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Market Power and Growth through Vertical and Horizontal Competition

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Abstract

I study the implications of innovators' market power to growth and welfare in a two-R&D-sector economy. In this framework either vertical or horizontal competition is binding in the price setting stage, depending on the model parameters and the implemented market-power policy. I consider two alternative policies that are commonly, yet separately, used in the literature to constraint innovators' market power: patent lagging-breadth protection and direct price controls. I show that (a) the alternative policies may have non-monotonic and contradicting effects on growth (b) unconstrained market power may yields either excessive or insufficient growth compared with social optimum and (c) the social optimum can be achieved by reducing innovators market power with the proper policy instrument, along with a corresponding flat rate R&D-subsidy.

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1 Introduction

There is, by now, an extensive literature on the implications of innovators' market power to R&D-based growth and welfare. However, this literature has focused on either quality-ladder models of vertical innovation (with a fix variety span), or models of increasing specialization through variety-expansion (with no vertical innovation)[1]. The present work re-examines the relation between innovators' market power, welfare and growth, in a two-R&D-sector economy that incorporates both vertical and horizontal innovation[2].

In this framework either vertical or horizontal competition binds innovators in the price setting stage. When innovators' market power is not constrained by policy, The type of prevailing price competition depends the relative magnitude of elasticity of substitution across varieties and the innovation cost: horizontal (vertical) competition is binding when the elasticity of substitution is sufficiently large (small) compared to quality-improvements costs[3][4]. The present study shows that the welfare and growth implications of constraining innovators' market power depend on the prevailing type of price competition and on the exact implemented policy.

I consider two policy instruments that are commonly, yet separately, used in the current literature: direct price regulation and patent lagging-breadth protection. Direct price regulation is commonly used in variety-expansion models with no vertical innovation, either as a concrete policy (See for example Evans et al. 2003 and Zheng et al. 2014) or as a proxy for patent breadth protection (See for example Goh and Olivier 2002, Iwaisako and Futagami 2013). Under both interpretations this policy defines a price (or markup) ceiling for the innovators. Lagging breadth protection against vertical imitation was commonly used in models with vertical innovation only (See for example Li 2001, Chu 2011, and Chu et al. 2012)[5].

I show that under vertical price competition, loosening patent breadth protection enhances quality growth and reduces products variety. However, restricting innovators' market power through direct price regulation slows quality growth, down to a minimal rate for which horizontal competition becomes binding, with ambiguous impact on products variety. Hence, under vertical price competition loosening patent breadth protection and setting effective price ceiling will have contradicting effects on growth. Under horizontal competition, price regulation has no effect on quality growth but it reduces variety span. Nonetheless, loosening patent breadth protection sufficiently would shift the markets to vertical price competition, under which the former policy effects apply.

[1] Based on the canonical models of endogenous R&D driven growth of Grossman and Helpman (1991) and Aghion and Howitt (1992), and Romer's (1990) respectively.

[2] The seminal two-R&D-sector models were pioneered by Dinopoulos and Thompson (1998), Peretto (1998), Young (1998), and Howitt (1999). The present analysis builds on Young's (1998) framework.

[3] Young's (1998) original analysis is confined to binding horizontal competition under unconstrained market power.

[4] The work of Li (2001), to which I refer in details below, considers the case of binding horizontal (vertical) competition as "drastic innovation" ("non-drastic innovation"). A drastic innovation is a quality improvement that is large enough to avoid binding vertical competition with the innovator of the previous top of the line quality of the same product variety. The same terminology is used in Aghion and Howitt (2009); See Chapter.4.2.6 on P. 90 there.

[5] By exception, Chu and Pan (2013) model patent policy as a mark-up ceiling in a quality-ladder model, abstracting vertical competition, whereas Chu et al. (2016), which I further consider below, employ this policy in a Two-R&D-sector model.

The welfare analysis reveals that while horizontal price competition always yields insufficient quality growth compared with the social optimum (as shown by Young 1998), vertical price competition may results in either excessive or insufficient quality growth. Nonetheless, I show that whenever the quality improvements rate is insufficient (excessive) the socially optimal outcome can be achieved by setting a proper incomplete patent breadth protection (price ceiling) along with a corresponding flat-rate R&D subsidy.

The present work is closely related the study of Li (2001), which has shown that the type of binding competition - horizontal or vertical - affects the welfare implications of R&D subsidies[6]. His work, however, studies a model of vertical innovation with a fixed variety span, and most of the analysis applies to exogenous innovation size with endogenous arrival rate. In the current work innovation size is endogenous and innovation is certain. Li's patent policy analysis with endogenous innovation size is confined to unit demand elasticity (i.e. Cob-Douglas preferences), for which only vertical price competition is binding.

Hence, the present work extends Li's (2001) study along three lines: (1) allowing for endogenous variety span, (2) studying the implications of patent lagging-breadth protection to growth for any demand elasticity, and (3) studying the implications of direct price controls as alternative policy instrument within this extended framework.

