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Dynamics of Human Capital Accumulation, IPR Policy, and Growth

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Abstract

We study the effect of IPR (Intellectual Property Rights) policy on growth, in a closed overlapping-generations economy, which undergoes transitional development phase of human capital accumulation. We show that the growth-maximizing policy is stage-dependent: in the early development phase, during which innovation cost is high relative to worker productivity, weak IPR protection can expedite economic growth and may be necessary to escape long run stagnation. Weaker IPR protection erodes monopolistic deadweight loss and, thereby, increases aggregate output and saving. However, it also shifts investment away from R&D activity towards the formation of physical capital. We show that the former (positive) effect is dominant during the early development phase. However, as human capital is further accumulated and labor productivity correspondingly increases, economic growth is maximized with stronger IPR protection.

JEL Classification: : O-31, O-34

Key-words: Stage-Dependent IPR, OLG, Human Capital, Development and Growth.

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†We started studying the current topic in an earlier circulated working paper, titled "Economic Development and Stage-Dependent IPR". That paper presented different modeling approach, which we abandoned after detecting it has some analytical flaws. We have benefitted from comments by Niloy Bose, Thomas Osang, Hyeongwoo Kim, and participants at the SEA 2015 meeting in New Orleans.

1 Introduction

The strength of intellectual property rights (IPR), is positively correlated with economic development worldwide (See Eicher and Newiak 2013, and Chu et al. 2014). This empirical observation may suggest that developing countries fail to efficiently strengthen domestic IPR due to political (institutional) shortcomings, or that the optimal strength of IPR is positively related to the stage of economic development.

In a recent study, Chu et al. (Stage-dependent intellectual property rights, *Journal of Development Economics* 2014) presented the first analysis of stage-dependent optimal IPR, based on a trade-off between imitation from foreign direct investment (FDI) and reliance on domestic innovation. In this work, we propose a complementary case for stage-dependent IPR policy, which is independent of the imitation motive. We study IPR policy for a closed overlapping-generations (OLG) economy in which R&D-based growth is initiated and boosted through human capital accumulation. We show that the growth-maximizing IPR policy corresponds the development stage of the economy, with increasing IPR protection along th course of human capital accumulation.

Although most developing economies engage in extensive international trade, capital flows from developed to developing economies are limited, as noted by Lucas (1990). More recently, Alfaro et al. (2008) found that low institutional quality is the leading causal variable explaining this phenomenon. Hence, they conclude, "policies aimed at strengthening the protection of property rights, reducing corruption, and increasing government stability, bureaucratic quality, and law and order should be a priority for policy-makers seeking to increase capital inflows to poor countries"(p.365).

Nonetheless, some major developing economies, such as China and India, deliberately impose severe restrictions on capital inflows[1]. Figure 1 shows that financial openness is positively correlated with IPR measures (one the left-hand side) and per-capita output (on the right-hand side)[2]. China and India share the second lowest value of the financial openness index. Figure 2 presents the net inflows (% of GDP) and Gross capital formation (% of GDP) for China and India[3]. Over the entire period, the average percentage of FDI relative to gross domestic investment was 10% in China and only 3.7% in India. Taking those restrictions on capital flows as given, we study IPR policy for developing economies with closed capital markets.

[1] It was recently reported that China plans to relax its capital flows restrictions in coming years: *"Currently, although China permits cross-border flow of currency for foreign trade and investment in factories, the government restricts heavily funds entering and leaving the country for financial investments like stocks and bonds." (Wall Street Journal, May 28th, 2015).* The article is available at: http://www.wsj.com/articles/china-to-ease-limits-on-overseas-investments-1432841526

[2] The data cover 115 countries for the year 2005. Sources are: 1) “Index of Patent Rights” data from Park, W. (Research Policy, 2008), available (at:https://www.american.edu/cas/faculty/wgpark/upload/IPP-Research-Policy-May-2008-3.pdf). 2) Chinn-Ito Index is the measure of “Index of Financial Openness” introduced by Chinn and Ito (Journal of Development Economics, 2006), availabvle at: http://xt5bv6dq8y.scholar.serialssolutions.com/?sid=google&auinit=MD&aulast=Chinn&atitle=What+matters+for+financial+develo 3878). 3) GDP per capita (current US $) data from World Bank collection of Development Indicators, available at: http://data.worldbank.org/indicator/NY.GDP.PCAP.CD

[3] Here, Gross capital formation (% of GDP) acts as a proxy for domestic investment. Source: World Bank collection of Development Indicators (available at: http://data.worldbank.org/data-catalog/world-development-indicators)).

Figure 1:

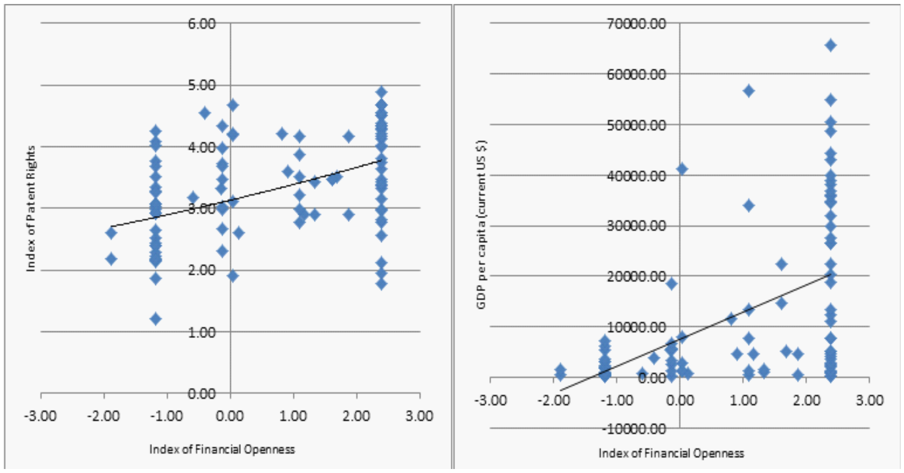
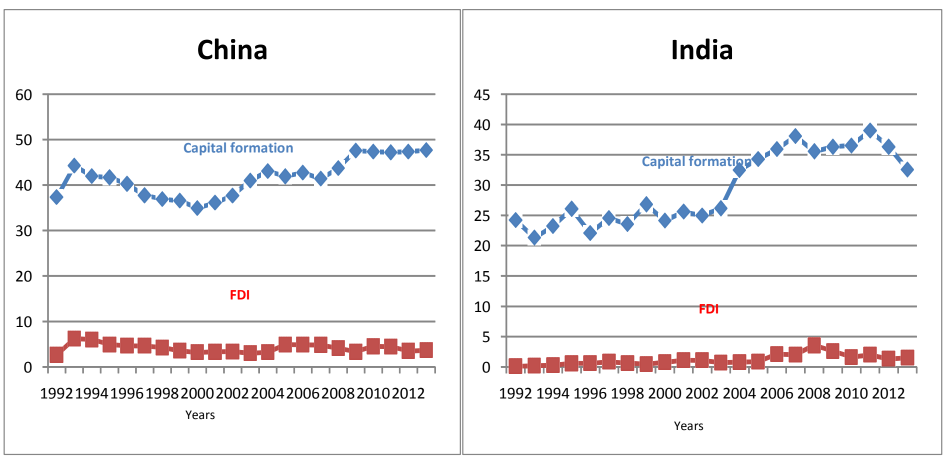


Figure 2:



The present analysis builds on the framework we have recently studied in Diwakar and Sorek (2016). In that study, we focus on the unique implications of the OLG demographic structure of finitely living agents to the effects of IPR policy on growth and welfare. We showed that they may contrast with those identified the counterpart model economy of infinitely living agents. In particular, we showed that in the OLG economy growth-maximizing IPR policy is subject to the following trade-off: weak IPR protection erodes monopolistic power over patented technologies, thereby increasing their utilization and, consequently, increasing also aggregate output, saving, and investment (this is a positive static effect of weak patents on growth)[4].

