Auburn University
Department of Economics
Working Paper Series

Forecasting Financial Stress Indices in Korea: A Factor Model Approach

Hyeongwoo Kim[†], Wen Shi[‡], and Hyun Hak Kim[*]

[†]Auburn University; [‡]Columbus State University;
[*]Kookmin University

AUWP 2016-10

Forecasting Financial Stress Indices in Korea: A Factor Model Approach

Hyeongwoo Kim*
Auburn University

Wen Shi†
Columbus State University

Hyun Hak Kim‡
Kookmin University

September 2016

Abstract

We propose factor-based out-of-sample forecast models for Korea's financial stress index and its 4 sub-indices that are developed by the Bank of Korea. We extract latent common factors by employing the method of the principal components for a panel of 198 monthly frequency macroeconomic data after differencing them. We augment an autoregressive-type model of the financial stress index with estimated common factors to formulate out-of-sample forecasts of the index. Our models overall outperform both the stationary and the nonstationary benchmark models in forecasting the financial stress indices for up to 12-month forecast horizons. The first common factor that represents not only financial market but also real activity variables seems to play a dominantly important role in predicting the vulnerability in the financial markets in Korea.

Keywords: Financial Stress Index; Principal Component Analysis; PANIC; In-Sample Fit; Out-of-Sample Forecast; Diebold-Mariano-West Statistic

JEL Classification: E44; E47; G01; G17

*Department of Economics, Auburn University, 0339 Haley Center, Auburn, AL 36849. Tel: +1-334-844-2928. Fax: +1-334-844-4615. Email: gmmkim@gmail.com.

†Contact Author: Wen Shi, Department of Accounting & Finance, Turner College of Business, Columbus State University, Columbus, GA 31907. Tel: (706) 507-8155. Fax: (706) 568-2184. Email: shi_wen@columbusstate.edu.

‡Department of Economics, Kookmin University, Seongbuk-Gu, Seoul, Korea. Tel: +82-70-3787-3318. Email: hyunhak.kim@kookmin.ac.kr.

1 Introduction

The bankruptcy of Lehman Brothers on September 15, 2008 has triggered the collapse of financial markets not only in the US but also in other countries including Korea. As we can see in this episode, financial market crises often occur abruptly, and quickly spread to other sectors of the economy, even to other countries. That is, financial market crises tend to come to a surprise realization with no systemic warnings. Since financial crises have harmful long-lasting spill-over effects on real activities even after the financial system becomes stabilized, it would be useful to have forecasting algorithms such as an Early Warning Signal (EWS), which can provide timely information on the vulnerability in financial markets that might be materialized in the near future.

There's an array of research works that attempt to predict financial crises in the current literature. For instance, Frankel and Saravelos [2012], Eichengreen et al. [1995], and Sachs et al. [1996] used linear regressions to test the statistical significance of various economic variables on the occurrence of crises. Some others employed discrete choice model approaches, either parametric probit or logit regressions (Frankel and Rose [1996]; Cipollini and Kapetanios [2009]) or nonparametric signal detection approaches (Kaminsky et al. [1998]; Brüggemann and Linne [1999]; Edison [2003]; Berg and Pattillo [1999]; Bussiere and Mulder [1999]; Berg et al. [2005]; EI-Shagi et al. [2013]; Christensen and Li [2014]).

It is crucial to find a proper measure of financial market vulnerability, which quantifies the potential risk that prevails in financial markets. One popularly used measure in the current literature is the Exchange Market Pressure (EMP) index. Since the seminal work of Girton and Roper [1977], many researchers have used the EMP index to develop EWS mechanisms in order to detect the turbulence in the money market across countries. See Tanner [2002] for a review.

One alternative measure that is rapidly gaining popularity is financial stress index (FSI). Unlike the EMP index that is primarily based on changes in exchange rates and international reserves, FSI's are typically constructed using a broad range of financial market variables. Currently, there are 12 FSI's available for the US financial market (Oet et al. [2011]) including 4 indices that are reported by the US Federal Reserve system.[1]

Some recent studies investigate what economic variables help predict financial market vulnerability using FSI's. For instance, Christensen and Li [2014] propose a model to fore-

[1]For some of FSI's in the Euro, see Grimaldi [2010], Grimaldi [2011], Hollo et al. [2012], and Islami and Kurz-Kim [2013]. There are FSI's for individual countries: Greece (Louzis and Vouldis [2011]), Sweden (Sandahl et al. [2011]), Canada (Illing and Liu [2006]), Denmark (Hansen [2006]), Switzerland (Hanschel and Monnin [2005]), Germany (van Roye [2011]), Turkey(Cevik et al. [2013]), Colombia (Morales and Estrada [2010]), and Hong Kong (S.Yiu et al. [2010]).

cast the FSI's developed by the IMF for 13 OECD countries, utilizing 12 economic leading indicators and three composite indicators. They used the signal extraction approach proposed by Kaminsky et al. [1998]. Slingenberg and de Haan [2011] constructed their own FSI's for 13 OECD countries and investigated what economic variables have predictive contents for the FSI's via linear regression models. Unfortunately, they fail to find any clear linkages between economic variables and those FSI's.[2]

The present paper proposes a new forecasting model for the financial market vulnerability in Korea using a broad range of time series macroeconomic data. We use the financial stress index and its four sub-indices developed by the Bank of Korea.[3,4] We estimate multiple latent common factors by employing the method of the principal components (Stock and Watson [2002]) for a panel of 198 monthly frequency time series data from October 2000 to December 2013.[5] We augment an autoregressive-type model of the financial stress index with estimated common factors, then formulate out-of-sample forecasts of the index for up to 12-month forecast horizons. We evaluate the out-of-sample forecast predictability of our models in comparison with two benchmark models, the nonstationary random walk (RW) and a stationary autoregressive (AR) model using the ratio of the root mean square prediction errors ($RRMSPE$) and the Diebold-Mariano-West (DMW) test statistics.

Our major findings are as follows. First, our factor models overall outperform the benchmark models. Our models performed especially well for the foreign exchange market sub-index. $RRMSPE$ was substantially greater than one (smaller mean squared prediction errors of our models) and the DMW test rejects the null of equal predictability for majority cases from 1 to 12-month forecast horizons. Second, parsimonious models with just one single factor perform as well as bigger models that include up to 8 common factors. Augmenting the AR-type model of the FSI with the first common factor seems to be sufficient to beat the benchmark models. Third, fixed-size rolling window methods performed overall similarly well as the recursive approach, which implies the stability of our models over time. We note that the first common factor, which plays a dominantly important role in predicting the FSI's, represents not only financial market but also real activity variables. That is, our findings suggest that real sector variables also contain substantial predictive contents for the

[2] Misina and Tkacz [2009] investigated the predictability of credit and asset price movements for financial market stress in Canada. Kim and Shi [2015] implemented forecasting exercises for the FSI in the US using a similar methodologies used in this paper.

[3] The 4 sub-indices are for the foreign exchange market, the stock market, the bond market, and the financial industry in Korea.

[4] The data is not publicly available and is for internal use only. We express our gratitude to give permission to use the data.

[5] We categorized these 198 variables into 13 groups that include an array of nominal and real activity variables.

financial market vulnerability in Korea.

The rest of the paper is organized as follows. Section 2 describes the econometric model and the out-of-sample forecasts schemes used in the present paper. We also explain our evaluation methods for our models. In Section 3, we provide data descriptions and preliminary analyses for latent common factor estimates. Section 4 reports our major findings from in-sample fit analyses and out-of-sample forecast exercises. Section 5 concludes.

2 The Econometric Model

Let $x_{i,t}$ be a macroeconomic variable $i \in \{1, 2, .., N\}$ at time $t \in \{1, 2, .., T\}$. Assume that $x_{i,t}$ has the following factor structure.

$$x_{i,t} = c_i + \lambda_i^{'} \mathbf{F}_t + e_{i,t}, \tag{1}$$

where c_i is a fixed effect intercept, $\mathbf{F}_t = [F_{1,t} \ \cdots \ F_{r,t}]^{'}$ is an $r \times 1$ vector of *latent* common factors, and $\lambda_i = [\lambda_{i,1} \ \cdots \ \lambda_{i,r}]^{'}$ denotes an $r \times 1$ vector of factor loading coefficients for $x_{i,t}$. $e_{i,t}$ is the idiosyncratic error term.

Estimation is carried out via the method of the principal components for the first-differenced data. As Bai and Ng [2004] show, the principal component analysis estimators for $\mathbf{F}_t$ and λ_i are consistent irrespective of the order of $\mathbf{F}_t$ as long as $e_{i,t}$ is stationary. However, if $e_{i,t}$ is an integrated process, a regression of $x_{i,t}$ on $\mathbf{F}_t$ is spurious. To avoid this problem, we apply the method of the principal components after differencing the data. Lag (1) by one period then subtract it from (1) to get,

$$\Delta x_{i,t} = \lambda_i^{'} \Delta \mathbf{F}_t + \Delta e_{i,t} \tag{2}$$

for $t = 2, \cdots, T$. Let $\Delta \mathbf{x}_i = [\Delta x_{i,1} \ \cdots \ \Delta x_{i,T}]^{'}$ and $\Delta \mathbf{x} = [\Delta \mathbf{x}_1 \ \cdots \ \Delta \mathbf{x}_N]$. We first normalize the data before the estimations, since the method of the principal components is not scale invariant. Employing the principal components method for $\Delta \mathbf{x} \Delta \mathbf{x}^{'}$ yields factor estimates $\Delta \hat{\mathbf{F}}_t$ along with their associated factor loading coefficient estimates $\hat{\lambda}_i$. Estimates for the idiosyncratic components are naturally given by the residuals $\Delta \hat{e}_{i,t} = \Delta x_{i,t} - \hat{\lambda}_i^{'} \Delta \hat{\mathbf{F}}_t$. Level variables are then recovered by re-integrating these estimates,

$$\hat{e}_{i,t} = \sum_{s=2}^{t} \Delta \hat{e}_{i,s} \tag{3}$$

for $i = 1, 2, ..., N$. Similarly,

$$\hat{\mathbf{F}}_t = \sum_{s=2}^{t} \Delta\hat{\mathbf{F}}_s \tag{4}$$

After obtaining latent factor estimates, we augment an AR-type model for the financial stress index (fsi_t) with $\Delta\hat{\mathbf{F}}_t$. Abstracting from deterministic terms,

$$fsi_{t+j} = \beta_j' \Delta\hat{\mathbf{F}}_t + \alpha_j fsi_t + u_{t+j},\ j = 1, 2, .., k \tag{5}$$