The negative effect of patent breadth protection on quality growth reported here generalizes Li's (2001) finding for the current extended framework[7]. However, in Li's (2001) framework the endogenous innovation size is always lower than the socially optimal level, and efficiency is restored with a combination of incomplete patent breadth and R&D subsidy[8]. As reported above, in the present analysis vertical competition may result also in excessive quality improvements, which can be fixed with a combination of price controls and R&D subsidy.

The present work is related also to a recent study by Chu et al. (2016), which provides the first analysis of patent breadth policy in a two-R&D-sector model. They show that the positive direct effect of innovators' markups on R&D investment and quality growth, induces a secondary, contradicting, effect: the increased profitability attracts entry by innovators of new product varieties (horizontal innovation), which by itself erodes the market shares of existing industries. This negative market-size effect decreases profitability and thereby slows down quality improvements.

Chu et al. (2016) find that the relative strength of these contradicting effects depends on the assumed cost structure. Under their preferred cost structure the direct positive effect is dominant in the short run, whereas the secondary (market entry) negative effect dominates in the long run.

An earlier work by Cozzi and Spinesi (2006) studies the implications of intellectual appropriability to growth in Howitt's (1999) two-R&D-sector model, where vertical innovation is subject

[6]More specifically his work showed that the assumed innovation size, and its corresponding price-competition regime, determine whether quality growth rate in equilibrium is excessive or insufficient compared with social optimum and, therefore, whether should R&D effort should be subsidized or taxed.

[7]The analysis of this effect in the current extended framework also provides a clear intuition for this effect that is missing in Li's work.

[8]See equations (17)-(18) on pp. 175-176 there. In Li's (2001) analysis of exogenous innovation size, small innovations should be taxed and large innovations should be subsidized, See Subsection 2.2 on pp. 171-173 there.

to imitation risk. They find that increased intellectual appropriability (i.e. enhanced patent enforcement) spurs quality growth and reduces variety span, by deterring espionage activity and imitation.

Both studies (Chu et al. 2016, and Cozzi and Spinesi 2006) assume prices are set through horizontal competition, whereas the present work shows that the implications of constraining innovators' market power depend on the interaction between the policy instrument and the type of price competition that prevails in the Two-R&D-sector economy.

The remainder of the paper is organized as follows. Section 2 outlines the model. Section 3 presents the different growth presents under vertical and horizontal competition, Section 4 analyzes the implications of patent breadth protection and direct price controls to Schumpeterian growth, Section 5 provides welfare analysis and Section 6 concludes this study.

2 Model

I employ Young's (1998) two-R&D-sector model, where time is discrete and the economy is population with L infinitely lived consumers.

2.1 Preferences

The consumer's lifetime utility is given by

$$U = \sum_{t=0}^{\infty} \beta^t \ln(c_t) \tag{1}$$

where $\beta \in (0, 1)$ is the time preference. The per capita utilization level of consumption in (1), denoted c, is derived from M differentiated products (i.e., "varieties"), denoted c_i, subject to a CES utility function

$$c_t = \left(\sum_{i=1}^{M_t} c_{i,t}^{\frac{1}{\varepsilon}} \right)^{\varepsilon} \tag{1a}$$

where $\varepsilon = \frac{s}{s-1}$, and s is the elasticity of substitution across all varieties. The consumption level of each variety is defined as $c_i = q_i x_i$, where x_i and q_i designate utilized quantity and quality, respectively.

The assumed preferences imply the following instantaneous aggregate demand for each variety, denoted $X_{i,t}^d$:

$$X_{i,t}^d = q_{i,t}^{s-1} p_{i,t}^{-s} C_t \lambda_t \tag{1b}$$

where $C_t = c_t L$ and λ_t is the *Lagrange* multiplier from the instantaneous utility maximization - i.e. the shadow value of spending consumer spending denoted e_t.

Consumers maximize their lifetime utility (1) subject to the dynamic budget constraint

$$a_{t+1} = (1 + r_{t+1})a_t + w_t - e_t \tag{2}$$

where a denotes the consumer's assets (which take the form of patents ownership), $1 + r_{t+1}$ is the (gross) interest rate earned between periods t and $t+1$, and w_t is labor income. The maximization of (1) subject to the dynamic budge constraint yields the familiar Euler condition $\frac{e_{t+1}}{e_t} = \beta(1+r_{t+1})$, which can be also written in terms of aggregate spending, denoted E:

$$\frac{E_{t+1}}{E_t} = \beta(1 + r_{t+1}) \tag{3}$$

2.2 Production and innovation

Labor is the sole input for production and innovation, and the wage rate is normalized to one. One unit of labor produces one consumption good (regardless of its variety and quality). Innovation is certain and is subject to the following cost function

$$f(q_{i,t+1}, \overline{q}_t) = \exp\left(\phi \frac{q_{i,t+1}}{\overline{q}_t}\right) \tag{4}$$

