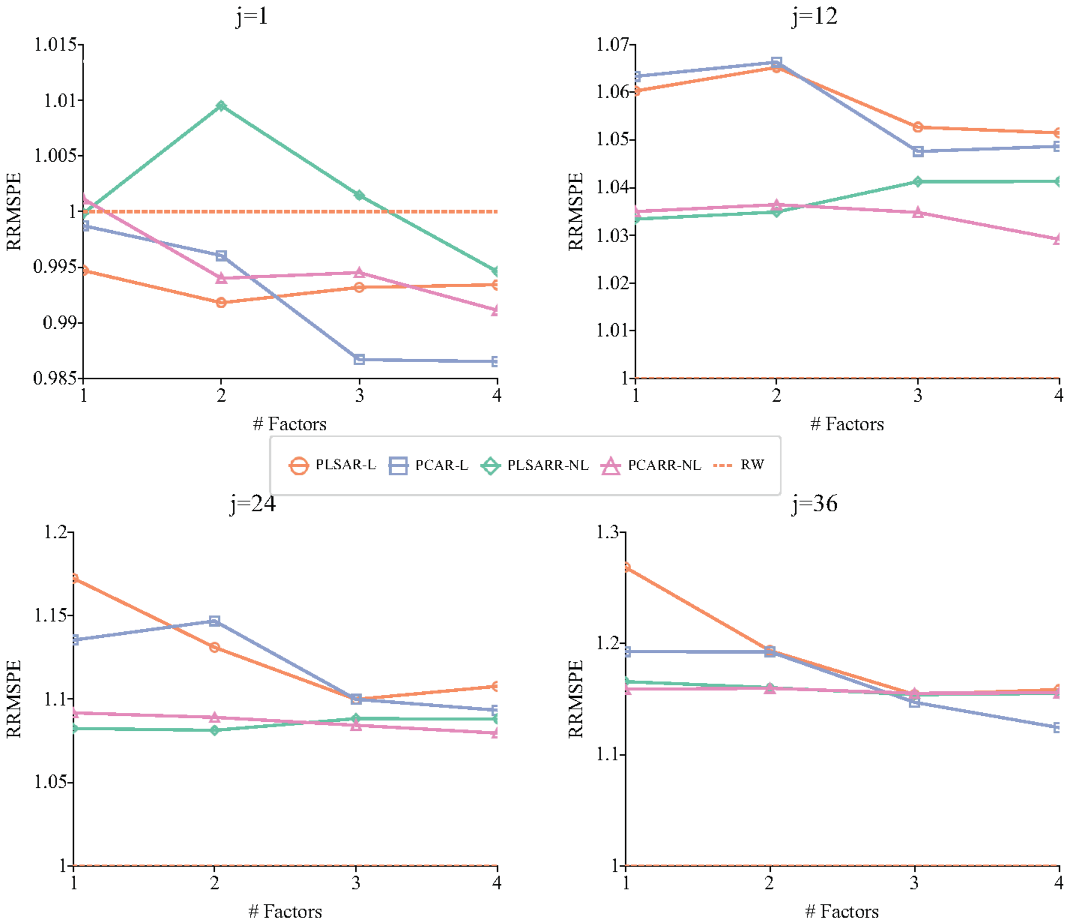
Note: We report the *RRMSPE* statistics obtained via a rolling window scheme with a 50% sample split point. *RRMSPE* denotes the ratio of the root mean squared prediction errors, which is the mean squared prediction error (*RMSPE*) from the benchmark random walk (RW) model divided by the *RMSPE* from each competing model. By construction, the *RRMSPE* of the RW model is one and the values that exceed 1 imply that the model outperforms the RW model.

Figure 8. j**-Period ahead Out-of-Sample Predictability: Broad Currencies** (q_t^B)