That is, we implement *direct* forecasting regressions for the j-period ahead financial stress index (fsi_{t+j}) on (differenced) common factor estimates ($\Delta\hat{\mathbf{F}}_t$) and the current value of the index (fsi_t), which belong to the information set (Ω_t) at time t.[6] Note that (5) is an AR(1) process for $j = 1$, extended by exogenous common factor estimates $\Delta\hat{\mathbf{F}}_t$. This formulation is based on our preliminary unit-root test results for the FSI's that show strong evidence of stationarity.[7] Applying the ordinary least squares (OLS) estimation for (5), we obtain the following j-period ahead forecast for the financial stress index.

$$\widehat{fsi}^{F}_{t+j|t} = \hat{\beta}_j' \Delta\hat{\mathbf{F}}_t + \hat{\alpha}_j fsi_t \tag{6}$$

To statistically evaluate our factor models, we employ the following *nonstationary* random walk (RW) model as a (no change) benchmark model.

$$fsi_{t+1} = fsi_t + \varepsilon_{t+1} \tag{7}$$

It is straightforward to show that (7) yields the following j-period ahead forecast.

$$\widehat{fsi}^{RW}_{t+j|t} = fsi_t, \tag{8}$$

where fsi_t is the current value of the financial stress index.

In addition to the RW model, we also employ the following *stationary* AR(1) model as the second benchmark model.

$$fsi_{t+j} = \alpha_j fsi_t + \varepsilon_{t+1}, \tag{9}$$

where α_j is the coefficient on the current FSI in the direct regression for the j-period ahead

[6] Alternatively, one may use a *recursive* forecasting regression model that replaces α_j with α^j, where α is the coefficient from an AR(1) model.

[7] ADF test results are available upon request.

FSI variable. This model specification yields the following j-period ahead forecast.

$$\widehat{fsi}^{AR}_{t+j|t} = \hat{\alpha}_j fsi_t, \tag{10}$$

where $\hat{\alpha}_j$ is the least squares estimate for α_j.

For evaluations of the prediction accuracy of our models, we use the ratio of the root mean squared prediction error ($RRMSPE$), that is, $RMSPE$ from the benchmark model divided by $RMSPE$ from the factor model. Note that our factor model outperforms the benchmark model when $RRMSPE$ is greater than 1.

Also, we employ the Diebold-Mariano-West (DMW) test for further statistical evaluations of our models. For the DMW test, we define the following loss differential function.

$$d_t = L(\varepsilon^A_{t+j|t}) - L(\varepsilon^F_{t+j|t}), \tag{11}$$

where $L(\cdot)$ is a loss function from forecast errors under each model, that is,

$$\varepsilon^A_{t+j|t} = fsi_{t+j} - \widehat{fsi}^A_{t+j|t}\ (A = RW, AR),\ \varepsilon^F_{t+j|t} = fsi_{t+j} - \widehat{fsi}^F_{t+j|t} \tag{12}$$

One may use either the squared error loss function, $(\varepsilon^j_{t+j|t})^2$, or the absolute loss function, $|\varepsilon^j_{t+j|t}|$.

The DMW test statistic tests the null of equal predictive accuracy, $H_0 : Ed_t = 0$, and is defined as follows.

$$DMW = \frac{\bar{d}}{\sqrt{\widehat{Avar}(\bar{d})}}, \tag{13}$$

where $\bar{d}$ is the sample mean loss differential, $\bar{d} = \frac{1}{T-T_0}\sum_{t=T_0+1}^{T} d_t$, and $\widehat{Avar}(\bar{d})$ denotes the asymptotic variance of $\bar{d}$,

$$\widehat{Avar}(\bar{d}) = \frac{1}{T-T_0}\sum_{i=-q}^{q} k(i,q)\hat{\mathbf{\Gamma}}_i \tag{14}$$

$k(\cdot)$ is a kernel function where T_0/T is the split point in percent, $k(\cdot) = 0,\ j > q$, and $\hat{\Gamma}_j$ is j^{th} autocovariance function estimate.[8] Note that our factor model (5) nests the stationary benchmark model in (9) with $\beta_j = \mathbf{0}$. Therefore, we use critical values obtained with re-centered distributions of the test statistic for nested models (McCracken [2007]). For the DMW statistic with the random walk benchmark (7), which is not nested by (5), we use the asymptotic critical values, which are obtained from the standard normal distribution.

[8]Following Andrews and Monahan [1992], we use the quadratic spectral kernel with automatic bandwidth selection for our analysis.

3 Data Descriptions and Factor Estimations

3.1 Data Descriptions

We use the financial stress index (FSI) data to assess the degree of financial stress or the vulnerability in financial markets in Korea to a financial crisis. Financial Condition Indices (FCI) share similar information as FSI's in the sense that they all measure the current financial conditions in the economy, though FCI's focus more on how financial variables react to changes in the market conditions.

Earlier attempts to develop an FSI were done by the Bank of Canada in 2003 and the Swiss National Bank in 2004, while the Kansas City Fed and the St. Louis Fed in the US also began using FSI's since 2008. In Korea, the Bank of Korea developed FSI's in 2007 and started to report the indices on a yearly basis in their Financial Stability Report. We obtained monthly frequency data which have been transformed from daily frequency raw data. The data are in principle for internal use only.[9]

The Korea's FSI data is based on 4 sub-indices for the bond market (FSI-Bond), the foreign exchange market (FSI-FX), the stock market (FSI-Stock), and the financial industry (FSI-Industry). Each sub-index is constructed as follows. FSI-Bond is based on a variety of credit spreads, long-short interest rate spreads, and covered interest rate differentials (CID). FSI-FX is obtained by utilizing the volatility and the growth rate of the Korean Won-US Dollar exchange rate as well as the growth rate of Korea's foreign exchange reserves. FSI-Stock is constructed based on the volatility and the growth rate of KOSPI (Korea Composite Stock Price Index), and the volatility and growth rate of the KOSPI trade volume. Lastly, FSI-Industry is based on the volatility and the β's of financial intermediaries' stocks, and the spread between the average bond yields issued by financial intermediaries and the treasury bond yield.

As we can see in Figure 1, all sub-indices show overall similar movements as FSI total index. FSI-Bond exhibits much lower volatility than FSI, while FSI-Stock shows the highest volatility. All indices imply extremely high degree vulnerability during the recent financial crisis that began in 2008.

Figure 1 around here

We obtained all macroeconomic time series data from Kim [2013], which are used to extract latent common factors for our out-of-sample forecast exercises. Observations are

[9] We obtained permission from the Bank of Korea to use the data for this research.

monthly frequency and span from October 2000 to December 2013. All variables other than those in percent (e.g., interest rates and unemployment rates) are log-transformed prior to estimations. We categorized 198 time series data into 13 groups as summarized in Table 1.

Group #1 that includes 14 time series data represents a set of nominal interest rates. Groups #2 through #4 include prices and monetary aggregate variables, while group #5 covers an array of bilateral nominal exchange rates. Note that these groups overall represent the nominal sector variables. On the contrary, group #6 through #11 entail various kinds of real activity variables such as manufacturers' new orders, inventory, capacity utilizations, and industrial production indices. The last two groups represent business condition indices and stock indices in Korea, respectively.

Table 1 around here

3.2 Latent Factors and their Characteristics

We estimate up to 8 latent common factors by applying the method of the principal components to 198 macroeconomic data series after differencing and normalizing them. Estimated (differenced) common factors, $\Delta\hat{F}_1, \Delta\hat{F}_2, ..., \Delta\hat{F}_8$, are reported in Figure 2. We also report level common factor estimates $\hat{F}_1$ through $\hat{F}_8$ in Figure 3 that are obtained by re-integrating differenced common factors.

We note a dramatic decline in the first common factor estimate $\hat{F}_1$ around the beginning of the Great Recession in 2008. Similarly, the second common factor estimates $\hat{F}_2$ exhibits an abrupt downward movement about the same time. All estimated common factors *in levels* exhibit highly persistent dynamics, indicating a nonstationary stochastic process. Therefore, it seems to be appropriate to employ the method of the principal components to the data after differencing them to ensure the stationarity of the data (Bai and Ng [2004]).

Figures 2 and 3 around here

To see what variables represent each of these latent factors more closely, we report the factor loading coefficient ($\hat{\lambda}_i$) estimates in Figure 4. We also provide the marginal R^2 analysis by regressing each $x_{i,t}$ on each common factor estimate $\Delta\hat{F}_i$ to get R^2 values. Results are reported in Figure 5.

Figures 4 and 5 around here

To investigate the nature of the first common factor, we plot $\hat{F}_1$ and its associated factor loading coefficients ($\hat{\lambda}_{i,1}$) together on Figure 6. We first note that the factor loading coefficients for the first four groups and the last three groups are positively associated with $\hat{F}_1$, while variables in groups #5, #6, and #8 are mostly negatively associated with it. Overall, $\hat{F}_1$ represents not only the nominal variables (interest rates, prices, nominal exchange rates, and stock prices) but also real activity variables (new orders, industrial production, and business condition indices). Factor loading coefficients imply positive associations between Interest rates and prices (inflation rates), which seems to be consistent with the Fisher effect. Domestic prices are negatively related with nominal exchange rates (relative prices of the domestic currency), because domestic inflation is likely to be associated with depreciation of the home currrency.

Figure 6 around here

As we can see in Figure 7, the second common factor seems to closely represent variables in groups #5, #6, and #9 through #12, which are real sector variables with an exception of group #5. $\hat{F}_2$ is overall positively related with the majority of the variables in these groups. For instance, the factor loading coefficients ($\hat{\lambda}_{i,2}$) for nominal exchange rates (group #5) are positive, which implies that a depreciation of Korean wons ($\Delta x_{i,t} > 0$) are associated with an increase in real activities ($\Delta \hat{F}_2 > 0$) in Korea, that is, $\hat{\lambda}_{i,2} \Delta \hat{F}_2 > 0$. Similarly, new orders, sales, industrial production, and business condition indices groups have positive coefficients. Among the variables in group #10, unemployment variables have negative coefficients, while employment variables tend to exhibit positive ones. Overall, $\hat{F}_2$ seems to represent real sector variables.

