Auburn University
Department of Economics
Working Paper Series

London Calling: Nonlinear Mean Reversion across National Stock Markets

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AUWP 2017-05

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May 2017

Abstract

This paper revisits empirical evidence of mean reversion of relative stock prices in international stock markets. We implement a strand of univariate and panel unit root tests for linear and nonlinear models of 18 national stock indices during the period 1969 to 2012. Our major findings are as follows. First, we find little evidence of linear mean reversion irrespective of the choice of a reference country. Employing panel tests yields the same conclusion once the cross-section dependence is controlled. Second, we find strong evidence of nonlinear mean reversion when the UK serves as a reference country, calling attention to the stock index in the UK. Choosing the US as a reference yields very weak evidence of nonlinear stationarity. Third, via extensive Monte Carlo simulations, we demonstrate a potential pitfall in using panel unit root tests with cross-section dependence when a stationary common factor dominates nonstationary idiosyncratic components in small samples.

Key words: Unit Root Test; Exponential Smooth Transition Autoregressive (ESTAR) Unit Root Test; Nonlinear Panel unit root test; Panel Analysis of Nonstationarity in Idiosyncratic and Common Components (PANIC)

JEL Classification: C22, G10, G15

*Special thanks go to Randy Beard and Michael Stern for helpful suggestions. We also thank seminar participants at Auburn University, the 78th MEA annual meeting, the 84th SEA annual meeting and Cleveland State University.

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1 Introduction

It is an interesting question in international finance whether asset arbitrage in international stock markets implies that deviations between stock indices are short-lived. If so, one can diversify international portfolios by short-selling well performing assets and purchasing poorly performing assets to obtain excess returns as shown by Balvers et al. (2000). Such a strategy is called the contrarian investment strategy, and it may imply that stocks become less risky in the long run and are attractive for long-term investors (Spierdijk et al. (2012)). On the contrary, if deviations are permanent, one should short worse performing assets while buying better performing ones, because winner-loser reversals are not likely to happen. This is called the momentum strategy.

Since the end of the 1980s, a lot of research work has examined mean reversion in international stock markets. Fama and French (1988) and Poterba and Summers (1988) were the first to provide the evidence in favor of mean reversion. Fama and French state that 25-40% of the variation in 3-5 year stock returns can be attributed to negative serial correlation. Poterba and Summers (1988) found that a substantial part of the variance of the US stock returns is due to a transitory component. However, Richardson and Smith (1991) showed that if the small-sample bias is controlled, there is be no evidence for long-term mean reversion. Kim et al. (1991) report very weak evidence of mean reversion in the post-war era. Jegadeesh (1991) shows that mean reversion in stock prices is entirely concentrated in January.

An array of researchers investigated possible cointegration properties of the stock indices and their fundamental variables. For example, Campbell and Shiller (2001) examine the mean-reverting behavior of the dividend yield and price-earnings ratio over time. If stock prices are high in comparison to company fundamentals, it is expected that adjustment toward an equilibrium will be made. They find that stock prices contribute most to adjusting the ratios towards an equilibrium level.

Balvers et al. (2000) considered relative stock price indices of eighteen OECD countries compared to a world index to get around the difficult task of specifying a fundamental or trend path. Under the assumption that the difference between the trend path of one country's stock price index and that of a reference index is stationary, and that the speeds of mean reversion in different countries are similar, they found substantial evidence of mean reversion of relative stock price indices with a half-life of approximately 3.5 years. Similar evidence has been reported by Chaudhuri and Wu (2004) for 17 emerging equity markets.

The assumption of a constant speed of mean reversion may be too restrictive, however, since the speed of mean reversion may depend on the economic and political environment, and also it may change over time. For example, Kim et al. (1991) conclude that mean reversion is a pre-World War II phenomenon only. Poterba and Summers (1988) find that the Great Depression had a significant influence on the speed of mean reversion. Additionally, their panel unit root test may have a serious size distortion problem in the presence of cross-section dependence (Phillips and Sul (2003)). Controlling for cross-section dependence, Kim (2009) reports much weaker evidence of mean reversion of relative stock prices across international stock markets.

In recent work, Spierdijk et al. (2012) employed a wild bootstrap method to get the median unbiased estimation and a rolling window approach to a long horizon data (1900-2009) for their analysis. They find that stock prices revert more rapidly to their fundamental value in periods of high economic uncertainty, caused by major economic and political events such as Great Depression and the start of World War II. They report a statistically significant mean reversion for most of their sub-sample periods, but their panel test results don't seem to match their univariate test results very well.[1] Also in their panel model, they assumed that the speed of mean reversion is constant as in Balvers et al. (2000) which contradicts

Wälti (2011) studied the relationship between stock market co-movements and monetary integration. He reports that greater trade linkages and stronger financial integration contribute to greater stock market co-movements.[2]

In the present paper, we revisit the findings by Balvers et al. (2000). We re-examine the mean reversion of the relative stock price in international stock markets by using nonlinear unit root tests in addition to linear tests. Nonlinear models have been widely used in the study of financial data to account for state-dependent stochastic behavior due to market frictions such as transaction costs; examples include, for exchange rates and law of one price, Obstfeld and Taylor (1997), Taylor et al. (2001), Lo and Zivot (2001), Sarno et al. (2004) and Lee and Chou (2013), and for stock prices or returns, Bali et al. (2008), Zhu and Zhu (2013) and Kim and Ryu (2015). Nonlinear models are also used for the study of commodity prices (for example, Balagtas and Holt (2009), Holt and Craig (2006), and Goodwin et al. (2011)) to address nonlinear adjustments towards the equilibrium due to costly transactions, government interventions, or different expectations by individuals (Arize (2011)).

Using a nonlinear unit root test (ESTAR), we find strong evidence of nonlinear mean reversion of relative stock prices when the UK serves as the reference country. We find

[1]For example, with the US benchmark, only France shows mean reversion with the univariate test but there is a solid stationarity with the panel test.

[2]Also the author concludes that lower exchange rate volatility and joint EMU membership are associated with stronger stock market comovements. The joint significance of these two variables indicates that monetary integration raises return correlations by reducing transaction costs coming from exchange rate uncertainty, and through the common monetary policy and the convergence of inflation expectations leading to more homogeneous valuations.

very little evidence of linear mean reversion irrespective of the choice of the reference index. In addition, we employ a series of panel unit roots tests: the linear panel unit root test (Pesaran (2007)) and a newly developed nonlinear panel (PESTAR) unit root test (Cerrato et al. (2011)). These tests allow different mean reversion rates across countries and also allow for cross sectional dependence. Thus, our approach is less restrictive than Balvers et al. (2000) and should give more statistically reliable results.