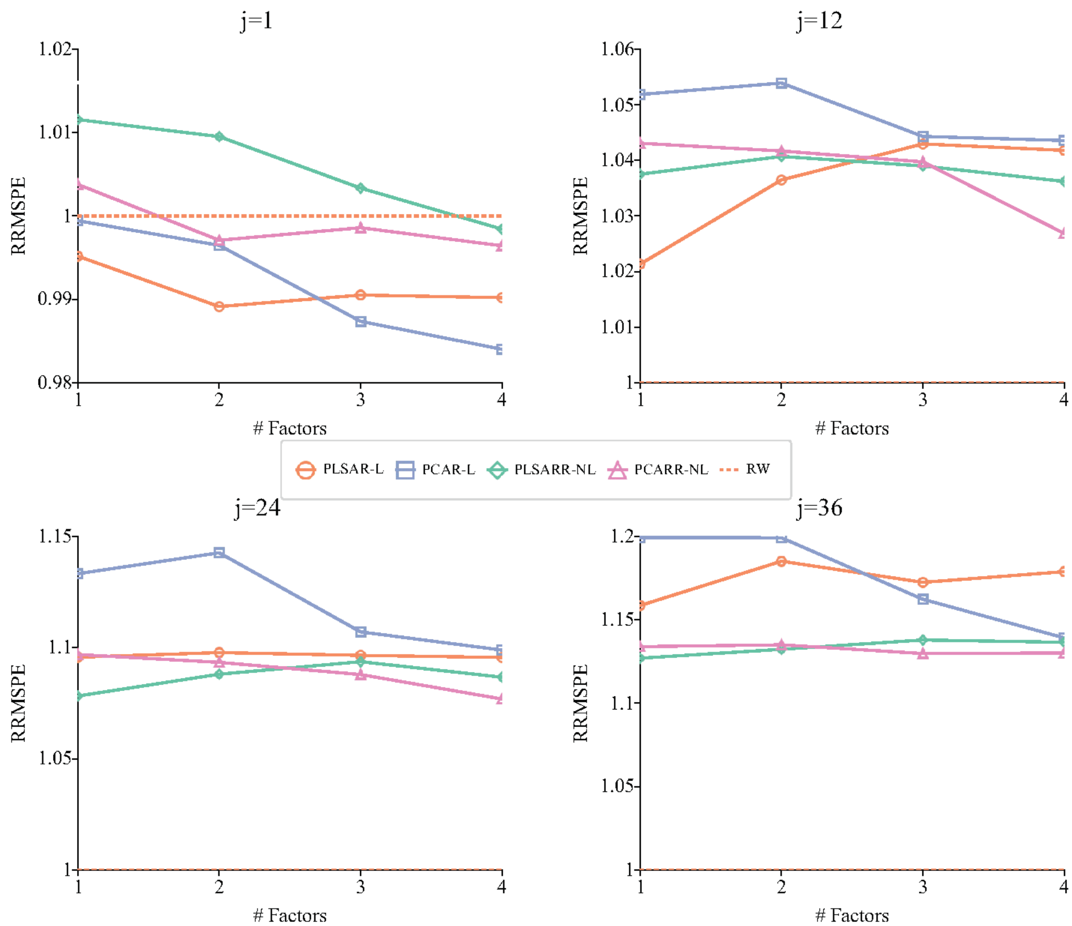
Note: We report the $RRMSPE$ statistics obtained via a rolling window scheme with a 50% sample split point. $RRMSPE$ denotes the ratio of the root mean squared prediction errors, which is the mean squared prediction error ($RMSPE$) from the benchmark random walk (RW) model divided by the $RMSPE$ from each competing model. By construction, the $RRMSPE$ of the RW model is one and the values that exceed 1 imply that the model outperforms the RW model.

Figure 9. j**-Period ahead Out-of-Sample Predictability: Labor Factors** (q_t^M)



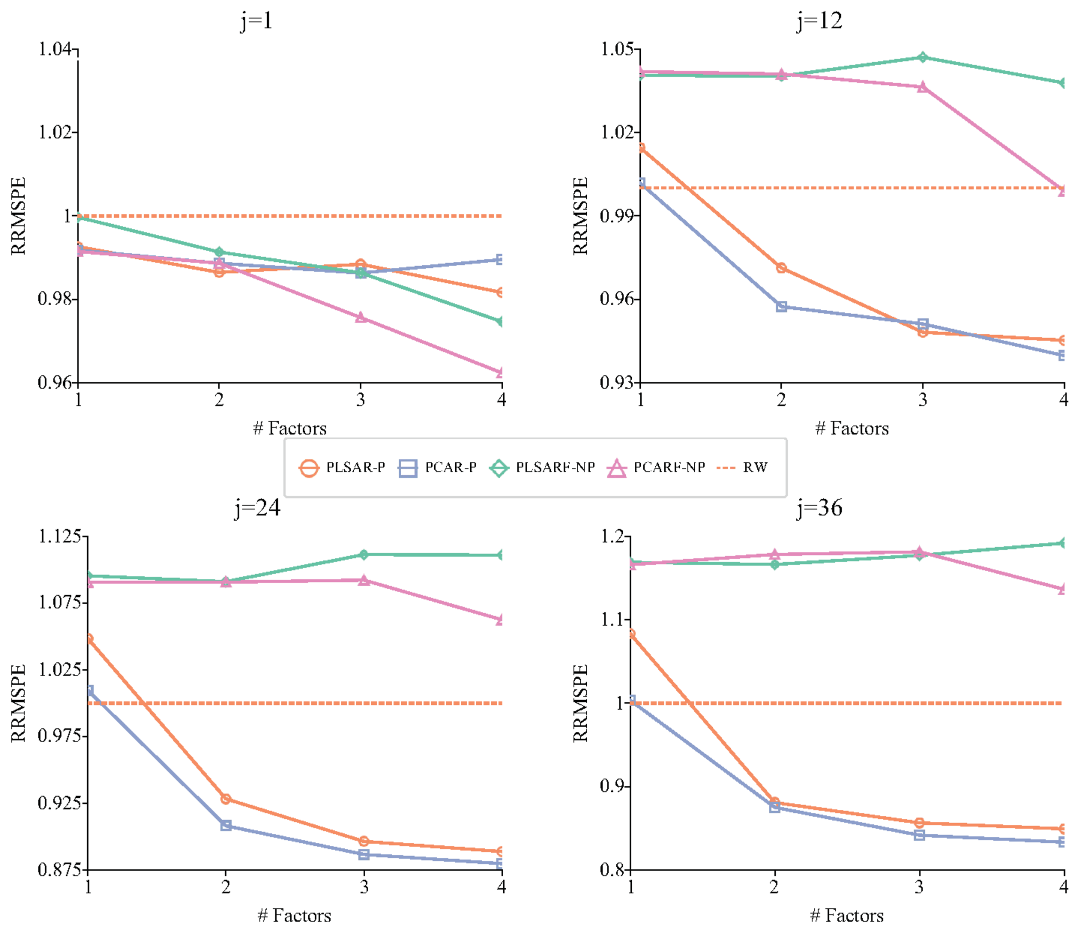
Note: We report the $RRMSPE$ statistics obtained via a rolling window scheme with a 50% sample split point. $RRMSPE$ denotes the ratio of the root mean squared prediction errors, which is the mean squared prediction error ($RMSPE$) from the benchmark random walk (RW) model divided by the $RMSPE$ from each competing model. By construction, the $RRMSPE$ of the RW model is one and the values that exceed 1 imply that the model outperforms the RW model.