Figure 7 around here

In what follows, our in-sample-fit analysis demonstrates a non-negligible role of the fourth common factor estimate $\hat{F}_4$ in explaining FSI's. So we report $\hat{F}_4$ and its associated factor loading coefficients in Figure 8. Estimates of $\lambda_{i,4}$ imply that $\hat{F}_4$ is more closely related with nominal variables in groups #1 through #4, while some variables among real variable groups #7 and #8 are also still closely related with $\hat{F}_4$.

Figure 8 around here

4 Forecasting Exercises

4.1 In-Sample Fit Analysis

We implement an array of least squares estimations for the following equation, employing alternative combinations of estimated common factors $\left\{\Delta\hat{F}_1, \Delta\hat{F}_2, ..., \Delta\hat{F}_8\right\}$ as explanatory variables.

$$fsi_{t+j} = \beta_j' \Delta\hat{\mathbf{F}}_t + u_{t+j}, \; j = 0, 1, 2, .., k \tag{15}$$

We report our in-sample fit analyses in Table 2 for the contemporaneous case ($j = 0$).[10]

We employed an R^2-based selection method from one-factor models to an 8-factor full model to find the best combination of explanatory variables. It turns out that the first common factor estimate $\Delta\hat{F}_1$ plays the most important role in explaining variations in all FSI indices with an exception of FSI-Bond. The second common factor estimate $\Delta\hat{F}_2$ explains negligibly small portion of variations in FSI indices.

Since R^2 increases as more variables are included, this R^2-based selection method always picks the full model as the best one. So, we considered two alternative selection methods. The adjusted R^2 selection method chose a 7-factor model, while a step-wise selection method (Specific-to-General rule) selected a 6-factor model for FSI. These methods chose a 5-factor model for FSI-FX. It should be noted, however, that maginal gains from adding more factors are often small, which implies that small dimension models with just one or two factors are sufficient to obtain a good in-sample fit for each financial stress index. In what follows, we demonstrate that small models perform well in out-of-sample forecast exercises as well.

Table 2 around here

4.2 Out-of-Sample Forecast Exercises and Model Evaluations

We implement out-of-sample forecast exercises using the following two schemes. First, we employ a recursive forecast method. We start formulating $k-$period ahead out-of-sample forecasts of FSI's (fsi_{T_0+k}) using the initial T_0 observations.[11] That is, we extract common factors from $\{x_{i,t}\}_{i=1,..,N}^{t=1,...,T_0}$ after differencing. Then, we obtain our factor model forecasts via (6). Next, we add one new set of observations to the sample and implement next forecast for fsi_{T_0+1+k} using this expanded set of observations $\{x_{i,t}\}_{i=1,..,N}^{t=1,...,T_0+1}$. We repeat this procedure

[10] Regressions for the 1-, 3-, and 6-month ahead FSI indices yield similar results.

[11] We used 70% initial observations.

until we forecast the last observation fsi_T. We implement this scheme for up to 12-month forecast horizons, $j = 1, 3, 6, 9, 12$.

The second scheme is a fixed-size rolling window method that repeats forecasting by adding one additional observation with the same split point (T_0/T) but dropping one earliest observation, maintaining the same sample size.

For statistical evaluations of our factor model, we employ the two benchmark models, the random walk (RW, no change) model and a stationary AR(1) model, and formulate forecasts via the equations (8) and (10), respectively. We evaluate our factor model forecasting performances relative to these benchmark models using the following two popular measures.

First, we report the ratio of the root mean square prediction error, $RRMSPE$, of each of the benchmark models to that of our factor models. Note that the factor model outperforms the benchmark model when the $RRMSPE$ is greater than one. Second, we employ the DMW statistics with asymptotic critical values when the random walk model is used, while the critical values from McCracken (2007) were used when the AR model is used because the AR model is nested by our factor models.

Our forecast exercise results for FSI total index are reported in Table 3. To save space, we report results with three 1-factor models, two two-factor models, and one three-factor model, which are chosen based on our in-sample fit analyses in previous section.

We note that our factor models outperform the RW model for all forecast horizons from 1-month to 1-year. $RRMSPE$ is greater than one for all cases. Our factor models outperform the benchmark model with the DMW test for majority cases. For example, the DMW test rejects the null of equal predictability at the 10% significance level for 24 out of 30 cases both with the recursive method and the rolling window method. We find especially strong out-of-sample forecast performances when the forecast horizon is equal to or greater than 3 months.

It turns out that our factor models also perform reasonably well in comparison with the stationary AR(1) benchmark model. $RRMSPE$ is greater than one for majority cases when the recursive method is employed, whereas our models perform relatively poorly when the rolling window method is used. Interestingly, the 1-factor model with $\Delta\hat{F}_4$, which is more closely related with nominal variables, performs consistently poorly. We note that the DMW test rejects the null of equal predictability for 8 out of 12 1-period ahead forecasts, while $RRMSPE$ is greater than 1 for 10 out of 12 cases. This is a good property because out-of-sample forecast exercises are more useful when it demonstrates superior predictability for short forecast horizon, as financial turmoils often occur suddenly without systematic warnings.

Table 3 around here

Table 4 reports out-of-sample forecast exercise results for FSI-Bond. Irrespective of its poor in-sample fit as seen in previous section, our factor model beats the RW model again for most cases by $RRMSPE$ criteria. The DMW test rejects the null of equal predictability for most cases when $j = 3, 6, 9, 12$ at the 10% significance level. With the AR model as the benchmark, our factor models overall perform well especially when $j = 3, 6, 9, 12$. Recall that $\Delta\hat{F}_4$ explains the most of variations in FSI-Bond as can be seen in Table 2. It is interesting to see that $\Delta\hat{F}_4$ exhibit the best out-of-sample predictability even when all other models perform poorly in comparison with the AR model.

Table 4 around here

Our factor models perform overall the best for FSI-FX, especially when the rolling window method is employed. $RRMSPE$ is greater than one for all cases with the random walk benchmark model. The DMW test rejects the null for all cases when the rolling window scheme is employed. Our models exhibit very good one-period ahead forecast performances with the AR benchmark whenever $\Delta\hat{F}_1$ is used.

Table 5 around here

Out-of-sample forecast performances for FSI-Stock are reported in Table 6. $RRMSPE$ is greater than 1 in most cases with the RW benchmark model, which implies our factor model outperform the model. However, the DMW test rejects the null of equal predictability only when $j = 12$. With the AR model, factor models performed well only in a few cases, though the DMW test rejects the null for 5 out of 6 cases when the rolling window scheme is used for one-period ahead forecasts.

Table 6 around here

Finally, we report forecast exercise results for FSI-Industry in Table 7. Our factor models performed better than the random walk model only when the forecast horizon is longer than a half year. $RRMSPE$ was often less than one when $j = 1, 3$. Forecast performances were worse especially when the AR model serves as the benchmark. Overall, our factor models perform the worst for FSI-Industry.

Table 7 around here

5 Concluding Remarks

This paper proposes an out-of-sample forecast model for the financial stress index developed by the Bank of Korea (BOK). We use the BOK's financial stress index and its 4 sub-indices to measure the vulnerability in financial markets in Korea. To reduce data dimensionality, we employ a parsimonious method to extract latent common factors from a panel of 198 time series macroeconomic variables that include not only nominal but also real activity variables. Following Bai and Ng [2004], we apply the method of the principal components to these variables after differencing them to estimate the common factors consistently. Our in-sample fit analyses demonstrate that estimated factors explain substantial shares of variations of all financial stress indices with an exception of FSI-Bond.

We implement out-of-sample forecast exercises using the recursive and the fixed size rolling window schemes with the two benchmark models, the random walk and a stationary AR(1) models. We evaluate out-of-sample predictability of our factor models using the ratio of root mean square prediction errors ($RRMSPE$) and the DMW test statistics.

Our findings imply that there exists a tight linkage between the Korean FSI's and estimated common factors. Interestingly, we observe that not only nominal but also real activity variables, proxied especially by the first common factor estimate, seem to contain useful predictive contents for FSI's in Korea. Especially, our factor models demonstrate superior performance over the random walk benchmark model in most cases. Our models also show fairly good performances relative to the AR model in short forecast horizons, which can be practically useful because financial crises often occur abruptly. We also find parsimonious models that are based on a few common factors perform as well as other bigger models.

References

Donald W. K. Andrews and J. Christopher Monahan. An improved heteroskedasticity and autocorrelation consistent covariance matrix estimator. *Econometrica*, 60(4):953–966, 1992.

Jushan Bai and Serena Ng. A panic attack on unit roots and cointegration. *Econometrica*, 72:1127–1177, 2004.

Andrew Berg and Catherine Pattillo. Predicting currency crises: The indicators approach and an alternative. *Journal of International Money and Finance*, 18(4):561–586, 1999.

Andrew Berg, Eduardo Borensztein, and Catherine Pattillo. Assessing early warning systems: How have they worked in practice? IMF Working Paper 04/52. International Monetary Fund, 2005.

Axel Brüggemann and Thomas Linne. How good are leading indicators for currency and banking crises in central and eastern europe? an empirical test. IWH Discussion Papers, 95.Halle Institute for Economic Research., 1999.

Matthieu Bussiere and Christian Mulder. External vulnerability in emerging market economies - how high liquidity can offset weak fundamentals and the effects of contagion. IMF Working Papers 99/88, 1999.

Emrah Ismail Cevik, Sel Diboogu, and Turalay Kenc. Measuring financial stress in turkey. *Journal of Policy Modeling*, 35:370–383, 2013.

Ian Christensen and Fuchun Li. Predicting financial stress events: A signal extraction approach. Working Paper 2014-37. Bank of Canada., 2014.

Andrea Cipollini and George Kapetanios. Forecasting financial crises and contagion in asia using dynamic factor analysis. *Journal of Empirical Finance*, 16(2):188–200, 2009.

Hali J. Edison. Do indicators of financial crises work? an evaluation of an early warning system. *International Journal of Finance and Economics*, 8 (1):11–53, 2003.