We find no evidence of mean reversion from these panel unit root tests, a result which seems to be inconsistent with the univariate ESTAR test results with the UK as the reference index that provide strong evidence of mean reversion. To look into this seemingly conflicting statistical result, we conducted a principal component analysis via the PANIC method developed by Bai and Ng (2004). We note empirical evidence of stationarity of the estimated first common factor or cross-section means that served as proxy variables for the common factor in Pesaran (2007) and Cerrato et al. (2011) with the UK reference. When the stationary first common factor dominates idiosyncratic components that are quite persistent or even nonstationary, the panel unit root tests that filter out the stationary common factor may yield evidence against stationarity in the short-run, while the univariate test rejects the null of nonstationarity. Via Monte Carlo simulations, we confirm this conjecture.

In sum, our findings imply that the contrarian investment would be useful when national equity prices deviate sufficient from the UK stock index, while one may employ the momentum strategy with the US as a reference.

The rest of the paper is organized as follows. Section 2 constructs our baseline model of the relative stock indices. Sections 3 and 4 report univariate and panel unit root test results, respectively. Section 5 discusses our results using a dynamic factor analysis framework. Section 6 establishes and provides simulation results. Section 7 concludes.

2 The Baseline Model

We use a model of a stochastic process for national stock indices, employed in Kim (2009), that is a revised model of Balvers et al. (2000).

Let $p_{i,t}$ be the the national stock index and $f_{i,t}$ be its fundamental value in country i, all expressed in natural logarithms. We assume that $p_{i,t}$ and $f_{i,t}$ obey nonstationary stochastic processes. If $p_{i,t}$ and $f_{i,t}$ share a *unique* nonstationary component, deviations of $p_{i,t}$ from $f_{i,t}$ must die out eventually. That is, $p_{i,t}$ and $f_{i,t}$ are cointegrated with a known cointegrating vector $[1 \quad -1]$. Such a stochastic process can be modeled by the following error correction model,

$$\Delta(p_{i,t+1} - f_{i,t+1}) = a_i - \lambda_i(p_{i,t} - f_{i,t}) + \varepsilon_{i,t+1}, \tag{1}$$

where $0 < \lambda_i < 1$ represents the speed of convergence and $\varepsilon_{i,t}$ is a mean-zero stochastic process from an *unknown* distribution. The fundamental value $f_{i,t}$ is not directly observable, but is assumed to obey the following stochastic process,

$$f_{i,t} = c_i + p_{w,t} + \upsilon_{i,t}, \tag{2}$$

where c_i is a country-specific constant, $p_{w,t}$ denotes a reference stock index price, and $\upsilon_{i,t}$ is a zero-mean, possibly *serially correlated* stationary process from an *unknown* distribution.

Combining Eqs. (1) and (2) and after controlling for serial correlation, we obtain the following augmented Dickey-Fuller equation for the relative stock price, $r_{i,t} = p_{i,t} - p_{w,t}$, for country i,

$$r_{i,t} = \alpha_i + \rho_i r_{i,t-1} + \sum_{j=1}^{k} \beta_{i,j} \Delta r_{i,t-j} + \eta_{i,t}. \tag{3}$$[3]

That is, $r_{i,t}$ measures deviations of the stock index in country i from a reference index at time t. Note that $\rho_i \in (0, 1)$ is the persistence parameter of the stock index deviation for country i.

It is easy to see that Eq. (3) is equivalent to Eq. (4) in Balvers et al. (2000). It should be noted, however, that Eq. (3) does not require the homogeneity assumption for the convergence rate λ.[4] Furthermore, we do not need to impose any distributional assumptions on η_t.[5]

3 Univariate Unit Root Tests

3.1 Data

Following Balvers et al. (2000), we use a panel of yearly observations of the Morgan Stanley Capital International (MSCI) stock price indices for 18 Developed Market group countries during the period 1969 to 2012 to test for mean reversion. The observations are end-of-period (December) value-weighted gross index prices in US dollar terms that include dividends. Table 1 provides summary statistics for the deviations of the logarithm of the relative stock indices of 17 countries to the two reference countries, US and UK respectively.

The mean values of the index deviations relative to the US index range from -0.981 for

[3] Refer to Kim (2009) for derivation of the equation.

[4] In order to derive Eq.(4) in Balvers et al. (2000) from their Eq.(1), one has to assume $\lambda^i = \lambda^w$ where w refers to the reference country. Otherwise, the unobserved term P^{*i}_{t+1} in their Eq.(1) cannot be cancelled out and remains in their estimation equation.

[5] Balvers et al. (2000) use Andrews (1993)'s methodology to calculate the median unbiased estimates and the corresponding confidence intervals, which requires Gaussian error terms.

Italy to 1.583 for Hong Kong, and the standard deviations vary from 0.235 for UK to 0.709 for Japan. The mean values of the stock index deviations relative to the UK index range from -1.252 for Italy to 1.312 for Hong Kong, and the standard deviations vary from 0.235 for US to 0.639 for Japan. We also checked the normality of the data using the Jarque-Bera test. The test rejects the null hypothesis of normality at the 5% significance level for 3 and 6 countries with the US index and with the UK index, respectively.[6]

Table 1 around here

In the following two subsections for univariate tests we will drop country specific index i in the formulas for notational simplicity.

3.2 Linear Unit-Root Test Analysis

We first implement univariate linear unit root tests employing the following conventional augmented Dickey-Fuller (ADF) test,

$$r_t = \alpha + \psi t + \rho r_{t-1} + \sum_{j=1}^{k} \beta_j \Delta r_{t-j} + \eta_t, \tag{4}$$

where $\psi = 0$ for the ADF test with an intercept only. We implemented the test for the deviations of the logarithms of national stock price indices relative to that of the reference country (US or UK). Results are reported in Table 2.

When the US index serves as the reference, the test rejects the null of nonstationarity for 6 out of 17 countries at the 10% significance level when an intercept is included (Belgium, France, Germany, Hong Kong, Norway, and the UK). Allowing for trend stationarity, the test rejects for one additional country (Sweden) at the 5% level. When the UK index is used as the reference, the test rejects the null for 6 out of 17 countries when an intercept is included. Allowing the time trend, the test rejects for 3 additional countries (Italy, the Netherlands, and Sweden).

A rejection of the null hypothesis of nonstationarity implies that the national stock index tends to synchronize with that of the reference country, because deviations of the stock price from the reference index are not permanent. That is, short-selling a better-performing stock index and buying the other would generate financial gains on average. Put differently,

[6]The Jarque-Bera test tends to reject the null hypothesis more often for higher frequency financial data. The test unanimously rejects the null of normality when we use the monthly frequency data. All results are available from authors upon requests.

stationarity of r_t suggests that a contrarian strategy would perform well for the pair of the national stock index and the reference index.

Confining our attention only to such linear piecewise convergence, our findings imply limited evidence in favor of the contrarian strategy, even though we observe a little stronger evidence using the contrarian strategy when the UK index serves as the reference.