Figure 10. j**-Period ahead Out-of-Sample Predictability: Labor Factors** (q_t^B)



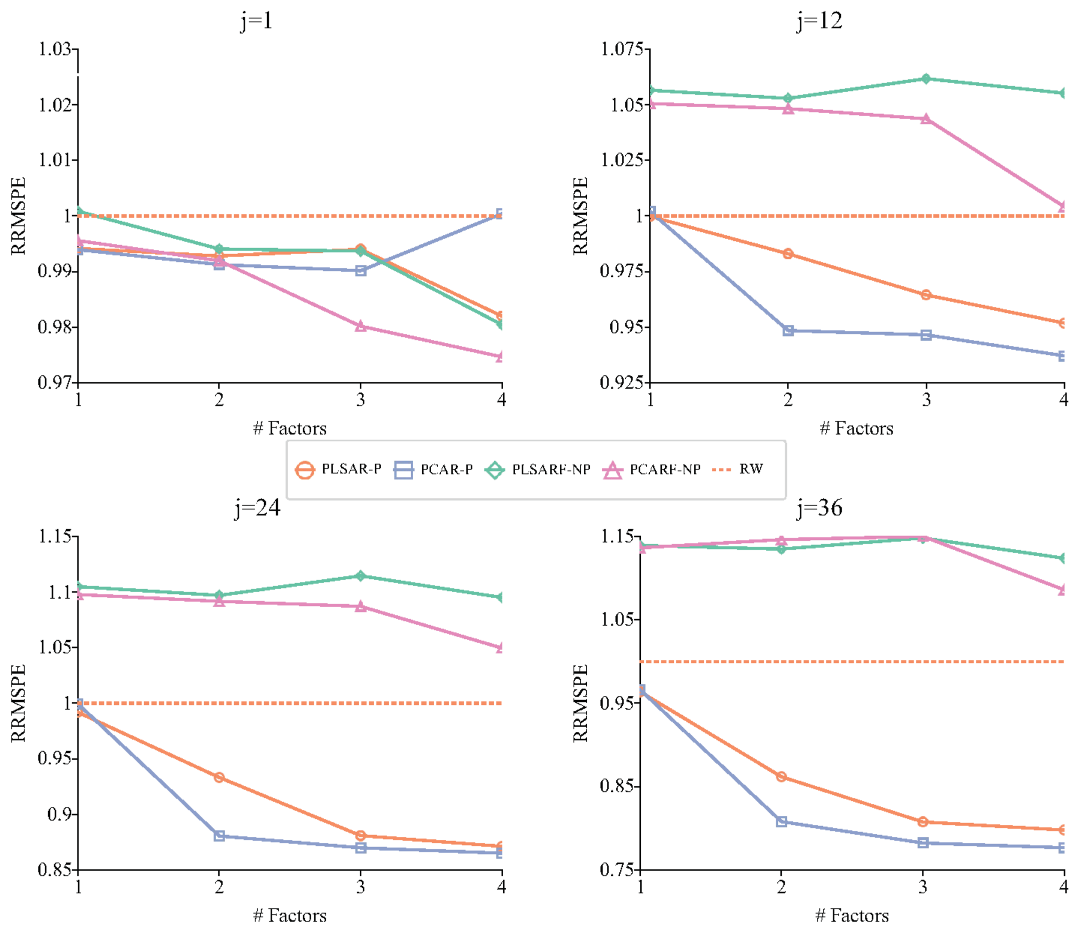
Note: We report the *RRMSPE* statistics obtained via a rolling window scheme with a 50% sample split point. *RRMSPE* denotes the ratio of the root mean squared prediction errors, which is the mean squared prediction error (*RMSPE*) from the benchmark random walk (RW) model divided by the *RMSPE* from each competing model. By construction, the *RRMSPE* of the RW model is one and the values that exceed 1 imply that the model outperforms the RW model.

Figure 11. j**-Period ahead Out-of-Sample Predictability: Price Factors** (q_t^M)



Note: We report the *RRMSPE* statistics obtained via a rolling window scheme with a 50% sample split point. *RRMSPE* denotes the ratio of the root mean squared prediction errors, which is the mean squared prediction error (*RMSPE*) from the benchmark random walk (RW) model divided by the *RMSPE* from each competing model. By construction, the *RRMSPE* of the RW model is one and the values that exceed 1 imply that the model outperforms the RW model.