M. EI-Shagi, T. Knedlik, and G. von Schweinitz. Predicting financial crises: The(statistical) significance of the signals approach. *Journal of International Money and Finance*, 35: 75–103, 2013.

Barry Eichengreen, Andrew K. Rose, and Charles Wyplosz. Exchange market mayhem: The antecedents and aftermath of speculative attacks. *Economic Policy*, 10 (21):249–312, 1995.

Jeffrey Frankel and George Saravelos. Can leading indicators assess country vulnerability? evidence from the 2008–09 global financial crisis. *Journal of International Economics*, 87: 216 –231, 2012.

Jeffrey A. Frankel and Andrew K. Rose. Currency crashes in emerging markets: an empirical treatment. *Journal of International Economics*, 41 (3/4):351–366, 1996.

Lance Girton and Don Roper. A monetary model of exchange market pressure applied to the postwar canadian experience. *American Economic Review*, 67:537–548, 1977.

Marianna B. Grimaldi. Detecting and interpreting financial stress in the euro area. ECB Working Paper Series No. 1214, European Central Bank, June 2010.

Marianna B. Grimaldi. Up for count? central bank words and financial stress. Sveriges Riksbank Working Paper Series 252, April 2011.

Elke Hanschel and Pierre Monnin. Measuring and forecasting stress in the banking sector: Evidence from switzerland. BIS Paper No. 22, Bank for International Settlements, April 2005.

Jakob L. Hansen. A risk index for euro-denominated assets. National Bank of Denmark Working Paper No. 36/2006,, May 2006.

Daniel Hollo, Manfred Kremer, and Marco Lo Duca. Ciss-a composite indicator of systemic stress in the financial system. ECB Working Paper Series No. 1426, March 2012.

Mark Illing and Ying Liu. Measuring financial stress in a developed country: An application to canada. *Journal of Financial Stability*, 2(3):243–65, 2006.

Mevlud Islami and Jeong-Rong Kurz-Kim. A single composite financial stress indicator and its real impact in the euro area. Discussion paper 31/2013,Deutsche Bundesbank, 2013.

Graciela Kaminsky, Saul Lizondo, and Carmen Reinhart. Leading indicators of currency crises. IMF Working Paper No. 45 International Monetary Fund, 1998.

Hyeongwoo Kim and Wen Shi. Forecasting financial market vulnerability in the u.s.: A factor model approach. Technical report, Auburn University, 2015.

Hyun Hak Kim. Forecasting macroeconomic variables using data dimension reduction methods: The case of korea. BOK Working Paper 2013-26, Bank of Korea, 2013.

Dimitrios P. Louzis and Angelos T. Vouldis. A financial systemic stress index for greece. Presented at the First Conference of the Macro-prudential Research (MaRs) network of the European System of Central Banks in Frankfurtam Main, October 2011.

Michael W. McCracken. Asymptotics for out of sample tests of granger causality. *Journal of Econometrics*, 140:719–752, 2007.

Miroslav Misina and Greg Tkacz. Credit, asset prices, and financial stress. *International Journal of Central Banking*, 5(95-122), 2009.

Miguel A. Morales and Dairo Estrada. A financial stability index for colombia. *Annals of Finance*, 6(4):555–81, 2010.

Mikhail V. Oet, Ryan Eiben, Timothy Bianco, Dieter Gramlich, and Stephen J. Ong. The financial stress index: Identification of system risk conditions. Working Paper No. 1130, Federal Reserve Bank of Cleveland, November 2011.

Jeffrey Sachs, Aaron Tornell, and Andres Velasco. Financial crises in emerging markets: The lessons from 1995. *Brookings Papers on Economic Activity*, 27(1):147–199, 1996.

Johannes Sandahl, Mia Holmfeldt, Anders Rydén, and Maria Strömqvist. An index of financial stress for sweden. *Sveriges Riksbank Economic Review*, 2:49–67, 2011.

Jan Willem Slingenberg and Jakob de Haan. Forecasting financial stress. DNB Working PaperNo. 292 De Nederlandsche Bank, April 2011.

James H. Stock and Mark W. Watson. Macroeconomic forecasting using diffusion indexes. *Journal of Business and Economic Statistics*, 20(2):147–162, 2002.

Matthew S.Yiu, Wai.-Yip Alex Ho, and Lu Jin. A measure of financial stress in hong kong financial market: The financial stress index. Hong Kong Monetary Authority Research Note No. 02/2010, March 2010.

Evan Tanner. Exchange market pressure, currency crises, and monetary policy; additional evidence from emerging markets. IMF Working Papers No. 02/14, January 2002.

Björn van Roye. Financial stress and economic activity in germany and the euro area. Kiel Working Paper No. 1743, Kiel Institute for the World Economy, November 2011.