[4]This is in line with Jones and Manuelli (1992) who showed that in neoclassical economy of finitely living agents, growth is bounded by the ability of the young generation to purchase capital held by the old. One of the remedies they consider to support sustained growth in such economy is direct income transfers from old to young. Similarly, Uhlig and Yanagawa (1996) showed that reliance on capital-income taxation can enhance growth.

Yet, at the same time, weak IPR protection also shifts investment away from innovation towards the formation of physical capital (this is a negative dynamic effect of weak patents on growth). Hence, IPR policy involves a trade-off between the level of aggregate investment and its allocation between R&D and the formation of physical capital. We found that, due to this trade off, long run growth is maximized with incomplete IPR protection. The analysis in our previous work, like in most related studies we review below, was confined to the balanced growth equilibrium.

In the present paper, we extend the previous framework by incorporating dynamics of human capital accumulation, which is central to the process of economic development, and we focus on the growth maximizing IPR policy along the transitional dynamics. We show that, human capital accumulation has a direct positive effect on economic growth due to the increase in effective labor supply and thereby output.

However, the effect of human capital on innovation involves two contradicting forces, which mirror the effects of IPR policy on innovation that we have considered above: on the positive side the direct increase in output increases overall investment. However, the increase in effective labor supply also increases demand for each machine variety, thereby shifting investment away from the development of new varieties (innovation) towards formation of more machines of each existing variety (physical capital formation).

We find that the relative strength of the former, positive, effect is increasing as human capital is accumulated. Therefore, growth is maximized with an increasing IPR protection along the transitional dynamics, which are driven by the accumulation of human capital. R&D-based growth is induced only when a certain threshold level of human capital accumulation is surpassed. Then, perpetual innovation takes place, in the form of expanding the variety span of differentiated inputs, and growth accelerates towards its stationary sustainable rate. If the critical level of human capital is not reached, weak IPR may be necessary to initiate and sustain positive and low R&D-based growth[5].

There is, by now, a large body of theoretical research on IPR policy, innovation and economic growth for closed economies; See, among others, Li (2001), Kwan and Lai (2003), O'Donoghue and Zweimuller (2004), Furukawa (2007), Acemoglu and Akcigit (2012) Cysne and Turchick (2012), Zheng et al. (2013), Iwaisako and Futagami (2013). However, this literature applies almost exclusively to economies with infinitely lived agents, following the canonical frameworks of Romer (1990), Grossman and Helpman (1991), and Aghion and Howitt (1992).

These closed-economy models commonly lack transitional dynamics. Hence, this literature has focused on the long-run IPR policy - along the balanced growth path - as did the few works that studied IPR policy for OLG economies, such as Chou and Shy (1993), Sorek (2011), and Diwakar and Sorek (2016)[6].

[5] In their recent empirical work, Mohtadi and Ruediger (2014) report on non-linear effect of IPR protection on growth in developing economies, depending on their human capital accumulation - consistently with our results.

[6] A related strand of literature studies the implications of IPR for international trade between economies at different development stages, in the "North-South" fashion of Helpman (1993). See Chu et al. (2014) for a recent compact summary of this literature.

Several studies in this literature have also accounted for the transitional effects of discrete and gradual change in patent policy, along the adjustment path from one steady state to another. See, for example, Chu (2009), Chu et al.(2012), and Cozzi and Galli (2014). However, the transitional dynamics quantified therein are due to changes in patent policy and are not related to an endogenous phase of economic development, whereas in the present study, we focus on patent policy that is set to correspond such transitional dynamics that are driven by endogenous human capital accumulation process.

Chu et al.(2014) recently presented a model of stage-dependent optimal IPR policy. The main message from their analysis is that in the early development stage, growth is maximized with weak IPR because it enables the imitation of foreign technologies through FDI. However, as the developing economy catches up with the global technological frontier, imitation opportunities are exhausted. Then, stronger IPR is required to support growth that is based on domestic innovation[7]. Chu et al. (2014) provided evidence that China's IPR policy over the last few decades followed such a path.

In this work we present a complementary case for stage-dependent IPR policy that is independent of the imitation motive. Another recent work, by Iwaisako (2013), proposed a welfare maximizing IPR that is positively related to the level of public spending, for closed developing economies where both R&D investment and public spending are engines of growth. This work, however, focuses on steady-state analysis with no transitional dynamics.

The remainder of the paper is organized as follows: Section 2 presents the OLG model of variety expansion and human capital accumulation. Section 3 presents the dynamics of innovation and growth in the model economy. Section 4 analyzes the growth-maximizing IPR policy, and Section 5 concludes this study.

2 The Model

The model presented below is identical to the one employed by Kwan and Lai (2003), Cysne and Turchick (2012) and Zeng et al. (2013) and Barro and Sala-I-Martin (2004, Ch. 6), except for its demographic structure, which follows Diamond's (1965) canonical OLG framework. A population of constant size composes two overlapping generations in each period - "young" and "old". Generation size is normalized to one, and each young agent is endowed with one unit of time. Old agents retire to consume their saving. First, we present the model under the assumption of complete IPR protection, meaning that all inventions are patented forever and patent holders can charge the unconstrained optimal monopolistic price. In Section 4, we study the effect of incomplete IPR protection on growth, and the implied growth-maximizing IPR policy.

[7] Chen and Puttitanum (2005) presented a similar argument based on a static analytical framework.

2.1 Production and Innovation

Final output that can be used for consumption and investment is produced by perfectly competitive firms with labor and differentiated inputs, which are investment goods - i.e., "machines" - subject to the CRTS technology[8]

$$Y_t = AH_t^{1-\alpha} \int_0^{N_t} K_{i,t}^{\alpha} \, \mathrm{d}i \quad \alpha \in (0,1) \tag{1}$$

where A is a productivity factor, and H_t is the aggregate effective labor supply at period t, to be defined in Subsection 2.2. $K_{i,t}$ is the utilization level of machine variety i in period t, and N_t measures the number of available machine varieties in period t[9]. The price of the final good is normalized to one. Under symmetric equilibrium, the utilization level for all machines is the same, i.e., $K_{i,t} = K_t \ \forall \ i$, and thus total output is

$$Y_t = AN_t {K_t}^{\alpha} H_t^{1-\alpha} \tag{1a}$$

The labor market is perfectly competitive, and therefore, the equilibrium wage and aggregate labor income are $w_t = A(1-\alpha)N_t {K_t}^{\alpha} H_t^{-\alpha}$ and $w_t H_t = A(1-\alpha)N_t {K_t}^{\alpha} H_t^{1-\alpha}$, respectively. The profit for the representative final-good producer is

$$\max_{K_{i,t}} \Pi_{i,t} = \pi_{i,t} = AH_{t,i}^{1-\alpha} \int_0^{N_t} K_{i,t}^{\alpha} \, \mathrm{d}i - \int_{i=1}^{N_t} p_{i,t} K_{i,t} \, \mathrm{d}i - w_t H_{t,i}$$

where $p_{i,t}$ is the per-unit price of machine variety i and $H_{t,i}$ is the level of effective labor employed with machine variety i. Profit maximization yields the demand for each machine: $K_{i,t}^d = A^{\frac{1}{1-\alpha}} H_t \left(\frac{\alpha}{p_{i,t}}\right)^{\frac{1}{1-\alpha}}$. The cost of producing each machine is one final output unit. Machines are formed through investment, one period ahead of usage, and they fully depreciate after one utilization period. Hence, given the demand for each machine-variety, $K_{i,t}^d$, the periodic surplus form holding a patent over machine i is $PS_{i,t} = \left[p_{i,t} - (1+r_t)\right] K_{i,t}^d$, where r_t is the net interest rate paid in period t. Under the assumed complete IPR protection, this surplus is maximized by the standard monopolistic price $p_{i,t} = \frac{1}{\alpha}(1+r_t)$, $\forall i$. Plugging this price into $K_{i,t}^d$ yields $K_{i,t}^d = A^{\frac{1}{1-\alpha}} H_t \left(\frac{\alpha^2}{1+r_t}\right)^{\frac{1}{1-\alpha}}$ $\forall i, t$. Then, plugging the latter expression back into (1a) yields the following expression for total output

$$Y_t = A^{\frac{1}{1-\alpha}} N_t H_t \left(\frac{\alpha^2}{1+r_t}\right)^{\frac{\alpha}{1-\alpha}} \tag{1b}$$