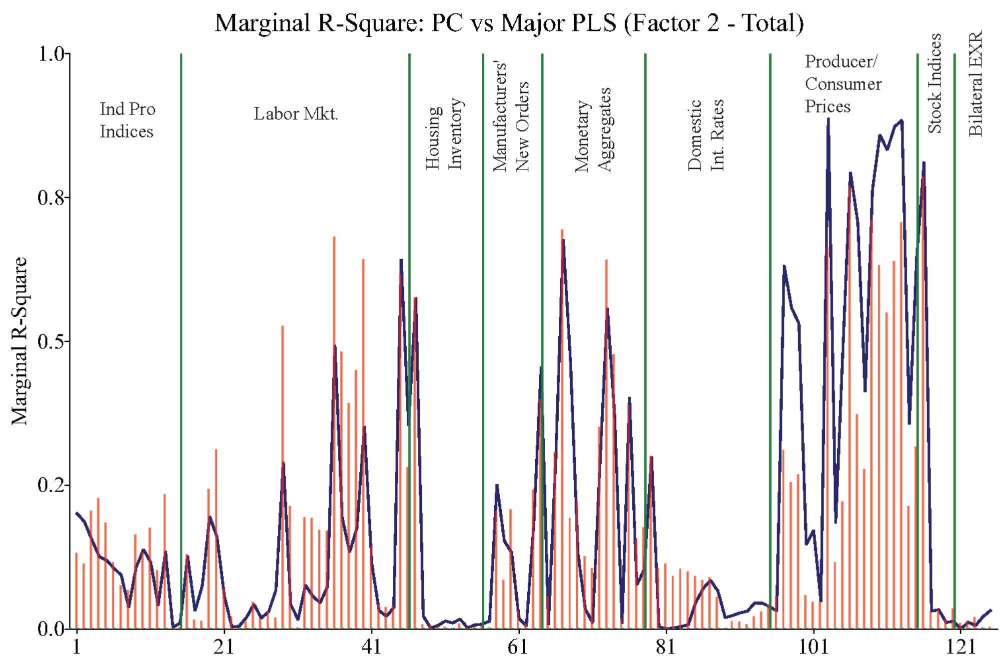
Figure 12. j-Period ahead Out-of-Sample Predictability: Price Factors (q_t^B)

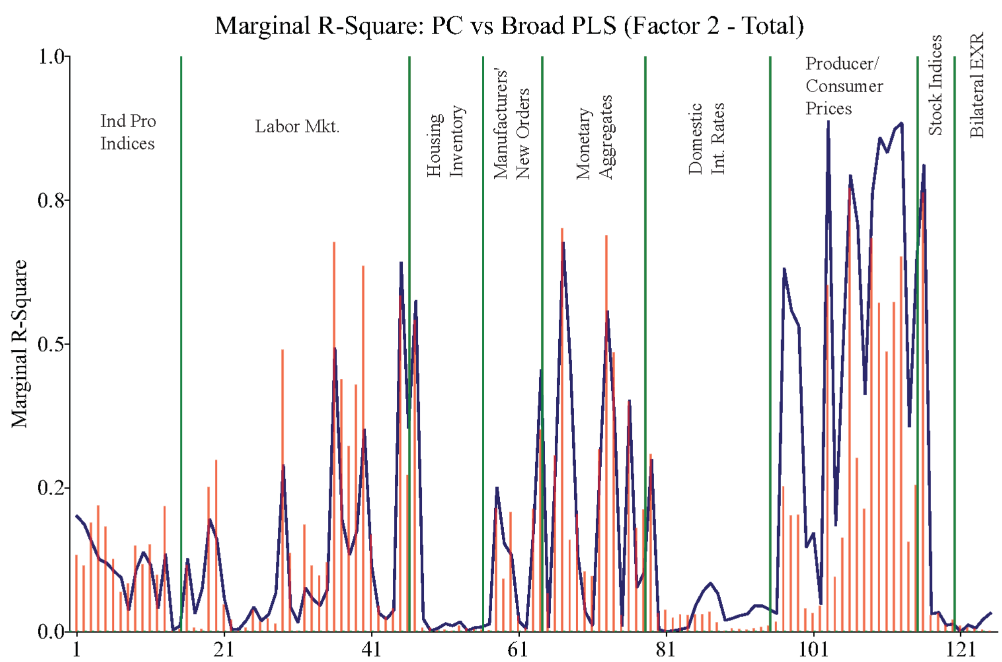


Note: We report the *RRMSPE* statistics obtained via a rolling window scheme with a 50% sample split point. *RRMSPE* denotes the ratio of the root mean squared prediction errors, which is the mean squared prediction error (*RMSPE*) from the benchmark random walk (RW) model divided by the *RMSPE* from each competing model. By construction, the *RRMSPE* of the RW model is one and the values that exceed 1 imply that the model outperforms the RW model.