Figure 1. Financial Stress Index (Dashed) and 4 Sub-Indices

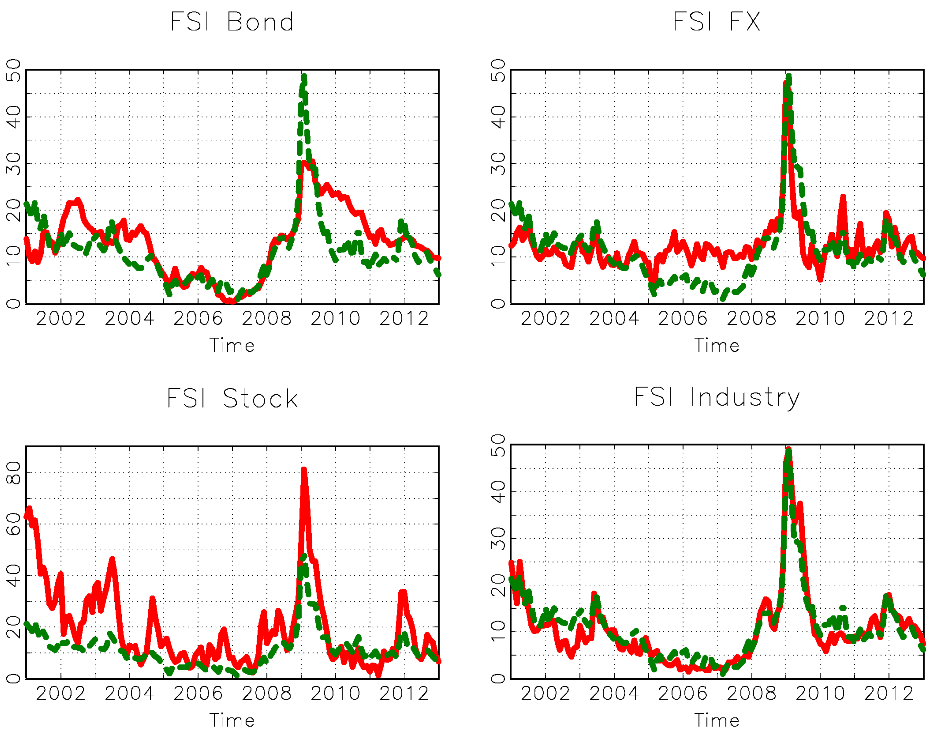


Figure 2. Factor Estimates: Differenced Factors

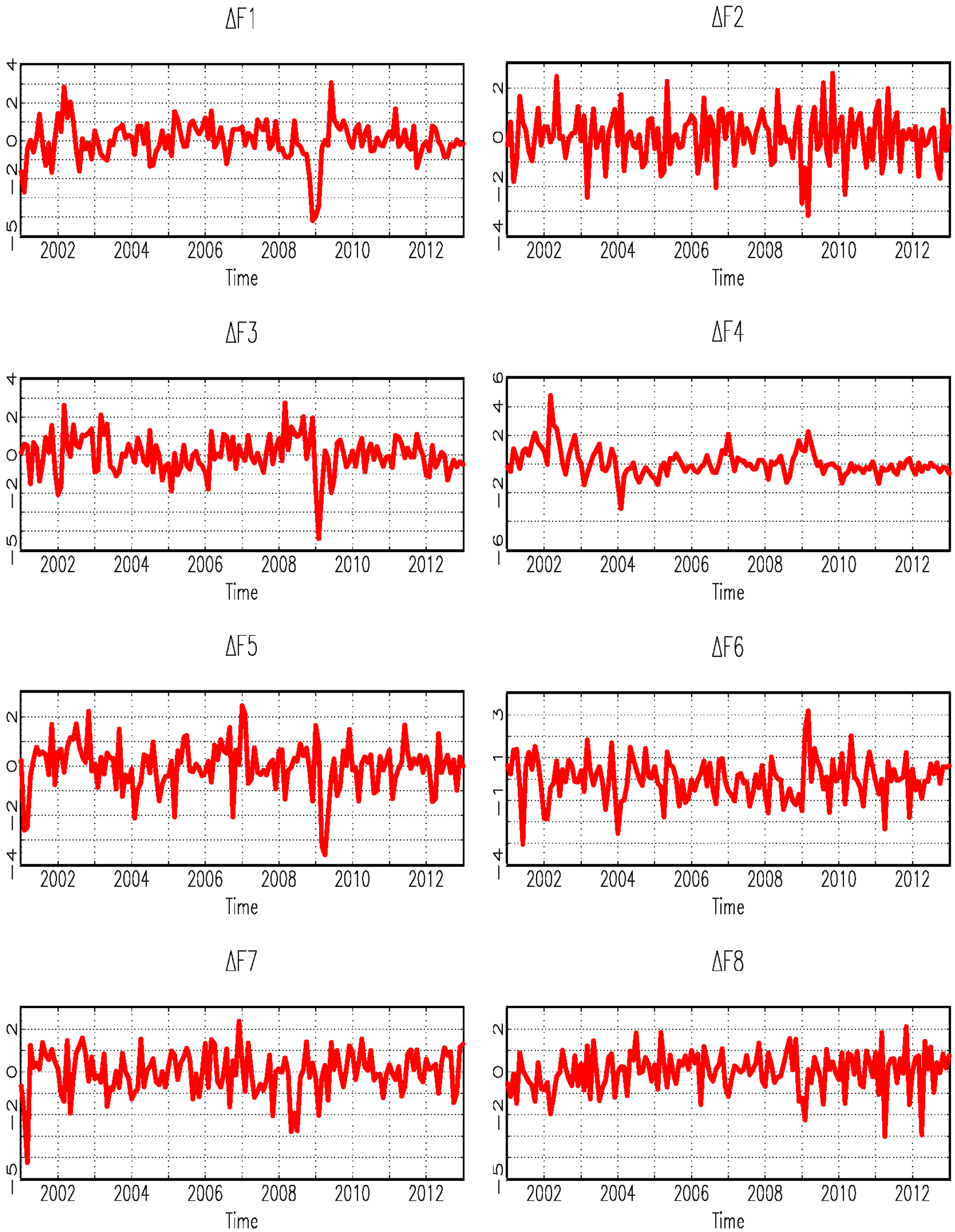


Figure 3. Factor Estimates: Level Factors

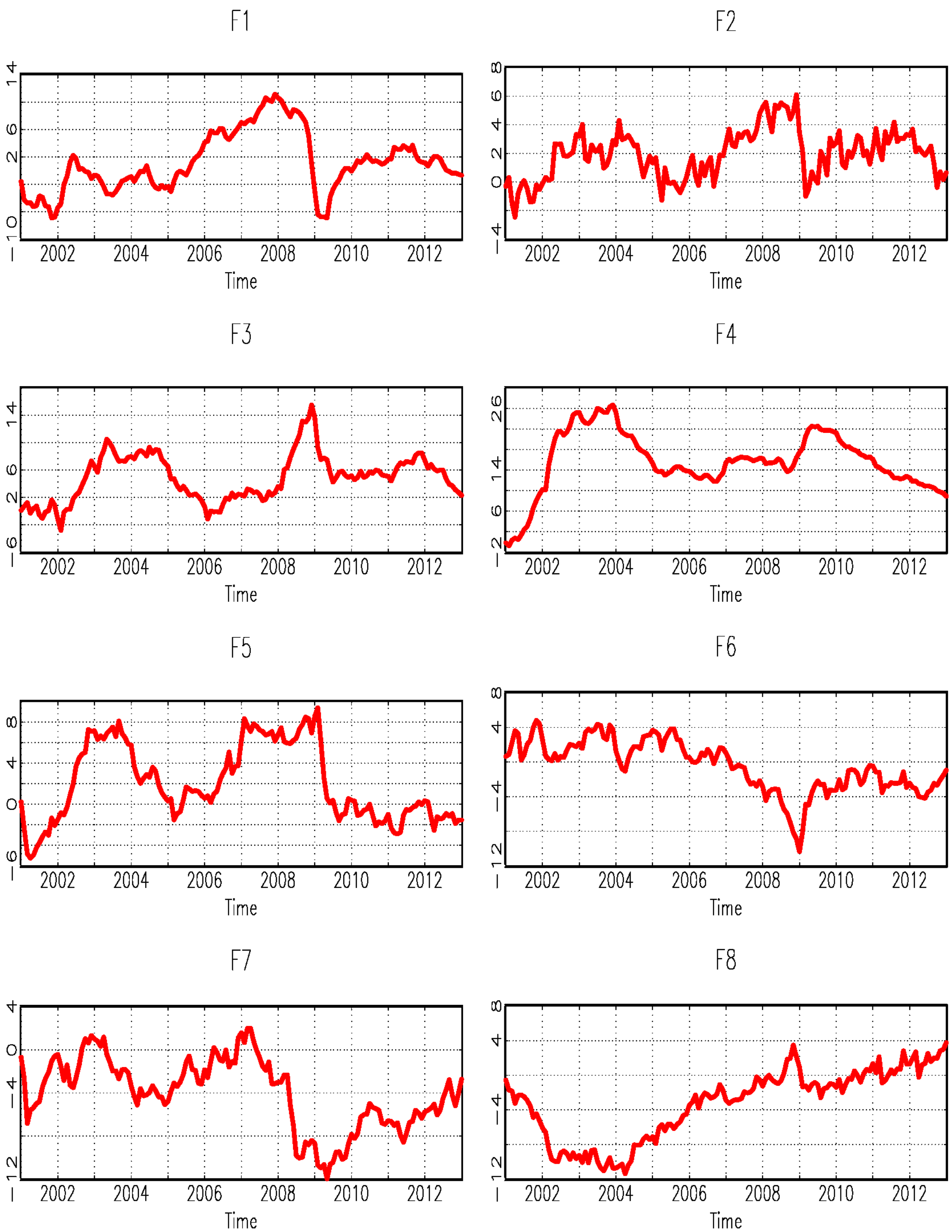


Figure 4. Factor Loading Coefficients Estimates