The innovation cost in sector i is increasing in the rate of quality improvement over the existing quality frontier – denoted $\overline{q}_t$, which is the highest quality already attained in the economy. I denote the rate of quality improvements $\kappa \equiv \frac{q_{i,t+1}}{\overline{q}_t}$ to rewrite the innovation cost

$$f_{i,t+1} \equiv f(\kappa_{i,t+1}) = \exp(\phi\kappa_{i,t+1}) \tag{4a}$$

3 Equilibrium and growth

Due to the assumed certain outcome of R&D investments, innovation takes exactly one period, and therefore the effective market lifetime of each quality improvement is one period as well. I will first present the equilibrium under binding horizontal innovation, as introduced by Young (1998).Then I will turn to present the equilibrium under vertical innovation. In this section lagging-breadth protection is complete, that is no imitation is allowed (so innovators can fully appropriate their incremental quality improvement), and innovators' price is not directly regulated. This will be our benchmark case for the policy analysis that follows, in Section 4.

3.1 Horizontal competition

Under horizontal competition each firm maximizes the following profit, denoted Π

$$\Pi_{i,t} = \frac{(p_{i,t+1} - 1)X^d_{i,t+1}}{1 + r_{t+1}} - f_{i,t+1} \tag{5}$$

Maximizing (5) for p yields the standard optimal monopolistic price[9] $p^* = \varepsilon$. The first-order condition for optimal quality choice, combined with a free-entry (zero-profit) condition that is imposed on (5), yields the equilibrium quality improvements rate $\kappa_h^e = \frac{s-1}{\phi}$. Hence

$$\forall_{i,t} : p_h^e = \varepsilon,\ \kappa_h^e = \frac{s-1}{\phi} \tag{5a}$$

Under horizontal competition price is decreasing, and quality growth rate is increasing, with the elasticity of substitution across varieties. This is because the elasticity of substitutions defines the intensity of horizontal competition, by defining demand's sensitivity to changes in relative price and quality.

Under symmetric equilibrium, demand for each variety is $X_t^d = \frac{E_t}{\varepsilon M_t} \forall_i$, and thus, imposing the free entry condition on (5) implies

$$\frac{(1-\frac{1}{\varepsilon})\frac{E_{t+1}}{M_t+1}}{f_h^e} = 1 + r_{t+1} \tag{6}$$

Combining (3) and (6) I obtain

$$E_t = \frac{M_{t+1} f_h^e}{(1-\frac{1}{\varepsilon})\beta} \tag{7}$$

The aggregate uses resources (or labor-market clearing) constraint requires that aggregate labor supply is fully employed in production and R&D activity:

$$L = \frac{E_t}{\varepsilon} + M_{t+1} f_h^e \tag{8}$$

Plugging (7) into (8) reveals that spending in the model economy is stationary, $\forall t : E = \frac{L}{\frac{1}{\varepsilon}+(1-\frac{1}{\varepsilon})\beta}$. Therefore, by (3), the interest rate is stationary as well, $\forall t : 1 + r = \beta^{-1}$. Using this result when plugging (6) into (8) yields the (stationary) equilibrium variety span

$$M_h^e = \frac{L}{f_h^e \left[\frac{1}{(\varepsilon-1)\beta} + 1\right]} \tag{9}$$

3.2 Vertical competition

The price and quality growth rate presented in (5a) can indeed prevail in equilibrium only as long as vertical competition with previous top of the line quality is not binding. Namely, under horizontal competition the quality-adjusted price of the recent developed quality must exceeds that of the previous leading quality, when the latter is sold at the marginal cost price (which equals one). Hence, vertical competition becomes binding, when $p_h^* > \kappa_h^* \Leftrightarrow \frac{s}{(s-1)^2} > \frac{1}{\phi}$, that is when the elasticity of substitution is sufficiently low relative to the innovation cost parameter.

When vertical competition is binding, the price of most recent product will be set through

[9] Throughout the analysis I use the asterisk superscript to denote the maximizers of individual value functions. The superscript "e" denotes equilibrium values. The subscripts "h" and "v" denote horizontal and vertical competition, respectively.

Bertrand competition with the product that was developed in the previous period, to satisfy[10] $p = \kappa$. Hence, the demand for each variety from (1b) modifies to $X_{i,t}^{d} = q_{i,t}^{s-1}\kappa_{i,t}^{-s}C_t\lambda_t = (q_{i,t-1}\kappa)^{s-1}\kappa_{i,t}^{-s}C_t\lambda_t$, and the (present value) profit function (5) modifies to

$$\Pi_{i,t} = \frac{(\kappa_{i,t+1}-1)(q_{i,t}\kappa_{i,t+1})^{s-1}(\kappa_{i,t+1})^{-s}C_{t+1}\lambda_{t+1}}{1+r_{t+1}} - f_{i,t+1} \tag{10}$$