Appendix

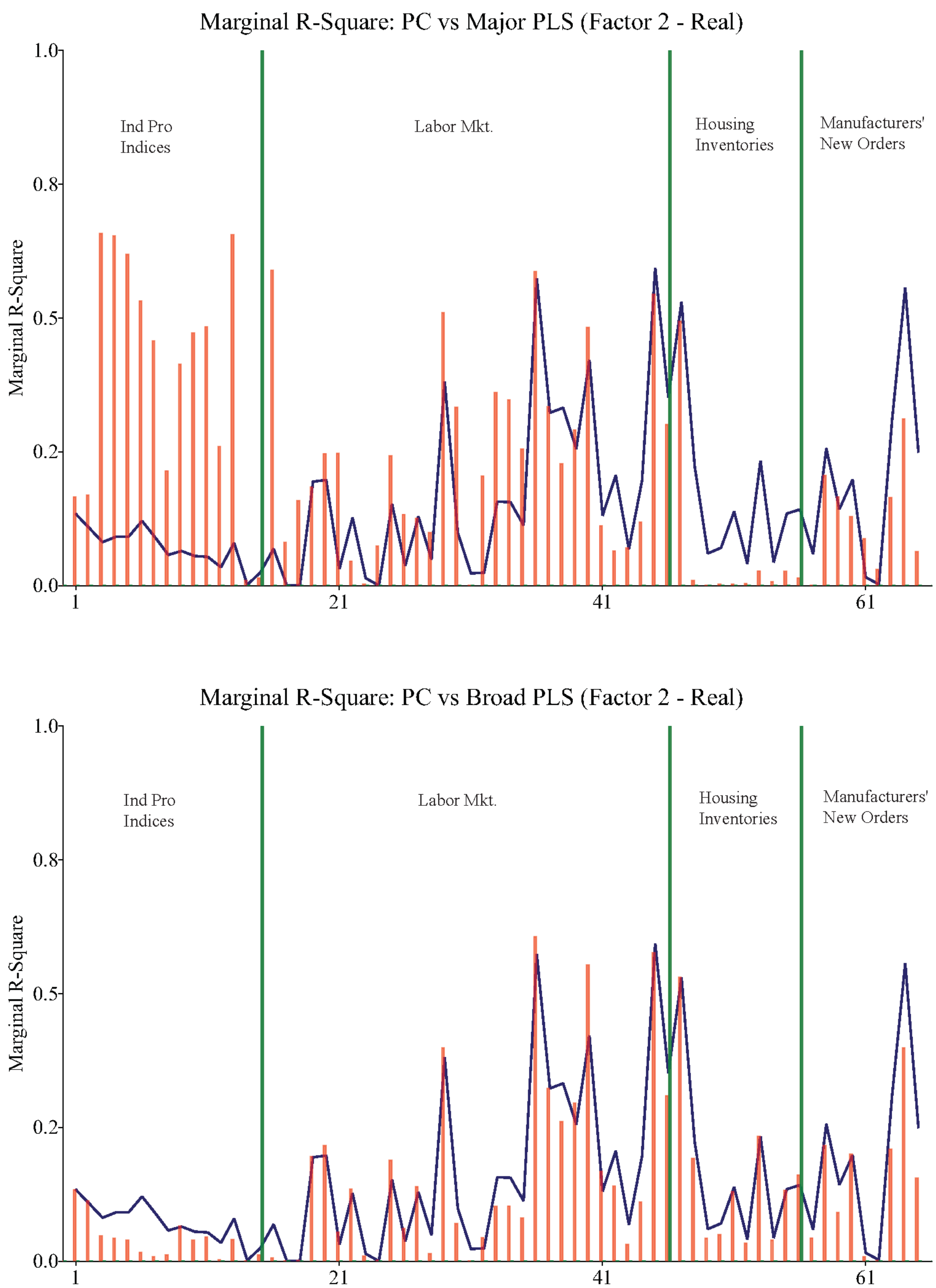
Figure A1. Marginal R^2 Analysis for PC (line) and PLS Factors (bar)





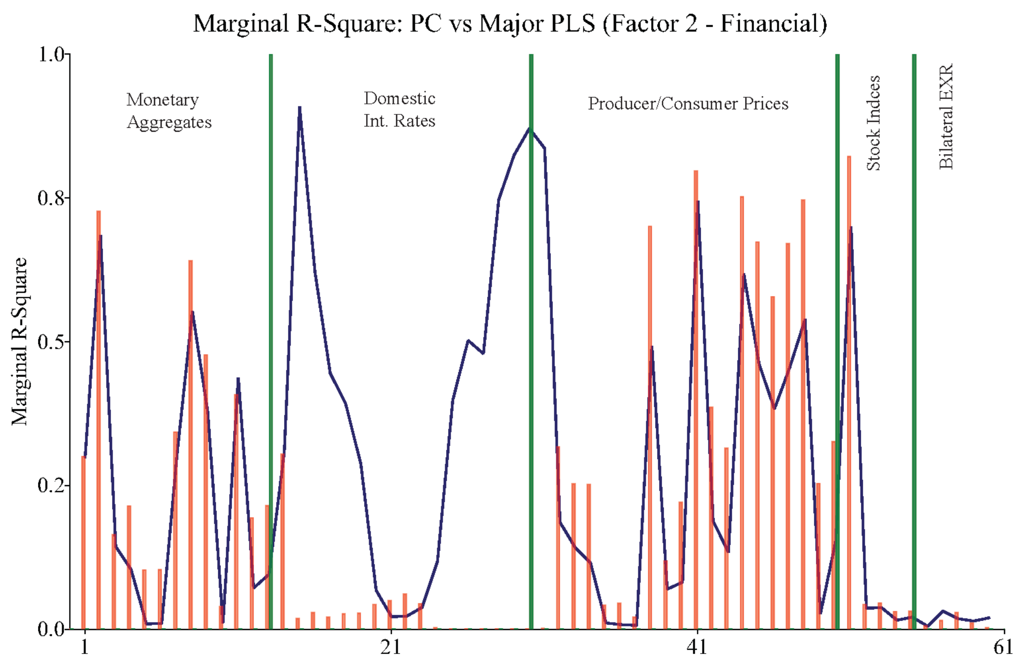
Note: We report the R^2 values that were obtained by regressing each predictor on the common factor estimate. That is, the horizontal axis is the predictor IDs.