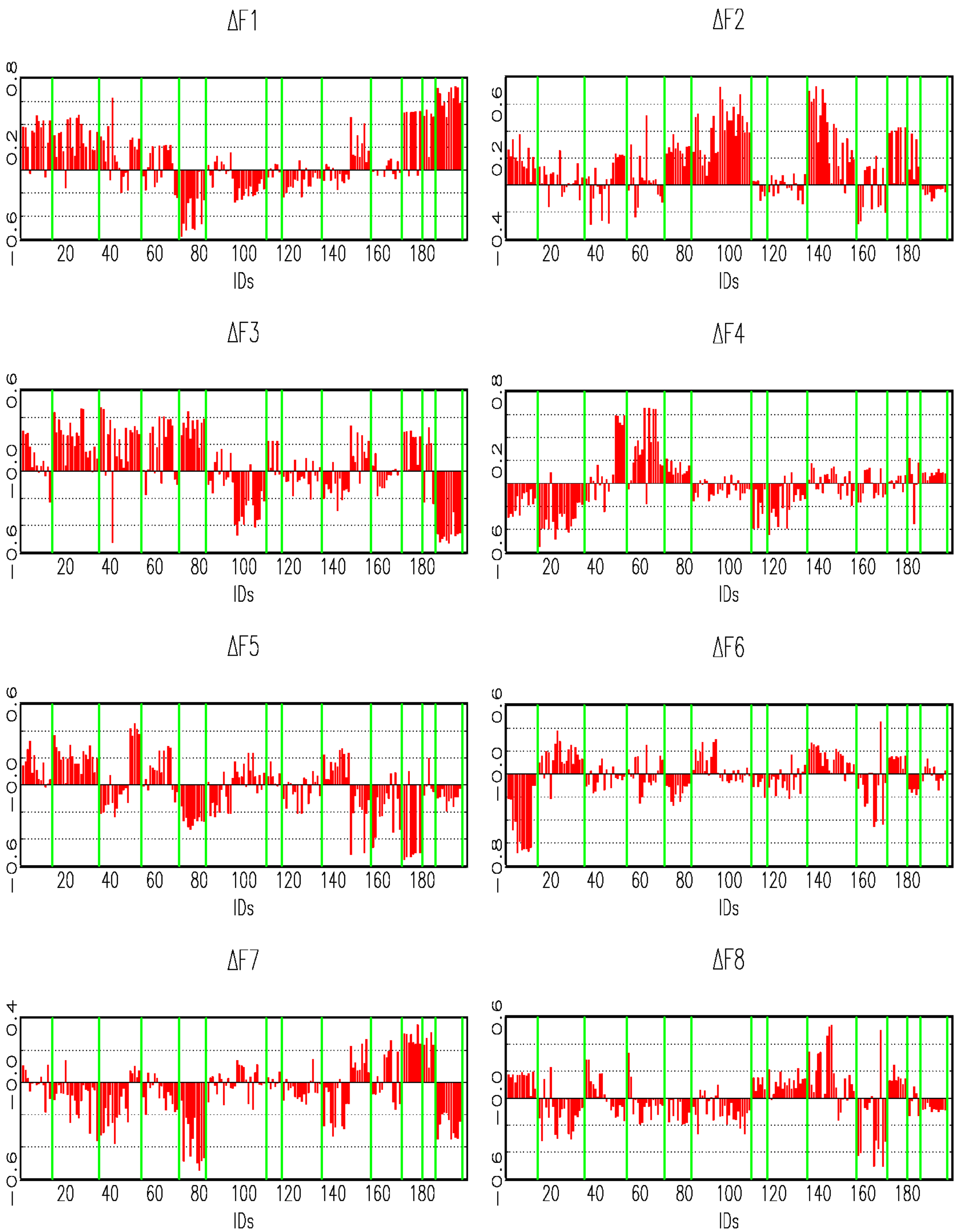


Figure 5. Marginal R^2 Analysis

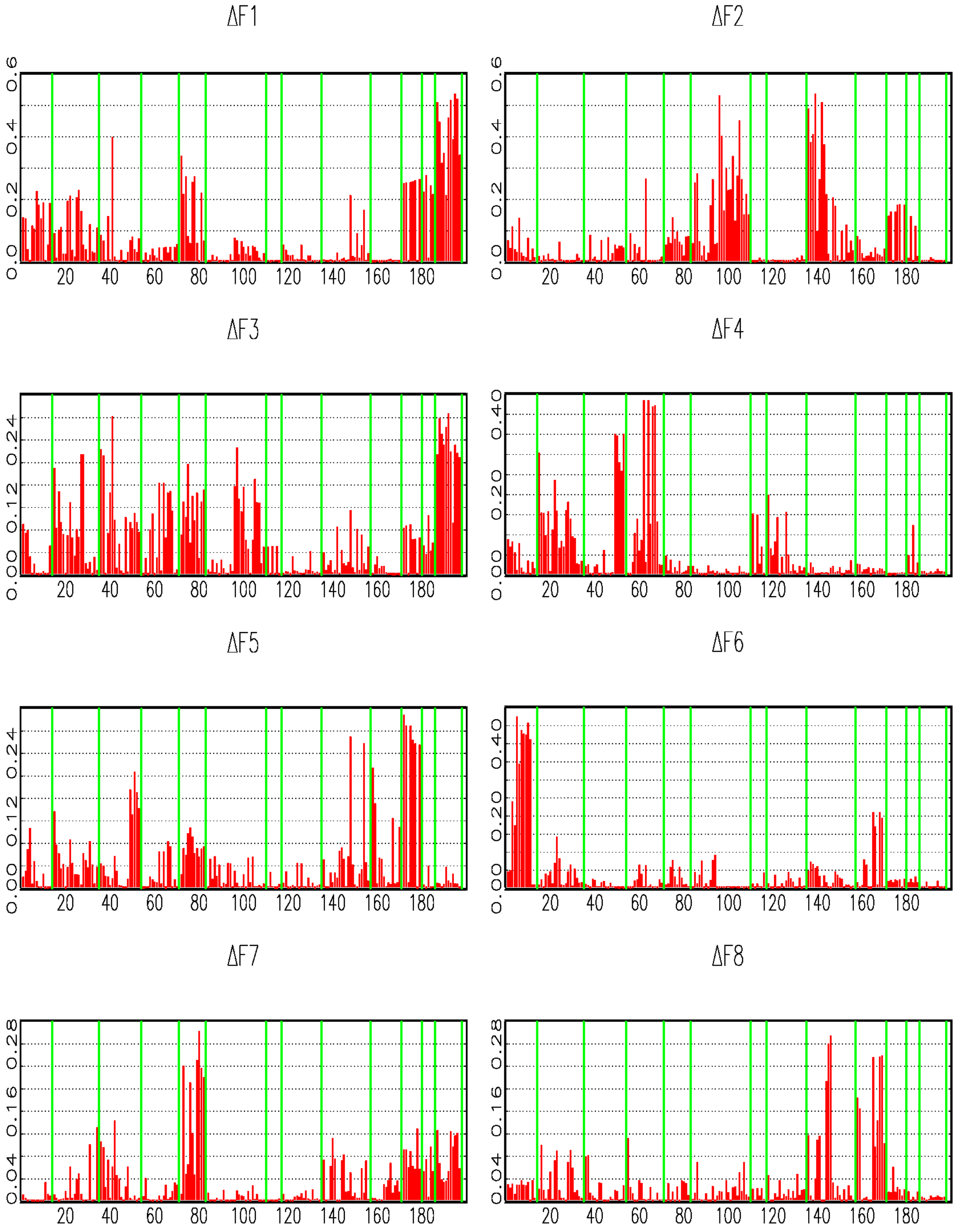


Figure 6. Common Factor #1 and its Factor Loading Coefficients

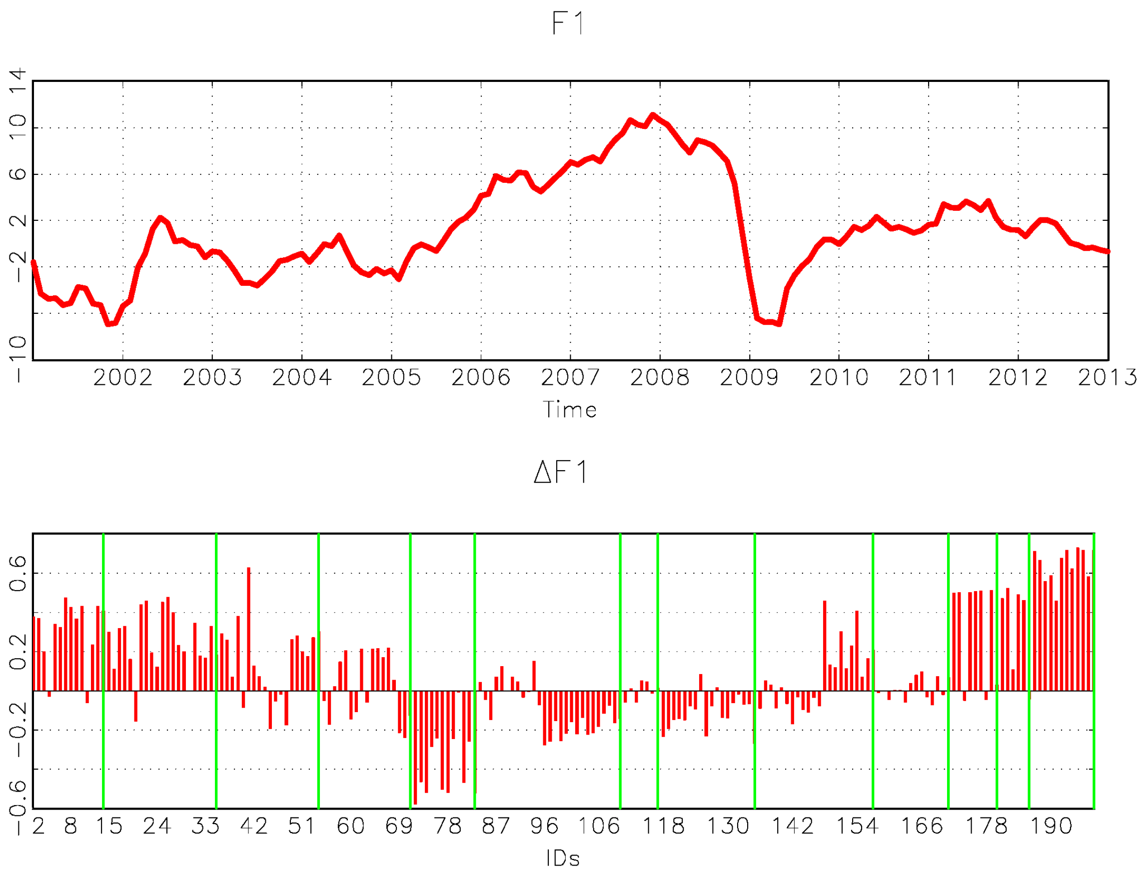


Figure 7. Common Factor #2 and its Factor Loading Coefficients

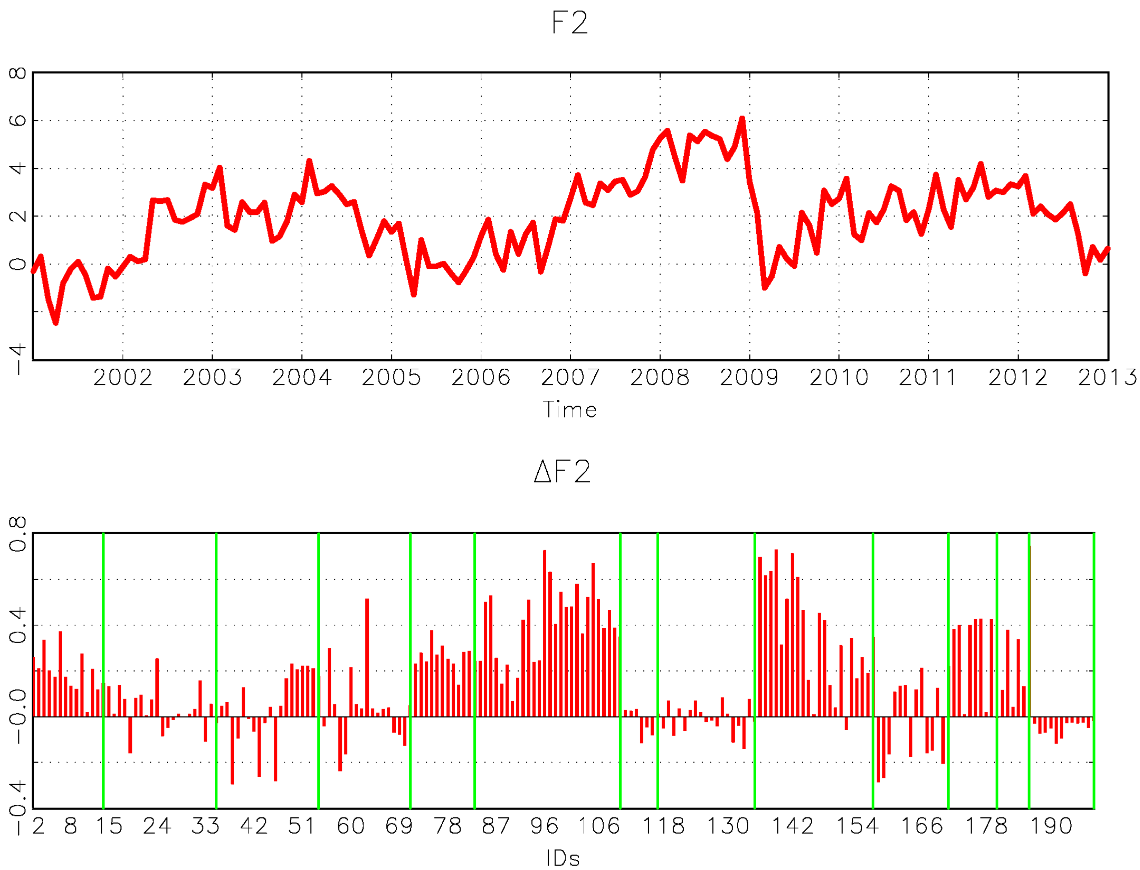
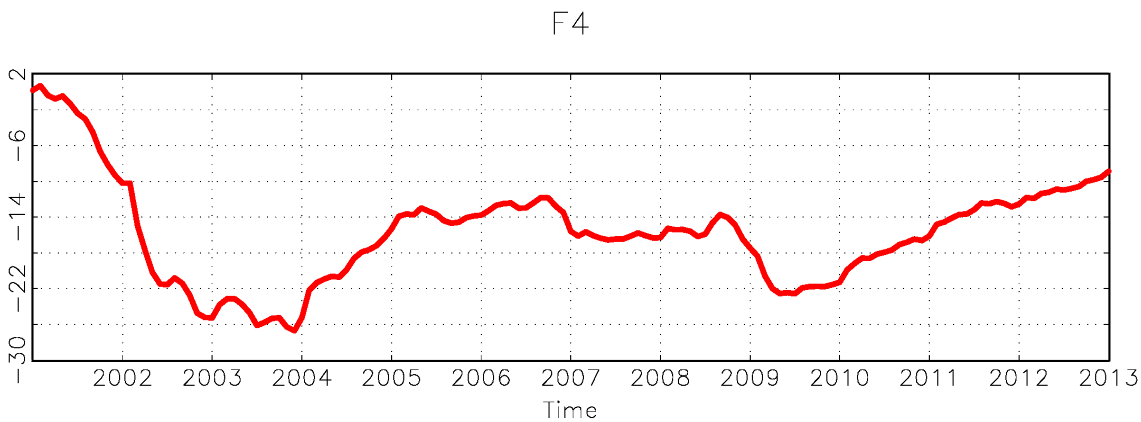