Table 2 around here

3.3 Nonlinear Unit-Root Test Analysis

It is known that the linear ADF test has low power when the true data generating process (DGP) is nonlinear. One way to get around this difficulty is to use a nonlinear unit-root test. For this purpose, we revise the linear model (4) to a nonlinear model by allowing transitions of the stock price deviation r_t between the stationary and the nonstationary regime. Stock prices may adjust to long-run equilibrium only when the deviation is big enough in the presence of a fixed transaction cost. Then, r_t may follow a unit root process locally around the long-run equilibrium value. We employ a variation of such stochastic processes that allows gradual transitions between the regimes. Specifically, we assume the following exponential smooth transition autoregressive process for r_t,

$$r_t = r_{t-1} + \xi r_{t-1}\{1 - \exp(-\theta r_{t-d}^2)\} + \epsilon_t, \tag{5}$$

where θ is a strictly positive scale parameter so that $0 < \exp(-\theta r_{t-d}^2) < 1$, ξ a geometric ergodicity, and d is a delay parameter. Note that when r_{t-d} is very big, that is, when national stock price indices substantially deviate from the reference index, $\exp(-\theta r_{t-d}^2)$ becomes smaller, converging to 0, which implies that the stochastic process (5) becomes a stationary AR(1) process ($1 + \xi = \rho < 1$). On the other hand, if r_{t-d} is close to zero, then r_t becomes a unit root process. Alternatively, Eq. (5) can be rewritten as

$$\Delta r_t = \xi r_{t-1}\{1 - \exp(-\theta r_{t-d}^2)\} + \epsilon_t. \tag{6}$$

Note that ξ is not identified under the unit root null hypothesis, which results in the so-called "Davies Problem." To deal with it, Kapetanios et al. (2003) transformed it using the first-order Taylor approximation as follows (assuming $d = 1$):

$$\Delta r_t = \delta r_{t-1}^3 + \epsilon_t. \tag{7}$$

They show that, under the unit root null, the least squares t-statistic for δ has the following asymptotic distribution,

$$\frac{\frac{1}{4}W(1)^2 - \frac{3}{2}\int_0^1 W(1)^2\,ds}{\sqrt{\int_0^1 W(1)^6\,ds}}, \tag{8}$$

where $W(s)$ is the standard Brownian motion defined on $s \in [0,1]$. When error terms (ε_t) are serially correlated, Eq. (7) can be augmented as follows:

$$\Delta r_t = \delta r_{t-1}^3 + \sum_{j=1}^{k} \beta_j \Delta r_{t-j} + \epsilon_t. \tag{9}$$

We tested the data for both when an intercept is included and when an intercept and time trend are included. Results are shown in Table 3.

Table 3 around here

With the US index, the test rejects the null hypothesis of nonstationarity only for two countries, Hong Kong and the UK. With the UK as the reference country, however, the test rejects the null hypothesis for 10 countries at the 10% significance level. Allowing a time trend, the test rejects the null for an additional 2 countries, the Netherlands and Sweden. In combination with the results from the linear test results, our empirical findings yield a maximum of 14 rejections out of 17 countries at the 10% significance level, while we obtained a maximum 7 rejections out of 17 when the US serves as a reference country.[7] These findings imply that the UK stock index may be used as an anchor index in constructing international equity portfolios. When deviations of national equity indices from the UK index are large, one may short better performing assets while buying worse performing assets, since winner-loser reversals are likely to happen. When the US stock index serves as the reference, one should employ the momentum strategy because deviations of equity prices seem to be permanent.

4 Panel Unit Root Tests with Cross-Section Dependence Consideration

It is known that the univariate ADF test has low power in small samples. In this section we employ a series of panel unit root tests that are known to increase power over the univariate

[7] Note that the linear test shows the relative prices of France and Norway vis-à-vis the UK are stationary, whereas the ESTAR does not. This may be due to the fact that the ESTAR test uses Taylor approximation and could miss some useful information. See Kim and Moh (2010) for some discussion on the issue.

tests (Taylor and Sarno (1998)).

As Phillips and Sul (2003) pointed out, however, the so-called first-generation panel unit root tests such as Maddala and Wu (1999), Levin et al. (2002), and Im et al. (2003) are known to be seriously over-sized (reject the null hypothesis too often) when the data are cross-sectionally dependent. We first test this issue by employing the statistic proposed by Pesaran (2004) described below in Eq. (10),

$$CD = \sqrt{\frac{2T}{N(N-1)}} \left(\sum_{i=1}^{N-1} \sum_{j=i+1}^{N} \hat{\rho}_{i,j} \right) \xrightarrow{d} N(0,1), \tag{10}$$

where $\hat{\rho}_{i,j}$ is the pair-wise correlation coefficients from the residuals of the ADF regressions (4). We report results in Table 4. The results imply a very strong degree cross-section dependence. In what follows, therefore, we employ available second-generation panel unit root tests with cross-section dependence consideration.

Table 4 around here

4.1 Linear Panel Unit-Root Test Analysis

We first employ Pesaran (2007)'s cross-sectionally augmented panel ADF (PADF) test given by

$$CIPS(N,T) = t_{N,T} = N^{-1} \sum_{i=1}^{N} t_i(N,T), \tag{11}$$

where $t_i(N,T)$ is the t-statistic for b_i from the following least squares regression,

$$\Delta r_{i,t} = a_i + b_i r_{i,t-1} + c_i \bar{r}_{t-1} + \sum_{j=0}^{p} d_{ij} \Delta \bar{r}_{t-j} + \sum_{j=1}^{p} \delta_{ij} \Delta r_{i,t-j} + e_{i,t}. \tag{12}$$

Here, $\bar{r}_t$ is the cross-section average at time t, which proxies the common factor component for $i = 1, ..., N$. Note that this is a cross-sectionally augmented version of the IPS (Im et al. (2003)) test.

We report test results in Table 5. In contrast to empirical evidence from Balvers et al. (2000), we obtain very weak panel evidence of stationarity even at the 10% significance level when we control for cross-section dependence, irrespective of the choice of the reference country. This implies that the strong evidence of stationarity in Balvers et al. (2000) could have been due to size distortion caused by a failure to account for the cross-section dependence.

Table 5 around here

4.2 Nonlinear Panel Unit-Root Test Analysis

We next explore the panel evidence of nonlinear stationarity by employing a test proposed by Cerrato et al. (2011). This test is an extension of the nonlinear ESTAR unit root test by Kapetanios et al. (2003) to a panel version test in combination with the methodology suggested by Pesaran (2007) to address the issue of cross-section dependence.