[8] Strulik et al. (2013) and Prettner (2014) follow the same modeling approach for the OLG framework. Kwan and Lai (2003), Cysne and Turchick (2012) and Zeng et al. (2013) consider the differentiated inputs as intermediate goods in the infinite horizon setup with no human capital accumulation, and so did Barro and Sala-I-Martin (2004) and Aghion and Howitt (2009) in their textbooks variety expansion models -(see Chapters 6.1 and 3.4, respectively). In Diwakar and Sorek (2016), we show that the definition of differentiated inputs as investment goods or intermediate goods, has immediate implications to the effect of IPR policy on growth, which are unique to the OLG framework.

[9] The elasticity of substitution between different varieties is $\frac{1}{\alpha}$.

Equation (1b) implies that the output growth rate, denoted g_Y, is defined by the variety expansion rate (g_N) and the growth rates of effective labor supply (g_H) and the gross interest rate (g_{1+r}):

$$1 + g_{Y,t+1} \equiv \frac{Y_{t+1}}{Y_t} = (1 + g_{N,t+1})(1 + g_{H,t+1})(1 + g_{1+r,t+1})^{\frac{-\alpha}{1-\alpha}} \tag{1c}$$

Notice that g_{1+r} here actually represents that growth rate of machines price, due to demand increase that is driven by human capital accumulation (as proven in Lemma 1 below). In Section 4, we will replace it with the more general term g_p, for the analysis of incomplete IPR protection. As cohort's size is constant, the aggregate output growth rate given in (1c) also defines the growth rate of per capita (and per-worker) output, denoted $1 + g_{y,t+1}$, which is the measure of economic growth.

The innovation technology follows the conventional "lab-equipment" specification[10]

$$\Delta N_{t+1} = (N_{t+1} - N_t) = \frac{R_t}{\phi} \tag{2}$$

where R is R&D investment in new varieties, and thus ϕ is the fixed cost per blue print for a new machine variety[11]. Each new variety is granted an eternal patent, which gives rise to inter-generational trade in patents, where the no-arbitrage condition implies that, in equilibrium with innovation, the value of old patents equals the cost of inventing new varieties; hence, the equilibrium interest rate is $1 + r_{t+1} = \frac{PS_{t+1}+\phi}{\phi}$, where we already know that $PS_{t+1} = [p_{t+1} - (1 + r_{t+1})] K^d_{t+1}$. Plugging the explicit form of PS_{t+1} into the interest rate equation yields the following implicit function for the equilibrium interest rate, denoted r^*

$$1 + r^*_{t+1} = \frac{PS_{t+1} + \phi}{\phi} = \frac{(1 + r^*_{t+1})^{\frac{-\alpha}{1-\alpha}} \left(\frac{1}{\alpha} - 1\right) A^{\frac{1}{1-\alpha}} H_{t+1} \alpha^{\frac{2}{1-\alpha}} + \phi}{\phi} \tag{3}$$

The above expression implies that the investment in old and new patents in physical capital ("machines") - yield the same rate of return.

Lemma 1 *There exists a unique equilibrium interest rate r^* that solves equation (3), and is increasing in effective labor supply, that is $\frac{\partial(1+r^*_{t+1})}{\partial H_{t+1}} > 0$.*

Proof. The left-hand side of (3) is increasing linearly in r^* from one to infinity. The right-hand side of (3) is a decreasing function of r^* , which ranges from $\frac{\left(\frac{1}{\alpha}-1\right)A^{\frac{1}{1-\alpha}}\alpha^{\frac{2}{1-\alpha}}H_{t+1}+\phi}{\phi} > 1$ (for $r^* = 0$) to one (as r^* approaches infinity). Hence, by the fixed point theorem, there exists a unique positive (net) interest rate, denoted r^*, which equalizes the returns on investment in old and young patents, and machines. This is the equilibrium interest rate. Applying the implicit function theorem to equation (3) we obtain $\frac{\partial(1+r^*_{t+1})}{\partial H_{t+1}} = \frac{\frac{(1+r_{t+1})^{\frac{-\alpha}{1-\alpha}}\left(\frac{1}{\alpha}-1\right)A^{\frac{1}{1-\alpha}}\alpha^{\frac{2}{1-\alpha}}}{\phi}}{\frac{\frac{\alpha}{1-\alpha}(1+r_{t+1})^{\frac{-1}{1-\alpha}}\left(\frac{1}{\alpha}-1\right)A^{\frac{1}{1-\alpha}}H_{t+1}\alpha^{\frac{2}{1-\alpha}}}{\phi}+1} > 0.$ ■

[10] This specification was used in previous analyses of patent policy by Kwan and Lai (2003) and Cysne and Turchick (2012), and Zheng et al. (2013), among others.

[11] In Section 4, we will consider the possible implications of alternative specification, where innovation cost is increasing with technological progress.

It can be shown that, as human capital is being accumulated, the overall effect of $(1+g_{H,t+1})(1+g_{1+r,t+1})^{\frac{-\alpha}{1-\alpha}}$ on output growth rate presented in (1c) is positive. This product combines the following growth factors: the direct positive effect of increase in effective labor supply on output, and the resulting increase in demand for machines which is partially offset by following increase in machines price. We show this explicitly in the proof of Lemma 1 (that is presented in Appendix A).

2.2 Human capital formation

Households can invest time in forming human capital, which enhances their labor productivity, subject to the human capital formation function

$$h_t = (\delta + \xi e_t) h_{t-1}^{\gamma} \tag{4}$$

Where $\gamma, \delta > 0$, and $\gamma < 1$. The parameter γ measures the degree of intergenerational spillover of human capital, ξ is a productivity factor in formation of new human capital, and δ reflects the basic human capital capacity each worker is born with [12]. The effective labor supply of the representative worker is the product of her productivity and her labor time, $l_s = 1 - e_t$

$$H_t = (1 - e_t)(\delta + \xi e_t) h_{t-1}^{\gamma} \tag{4a}$$

Workers' time uses are restricted to labor and human capital formation, which improves labor productivity. Therefore, the optimal investment in human capital should maximize the effective labor supply. Maximizing (4a) with respect to e_t yields the optimal investment, denoted $\overline{e}$, which is time invariant: $\overline{e} = \frac{1-\frac{\delta}{\xi}}{2}$.[13] Therefore, by (4a) and $\overline{e}$, effective labor supply evolves according to the dynamic equation[14]

$$H_t = \underbrace{(1 - \overline{e})^{1-\gamma}(\delta + \xi \overline{e})}_{\equiv \varphi} H_{t-1}^{\gamma} \equiv \varphi H_{t-1}^{\gamma} \tag{4b}$$

Hence, the effective labor supply grows at the rate

$$1 + g_{H,t} \equiv \frac{H_t}{H_{t-1}} = \varphi H_{t-1}^{\gamma-1} \tag{5}$$

For $\gamma < 1$ and $(\delta + \xi \overline{e})^{\frac{1}{1-\gamma}} = (\frac{\xi+\delta}{2})^{\frac{1}{1-\gamma}} > h_0$, effective labor supply grows at a decreasing rate, approaching the stationary (long-term) level $\widetilde{H} = \frac{1}{\xi}(\frac{\xi+\delta}{2})^{\frac{1}{1-\gamma}+1}$. Plugging $\overline{e}$ into φ yields: $\varphi(\overline{e}) = \left(\frac{1}{\xi}\right)^{1-\gamma} \left(\frac{\xi+\delta}{2}\right)^{2-\gamma}$. Notice that $\widetilde{H}$ is increasing in γ. For the case where $\gamma = 1$, which we abstract for now, human capital grows perpetually at a constant rate $\varphi = \left(\frac{\xi+\delta}{2}\right)$.[15]

[12] The specification in (4) is common to the literature on human capital and growth. See, for example, De-la-Croix and Deopke (2004). For $\delta, \gamma = 1$, equation (4) is identical to Lucas' (1988) seminal formulation.