Equation (10) shows that under vertical competition the effect of quality choice on innovator's profit composes the direct effect of quality on demand, which presents also in the horizontal competition case, and the additional effect of quality choice on the price. Notice, that for $s = 1$, which is the case studied in Li (2001), the effect of κ only the profit is confined to the price channel. The first order condition for maximizing (10) with respect to κ is

$$\frac{(q_{i,t})^{s-1}C_{t+1}\lambda_{t+1}}{1+r_{t+1}} = \left(\kappa_{i,t+1}^{*}\right)^2\phi f_{i,t+1}^{*} \tag{11}$$

and combining (11) with the free-entry (zero profit) condition that is imposed on (10) yields

$$\forall i,t : \frac{1}{\kappa_v^e(\kappa_v^e-1)} = \phi \tag{12}$$

The implicit expression of κ_v^e in (12) is a quadratic function with the positive root:

$$\kappa_v^e = \frac{1+\sqrt{1+\frac{4}{\phi}}}{2} > 1$$

Notice that, by (12), once vertical competition is binding in the price setting stage, the rate of quality improvements is independent of the elasticity of substitution across varieties s. It can be easily shown that vertical competition results in a higher quality and a lower price than the ones that would prevail under horizontal competition for same parameter values. That is for $\frac{s}{(s-1)^2} < \frac{1}{\phi}$: $\frac{s-1}{\phi} = \kappa_h^e < \kappa_v^e = p_v^e < \varepsilon = p_h^e$ (and for $\frac{s}{(s-1)^2} > \frac{1}{\phi} : \varepsilon < \kappa_v^e < \kappa_h^e$).

The economic intuition behind this result is the following: vertical competition brings innovators price below the one they would charge based on horizontal competition considerations. Hence choosing κ under vertical competition counts both for the direct effect of κ on demand (the horizontal competition effect), and its positive effect on the profit through the resulting price increase (as vertical competition presses the price below its profit maximizing level $p_h^e = \varepsilon$).

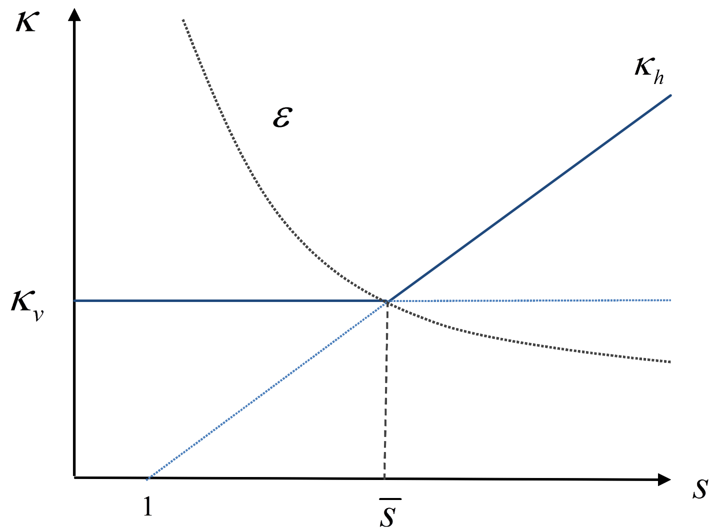
The higher quality and lower price under vertical competition bring the equilibrium variety span below the horizontal competition level:

$$M_v^e = \frac{L}{f_v^e\left[\frac{1}{(\kappa_v^e-1)\beta}+1\right]} \tag{12a}$$

Figure 1 illustrates the relation between the elasticity of substitution across varieties and the rate of quality improvements under the two alternative competition types.

[10] As in Grossman and Helpman (1991) canonical quality ladder model.

Figure 1



For a given innovation cost ϕ, the rate of quality improvements under horizontal competition is increasing linearly with s, and is independent of s under vertical competition. The degree of substitution $\overline{s}$, which satisfies $\frac{\overline{s}}{(\overline{s}-1)^2} = \phi$, is the critical level below (above) which vertical (horizontal) competition becomes binding. Above (below) this degree of substitution quality improvements rate is higher (lower) under horizontal competition. For $s = \overline{s}$ the quality improvement rates equal ε.

4 Market Power and Growth

I turn now to explore the effect of innovators' market power on quality growth and products variety, under each competition type. First, I will explore the effect of direct price control, which where commonly interpreted as patent breadth protection in models with horizontal innovation only. Then I will turn to study the implications of patent lagging-breadth protection in the way it was studied in models with vertical innovation only.

4.1 Price controls

I model direct price control as an effective price ceiling, denoted $\mu < p^e$. Starting with horizontal competition, I substitute the binding price ceiling $\mu \in (1, \varepsilon)$ in the profit function (5)

$$\frac{(\mu - 1)\left(\kappa_{i,t+1} q_t\right)_{i,t}^{s-1} \mu^{-s} C_{t+1} \lambda_{t+1}}{1 + r_{t+1}} - f_{i,t+1}^{*} \tag{13}$$

The first-order condition for optimal quality improvement, set to maximize (13), is

$$\frac{(s-1)(\mu - 1)\left(q_t\right)^{s-2} (\mu)^{-s} C_{t+1} \lambda_{t+1}}{1 + r_{t+1}} = \frac{\phi f_{i,t+1}^{*}}{\left(\kappa_{i,t+1}^{*}\right)^{s-2}} \tag{14}$$