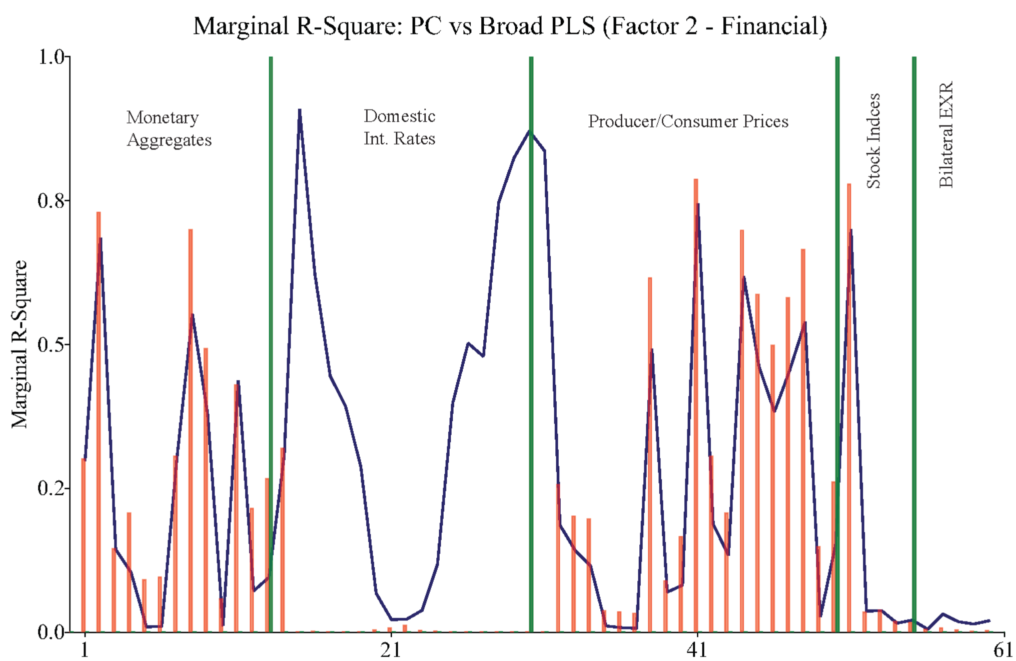
Figure A2. Marginal R^2 Analysis for PC (line) and PLS Factors (bar)



Note: We report the R^2 values that were obtained by regressing each predictor on the common factor estimate. That is, the horizontal axis is the predictor IDs.

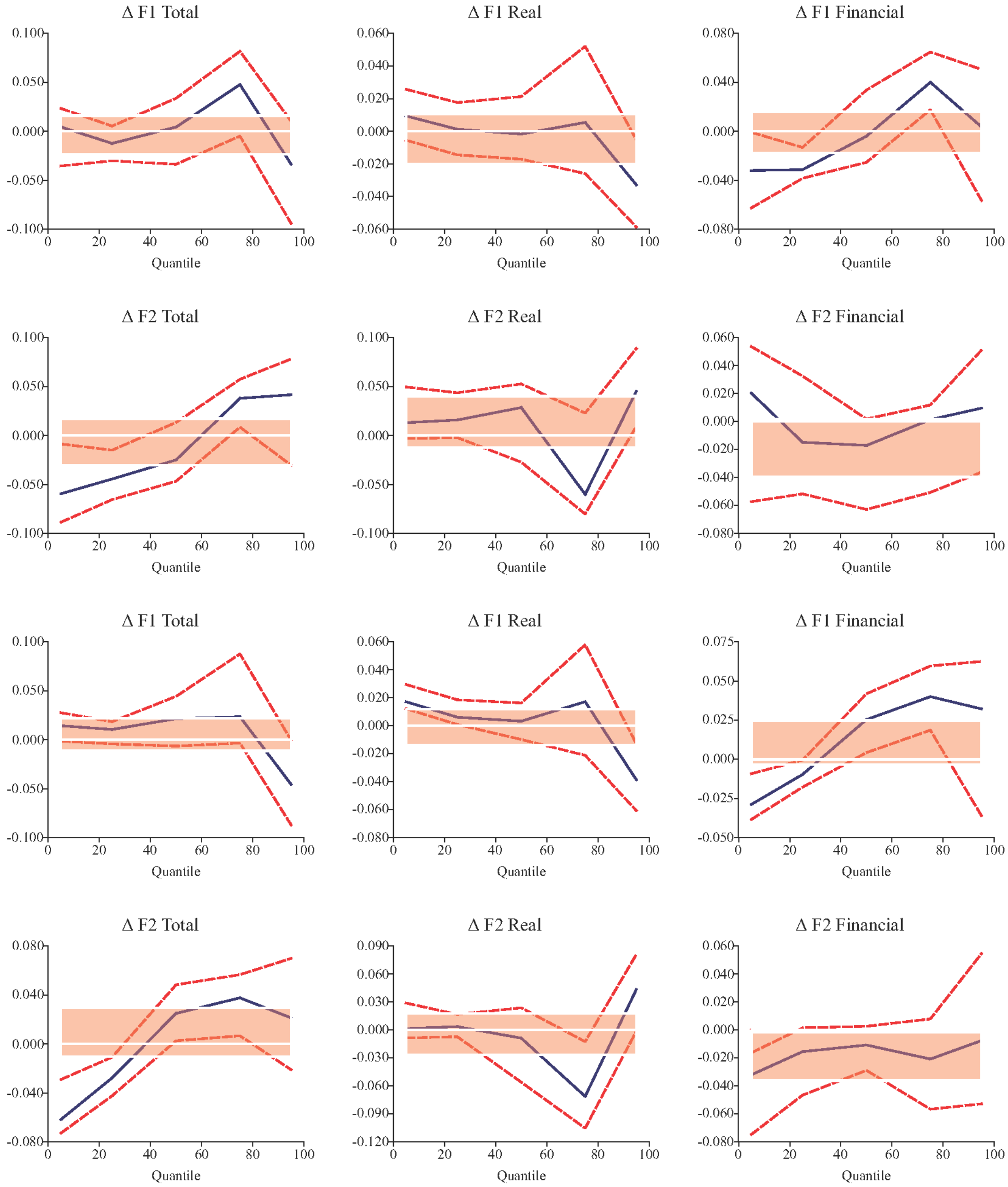
Figure A3. Marginal R^2 Analysis for PC (line) and PLS Factors (bar)