[13] We assume $\xi > \delta$ to ensure positive investment in human capital formation.

[14] Following that h_{t-1} in equation (4) can be written as $\frac{H_{t-1}}{1-\overline{e}}$.

[15] In Section 3 we show that under the assumed lab-equipment innovation technology output growth rate in this case is increasing perpetually ("exploding"). Nevertheless, in Section 4 we also show that allowing for increasing

2.3 Preferences

Lifetime utility from consumption, over two periods, for a worker who was born in period t follows the tractable logarithmic formulation[16]

$$u(c_{t,1}, c_{t,2}) = \ln c_{t,1} + \rho \ln c_{t,2} \tag{6}$$

where $\rho \in (0,1)$ is the subjective discount factor. Young agents allocate their labor income between consumption and saving, denoted s. The solution for optimal saving in this standard problem is $s_t = \frac{1}{1+\rho^{-1}} w_t H_t$. Substituting the explicit expression for w_t (and recalling the population size is normalized to one) we obtain aggregate saving

$$S_t = \frac{(1-\alpha) A^{\frac{1}{1-\alpha}} N_t H_t \left(\frac{\alpha^2}{1+r_t}\right)^{\frac{\alpha}{1-\alpha}}}{1+\rho^{-1}} \tag{7}$$

3 Equilibrium and Growth Dynamics

The savings out of labor income in (7) are allocated over three types of investment: buying patents on old technologies, inventing new machine varieties, and the formation of physical capital (i.e., specialized machines). As new and old varieties play equivalent roles in production, the market value of an old variety equals the cost of inventing a new one - ϕ_t. The market for specialized machines clears, as the supply of each variety equals the demand (as defined in Subsection 2.1). Hence, investments in each period -denoted I - satisfy

$$I_t = N_{t+1} \left[\phi + A^{\frac{1}{1-\alpha}} H_{t+1} \left(\frac{\alpha^2}{1+r_{t+1}} \right)^{\frac{1}{1-\alpha}} \right] \tag{8}$$

By equalizing (8) to (7), we impose the resource-uses equilibrium condition $I_t = S_t$ to obtain the dynamic equation, which governs the variety expansion rate

$$1 + g_{N,t+1} = \frac{(1-\alpha)}{1+\rho^{-1}} \frac{A^{\frac{1}{1-\alpha}} H_t \left(\frac{\alpha^2}{1+r_t}\right)^{\frac{\alpha}{1-\alpha}}}{\phi + A^{\frac{1}{1-\alpha}} H_{t+1} \left(\frac{\alpha^2}{1+r_{t+1}}\right)^{\frac{1}{1-\alpha}}} \tag{9}$$

It can be shown that the level effective labor supply (i.e. for a stationary H level) has a positive effect on variety expansion rate[17]. However, the increase in effective labor supply by itself works to slow down variety expansion rate, by increasing the denominator in (9), due to increase in demand

innovation cost, one can support a balanced growth along with perpetual human capital accumulation (i.e. with $\gamma = 1$), but such configuration greatly complicates the analysis of the transitional dynamics we are interested in.

[16] In Appendix C, we consider the more general CIES (Constant Inter temporal Elasticity of Substitution) instantaneous utility function.

[17] Therefore, the model presents a strong scale-effect, as all aforementioned references that follow the lab-equipment specification for the innovation cost.

for machines that shift investment away from innovation.

By substituting (9) into equation (1c), we write per-capita output growth rate as:

$$1+g_{y,t+1}=\frac{(1-\alpha)}{1+\rho^{-1}}\frac{\left(\frac{\alpha^2}{1+r_{t+1}}\right)^{\frac{\alpha}{1-\alpha}}}{\widehat{\phi}_t+\left(\frac{\alpha^2}{1+r_{t+1}}\right)^{\frac{1}{1-\alpha}}} \tag{10}$$

Where $\widehat{\phi}_t \equiv \frac{\phi}{A^{\frac{1}{1-\alpha}}H_t(1+g_{H,t+1})}$, which, following equations (4b) and (5) can be written as $\widehat{\phi}_t \equiv \frac{\phi}{A^{\frac{1}{1-\alpha}}H_{t+1}}$, is the effective per-worker innovation cost - augmented to labor productivity. To further analyze (10), we define $\psi \equiv \frac{\alpha}{p}=\frac{\alpha^2}{1+r}$, and rewrite it as

$$1+g_{y,t+1}=\frac{(1-\alpha)}{1+\rho^{-1}}\frac{\psi_{t+1}^{\frac{\alpha}{1-\alpha}}}{\widehat{\phi}_t+\psi_{t+1}^{\frac{1}{1-\alpha}}}=\frac{(1-\alpha)}{1+\rho^{-1}}\frac{1}{\frac{\widehat{\phi}_t}{\psi_{t+1}^{\frac{\alpha}{1-\alpha}}}+\psi_{t+1}} \tag{10a}$$

Lemma 2 *Growth rate is accelerating along the process of human capital accumulation, that is* $\frac{\partial(1+g_{y,t+1})}{\partial H_{t+1}}>0$

Proof. By (10a), ψ_{t+1} is decreasing in H_{t+1}, if $\frac{\widehat{\phi}_t}{\psi_{t+1}^{\frac{\alpha}{1-\alpha}}}$ is also decreasing in H_{t+1}, it is guaranteed that $\frac{\partial(1+g_{y,t+1})}{\partial H_{t+1}}>0$. In Appendix A we prove that this is indeed the relevant case here. ■

Let us also denote the long term stationary values of the different variables in our model with tilde, e.g. the long-run growth rates is $\widetilde{g}\equiv\lim_{t\to\infty}g_t$.

Proposition 1 *A sufficiently high value of γ guarantees the surpassing of a critical human capital level, H_c, above which the economy follows transitional dynamics along which growth accelerates up to the economy's finite long-run growth rate.*

Proof. Recall that per-capita output growth is given in equation (1c) by $1+g_{y,t+1}=(1+g_{N,t+1})(1+g_{H_{t+1}})(1+g_{1+r,t+1})^{\frac{\alpha}{1-\alpha}}$, whereas equation (5) implies that human-capital growth rate ($g_{H,t+1}$) approaches zero in the long run and is independent of the variety expansion rate defined in (9). Moreover, equation (3) and Lemma 1 imply that for $\widetilde{g}_H=0$, the interest rate is also stationary, i.e. $g_{1+r}=0$. Then, by equation (9), $\widetilde{g}_N,\widetilde{g}_y>0$ iff $\frac{1-\alpha}{1+\rho^{-1}}\left(\frac{\alpha^2}{1+\widetilde{r}}\right)^{\frac{\alpha}{1-\alpha}}-\left(\frac{\alpha^2}{1+\widetilde{r}}\right)^{\frac{1}{1-\alpha}}>\widetilde{\phi}$ .Then, a sufficiently high γ guarantees that $\widetilde{H}$ is large enough to bring the term $\widetilde{\phi}$ sufficiently low so that (10a) approaches $1+\widetilde{g}_y=\frac{\rho}{1+\rho}\frac{(1-\alpha)(1+\widetilde{r})}{\alpha^2}$. Since the interest rate $\widetilde{r}$ is increasing with $\widetilde{H}$ (which is increasing in γ), sufficiently large γ guarantees that the latter condition will hold. Alternatively having $\frac{\rho}{1+\rho}\frac{(1-\alpha)}{\alpha^2}>1$ is also sufficient to ensures positive long-run growth, that is $\widetilde{g}_y>0$. By Lemma 2, along the transitional dynamics growth is accelerating. ■

Notice that, for $\gamma=1$ the growth rate in (10) would increase perpetually: as $\widehat{\phi}_t\equiv\frac{\phi}{A^{\frac{1}{1-\alpha}}H_{t+1}}$ goes down to zero the growth rate in (10) approaches $1+g_{y,t+1}=\frac{(1-\alpha)}{1+\rho^{-1}}\frac{1+r_{t+1}}{\alpha^2}$ and, by Lemma

1, growth will accelerate perpetually due to the perpetual increase of the interest rate, driven by constantly increasing supply of effective labor.