For this, we rewrite Eq. (6) as the following set of equations,

$$\Delta r_{i,t} = \xi_i r_{i,t-1}\{1 - \exp(-\theta r_{i,t-d}^2)\} + \epsilon_{i,t}, \text{ and } \epsilon_{i,t} = \delta_i f_t + u_{i,t}, \tag{13}$$

where δ_i is a country-specific factor loading, f_t is a common factor, and $u_{i,t}$ is a (possibly serially correlated) idiosyncratic shock. Cerrato et al. (2011) suggest the following nonlinear cross-section augmented IPS-type statistics,

$$t_{N,T} = N^{-1} \sum_{i=1}^{N} t_i(N, T), \tag{14}$$

where $t_i(N, T)$ is the t-statistic for $\beta_{i,0}$ from the following least squares regression,

$$\Delta r_{i,t} = \alpha_i + \beta_{i,0} r_{i,t-1}^3 + \gamma_{i,0} \bar{r}_{t-1}^3 + \sum_{j=1}^{p} (\beta_{i,j} \Delta r_{i,t-j} + \gamma_{i,j} \Delta \bar{r}_{t-j}^3) + e_{i,t}, \tag{15}$$

where $\bar{r}_t$ is the cross-section average at time t, which proxies the common factor component for $i = 1, ..., N$. In the absence of cross-section dependence, $\gamma_{i,j} = 0$ for all i and j, and the test statistic is reduced to nonlinear ESTAR test in Eq. (9).

We report test results in Table 6. It is interesting to see that the test does not reject the null hypothesis for both reference cases at the 10% significance level. This is somewhat puzzling because we obtained strong evidence of nonlinear stationarity from the univariate ESTAR tests when the UK serves as the reference country. Since the panel test (14) has the alternative hypothesis that states that there are stationary $r_{i,t}$ for $i = 1, ..., N_1$ and $N_1 > 0$, and the univariate test rejects the null for 12 out of 17 countries, it would be natural to expect panel evidence of stationarity. Yet, we do not find it. To look into this apparent contradiction further, we turn to a dynamic factor analysis in what follows based on the following conjecture.

If the first common factor is stationary and has dominating effects on $r_{i,t}$ in the short-run, the stochastic properties of $r_{i,t}$ may resemble those of stationary variables even when the idiosyncratic component is nonstationary. Even though the nonstationary idiosyncratic component will dominate the stationary common factor in the long-run, unit root tests for finite horizon observations may reject the null of nonstationarity.

Table 6 around here

5 Dynamic Factor Analysis

In this section, we attempt to understand seemingly inconsistent statistical evidence from the univariate and the panel unit root test when the UK serves as the base country. We note that the panel unit root tests from the previous section control for the cross-section dependence by taking and including the first common factor in the regression. We employ the following factor structure motivated by the framework of the PANIC method by Bai and Ng (2004), described as follows. First we write

$$\begin{aligned} r_{i,t} &= a_i + \lambda_i' \mathbf{f}_t + e_{i,t}, \\ (1 - \alpha L)\mathbf{f}_t &= \mathbf{A}(\mathbf{L})\mathbf{u}_t, \\ (1 - \rho_i L) e_{i,t} &= B_i(L)\varepsilon_{i,t}, \end{aligned} \tag{16}$$

where a_i is a fixed effect intercept, $\mathbf{f}_t = [f_1 \ldots f_r]'$ is a $r \times 1$ vector of (latent) common factors, $\lambda_i = [\lambda_{i,1} \ldots \lambda_{i,r}]'$ denotes a $r \times 1$ vector of factor loadings for country i, and $e_{i,t}$ is the idiosyncratic error term. $\mathbf{A}(L)$ and $B_i(L)$ are lag polynomials. Finally, we assume that $\mathbf{u}_t, \varepsilon_{i,t}$, and λ_i are mutually independent.

Estimations are carried out by the method of principal components. When $e_{i,t}$ is stationary, $\mathbf{f}_t$ and λ_i can be consistently estimated irrespective of the order of $\mathbf{f}_t$. If $e_{i,t}$ is integrated, however, the estimator is inconsistent because a regression of $r_{i,t}$ on $\mathbf{f}_t$ is spurious. PANIC avoids such a problem by applying the method of principal components to the first-differenced data. That is,

$$\Delta r_{i,t} = \lambda_i' \Delta \mathbf{f}_t + \Delta e_{i,t} \tag{17}$$

for $t = 2, \cdots, T$. Let $\Delta \mathbf{r}_i = [\Delta r_{i,2} \cdots \Delta r_{i,T}]'$ and $\Delta \mathbf{r} = [\Delta \mathbf{r}_1 \cdots \Delta \mathbf{r}_N]$. After proper normalization, the method of principal components for $\Delta \mathbf{r} \Delta \mathbf{r}'$ yields estimated factors $\Delta \hat{\mathbf{f}}_t$, the associated factor loadings $\hat{\lambda}_i$, and the residuals $\Delta \hat{e}_{i,t} = \Delta r_{i,1} - \hat{\lambda}_i' \Delta \hat{\mathbf{f}}_t$. Re-integrating these, we obtain the following

$$\hat{\mathbf{f}}_t = \sum_{s=2}^{t} \Delta \hat{\mathbf{f}}_s, \quad \hat{e}_{i,t} = \sum_{s=2}^{t} \Delta \hat{e}_{i,s} \tag{18}$$

for $i = 1, \cdots, N$.

Bai and Ng (2004) show that when $k = 1$, the ADF test with an intercept can be used to test the null of a unit root for the single common component $\hat{\mathbf{f}}_t$. For each idiosyncratic component $\hat{e}_{i,t}$, the ADF test with no deterministic terms can first be applied. Then, a panel unit root test statistic for these idiosyncratic terms can be constructed as follows:

$$P_{\hat{e}} = \frac{-2\sum_{i=1}^{N} \ln p_{\hat{e}_i} - 2N}{2\sqrt{N}} \xrightarrow{d} N(0,1). \tag{19}$$

In Table 7, we report the linear and nonlinear unit root test for the estimated first common factor. The tests reject the null of nonstationarity only for the case with the UK, which implies that the first common factor is likely to be stationary.

Table 7 around here

In Figure 1, we plot the first five common factors and their relative portions of the stock price deviations with the UK as the reference. Starting with initial 50% observations, we use a recursive method to repeatedly estimate five common factors along with shares of variations explained by each common factor from each set of samples. The graph shows that the first common factor explains roughly about 45% of total variations, while other common factors play substantially smaller roles.[8] Put differently, the *stationary* first common factor seems to play a dominant role in determining the stochastic properties of $r_{i,t}$ in the short-run.[9] Also, we estimate idiosyncratic factor loading coefficients (λ_i) in Eq. (16) that measure country-specific degrees of dependence of $r_{i,t}$ on the common factor. Estimates are reported in Figure 2. The results show that the first common factor represents each of $r_{i,t}$ fairly well with a few exceptions of Hong Kong and Singapore.[10]

Figure 1 around here

Figure 2 around here

Note on the other hand that this first common factor resembles the dynamics of the proxy common factor (cross-section means) in Eqs. (12) and (15) as we can see in Figure 2.

Figure 3 around here

[8]Similar patterns were observed when the US is the reference country.

[9]It will be eventually dominated by nonstationary idiosyncratic component in the long-run.

[10]Similar patterns were again observed when the US serves as the reference country.