Notice that, at the individual firm level, for $s > 2$ the price ceiling has ambiguous effect on the optimal rate of quality improvement in (14). However, under symmetric equilibrium (for which $x_i^d = \frac{E_{t+1}}{\mu M_{t+1}}$), the above equilibrium condition yields a definite negative effect of price ceiling on the optimal rate of quality growth:

$$\forall i : \frac{(s-1)(1-\frac{1}{\mu})\frac{E_{t+1}}{M_{t+1}}}{1+r_{t+1}} = \kappa^*_{\mu,t+1}\phi f^*_{\mu,t+1} \tag{14a}$$

Then, combining the first-order condition (14) with the zero-profit condition, imposed on (13), reveals that the equilibrium quality-improvements rate remains unaffected by the price ceiling[11] $\kappa^e_{h,\mu} = \frac{s-1}{\phi}$. Hence, under horizontal competition, quality growth in the model economy is independent of the effective price ceiling, i.e. $\frac{\partial \kappa^e_{h,\mu}}{\partial \mu} = 0$. Finally, plugging the price ceiling into the resources-uses constraint (8) yields

$$M^e_{h,\mu} = \frac{L}{f^e_{h,\mu}\left[\frac{1}{(\mu-1)\beta}+1\right]} \tag{15}$$

As the quality-improvements rate does not change with the price ceiling, the R&D investment in (15) remains as in (9). Hence, products' variety is negatively affected by the price ceiling, that is $\frac{\partial M^e_h}{\partial \mu} > 0$. The direct negative effect of price ceiling on profitability and quality growth presented in condition (14a), pushes firms out of the market (as required by the zero-profit condition), eliminating existing product lines. These direct and secondary effects of price ceiling on quality growth correspond to the "short-run" positive effect and the "long term" considered in Chu et al. (2016), respectively[12]. As pointed out by Chu et al. (2016), under the current cost specification (with entrants cost equals incumbents R&D-cost) the two effects cancel out (See second paragraph on page 818 there).

Proposition 1 summarizes the above results.

Proposition 1 *Under horizontal competition price ceiling has no effect on the rate of quality improvements and it decreases variety span. That is* $\frac{\partial \kappa^e_h(\mu)}{\partial \mu} = 0$ *and* $\frac{\partial M^e_h}{\partial \mu} < 0$.

Suppose now that vertical competition is initially binding. In this case the price ceiling can yield two different outcomes. According to equation (13), under effective price ceiling the price is not a function of quality anymore, implying that innovators choose the same quality improvement rate they choose under horizontal competition: $\kappa_h = \frac{s-1}{\phi}$. However, this can be indeed the equilibrium outcome only if the price ceiling is set sufficiently low, such that $\mu < \frac{s-1}{\phi}$. Under this price ceiling vertical competition is not binding anymore and the economy switches into horizontal competition, with $\kappa^e_h = \frac{s-1}{\phi}$.

[11] Hereafter, I add the relevant policy instrument to the variables' subscript.

[12] Their model, which build on Peretto (2007,2011), exhibits gradual market entry (or exist) due to entry cost that is increasing with production scale.

However, for $\frac{s-1}{\phi} < \mu < \kappa_v^e$, vertical competition is still binding for $\kappa = \frac{s-1}{\phi}$. That is, the quality adjusted price $\frac{p}{q_{t+1}} = \frac{\mu}{q_t \frac{s-1}{\phi}}$ is too high for driving the previous leading quality product out of the market: $\frac{s-1}{\phi} < \mu \Rightarrow \frac{\mu}{q_t \frac{s-1}{\phi}} > \frac{1}{q_t}$. Hence, for any $\kappa < \mu$ innovators expect vertical competition to be binding in the price setting stage. However, once $\mu = \kappa$ additional increase in κ will not enable further price increase. Therefore, innovators choose the limit rate of quality improvement that is required to deter vertical competition, that is $\kappa_v^e(\mu) = \mu$, and thus $\frac{s-1}{\phi} < \mu < \kappa_v^e$: $\frac{\partial \kappa_v^e(\mu)}{\partial \mu} > 0$.

Plugging $p, \kappa = \mu$ into the aggregate resources-uses constraint yields

$$M_{v,\mu}^e = \frac{L}{f_{v,\mu}^e \left[\frac{1}{(\mu-1)\beta} + 1\right]} \tag{15a}$$

Equation (15a) reveals that when vertical competition is binding, the effect of price-controls on variety span is ambiguous because the price and R&D investment move in the same direction. Differentiating the right hand side of (15a) for μ shows that the sign of the derivative depends on whether μ is larger or smaller than $\frac{-1+\sqrt{1+\frac{\beta 4}{\phi}}}{2\beta}$.

Proposition 2 summarizes the latter results.