3.1 Equilibrium with no innovation

We turn now to characterize the equilibrium with no innovation. Consider the case with $\gamma = \gamma_L$ and the corresponding value of $\widetilde{H} = \widetilde{H}_L$, were γ_L and $\widetilde{H}_L$ are too low to initiate innovation. When there is no innovation, the economy remains with its initial variety span (N_0), and the value of old patents falls below the innovation cost to equalize the right hand side of equation (9) to one, implying a constant variety span. To understand that, notice that if the value of old patent does not go down below the innovation cost level, whenever the right hand side in equation (9) is below one the number of utilized machine-varieties is declining. This means that the value of some patented machines (those who are to be eliminated from the market) goes to zero while the patent value of the remaining machines is positive, which cannot be an equilibrium outcome. Hence, adding the subscript "L" to all other variables, we obtain the following two stationary-equilibrium conditions

$$1 + \widetilde{r}_L = \frac{(1+\widetilde{r}_L)^{\frac{-\alpha}{1-\alpha}} \left(\frac{1}{\alpha} - 1\right) A^{\frac{1}{1-\alpha}} \widetilde{H}_L \alpha^{\frac{2}{1-\alpha}} + \widetilde{\phi}_L}{\widetilde{\phi}_L}$$

$$1 = \frac{(1-\alpha)}{1+\rho^{-1}} \frac{A^{\frac{1}{1-\alpha}} \widetilde{H}_L \left(\frac{\alpha^2}{1+\widetilde{r}_L}\right)^{\frac{\alpha}{1-\alpha}}}{\widetilde{\phi}_L + A^{\frac{1}{1-\alpha}} \widetilde{H}_L \left(\frac{\alpha^2}{1+\widetilde{r}_L}\right)^{\frac{1}{1-\alpha}}} = \frac{(1-\alpha)}{1+\rho^{-1}} \frac{\left(\frac{\alpha^2}{1+\widetilde{r}_L}\right)^{\frac{\alpha}{1-\alpha}}}{\frac{\widetilde{\phi}_L}{A^{\frac{1}{1-\alpha}} \widetilde{H}_L} + \left(\frac{\alpha^2}{1+\widetilde{r}_L}\right)^{\frac{1}{1-\alpha}}}$$

where $\widetilde{\phi}_L < \phi$. Suppose further that γ is increasing to $\gamma_H > \gamma_L$, resembling an exogenous productivity improvement in the process of human capital formation. The increase in γ will initiate a transitional process of human capital accumulation, and if γ_H is sufficiently high (such that $\widetilde{H}_H > H_c$), at some point along the transitional dynamics innovation will take place. As long as the supply of effective labor is lower than the critical level, i.e. $H_t < H_c$, the value of old patents is lower than the cost of innovation and, thus innovation is not profitable. During this phase, the equilibrium value of old patents defines the following equilibrium interest rate,

$$1 + r_{t+1} = \frac{(1+r_{t+1})^{\frac{-\alpha}{1-\alpha}} \left(\frac{1}{\alpha} - 1\right) A^{\frac{1}{1-\alpha}} H_{t+1} \alpha^{\frac{2}{1-\alpha}} + \phi_{t+1}}{\phi_t}$$

while also equalizing the right hand side of (9) to one (as explained above):

$$1 = \frac{(1-\alpha)}{1+\rho^{-1}} \frac{A^{\frac{1}{1-\alpha}} H_t \left(\frac{\alpha^2}{1+r_t}\right)^{\frac{\alpha}{1-\alpha}}}{\phi_t + A^{\frac{1}{1-\alpha}} H_{t+1} \left(\frac{\alpha^2}{1+r_{t+1}}\right)^{\frac{1}{1-\alpha}}}$$

where $\phi_t < \phi$ is the value of old patents (as explained above). The interest rate in the expression above implies that, along the transitional dynamics, the equilibrium in each period relies on perfect foresight, as the interest rate set in today relies on the value of old patent that will prevail in the next period (ϕ_{t+1}). The complete analysis of this phase, of human capital accumulation with no innovation, is complicated and falls beyond the scope of this work. However, by Proposition 1, once effective labor supply is sufficiently high, the market value of old patents reaches the cost of inventing new machine varieties, ϕ, and innovation takes place.

Strulik et al. (2013) presented a unified growth model with endogenous fertility that has similar threshold properties. In their model, before achieving the critical human capital level, the economy employs a neoclassical technology. Once the threshold human capital level is reached, the economy switches to progressive technology of specialized capital to initiate sustained R&D-based growth. Nevertheless, Iwaisako (2002) demonstrated that reaching this threshold does not guarantee that private investors will choose to abundant the old traditional (constant returns to scale) technology for adopting the new progressive (Increasing returns to scale) technology.

4 Intellectual Property Rights

Next, we turn to study the growth-maximizing IPR policy for the innovating economy. We model IPR policy as the strength of patent breadth protection, designated by the parameter λ. The degree of patent breadth protection limits the ability of patent holders to charge the unconstrained monopolistic price: $p(\lambda) = \lambda p^* = \frac{\lambda(1+r)}{\alpha}$, where $\lambda \in (\alpha, 1)$, and thus $p(\lambda) \in \left(1+r, \frac{1+r}{\alpha}\right)$. One can think of $p(\lambda)$ as the maximal price a patent holder can set and still deter competition by imitators. Weaker breadth protection lowers the cost of imitation, thereby imposing a lower deterrence price on patent holders[18]. When $\lambda = 1$, patent breadth protection is complete and patent holders can charge the unconstrained monopolistic price $p = \frac{1+r}{\alpha}$. With zero protection $\lambda = \alpha$, patent holders completely lose their market power and thus sell at zero mark-up price, that is, $p = 1 + r$. Nonetheless, the interest rate itself is affected by the IPR. Modifying equation (3), we obtain the implicit expression for the interest rate as a function of IPR policy

$$1 + r^*_{t+1} = \frac{\left(1 + r^*_{t+1}\right)^{\frac{-\alpha}{1-\alpha}} \left(\frac{\lambda_{t+1}}{\alpha} - 1\right) \lambda_{t+1}^{\frac{-1}{1-\alpha}} A^{\frac{1}{1-\alpha}} H_{t+1} \alpha^{\frac{2}{1-\alpha}} + \phi}{\phi} \tag{11}$$

Lemma 3 *There exists an equilibrium interest rate r^*_t that is increasing with IPR protection and with effective labor supply. That is* $\frac{\partial r^*_{t+1}}{\partial \lambda_{t+1}}, \frac{\partial r^*_{t+1}}{\partial H_{t+1}} > 0$.