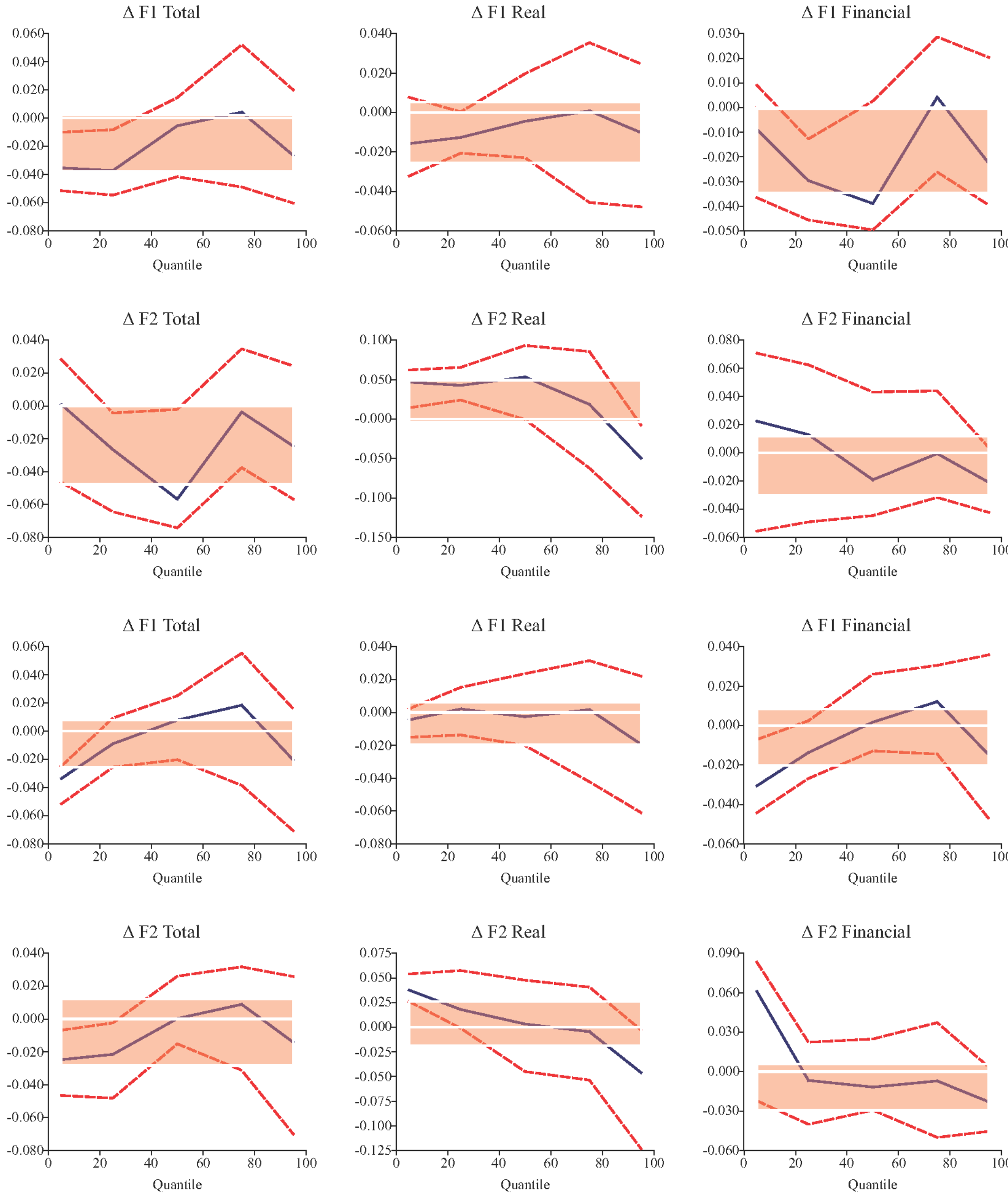
Note: We report the R^2 values that were obtained by regressing each predictor on the common factor estimate. That is, the horizontal axis is the predictor IDs.