In addition to evidence of the linear and nonlinear stationarity of the common factor with the UK shown in Table 7, we compare the speeds of transitions from the ESTAR model specification for the common factors with the US and with the UK. For this purpose, we estimate the scale parameter θ in Eq. (5) via the nonlinear least squares (NLLS) method to evaluate the speed of transitions across the stationarity and nonstationarity regimes. Note that we cannot estimate ξ and θ separately in Eq. (6). Following Kapetanios et al. (2003), we assume $\xi = -1$.

We report a sample transition function estimate along with the 95% confidence bands in Figure 3. We note that the transition function for the common factor with the US reference may be consistent with nonstationarity, because the 95% confidence band of θ hits the zero lower bound, and we cannot reject the possibility of a single regime, which is the nonstationarity regime.[11] With the UK, the confidence band of the transition function remains compact ($\hat{\theta}$ was 1.308 and the standard error was 0.570).

Figure 4 around here

This evidence explains why the panel unit root tests fail to reject the null of nonstationarity, even when the univariate test rejects the null for many countries. To control for cross-section dependence, the test procedures incorporated in Eqs. (12) and (15) take out the dominant stationary common component, but leave the nonstationary idiosyncratic components. Hence, the panel tests might fail to reject the null of nonstationarity. However, the univariate unit root tests may reject the null because the dominant stationary component overpower the idiosyncratic component. We confirm this conjecture via Monte Carlo simulations in the next section.

6 Further investigation on Panel Results: Monte Carlo Simulation Analysis

We implement an array of Monte Carlo simulations in this section to see how plausible our conjecture from the previous section is. For this purpose, we construct 17 time series that have a factor structure with a nonlinear stationary common factor motivated by our panel ESTAR model. We assume that each of the 17 idiosyncratic components is nonstationary. That is, 17 time series variables $x_{i,t}$ share the following common component,

[11] $\hat{\theta}$ was 1.746 and the standard error was 1.018, implying a negative value for the lower bound ($\hat{\theta}-1.96\cdot s.e.$). Since θ is bounded below zero, the estimate assumes 0 for the lower bound.

$$f_t = f_{t-1} + \xi f_{t-1}\{1 - \exp(-\theta f_{t-1}^2)\} + \mu_t, \tag{20}$$

where ξ is set at -1 following Kapetanios et al. (2003) and μ_t is a mean-zero i.i.d. process. The DGP assumes $\theta = 1.308$, which is the estimate from the previous section for the 17 relative stock price indices relative to the UK. In addition to Eq. (20), we generate 17 independent nonstationary idiosyncratic components that are to be added to the common factor to construct each time series as follows:

$$x_{i,t} = \lambda_i \mathbf{f}_t + \varepsilon_{i,t} \tag{21}$$

and

$$\varepsilon_{i,t} = \varepsilon_{i,t-1} + u_{i,t}, \tag{22}$$

where $u_{i,t} \sim N(0,1)$. We used factor loading estimates (λ_i) from the PANIC estimations in the previous section. Then we employ a nonlinear univariate unit root test and the panel nonlinear unit root test. Repeating this process many times, we expect to see strong evidence of stationarity from the univariate tests and weak evidence from the panel tests in small samples, but weak evidence of stationarity from both types of tests in large samples where nonlinear idiosyncratic components must eventually dominate the stationary common factor.

We ran 3,000 Monte Carlo simulations for five different numbers of observations: 50, 100, 200, 300, and 500. In Table 8, we report the percentage of the mean and the median of the the frequency of the rejections of the null of unit roots out of 17 at the 5% significance level for the univariate ESTAR tests. For the panel test, we report the rejection rate at the 5% level for each exercise.

We confirm our conjecture by these simulations. When the number of observation is small, e.g., 50, the univariate ESTAR test rejects the null for many series about 50% frequency on average. This tendency disappears quickly as the number of observation increases. For example, when the number of observations is 500, only about 1 rejections out of 17 variables were observed. For all cases, the panel ESTAR that removes the effect of the stationary common factor rejected the null with near 0.5% frequency. Therefore, our empirical evidence suggests that stock indices with the UK as the reference country possess a dominating common factor that is nonlinear stationary, which makes it possible to profitably utilize a contrarian strategy when deviations are big.

Table 8 around here

7 Concluding Remarks

We revisited the topic of mean reversion in national stock prices across international stock markets relative to the US and the UK using the Morgan Stanley Capital International annual gross stock index data for 18 developed countries. We found strong evidence of mean reversion for a maximum 14 out of 17 countries in the case of the UK (but not the US) as the reference country, while very weak evidence of linear mean reversion was observed irrespective of the choice of the reference country.

Implementing panel version linear unit root tests while controlling for cross-section dependence provided weak evidence of stationarity in the univariate tests. The panel nonlinear unit root test also failed to reject the null of nonstationarity even when the UK served as the reference country. The results appear inconsistent.

To resolve this seeming puzzle, we estimated a common factor, then tested the null of nonstationarity with linear and nonlinear stationarity alternatives. Our tests strongly favor the stationarity for the first common factor from the panel when the UK serves as the reference country. These results imply that the first common factor with the UK is stationary and dominates nonstationary idiosyncratic components in small samples. That is, when the first common factor dominates the nonstationary idiosyncratic component, the panel unit root test that removes the influence of the stationary common factor may yield evidence against stationarity even though it behaves as a stationary variable in finite samples, even though it will become dominated by nonstationary variables in the long-run. Our Monte Carlo simulation analysis confirms our conjecture.

Our empirical findings suggest that the UK equity index may be used as an anchor in managing international equity portfolios. Big deviations of national equity prices from the UK index may be accompanied by winner-loser reversal soon. Therefore, one may consider short-selling better performing assets while buying worse performing ones. On the contrary, one should employ the momentum strategy with the US index, because deviations of equity prices are more likely to be permanent. This might explain the steady strong performance of the US stock markets compared to those of other OECD countries.