Proof. The proof is identical to the one presented for Lemma 1. Differentiating the right hand side for λ yields a positive derivative for $\lambda < 1$. Hence the value of r^*, which solves (11), is increasing with patent breadth protection λ. ■

Lemma (3) implies that the effective price $p(\lambda) = \frac{\lambda[1+r(\lambda)]}{\alpha}$ is decreasing as IPR policy is loosened, and consequently, demand for each machine variety is increasing, according to $K^d = A^{\frac{1}{1-\alpha}} H \left[\frac{\alpha}{p(\lambda)}\right]^{\frac{1}{1-\alpha}}$. By plugging $p_t(\lambda_t)$ into (7), we modify the saving equation, to be a function of the IPR policy implemented in period t

[18] Our modeling approach is equivalent to the explicit price (ceiling) regulation considered by Zeng et al.(2013).

$$S_t = \frac{(1-\alpha)A^{\frac{1}{1-\alpha}} N_t H_t \left[\frac{\alpha}{p_t(\lambda_t)}\right]^{\frac{\alpha}{1-\alpha}}}{1+\rho^{-1}} \tag{12}$$

Modifying (8) correspondingly yields aggregate investment that is made in period t, as a function of the expected IPR policy in period $t+1$

$$I_t = N_{t+1}\left\{\phi + A^{\frac{1}{1-\alpha}} H_{t+1}\left[\frac{\alpha}{p_{t+1}\left(\lambda_{t+1}\right)}\right]^{\frac{1}{1-\alpha}}\right\} \tag{13}$$

By equalizing (12) and (13), we impose the aggregate resource-use constraint to obtain the variety expansion rate as function of IPR policy

$$1+g_{N,t+1} = \frac{(1-\alpha)}{1+\rho^{-1}} \frac{A^{\frac{1}{1-\alpha}} H_t \left[\frac{\alpha}{p_t(\lambda_t)}\right]^{\frac{\alpha}{1-\alpha}}}{\phi + A^{\frac{1}{1-\alpha}} H_{t+1}\left[\frac{\alpha}{p_{t+1}(\lambda_{t+1})}\right]^{\frac{1}{1-\alpha}}} \tag{14}$$

Equation (14) presents a form of the conventional trade-off between static and dynamic efficiency faced by the IPR policy maker. The expression in the numerator is decreasing with IPR protection, as weak IPR boost aggregate saving, as defined by (12), for a given variety level N_t. This is the negative static effect of strong IPR on the growth rate. However the denominator in (13) is also decreasing with λ_{t+1} through the positive effect of IPR protection on investment in variety expansion, following (13). This is the positive dynamic effect of strong IPR on innovation rate.

Under incomplete IPR protection the output growth rate expression (1c) modifies accordingly to $1+g_{Y,t+1} \equiv \frac{Y_{y+1}}{Y_t} = (1+g_{N,t+1})\,(1+g_{H,t+1})(1+g_{p(\lambda),t+1})^{\frac{-\alpha}{1-\alpha}}$. Then, plugging (14) into the latter expression yields the output growth rate as a function of IPR policy

$$1+g_{y,t+1} = \frac{1-\alpha}{1+\rho^{-1}} \frac{\psi_{t+1}^{\frac{\alpha}{1-\alpha}}}{\widehat{\phi}_t + \psi_{t+1}^{\frac{1}{1-\alpha}}} \tag{14a}$$

Recall that $\widehat{\phi}_t \equiv \frac{\phi}{A^{\frac{1}{1-\alpha}} H_{t+1}}$ and $\psi \equiv \frac{\alpha}{p(\lambda)} = \frac{\alpha^2}{\lambda[1+r(\lambda)]}$, where $\frac{\partial \psi}{\partial \lambda} < 0$. We turn now to characterize the IPR policy that maximizes average growth along the transitional dynamics and the stationary equilibrium, denoted λ^*_{t+1} and $\tilde{\lambda}^*$, respectively.

Notice that, although economic growth along the transitional dynamics is driven by both human capital accumulation and variety expansion, the growth-maximizing IPR policy aims at maximizing variety expansion. This is because the human capital accumulation process is independent of IPR policy.

Proposition 2 *Along the transitional dynamics, growth is maximized by increasing IPR protection, and long run growth is maximized with incomplete IPR protection,that is* $\frac{\partial \lambda^*_{t+1}}{H_{t+1}} > 0$ *and* $\tilde{\lambda}^* < 1$.

Proof. Differentiating (14a) for ψ reveals that $\frac{\partial(1+g_{y,t+1})}{\partial\psi_{t+1}} > 0$, as long as $\frac{\alpha}{\psi_{t+1}} > \frac{\psi_{t+1}^{\frac{\alpha}{1-\alpha}}}{\widehat{\phi}_t+\psi_{t+1}^{\frac{1}{1-\alpha}}}$. Hence, because $\frac{\partial\psi_{t+1}}{\partial\lambda_{t+1}} < 0$, under this condition growth is increasing with loosening IPR protection. Rearranging the latter inequality yields the following implicit condition for the growth maximizing IPR policy $\frac{\alpha}{1-\alpha}\frac{\widehat{\phi}_t}{\psi_{t+1}^{\frac{1}{1-\alpha}}(\lambda_{t+1}^*)} = 1$. Appendix B proves that $\frac{\widehat{\phi}_t}{\left[\psi_{t+1}(\lambda_{t+1}^*)\right]^{\frac{1}{1-\alpha}}}$ is decreasing in H_{t+1}, and thus the latter equality implies that λ_{t+1}^* is increasing in H_{t+1} (and over time). In the long-run, the stationary growth-maximizing policy should satisfy $\frac{\alpha}{1-\alpha}\frac{\tilde{\phi}}{\tilde{\psi}^{\frac{1}{1-\alpha}}(\tilde{\lambda}^*)} = 1$. Appendix B shows also that the latter condition implies $\tilde{\lambda}^* < 1$. ■

The second part of Proposition 2, regarding the long-run growth maximizing policy was already established in Diwakar and Sorek (2016). Combining this result with Proposition 1 implies that for intermediate values of γ (and corresponding value of $\widetilde{H}$) a perpetual R&D-based growth will not take place under complete (or excessive) IPR protection. In this case, incomplete IPR protection is necessary for sustained R&D-based growth.

4.1 Varying innovation cost

Thus far, we have studied a lab-equipment innovation process of constant cost. However, alternative modeling approaches propose that the per-variety invention cost may change along the innovation process and in level of R&D investment. Two main effects that were discussed and implemented in the literature are the inter temporal knowledge spillover which works to decrease per-variety invention cost, and the stepping on toes ("fishing out") effect which works to increase invention cost as R&D efforts increase. See Jones (1995) and Segerstrom (1998), for example[19].

Following these notions, we turn now to explore possible implications of incorporating varying innovation cost in our model. We consider the case where innovation cost is increasing with innovation rate - $\phi_t = \phi_t(g_{N,t+1})$, which resembles reduced form of the fishing-out effect. Allowing also for perpetual human capital accumulation, that is assuming $\gamma = 1$, we rewrite the corresponding growth-rate equations (14)-(14a), respectively

$$1 + g_{N,t+1} = \frac{1-\alpha}{1+\rho^{-1}} \frac{A^{\frac{1}{1-\alpha}} H_t \left[\frac{\alpha}{p_t(\lambda_t)}\right]^{\frac{\alpha}{1-\alpha}}}{\phi_t(g_{N,t+1}) + A^{\frac{1}{1-\alpha}} H_{t+1} \left[\frac{\alpha}{p_{t+1}(\lambda_{t+1})}\right]^{\frac{1}{1-\alpha}}}$$

$$1 + g_{y,t+1} = \frac{1-\alpha}{1+\rho^{-1}} \frac{\psi_{t+1}^{\frac{\alpha}{1-\alpha}}}{\widehat{\phi}_t + \psi_{t+1}^{\frac{1}{1-\alpha}}}$$

Where $\widehat{\phi}_t \equiv \frac{\phi_t(g_{N,t+1})}{A^{\frac{1}{1-\alpha}} H_{t+1}}$, $p(\lambda) = \frac{\lambda[1+r(\lambda)]}{\alpha}$, and $\psi_{t+1} = \frac{\alpha}{p_{t+1}(\lambda_{t+1})}$, as before. The interest rate from (11) is modified accordingly to

[19] Jones (1999) provides a compact summary of the topic.

$$1 + r_{t+1} = \frac{(1 + r_{t+1})^{\frac{-\alpha}{1-\alpha}} \left(\frac{\lambda_{t+1}}{\alpha} - 1\right) \lambda_{t+1}^{\frac{-1}{1-\alpha}} A^{\frac{1}{1-\alpha}} H_{t+1} \alpha^{\frac{2}{1-\alpha}} + \phi_{t+1}(g_{N,t+2})}{\phi_t(g_{N,t+1})}$$

.