Proposition 2 *Under vertical competition, an effective price ceiling slows the rate of quality improvements, i.e.* $\frac{\partial \kappa_v^e(\mu)}{\partial \mu} > 0$ *with* $\kappa_\mu^e = \mu$*, down to the level* $\mu = \frac{s-1}{\phi}$ *below which the market switches the horizontal competition with* $\frac{\partial \kappa_v^e(\mu)}{\partial \mu} > 0$ *and* $\kappa_\mu^e = \frac{s-1}{\phi}$*. The effect of price ceiling on variety span is ambiguous.*

4.2 Patent Breadth

I turn now to explore the implications of patent breadth policy to quality growth and product varieties range. Following Li (2001)[13], I consider patent lagging-breadth policy that defines protection against vertical imitation. The patent breadth parameter $\theta \in \left(\frac{1}{\kappa}, 1\right)$ permits the industry leader of quality q_{t+1} to prohibit the producer of the second-highest quality goods from producing quality above $\frac{q_{t+1}}{\theta}$. Hence, setting $\theta = \frac{1}{\kappa}$ provides complete lagging-breadth protection and for $\theta = 1$ full imitation is allowed (so there is no lagging-breadth protection).

Proposition 3 *Under horizontal competition, loosening patent breadth protection has no effect on market outcomes as long as* $\theta > \frac{s\phi}{(s-1)^2}$*. A lower breadth protection switches the market from horizontal to vertical competition.*

Proof. For $\theta > \frac{s\phi}{(s-1)^2}$ horizontal competition is still binding because $p_h^* < \theta \kappa_h^* \Longleftrightarrow \theta > \frac{s\phi}{(s-1)^2}$. Hence, loosening patent lagging-breadth protection within this range does not yet impose vertical competition and therefore has no effect on market outcomes. ■

Once vertical competition is effective the profit function (10) modifies to

[13] See subsection 3.1 on p. 174 there.

$$\Pi_{i,t} = \frac{(\theta\kappa_{i,t} - 1)(q_{i,t}\kappa_{i,t})^{s-1}(\theta\kappa_{i,t})^{-s} C_{t+1}\lambda_{t+1}}{1 + r_{t+1}} - f_{i,t+1} \tag{16}$$

The first order condition for maximizing (16) with respect to κ is[14]

$$\forall_i : \frac{q_t^{s-1}\theta^{-s}C_{t+1}L}{1 + r_{t+1}} = \phi\,(\kappa_t^*)^2 f_{t+1}^* \tag{17}$$

Equation (17) shows that, given the aggregate variables, each individual innovator finds it optimal to increase innovation size in response to a decrease in patent breadth protection. To get the economic intuition for this result, recall (from Section 3) that vertical competition brings the price below the level that is optimal based on demand's price elasticity. Therefore, quality improvement under vertical competition combines a direct effect on profit (through increasing demand) and additional positive effect through increasing the price innovators can set. For a weaker lagging-breadth protection the price innovators can charge (for a given κ) falls further below the profit maximizing price and thus the price effect of κ on the profit is getting stronger.

Combining the first order condition (17) with the zero-profit condition, imposed on (16), yields

$$\forall_{i,t} : \frac{1}{\kappa_v^e\,(\theta\kappa_v^e - 1)} = \phi \tag{18}$$

Equation (18) implies that the rate of quality improvements under vertical competition is decreasing with the degree of patent lagging-breadth protection. Equation (18) implies also that the equilibrium price under vertical competition, $p_{v,\theta}^e = \theta\kappa_v^e$, is increasing with lagging-breadth protection[15]. That is, the increase in the quality improvement rate does not fully compensate for the direct decrease in price due to a lower θ. The above equilibrium price and quality growth rate support the variety span

$$M_{v,\theta}^e = \frac{L}{f_{v,\theta}^e\left[\frac{1}{(\theta\kappa_v^e - 1)\beta} + 1\right]} \tag{18a}$$

Proposition 4 *Under vertical competition, loosening patent breadth protection increases the rate of quality growth and decreases variety span, that is* $\frac{\partial\kappa_v^e}{\partial\theta} < 0$ *and* $\frac{\partial M_v^e}{\partial\theta} > 0$

Proof. By (18): $\frac{\partial\kappa_v^e}{\partial\theta} < 0$ and $\frac{\partial(\theta\kappa_v^e)}{\partial\theta} > 0$. Therefore, loosening lagging-breadth protection (i.e. decreasing θ) works to decreases the denominator in (18a) through both the price and R&D-investment effects, hence $\frac{\partial M_v^e}{\partial\theta} > 0$. ■

[14] Notice that (15) can be more conveniently written as $\Pi_{i,t} = \frac{(\theta - \frac{1}{\kappa})(q_{i,t})^{s-1}\theta^{-s}C_{t+1}L}{1+r_{t+1}} - f(\kappa)$.

[15] This shows more easily after re-writing (19) as $\frac{1}{\kappa_v^e\phi} + 1 = \theta\kappa_v^e$. For a higher θ the right hand side is increasing, as κ_v^e decreases, but in equilibrium the right hand side equals the price $\theta\kappa_v^e$.