Figure A4. Quantile Regression Results - 12 Month Forecast



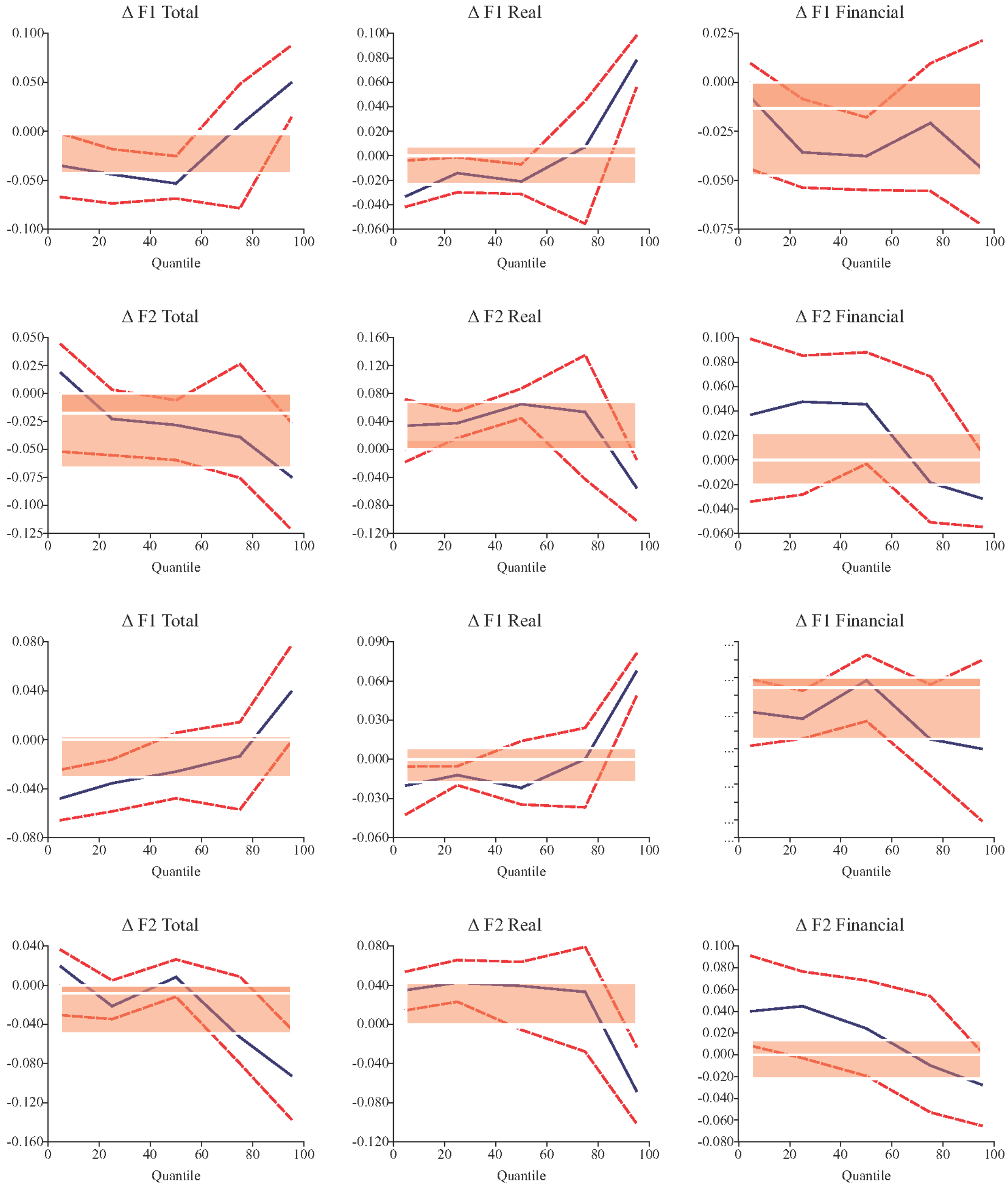
Note: We report the quantile regression coefficient estimate of the first common factor with the 90% confidence band that are obtained via nonparametric bootstrap. We chose 5, 25, 50, 75, and 95 percentiles. The shaded areas indicate the 90% confidence bands of the ordinary least squares coefficient estimate.

Figure A5. Quantile Regression Results - 24 Month Forecast



Note: We report the quantile regression coefficient estimate of the first common factor with the 90% confidence band that are obtained via nonparametric bootstrap. We chose 5, 25, 50, 75, and 95 percentiles. The shaded areas indicate the 90% confidence bands of the ordinary least squares coefficient estimate.

Figure A6. Quantile Regression Results - 36 Month Forecast



Note: We report the quantile regression coefficient estimate of the first common factor with the 90% confidence band that are obtained via nonparametric bootstrap. We chose 5, 25, 50, 75, and 95 percentiles. The shaded areas indicate the 90% confidence bands of the ordinary least squares coefficient estimate.