Inspection of the above equations reveals that, a finite stationary growth rate can be reached only if the innovation cost ϕ_t increases at the same rate that human capital is being accumulated. For such equilibrium the second part of Proposition 2, which implies $\tilde{\lambda}^* < 1$, still holds. However, the complete analysis of the transitional dynamics here, gets complicated as the equilibrium in each period relies on perfect foresight equilibrium, as in the case of equilibrium with no-innovation we have considered in Section 3, and is left for future research.

5 Conclusions

In this work, we studied growth-maximizing IPR policy for a closed OLG economy where R&D-based growth is stimulated and boosted through the accumulation of human capital. We have shown that once the economy surpasses a critical level of human capital accumulation, R&D-based growth is initiated. Then, the economy follows transitional dynamics of accelerating growth, which converges to a steady long-run growth rate.

We have characterized a stage-dependent IPR policy that maximizes output growth, which tightens IPR protection along the transitional dynamics of economic development. Moreover, we have shown that if the critical level of human capital accumulation is not reached, weak IPR protection may be necessary for the economy to initiate R&D-based growth.

The demographic structure of the OLG model economy restricts saving and investment to rely on labor income. This property, in turn, implies unique form of trade-off to be considered by the IPR policy maker: weaker IPR protection increases output and thereby labor income and aggregate saving, for a given level of the machine varieties span. This positive impact is due to the regular effect of limiting monopolistic prices on output. However, weaker IPR protection shifts investment away from R&D activity towards increased formation of physical capital.

We have shown that the net impact of these two countervailing effects changes along the course of economic development, in accordance with the change in the ratio between labor productivity and innovation cost. Along the transitional dynamics, growth acceleration, which characterizes the escape from stagnation and the phase of rapid economic development, the growth-maximizing policy is gradual tightening of IPR protection.

Our theoretical results provide a novel normative explanation for the observed positive correlation between IPR and economic development, across states and over time. This explanation aligns with the actual IPR policy implemented in developing economies as China in recent decades, and is complementary to that proposed by Chu et al. (2014) for an open developing economy.

Appendix A: Proving Lemma 2

To complete the proof of Lemma 2 we should show that the expression $\frac{\widehat{\phi}_t}{\psi_{t+1}^{\frac{\alpha}{1-\alpha}}}$ is decreasing in H_{t+1}, where $\widehat{\phi}_t \equiv \frac{\phi}{A^{\frac{1}{1-\alpha}} H_{t+1}}$ and $\psi_{t+1} = \frac{\alpha^2}{1+r_{t+1}}$. First, we write the explicit expression:

$$\frac{\widehat{\phi}_t}{\psi_{t+1}^{\frac{\alpha}{1-\alpha}}} = \frac{\phi[1 + r_{t+1}(H_{t+1})]^{\frac{\alpha}{1-\alpha}}}{\alpha^{\frac{2\alpha}{1-\alpha}} A^{\frac{1}{1-\alpha}} H_{t+1}} \tag{A.1}$$

Because $1 + r_{t+1}$, given in equation (11) is an implicit function of H_{t+1}, we cannot prove that $\frac{\widehat{\phi}_t}{\psi_{t+1}^{\frac{\alpha}{1-\alpha}}}$ is decreasing in H_{t+1} directly through differentiating (with respect to H_{t+1}). Instead, we will prove it indirectly by using the following expression, which we obtain by subtracting one from the right hand side of equation (11):

$$1 + \check{r}_{t+1} = \frac{(1 + \check{r}_{t+1})^{\frac{-\alpha}{1-\alpha}} \left(\frac{1}{\alpha} - 1\right) A^{\frac{1}{1-\alpha}} H_{t+1} \alpha^{\frac{2}{1-\alpha}}}{\phi} \tag{A.2}$$

Rearranging this expression yields the following explicit form of the (hypothetical) reference interest rate $\check{r}$:

$$1 + \check{r}_{t+1} = A\alpha^2 \left[\frac{\left(\frac{1}{\alpha} - 1\right) H_{t+1}}{\phi}\right]^{1-\alpha} \tag{A.3}$$

Plugging (A.3) into (A.1) reveals that, for $\check{r}$ the expression $\frac{\widehat{\phi}_t}{\psi_{t+1}^{\frac{\alpha}{1-\alpha}}}$ is decreasing in H_{t+1}: $\frac{\phi^{1-\alpha}\left(\frac{1-\alpha}{\alpha}\right)^{\alpha}}{AH_{t+1}^{1-\alpha}}$. Nevertheless, we will show now, that the actual interest rate, given in (11), increases at a slower rate in H_{t+1} than the one in (A.2), implying that (A.1) is decreasing in the actual interest rate r_{t+1}. Applying the implicit function theorem to (11) and (A.2) we obtain the following increase rates:

$$\frac{\frac{\partial(1+\check{r}_{t+1})}{\partial H_{t+1}}}{1 + \check{r}_{t+1}} = \frac{\left[\frac{\frac{(1+\check{r}_{t+1})^{\frac{-\alpha}{1-\alpha}}\left(\frac{1}{\alpha}-1\right)A^{\frac{1}{1-\alpha}}\alpha^{\frac{2}{1-\alpha}}}{\phi}}{\frac{\frac{\alpha}{1-\alpha}(1+\check{r}_{t+1})^{\frac{-\alpha}{1-\alpha}-1}\left(\frac{1}{\alpha}-1\right)A^{\frac{1}{1-\alpha}}H_{t+1}\alpha^{\frac{2}{1-\alpha}}}{\phi}+1}\right]}{1 + \check{r}_{t+1}} \tag{A.4}$$

$$\frac{\frac{\partial(1+r_{t+1})}{\partial H_{t+1}}}{1 + r_{t+1}} = \frac{\left[\frac{\frac{(1+r_{t+1})^{\frac{-\alpha}{1-\alpha}}\left(\frac{1}{\alpha}-1\right)A^{\frac{1}{1-\alpha}}\alpha^{\frac{2}{1-\alpha}}}{\phi}}{\frac{\frac{\alpha}{1-\alpha}(1+r_{t+1})^{\frac{-\alpha}{1-\alpha}-1}\left(\frac{1}{\alpha}-1\right)A^{\frac{1}{1-\alpha}}H_{t+1}\alpha^{\frac{2}{1-\alpha}}}{\phi}+1}\right]}{1 + r_{t+1}} \tag{A.5}$$

As $r_{t+1} > \widetilde{r}$, if the numerator in (A.5) is lower than in (A.4) the right hand side in the latter must by smaller than in the former. The common numerator in the above expressions can be written as:

$$\frac{\frac{\left(\frac{1}{\alpha}-1\right)A^{\frac{1}{1-\alpha}}\alpha^{\frac{2}{1-\alpha}}}{\phi}}{\frac{\frac{\alpha}{1-\alpha}(1+r_{t+1})^{-1}\left(\frac{1}{\alpha}-1\right)A^{\frac{1}{1-\alpha}}H_{t+1}\alpha^{\frac{2}{1-\alpha}}}{\phi}+(1+r_{t+1})^{\frac{\alpha}{1-\alpha}}} \tag{A.6}$$