Figure 8. Common Factor #4 and its Factor Loading Coefficients



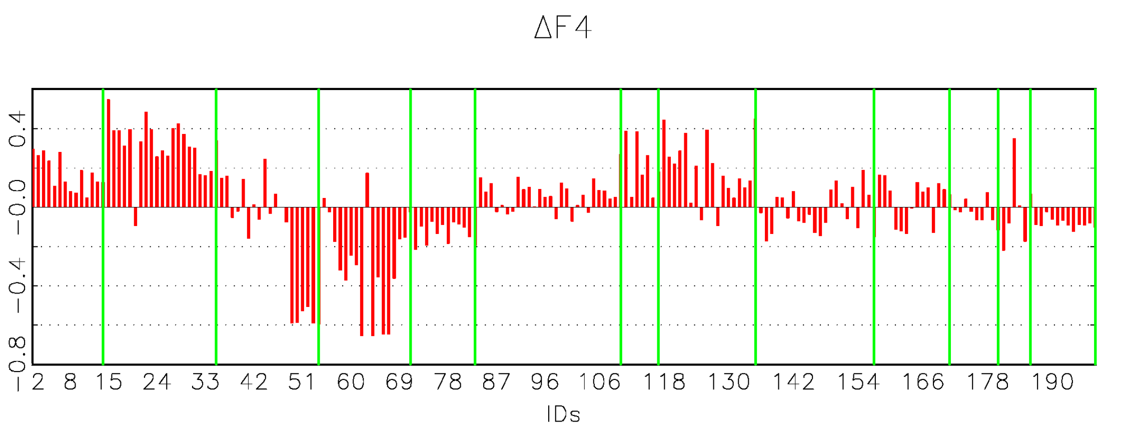


Table 1. Macroeconomic Data Descriptions

Group ID	Data ID	Data Descriptions
#1	1-14	Domestic and World Interest Rates
#2	15-35	Exports/Imports Prices
#3	36-54	Producer/Consumer/Housing Prices
#4	55-71	Monetary Aggregates
#5	72-83	Bilateral Exchange Rates
#6	84-110	Manufacturers'/Construction New Orders
#7	111-117	Manufacturers' Inventory Indices
#8	118-136	Housing Inventories
#9	136-157	Sales and Capacity Utilizations
#10	158-171	Unemployment/Employment/Labor Force Participation
#11	172-180	Industrial Production Indices
#12	181-186	Business Condition Indices
#13	187-198	Stock Indices

Table 2. In-Sample Fit Analysis for Selection of Factors

	Financial Stress Index	
#Factors	Factors	R^2
1	$\Delta\hat{F}1$	0.233
2	$\Delta\hat{F}1$, $\Delta\hat{F}4$	0.331
3	$\Delta\hat{F}1$, $\Delta\hat{F}4$, $\Delta\hat{F}5$	0.365
4	$\Delta\hat{F}1$, $\Delta\hat{F}4$, $\Delta\hat{F}5$, $\Delta\hat{F}8$	0.388
5	$\Delta\hat{F}1$, $\Delta\hat{F}4$, $\Delta\hat{F}5$, $\Delta\hat{F}7$, $\Delta\hat{F}8$	0.409
$6^{\dagger}$	$\Delta\hat{F}1$, $\Delta\hat{F}2$, $\Delta\hat{F}4$, $\Delta\hat{F}5$, $\Delta\hat{F}7$, $\Delta\hat{F}8$	0.421
7^{*}	$\Delta\hat{F}1$, $\Delta\hat{F}2$, $\Delta\hat{F}3$, $\Delta\hat{F}4$, $\Delta\hat{F}5$, $\Delta\hat{F}7$, $\Delta\hat{F}8$	0.426
8	$\Delta\hat{F}1$, $\Delta\hat{F}2$, $\Delta\hat{F}3$, $\Delta\hat{F}4$, $\Delta\hat{F}5$, $\Delta\hat{F}6$, $\Delta\hat{F}7$, $\Delta\hat{F}8$	0.429

	Financial Stress Index - Bond	
#Factors	Factors	R^2
1	$\Delta\hat{F}4$	0.036
2	$\Delta\hat{F}4$, $\Delta\hat{F}5$	0.054
$3^{\dagger}$	$\Delta\hat{F}1$, $\Delta\hat{F}4$, $\Delta\hat{F}5$	0.068
4^{*}	$\Delta\hat{F}1$, $\Delta\hat{F}4$, $\Delta\hat{F}5$, $\Delta\hat{F}8$	0.079
5	$\Delta\hat{F}1$, $\Delta\hat{F}2$, $\Delta\hat{F}4$, $\Delta\hat{F}5$, $\Delta\hat{F}8$	0.083
6	$\Delta\hat{F}1$, $\Delta\hat{F}2$, $\Delta\hat{F}4$, $\Delta\hat{F}5$, $\Delta\hat{F}6$, $\Delta\hat{F}8$	0.084
7	$\Delta\hat{F}1$, $\Delta\hat{F}2$, $\Delta\hat{F}3$, $\Delta\hat{F}4$, $\Delta\hat{F}5$, $\Delta\hat{F}6$, $\Delta\hat{F}8$	0.085
8	$\Delta\hat{F}1$, $\Delta\hat{F}2$, $\Delta\hat{F}3$, $\Delta\hat{F}4$, $\Delta\hat{F}5$, $\Delta\hat{F}6$, $\Delta\hat{F}7$, $\Delta\hat{F}8$	0.085

	Financial Stress Index: Foreign Exchange	
#Factors	Factors	R^2
1	$\Delta\hat{F}1$	0.324
2	$\Delta\hat{F}1$, $\Delta\hat{F}4$	0.373
3	$\Delta\hat{F}1$, $\Delta\hat{F}4$, $\Delta\hat{F}7$	0.395
4	$\Delta\hat{F}1$, $\Delta\hat{F}4$, $\Delta\hat{F}6$, $\Delta\hat{F}7$	0.405
$5^{*\dagger}$	$\Delta\hat{F}1$, $\Delta\hat{F}4$, $\Delta\hat{F}5$, $\Delta\hat{F}6$, $\Delta\hat{F}7$	0.414
6	$\Delta\hat{F}1$, $\Delta\hat{F}2$, $\Delta\hat{F}4$, $\Delta\hat{F}5$, $\Delta\hat{F}6$, $\Delta\hat{F}7$	0.417
7	$\Delta\hat{F}1$, $\Delta\hat{F}2$, $\Delta\hat{F}4$, $\Delta\hat{F}5$, $\Delta\hat{F}6$, $\Delta\hat{F}7$, $\Delta\hat{F}8$	0.419
8	$\Delta\hat{F}1$, $\Delta\hat{F}2$, $\Delta\hat{F}3$, $\Delta\hat{F}4$, $\Delta\hat{F}5$, $\Delta\hat{F}6$, $\Delta\hat{F}7$, $\Delta\hat{F}8$	0.419

Note: $*$ and $\dagger$ denote the chosen model by the adjusted R^2 method and the specific to general rule, respectively.

Table 2. Continued

Financial Stress Index: Stock		
#Factors	Factors	R^2
1	$\Delta\hat{F}1$	0.235
2	$\Delta\hat{F}1$, $\Delta\hat{F}4$	0.357
3	$\Delta\hat{F}1$, $\Delta\hat{F}4$, $\Delta\hat{F}8$	0.388
4	$\Delta\hat{F}1$, $\Delta\hat{F}4$, $\Delta\hat{F}7$, $\Delta\hat{F}8$	0.417
5	$\Delta\hat{F}1$, $\Delta\hat{F}4$, $\Delta\hat{F}6$, $\Delta\hat{F}7$, $\Delta\hat{F}8$	0.438
6	$\Delta\hat{F}1$, $\Delta\hat{F}4$, $\Delta\hat{F}5$, $\Delta\hat{F}6$, $\Delta\hat{F}7$, $\Delta\hat{F}8$	0.456
7	$\Delta\hat{F}1$, $\Delta\hat{F}2$, $\Delta\hat{F}4$, $\Delta\hat{F}5$, $\Delta\hat{F}6$, $\Delta\hat{F}7$, $\Delta\hat{F}8$	0.471
$8^{*\dagger}$	$\Delta\hat{F}1$, $\Delta\hat{F}2$, $\Delta\hat{F}3$, $\Delta\hat{F}4$, $\Delta\hat{F}5$, $\Delta\hat{F}6$, $\Delta\hat{F}7$, $\Delta\hat{F}8$	0.479

Financial Stress Index: Financial Industry		
#Factors	Factors	R^2
1	$\Delta\hat{F}1$	0.189
2	$\Delta\hat{F}1$, $\Delta\hat{F}4$	0.260
3	$\Delta\hat{F}1$, $\Delta\hat{F}4$, $\Delta\hat{F}5$	0.322
4	$\Delta\hat{F}1$, $\Delta\hat{F}4$, $\Delta\hat{F}5$, $\Delta\hat{F}7$	0.352
5	$\Delta\hat{F}1$, $\Delta\hat{F}4$, $\Delta\hat{F}5$, $\Delta\hat{F}7$, $\Delta\hat{F}8$	0.378
6	$\Delta\hat{F}1$, $\Delta\hat{F}2$, $\Delta\hat{F}4$, $\Delta\hat{F}5$, $\Delta\hat{F}7$, $\Delta\hat{F}8$	0.395
7	$\Delta\hat{F}1$, $\Delta\hat{F}2$, $\Delta\hat{F}3$, $\Delta\hat{F}4$, $\Delta\hat{F}5$, $\Delta\hat{F}7$, $\Delta\hat{F}8$	0.410
$8^{*\dagger}$	$\Delta\hat{F}1$, $\Delta\hat{F}2$, $\Delta\hat{F}3$, $\Delta\hat{F}4$, $\Delta\hat{F}5$, $\Delta\hat{F}6$, $\Delta\hat{F}7$, $\Delta\hat{F}8$	0.421

Note: $*$ and $\dagger$ denote the chosen model by the adjusted R^2 method and the specific to general rule, respectively.