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Table 1. Summary Statistics

				Base Country: US				
ID	Country	Mean	Std Dev	Skewness	Kurtosis	JB	Min	Max
1	Aus	−0.414	0.341	−0.363	2.443	1.500	−1.239	0.132
2	Aut	0.179	0.521	0.163	2.834	0.241	−0.927	1.047
3	Bel	0.683	0.352	−0.503	3.464	2.196	0.000	1.267
4	Can	0.013	0.357	−0.202	2.761	0.394	−0.915	0.597
5	Den	0.729	0.363	−0.265	2.784	0.585	−0.144	1.420
6	Fra	0.179	0.240	0.035	3.572	0.594	−0.295	0.565
7	Ger	0.150	0.255	0.748	3.570	4.596	−0.338	0.699
8	HK	1.583	0.531	−0.312	4.125	2.962	0.000	2.615
9	Ita	−0.981	0.436	0.687	3.712	4.286	−1.715	0.000
10	Jap	0.770	0.709	0.376	2.979	1.014	−0.191	2.345
11	Net	0.718	0.404	−0.088	2.054	1.659	−0.224	1.260
12	Nor	0.469	0.403	0.654	4.778	8.733	−0.465	1.237
13	Sing	0.767	0.514	0.287	4.215	3.235	−0.253	1.843
14	Spa	−0.282	0.457	−0.110	3.305	0.254	−1.361	0.851
15	Swe	0.819	0.518	0.117	1.906	2.243	−0.258	1.732
16	Swi	0.418	0.258	−0.061	2.869	0.057	−0.189	0.833
17	UK	0.271	0.235	−0.593	4.944	9.298	−0.461	0.694
				Base Country: UK				
ID	Country	Mean	Std Dev	Skewness	Kurtosis	JB	Min	Max
1	Aus	−0.684	0.375	−0.225	2.247	1.378	−1.413	0.063
2	Aut	−0.092	0.541	0.910	4.223	8.615	−1.048	1.508
3	Bel	0.412	0.294	0.112	3.740	1.071	−0.169	1.165
4	Can	−0.258	0.440	−0.086	3.192	0.120	−1.121	0.859
5	Den	0.458	0.362	0.347	3.084	0.877	−0.304	1.358
6	Fra	−0.091	0.214	0.388	3.930	2.629	−0.549	0.602
7	Ger	−0.121	0.281	1.631	8.696	77.207	−0.539	0.867
8	HK	1.312	0.424	−0.024	3.317	0.185	0.000	2.077
9	Ita	−1.252	0.515	0.291	3.436	0.949	−1.991	0.141
10	Jap	0.499	0.639	0.491	3.038	1.728	−0.467	1.741
11	Net	0.447	0.302	−0.526	5.337	11.766	−0.384	0.832
12	Nor	0.198	0.426	0.988	5.015	14.271	−0.670	1.503
13	Sing	0.496	0.500	0.726	5.758	17.404	−0.458	1.479
14	Spa	−0.553	0.573	0.204	4.719	5.596	−1.576	1.312
15	Swe	0.548	0.457	−0.121	4.193	2.654	−0.293	1.404
16	Swi	0.147	0.278	0.355	4.238	3.653	−0.363	0.816
17	US	−0.271	0.235	0.593	4.944	9.298	−0.694	0.461

Note: JB refers the Jarque-Bera statistics, which has asymptotic χ^2 distribution with 2 degrees of freedom. For the US reference, most of the stock index deviation shows normality except for Norway and the UK whereas the stock index deviations for 6 countries (Austria, Germany, the Netherlands, Norway, Singapore and the US) show non-normality with the UK reference.

Table 2. Univariate Linear Unit Root Tests

	US		UK	
	ADF_c	ADF_t	ADF_c	ADF_t
Aus	-1.895	-1.722	-2.042	-1.499
Aut	-1.651	-2.268	-1.746	-2.659
Bel	-2.526*	-2.420	-3.099†	-3.098*
Can	-1.260	-1.247	-1.537	-1.466
Den	-2.060	-2.401	-2.285	-2.422
Fra	-2.663*	-2.804	-3.592‡	-3.553*
Ger	-2.667*	-2.671	-3.036†	-3.326*
HK	-3.275†	-3.692‡	-3.621‡	-4.204‡
Ita	-2.278	-2.735	-2.452	-3.067*
Jap	-1.030	-1.992	-1.172	-2.627
Net	-1.717	-1.416	-2.047	-3.394†
Nor	-3.013†	-3.005	-2.963†	-3.084
Sing	-2.268	-2.666	-2.139	-2.865
Spa	-1.840	-1.806	-1.794	-1.696
Swe	-1.303	-3.452†	-1.773	-3.866†
Swi	-2.161	-2.726	-2.270	-2.380
UK	-2.637*	-2.680	-	-
US	-	-	-2.637*	-2.680

Note: ADF_c and ADF_t denote the augmented Dickey-Fuller test statistic when an intercept and when both an intercept and time trend are present, respectively. *, †, and ‡ denote significance levels at the 10%, 5%, and 1% level, respectively.

Table 3. Univariate Nonlinear Unit Root Tests

	US		UK	
	$NLADF_c$	$NLADF_t$	$NLADF_c$	$NLADF_t$
Aus	-1.094	-1.078	-2.321	-1.440
Aut	-1.488	-2.113	-2.788*	-3.448†
Bel	-1.904	-1.763	-3.076†	-3.211*
Can	-1.883	-1.971	-3.469†	-2.978
Den	-1.930	-2.725	-3.923‡	-3.321*
Fra	-2.502	-2.503	-2.484	-2.473
Ger	-2.508	-2.541	-2.697*	-2.798
HK	-2.641*	-2.817	-2.639*	-3.495†
Ita	-2.180	-2.470	-3.095†	-4.288‡
Jap	-1.244	-1.864	-1.717	-2.044
Net	-1.401	-1.344	-1.899	-3.889†
Nor	-1.906	-1.962	-2.585	-2.724
Sing	-2.132	-2.500	-2.221	-3.007
Spa	-2.148	-2.166	-2.613*	-2.764
Swe	-1.416	-2.253	-1.299	-3.246*
Swi	-1.881	-2.628	-2.993†	-2.929
UK	-4.853‡	-4.858‡	-	-
US	-	-	-4.853‡	-4.858‡

Note: $NLADF_c$ and $NLADF_t$ denote the ESTAR test statistic (Kapetanios et al., 2003) when an intercept and when both an intercept and time trend are present, respectively. *, †, and ‡ denote significance levels at the 10%, 5%, and 1% level, respectively. Asymptotic critical values were obtained from Kapetanios et al. (2003).

Table 4. Cross-Section Dependence Test

	CSD	p-value
US	19.753	0.000
UK	24.586	0.000

Note: This test is proposed by Pesaran (2004).

Table 5. Panel Linear Unit Root Test Results

	$PADF_c$	$PADF_t$
US	-2.056	-2.573
UK	-2.012	-2.523

Note: Critical values were obtained from Pesaran (2007). The test fails to reject the null of nonstationarity for both reference countries.

Table 6. Panel Nonlinear Unit Root Test Results

	$NLPADF_c$	$NLPADF_t$
US	-1.345	-1.481
UK	-1.471	-1.588

Note: Critical values were obtained from Cerrato et al. (2011). The test fails to reject the null of nonstationarity for both reference countries.