Hence if $\frac{\frac{\alpha}{1-\alpha}(1+r_{t+1})^{-1}\left(\frac{1}{\alpha}-1\right)A^{\frac{1}{1-\alpha}}H_{t+1}\alpha^{\frac{2}{1-\alpha}}}{\phi}+(1+r_{t+1})^{\frac{\alpha}{1-\alpha}}$, which is the denominator in (A.6), is increasing in $(1+r_{t+1})$, the rate of increase given in (A.4) is lower than in (A.3). Straightforward differentiation reveals that the derivative is non-negative for $1+r_{t+1}\geq A\alpha^2\left[\frac{\left(\frac{1}{\alpha}-1\right)H_{t+1}}{\phi}\right]^{1-\alpha}$. This condition coincides with (A.3) when holding in equality. However, as the actual equilibrium interest-rate from (11) is higher than the one in (A.3), the latter condition holds with inequality. This implies that the numerator in (A.5) is smaller than in (A.4), which in turn implies that the rate of increase given in (A.4) is lower than in (A.3). Therefore, the expression in (A.1) is decreasing in H_{t+1} when evaluated for the equilibrium interest rate (11).

Notice also, that for the references interest rate demand for each machine is given by $K_{i,t}^d = A^{\frac{1}{1-\alpha}}H_t\left(\frac{\alpha^2}{1+\check{r}_t}\right)^{\frac{1}{1-\alpha}} = \frac{\phi}{\left(\frac{1}{\alpha}-1\right)}$. Hence, under the reference interest rate demand for each machine variety is stationary along the process of human capital accumulation. This is because the increase in the reference interest rate, and the corresponding increase in the price of machines, completely offsets the direct increase in demand the follows the increase in effective labor supply. However, as we have just proved, the actual interest rate grows at a slower rate due to increase in the effective labor supply, and therefore the actual price effect does completely offset the direct positive effect of increasing effective labor supply on the demand for machines (this conclusion was considered at the end of Subsection 2.1).

Appendix B: Proving Proposition 2

First, we have to show that the expression $\frac{\widehat{\phi}_t}{\psi_{t+1}^{\frac{1}{1-\alpha}}} = \frac{\phi}{A^{\frac{1}{1-\alpha}}H_{t+1}}\left(\frac{1+r_{t+1}}{\alpha^2}\right)^{\frac{1}{1-\alpha}}$ is decreasing in H_{t+1}. Following the same steps presented in Appendix A, we modify the reference (hypothetical) interest rate, to account for incomeplete patent protection $1+\check{r}_{t+1} = \frac{\left[\left(\frac{\lambda_{t+1}}{\alpha}-1\right)H_{t+1}\right]^{1-\alpha}\lambda_{t+1}^{-1}A\alpha^2}{\phi^{1-\alpha}}$. For this interest rate, it turns out that the expression $\frac{\widehat{\phi}_t}{\check{\psi}_{t+1}^{\frac{1}{1-\alpha}}}$ is independent of H_{t+1}: $\frac{\widehat{\phi}_t}{\check{\psi}_{t+1}^{\frac{1}{1-\alpha}}} = \left(\frac{\lambda_{t+1}}{\alpha}-1\right)\lambda_{t+1}^{-\frac{1}{1-\alpha}}$. However, we have alread proved in Appendix A that the actual interest rate is increasing in H_{t+1} at slower rate than the reference interest rate. This proof still holds for any given λ_{t+1}. Therefore, under the actual interest rate the expression $\frac{\widehat{\phi}_t}{\psi_{t+1}^{\frac{1}{1-\alpha}}}$ is decreasing in H_{t+1}. To complete the proof of Proposition 2 we should show that the equality $\frac{\alpha}{1-\alpha}\frac{\tilde{\phi}}{\tilde{\psi}^{\frac{1}{1-\alpha}}(\tilde{\lambda}^*)} = 1$ holds only for $\tilde{\lambda}^* < 1$. Using the explicit expressions for $\tilde{\phi}$ and $\tilde{\psi}^{\frac{1}{1-\alpha}}(\tilde{\lambda}^*)$ we obtain A.1-A.3 from the proof of Lemma 2, in Appendix A above, we obtain:

$$\frac{\alpha}{1-\alpha}\frac{\tilde{\phi}}{\tilde{\psi}^{\frac{1}{1-\alpha}}(\tilde{\lambda}^{*})}=\frac{\alpha}{1-\alpha}\frac{\phi}{A^{\frac{1}{1-\alpha}}\tilde{H}\left[\frac{\alpha^{2}}{\lambda[1+\tilde{r}(\lambda^{*},\tilde{H})]}\right]^{\frac{1}{1-\alpha}}}=1 \tag{B.1}$$

Because the interest in B.1 is given as an implicit expression in equation (11), we will follow the indirect proving approache we have employed to prove Lemma 2, in Appendix A above. We modify the hypothetical interest rate given in (A.3) to account for incomplete patent breadth in Appendix A above:

$$1+\check{r}=\frac{A\alpha^{2}}{\lambda^{*}}\left[\frac{\left(\frac{\lambda^{*}}{\alpha}-1\right)\widetilde{H}}{\phi}\right]^{1-\alpha} \tag{B.2}$$

Notice that by the construction of $\check{r}$, as explained in Appendix A, it is lower than the actual interest rate presented in (B.1), as given in (11). That is, $\check{r}<\tilde{r}$. Plugging B.2 into B.1 reveals that for the long term interest rate $\check{r}$ growth is maximized with $\lambda^{*}=1$. However, for $\lambda^{*}=1$ in both interest rate expression, we have $\check{r}<\tilde{r}$. Hence, by equation B.1, under the actual interest rate $\tilde{r}$ the growth is maximized with incomplete IPR protection, that is with is $\lambda^{*}<1$.

Appendix C: CIES utility function

Here, we consider a deviation from logarithmic utility function to the general CIES formulation

$$U(c_{t},c_{t+1})=\frac{c_{t}^{1-\theta}}{1-\theta}+\rho\frac{c_{t+1}^{1-\theta}}{1-\theta} \tag{C.1}$$

where $\frac{1}{\theta}$ is the inter-temporal elasticity of substitution ($\theta>0$). Hence, the indirect lifetime utility function is

$$U(c_{t},c_{t+1})=\frac{(w_{t}-s_{t})^{1-\theta}}{1-\theta}+\rho\frac{[s_{t}(1+r_{t+1})]^{1-\theta}}{1-\theta} \tag{C.2}$$

and optimal saving is given by

$$s_{t}=\frac{w_{t}}{1+\rho^{-\frac{1}{\theta}}(1+r_{t+1})^{\frac{\theta-1}{\theta}}} \tag{C.3}$$

Equalizing the corresponding aggregate saving expression to aggregate investment yields the growth rate

$$1+g_{y,t+1}=\frac{1-\alpha}{1+\rho^{-1}}\frac{\psi_{t+1}^{\frac{\alpha}{1-\alpha}}}{\left[\widehat{\phi}_{t}+\psi_{t+1}^{\frac{1}{1-\alpha}}\right]\left[1+\rho^{-\frac{1}{\theta}}\left(1+r_{t+1}(\lambda_{t+1})\right)^{\frac{\theta-1}{\theta}}\right]} \tag{C.4}$$

Clearly, the analysis of IPR policy is more complicated here, as it should account for the additional effect of IPR-policy on aggregate saving, which depends on the inter-temporal elasticity of substitution, i.e. the preference parameter θ. Nonetheless, for $\theta<1$ ($\theta>1$), changes in IPR will have a stronger (weaker) impact on growth (compared with the case in which $\theta=1$) due to the additional (countervailing) interest-rate channel effect.

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