5 Welfare Analysis

The socially-optimal quality growth rate and products variety span are defined by allocating the labor force over R&D and production activity, as to maximize the lifetime utility (1). Young (1998) have shown that[16] these socially-optimal values[17] are

$$\kappa^{**} = \frac{s-1}{\phi\,(1-\beta)},\ M^{**} = \frac{L}{f\,(\kappa^{**})\left[\frac{1}{(\varepsilon-1)\beta}+1\right]} \tag{19}$$

Clearly, by (5a), the quality growth rate under horizontal competition in the benchmark economy (from Section 3) is lower than the welfare maximizing one: $\kappa^{**} = \frac{s-1}{\phi(1-\beta)} > k_h^e = \frac{s-1}{\phi}$. However, comparing κ^{**} with κ_v^e reveals that vertical competition in the benchmark economy may yield insufficient or excessive quality growth, as illustrated in Figure 2.

Figure 2: Efficiency in Quality improvement rate

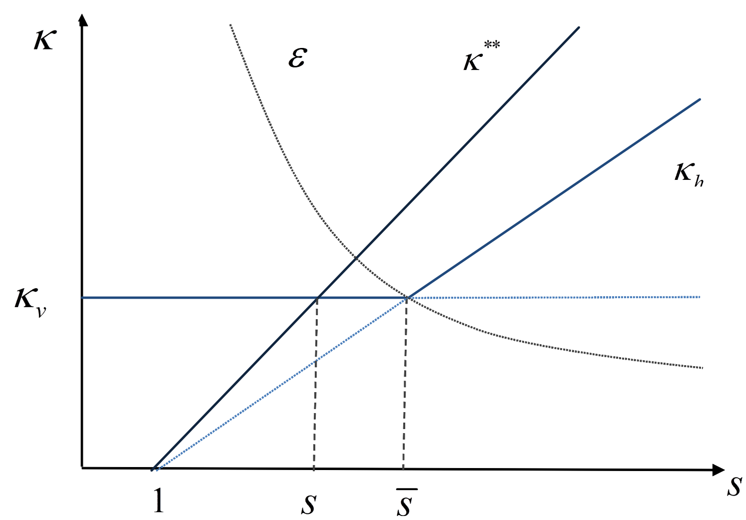


Figure 2 adds κ^{**} to the diagram presented in Figure 1 from Section 3, where $\underline{s} = \frac{(1-\beta)\phi\left(1+\sqrt{1+\frac{4}{\phi}}\right)}{2} + 1 > 1$. Hence, for any $s \in (1, \underline{s})$ vertical competition in the benchmark economy yields excessive quality growth and for $s \in (\underline{s}, \overline{s})$ quality growth is insufficient. This result is highlighted in the following proposition.

Proposition 5 *Under vertical competition the rate of quality improvements can exceed or fall short of the social optimum.*

Young (1998) shows that under horizontal competition quality growth can be enhanced to the socially optimal rate with a proper combination of a increasing-rate subsidy and a flat rate tax on

[16] See equations (30)-(33) on pp. 57-58 there.
[17] Denoted with double asterisk super script.

R&D (See P. 59 and footnote#16 there). The present analysis focuses on policies that combine a flat rate R&D subsidy with constraining innovators market power through patent breadth or price controls.

Under the present cost specification, a flat rate R&D subsidy has no effect on the equilibrium rate of quality improvements. However, such a subsidy affects the equilibrium variety span. Repeating the calculations from equation (1) to equation (9), after adding a flat rate R&D subsidy, denoted σ, that is funded through a corresponding (balanced budget) income lump-sum tax (or flat rate labor income tax)[18] yields the following variety span

$$M^e = \frac{L}{\left[\frac{(1-\sigma)}{(p^e-1)\beta} + 1\right]} \tag{20}$$

where p^e and κ^e are the equilibrium price and quality growth rate. By equation (18), loosening patent breadth protection to the proper degree, denoted θ^{**}, will bring the quality improvements rate to the efficient level by satisfying: $\theta^{**} \equiv \frac{1}{\kappa^{**}}\left(\frac{1}{\kappa^{**}\phi} + 1\right)$. Clearly, $\theta^{**} \in \left(\frac{1}{\kappa^{**}}, 1\right)$, hence there exists a positive and incomplete degree of patent breadth protection that supports the efficient rate of quality improvements, through vertical competition.

However, the equilibrium price under this lagging-breadth protection is $p^e = \theta^{**}\kappa^{**}$, that is smaller than ε. Therefore, under this patent policy θ^{**}, which supports κ^{**}, variety span is smaller than the efficient one. Plugging $\kappa^e = \kappa^{**}$ and $p^e = \theta^{**}\kappa^{**}$ into (20) and equalizing it to M^{**}, reveals that the efficient variety span can be supported with the subsidy rate $\sigma^{**} = \beta$.

If $\kappa_v^e < \kappa^{**}$ the efficient quality improvements rate can be achieved just like in the previous case (where $\kappa_h^e < \kappa^{**}$), i.e. with the incomplete patent breadth protection θ^{**} and a flat rate subsidy $\sigma^{**} = \beta$.