Table 7. Test for the First Common Factor

	Linear		*Nonlinear*	
	ADF_c	ADF_t	$NLADF_c$	$NLADF_t$
US	-2.177	-2.499	-1.289	-1.804
UK	-2.845*	-2.967†	-3.728†	-3.919†

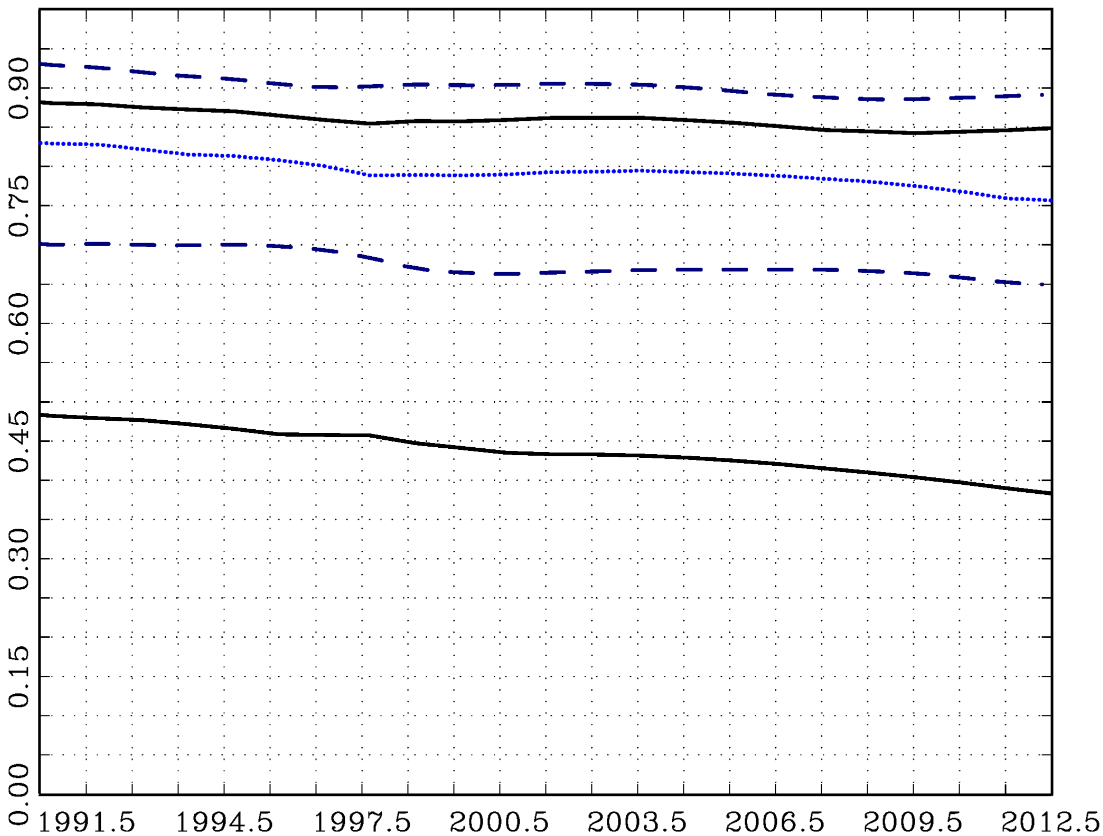
Note: *, †, and ‡ denote significance levels at the 10%, 5%, and 1% level, respectively.

Table 8. Simulation Results

		Univariate ESTAR		*Panel ESTAR*	
		$NLADF_c$	$NLADF_t$	$NLPADF_c$	$NLPADF_t$
nob=50	Median	52.9%	58.8%	1.4%	0.7%
	Mean	52.9%	60.0%		
nob=100	Median	41.2%	47.1%	0.7%	0.6%
	Mean	39.4%	44.7%		
nob=200	Median	23.5%	29.4%	0.2%	0.1%
	Mean	26.5%	30.0%		
nob=300	Median	17.6%	23.5%	0.5%	0.1%
	Mean	20.6%	5.9%		
nob=500	Median	17.6%	17.6%	0.2%	0.0%
	Mean	15.9%	17.1%		

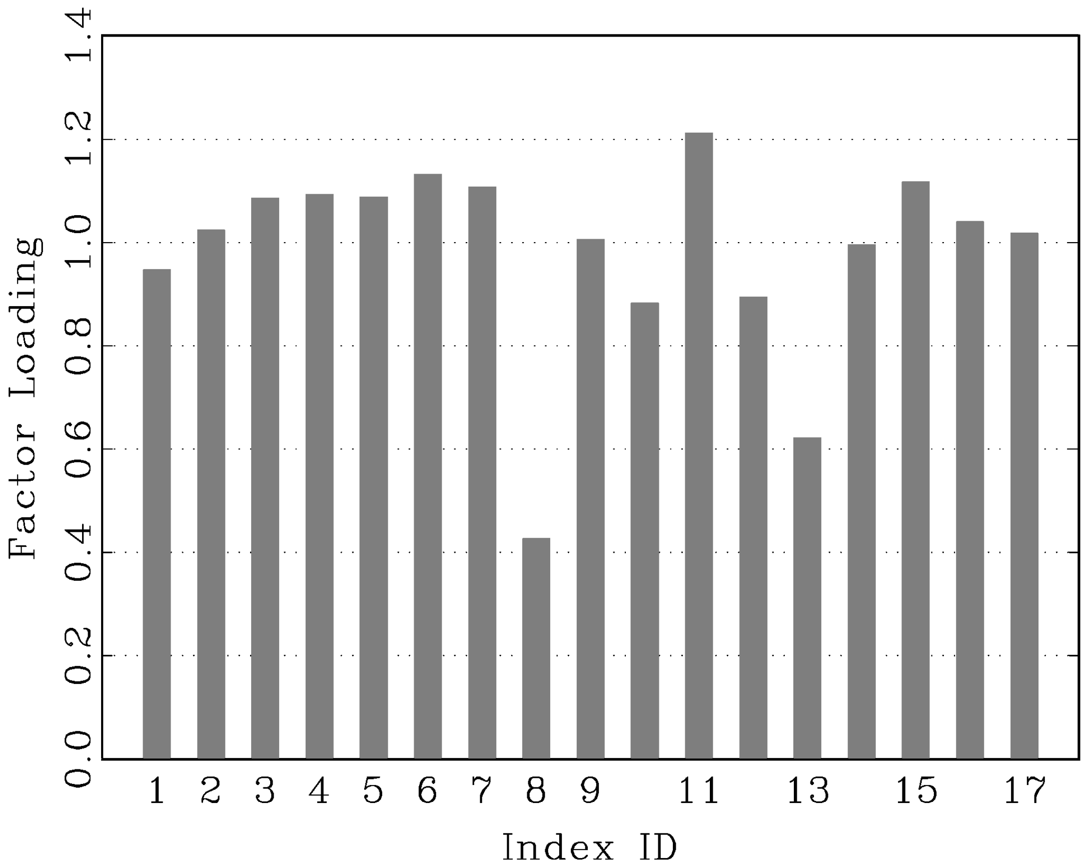
Note: The table shows simulation results. Numbers in the Univariate ESTAR section represent percentage of the mean and median of the frequency of rejections of the null of unit roots when univariate ESTAR test is employed for the 3000 iterations. Numbers in the Panel ESTAR section represent percentage of rejections of the null of unit roots when the Panel ESTAR test is employed for the 3000 iterations.

Figure 1. Cumulative Share of Variation by Five Common Factors: UK



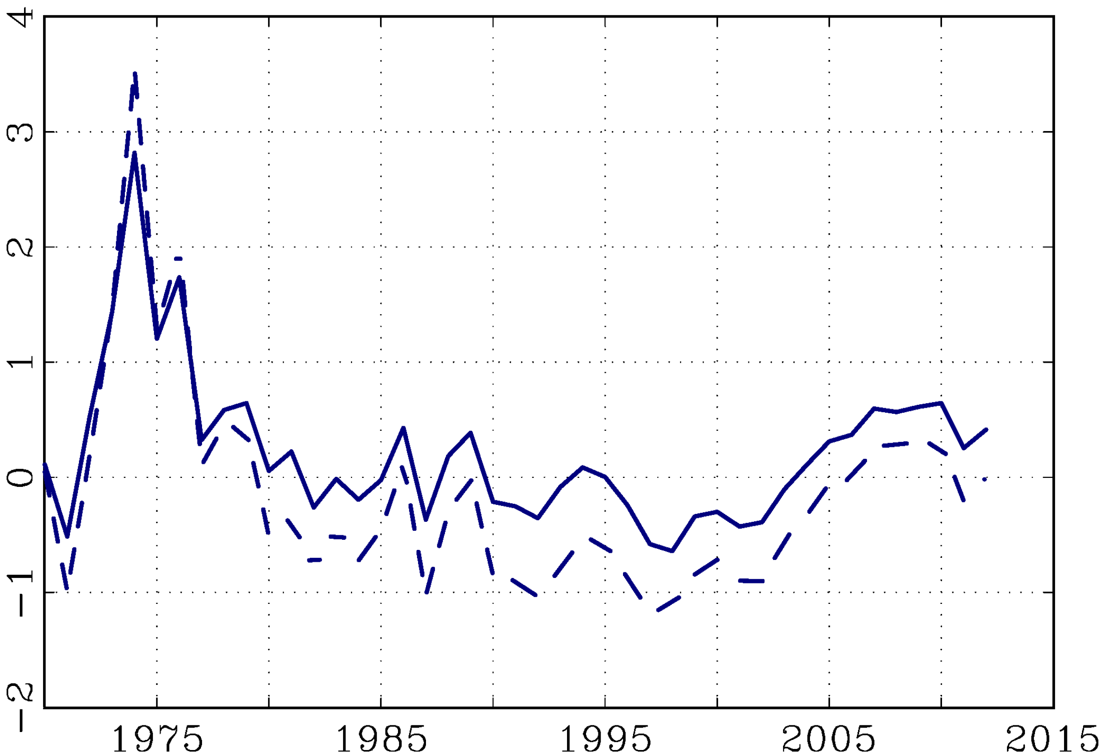
Note: A recursive method is used to repeatedly estimate the first five common factors using the initial 50% observations as the split point. We report shares of variations explained by the common factors from each set of samples.

Figure 2. Factor Loading Coefficients Estimation: UK



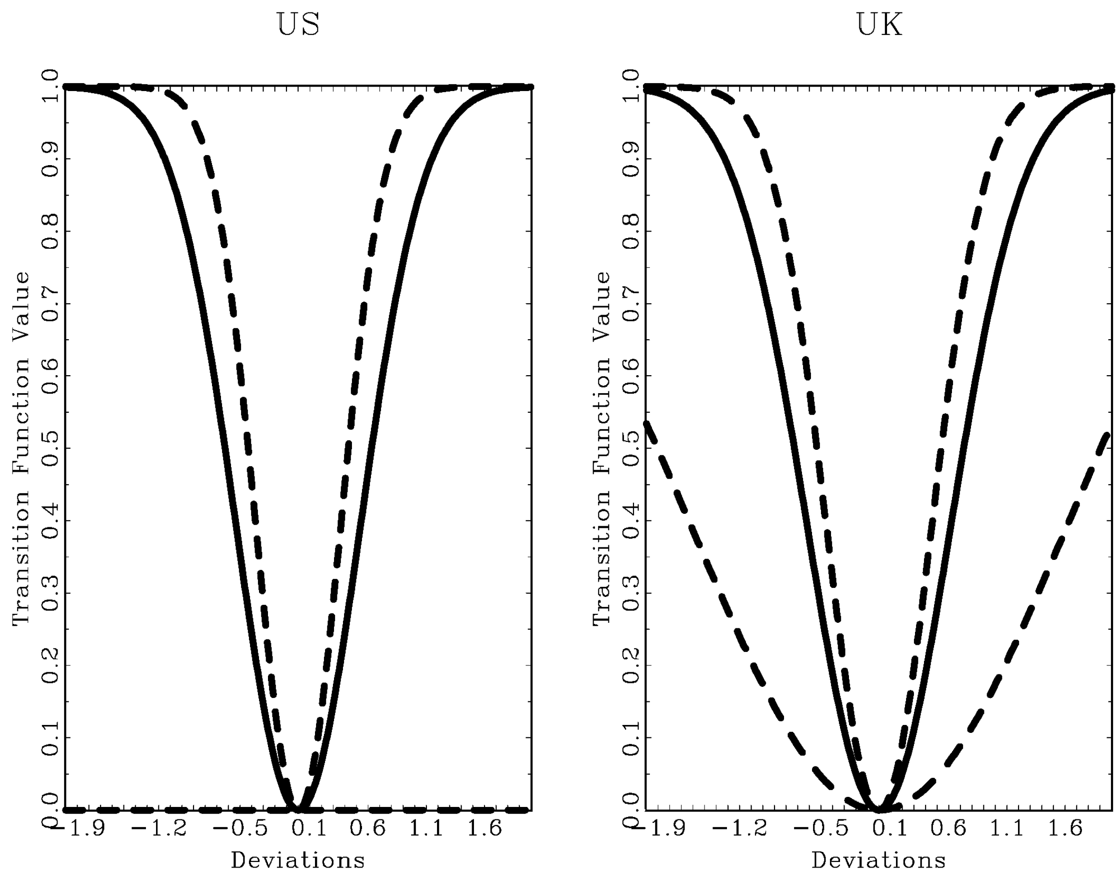
Note: We report factor loading coefficients (λ_i) in Equation (16). They represent the country-specific dependence on the common factor.

Figure 3. First Common Factor Estimates: UK



Note: We report two measures of the common factor: the first common factor (dashed line) via the PANIC (Bai and Ng, 2004) and the cross section mean (solid) as in Pesaran (2007).

Figure 4. Transition Function Estimates



Note: We report graphs of one minus the exponential transition function $1 - \exp(-\theta x^2)$ for the common factor estimates with the US and the UK. We used $\theta = 1.746$ for the US reference and $\theta = 1.308$ for the UK reference, obtained from the data. Dashed lines are 95% confidence bands. The lower bound for the US is negative, so we used 0 because θ is bounded below zero.