If the initial rate of quality improvements is too high, the optimal quality growth rate can be achieved with direct price ceiling that is set at $\mu = \kappa^{**}$. Then, imposing $\mu = \kappa^{**} = p^e = \kappa^e$ in (18) and equalizing it to M^{**} reveals that the subsidy rate required to support the optimal variety span should satisfy: $1 - \sigma^{**} = \frac{\kappa^{**}-1}{\varepsilon-1}$. This subsidy rate is positive for $\kappa^{**} < \varepsilon$, which always holds in the relevant case (when price regulation is welfare improving)[19].

The latter results are summarized in the following proposition.

Proposition 6 *The socially optimal growth rate and variety span can be restored with a combination of market power limitation and a flat R&D subsidy. For $\kappa^e < \kappa^{**}$ there exists incomplete lagging-breadth protection level θ^{**} for which $\kappa_v^e(\theta^{**}) = \kappa^{**}$,and $\sigma^{**} = \beta$ supports the efficiency variety span. For $\kappa^e > \kappa^{**}$ efficiency is achieved by a price ceiling $\mu = \kappa^{**}$ and the R&D subsidy rate $\sigma^{**} = 1 - \frac{\kappa^{**}-1}{\varepsilon-1} > 0$.*

[18] Imposed on equation (2).

[19] To see that, recall that as for $s = \widetilde{s}$ we have $\kappa_v = \kappa_h = \varepsilon$. However, for any $s < \widetilde{s}$ we have $\varepsilon > \kappa_v$, as ε is decreasing with s. Therefore, in the range where price control can enhance welfare, i.e. for which $\kappa_v > \kappa^{**}$, it must be also true that $\varepsilon > \kappa^{**}$.

6 Conclusion

In the two-R&D-sector economy either horizontal or vertical competition may be binding innovators' price setting. This study emphasizes that the type of binding competition has immediate implications to the effectiveness of alternative policies in promoting growth and welfare.

Under horizontal competition the rate of quality growth in the model economy is insufficient, whereas under vertical competition it may be either insufficient or excessive. In either case of insufficient quality growth, the socially optimal growth rate can be achieved by setting a proper degree of incomplete patent lagging-breadth protection. Excessive rate of quality growth can be offset by a proper price ceiling. In both cases, supporting the socially optimal variety span requires additional policy intervention, which is a corresponding flat rate R&D subsidy.

References

[1] Aghion P., Peter H., 1992. A model of growth through creative destruction. Econometrica 60, 323–51

[2] Aghion P., Howitt P., 2009. The economics of growth. MIT Press. Cambridge. MA

[3] Chu A., 2011. The welfare cost of one-size-fits-all patent protection. Journal of Economic Dynamics and Control 35, 876–90

[4] Chu A., Furukawa Y., Ji L., 2016. Patents, R&D subsidies, and endogenous market structure in a Schumpeterian economy. Southern Economic Journal 82, 809–825

[5] Chu A., Pan S., 2013. The escape-infringement effect of blocking patents on innovation and economic growth. Macroeconomic Dynamics 17, 955-969

[6] Cozzi G., Spinesi L, 2006. Intellectual appropriability, product differentiation, and growth. Macroeconomic Dynamics 10, 39–55

[7] Dinopoulos E., Thompson P., 1998. Schumpeterian growth without scale effects. Journal of Economic Growth 3, 313-335

[8] Evans L., Quigley N., Zhang J., 2003. Optimal price regulation in a growth model with monopolistic suppliers of intermediate goods. Canadian Journal of Economics 36, 463–474

[9] Grossman G.M., Helpman E., 1991. Quality ladders in the theory of growth. Review of Economic Studies 58, 43–61

[10] Howitt P., 1999. Steady endogenous growth with population and R&D inputs growing. Journal of Political Economy 107, 715–30

[11] Goh A.T., Olivier J., 2002. Optimal patent protection in a two-sector economy. International Economic Review 43, 1191–1214

[12] Iwaisako T., Futagami K., 2013. Patent protection, capital accumulation, and economic growth. Economic Theory 52, 631–68

[13] Li, C-W., 2001. On the policy implications of endogenous technological progress. The Economic Journal 111, 164–179.

[14] Peretto P., 1998. Technological change and population growth. Journal of Economic Growth 3:283–311.

[15] Peretto P., 2007. Corporate taxes, growth and welfare in a Schumpeterian economy. Journal of Economic Theory 137, 353–82

[16] Peretto P., 2011. The growth and welfare effects of deficit-financed dividend tax cuts. Journal of Money, Credit and Banking 43, 835–69

[17] Romer P., 1990. Endogenous technological progress. Journal of Political Economy 98, 71–102

[18] Young A., 1998. Growth without scale effects. Journal of Political Economy 106, 41-63

[19] Zeng, J., Zhang J., Fung M., 2014. Patents and price regulation in an R&D growth model. Macroeconomic Dynamics 18, 1–22