Table 3. j-Period Ahead Out-of-Sample Forecast: FSI

Recursive Method: RRMSPE vs. Random Walk

Factors	$j=1$	$j=3$	$j=6$	$j=9$	$j=12$
$\Delta\hat{F}1$	1.040	1.097†	1.251‡	1.344‡	1.398‡
$\Delta\hat{F}4$	1.031	1.084†	1.217†	1.296‡	1.415‡
$\Delta\hat{F}5$	1.049†	1.128†	1.246†	1.331‡	1.392‡
$\Delta\hat{F}1$, $\Delta\hat{F}4$	1.039	1.076*	1.235†	1.302‡	1.416‡
$\Delta\hat{F}1$, $\Delta\hat{F}5$	1.049	1.100*	1.279‡	1.357‡	1.397‡
$\Delta\hat{F}1$, $\Delta\hat{F}4$, $\Delta\hat{F}5$	1.047	1.079	1.270†	1.315‡	1.413‡

Rolling Window Method: RRMSPE vs. Random Walk

Factors	$j=1$	$j=3$	$j=6$	$j=9$	$j=12$
$\Delta\hat{F}1$	1.050	1.106†	1.280‡	1.378‡	1.438‡
$\Delta\hat{F}4$	1.021	1.085*	1.219†	1.354‡	1.432‡
$\Delta\hat{F}5$	1.039*	1.111†	1.279†	1.348‡	1.442‡
$\Delta\hat{F}1$, $\Delta\hat{F}4$	1.036	1.084†	1.244‡	1.371‡	1.437‡
$\Delta\hat{F}1$, $\Delta\hat{F}5$	1.057	1.090*	1.331‡	1.374‡	1.437‡
$\Delta\hat{F}1$, $\Delta\hat{F}4$, $\Delta\hat{F}5$	1.043	1.071	1.295‡	1.377‡	1.435‡

Recursive Method: RRMSPE vs. Autoregressive

Factors	$j=1$	$j=3$	$j=6$	$j=9$	$j=12$
$\Delta\hat{F}1$	1.008	0.975	1.015‡	1.014‡	1.003
$\Delta\hat{F}4$	0.999	0.963	0.987	0.977	1.015†
$\Delta\hat{F}5$	1.017‡	1.003	1.010	1.004	0.999
$\Delta\hat{F}1$, $\Delta\hat{F}4$	1.007	0.956	1.001	0.982	1.016‡
$\Delta\hat{F}1$, $\Delta\hat{F}5$	1.017*	0.977	1.037†	1.023‡	1.002
$\Delta\hat{F}1$, $\Delta\hat{F}4$, $\Delta\hat{F}5$	1.015*	0.959	1.030†	0.992	1.014†

Rolling Window Method: RRMSPE vs. Autoregressive

Factors	$j=1$	$j=3$	$j=6$	$j=9$	$j=12$
$\Delta\hat{F}1$	1.020*	0.981	1.028‡	1.015‡	0.998
$\Delta\hat{F}4$	0.992*	0.962	0.979	0.997	0.994
$\Delta\hat{F}5$	1.009‡	0.986	1.027†	0.993	1.001
$\Delta\hat{F}1$, $\Delta\hat{F}4$	1.007*	0.962	0.999	1.009*	0.998
$\Delta\hat{F}1$, $\Delta\hat{F}5$	1.027*	0.967	1.069‡	1.012†	0.998
$\Delta\hat{F}1$, $\Delta\hat{F}4$, $\Delta\hat{F}5$	1.013	0.950†	1.040†	1.014†	0.997

Note: *RRMSPE* denotes the mean square error from the random walk model relative to the mean square error from our factor model. Therefore, when *RRMSPE* is greater than one, our factor models perform better than the benchmark model.

Table 4. j-Period Ahead Out-of-Sample Forecast: FSI-Bond

Recursive Method: RRMSPE vs. Random Walk					
Factors	$j=1$	$j=3$	$j=6$	$j=9$	$j=12$
$\Delta\hat{F}4$	1.025	$1.154^{\ddagger}$	$1.204^{\ddagger}$	$1.365^{\ddagger}$	$1.547^{\ddagger}$
$\Delta\hat{F}5$	0.998	$1.084^{\dagger}$	$1.154^{\ddagger}$	$1.243^{\ddagger}$	$1.392^{\ddagger}$
$\Delta\hat{F}1$	1.000	1.047	$1.141^{\ddagger}$	$1.239^{\ddagger}$	$1.400^{\ddagger}$
$\Delta\hat{F}4$, $\Delta\hat{F}5$	1.008	$1.164^{\ddagger}$	$1.218^{\ddagger}$	$1.375^{\ddagger}$	$1.549^{\ddagger}$
$\Delta\hat{F}4$, $\Delta\hat{F}1$	1.005	1.114^{*}	$1.207^{\ddagger}$	$1.368^{\ddagger}$	$1.537^{\ddagger}$
$\Delta\hat{F}1$, $\Delta\hat{F}4$, $\Delta\hat{F}5$	0.987	1.117^{*}	$1.218^{\ddagger}$	$1.374^{\ddagger}$	$1.543^{\ddagger}$

Rolling Window Method: RRMSPE vs. Random Walk					
Factors	$j=1$	$j=3$	$j=6$	$j=9$	$j=12$
$\Delta\hat{F}4$	1.033	$1.158^{\ddagger}$	$1.264^{\ddagger}$	$1.402^{\ddagger}$	$1.639^{\ddagger}$
$\Delta\hat{F}5$	1.011	$1.120^{\ddagger}$	$1.257^{\ddagger}$	$1.360^{\ddagger}$	$1.594^{\ddagger}$
$\Delta\hat{F}1$	1.005	$1.085^{\dagger}$	$1.245^{\ddagger}$	$1.335^{\ddagger}$	$1.599^{\ddagger}$
$\Delta\hat{F}4$, $\Delta\hat{F}5$	1.021	$1.170^{\ddagger}$	$1.298^{\ddagger}$	1.447	$1.669^{\ddagger}$
$\Delta\hat{F}4$, $\Delta\hat{F}1$	1.009	$1.136^{\dagger}$	$1.298^{\ddagger}$	$1.437^{\ddagger}$	$1.679^{\ddagger}$
$\Delta\hat{F}1$, $\Delta\hat{F}4$, $\Delta\hat{F}5$	0.998	$1.142^{\dagger}$	$1.331^{\ddagger}$	$1.475^{\ddagger}$	$1.697^{\ddagger}$

Recursive Method: RRMSPE vs. Autoregressive					
Factors	$j=1$	$j=3$	$j=6$	$j=9$	$j=12$
$\Delta\hat{F}4$	1.011^{*}	$1.076^{\ddagger}$	$1.057^{\ddagger}$	$1.108^{\ddagger}$	$1.116^{\ddagger}$
$\Delta\hat{F}5$	0.984	1.011^{*}	$1.013^{\dagger}$	$1.009^{\ddagger}$	$1.003^{\dagger}$
$\Delta\hat{F}1$	0.986	0.976	1.002	1.006	1.009
$\Delta\hat{F}4$, $\Delta\hat{F}5$	0.994	$1.086^{\ddagger}$	$1.070^{\ddagger}$	$1.116^{\ddagger}$	$1.117^{\ddagger}$
$\Delta\hat{F}4$, $\Delta\hat{F}1$	0.990	$1.039^{\dagger}$	$1.060^{\ddagger}$	$1.110^{\ddagger}$	$1.108^{\ddagger}$
$\Delta\hat{F}1$, $\Delta\hat{F}4$, $\Delta\hat{F}5$	0.973	$1.042^{\dagger}$	$1.069^{\ddagger}$	$1.115^{\ddagger}$	$1.113^{\ddagger}$

Rolling Window Method: RRMSPE vs. Autoregressive					
Factors	$j=1$	$j=3$	$j=6$	$j=9$	$j=12$
$\Delta\hat{F}4$	1.011^{*}	$1.050^{\ddagger}$	$1.036^{\ddagger}$	$1.067^{\ddagger}$	$1.052^{\ddagger}$
$\Delta\hat{F}5$	0.990	$1.015^{\dagger}$	$1.031^{\ddagger}$	$1.035^{\ddagger}$	$1.024^{\dagger}$
$\Delta\hat{F}1$	0.983	0.983	$1.021^{\ddagger}$	$1.016^{\dagger}$	$1.027^{\ddagger}$
$\Delta\hat{F}4$, $\Delta\hat{F}5$	0.999	$1.061^{\ddagger}$	$1.064^{\ddagger}$	$1.101^{\ddagger}$	$1.072^{\ddagger}$
$\Delta\hat{F}4$, $\Delta\hat{F}1$	0.987	$1.029^{\dagger}$	$1.064^{\ddagger}$	$1.094^{\ddagger}$	$1.078^{\ddagger}$
$\Delta\hat{F}1$, $\Delta\hat{F}4$, $\Delta\hat{F}5$	0.976	$1.035^{\ddagger}$	$1.091^{\ddagger}$	$1.122^{\ddagger}$	$1.090^{\ddagger}$

Note: *RRMSPE* denotes the mean square error from the random walk model relative to the mean square error from our factor model. Therefore, when *RRMSPE* is greater than one, our factor models perform better than the benchmark model.

Table 5. j-Period Ahead Out-of-Sample Forecast: FSI-Foreign Exchange

Recursive Method: RRMSPE vs. Random Walk

Factors	$j=1$	$j=3$	$j=6$	$j=9$	$j=12$
$\Delta\hat{F}1$	1.120	1.293	$1.372^{\ddagger}$	$1.617^{\dagger}$	$1.658^{\dagger}$
$\Delta\hat{F}4$	1.095	$1.237^{\dagger}$	$1.385^{\ddagger}$	$1.584^{\dagger}$	$1.614^{\ddagger}$
$\Delta\hat{F}7$	1.088	$1.359^{\dagger}$	$1.312^{\dagger}$	$1.666^{\dagger}$	$1.623^{\dagger}$
$\Delta\hat{F}1$, $\Delta\hat{F}4$	1.118	$1.245^{\dagger}$	$1.426^{\ddagger}$	$1.590^{\dagger}$	$1.652^{\dagger}$
$\Delta\hat{F}1$, $\Delta\hat{F}7$	1.109	1.349	1.336	1.675	1.657
$\Delta\hat{F}1$, $\Delta\hat{F}4$, $\Delta\hat{F}7$	1.107	$1.299^{\dagger}$	$1.402^{\ddagger}$	$1.662^{\dagger}$	$1.651^{\ddagger}$

Rolling Window Method: RRMSPE vs. Random Walk

Factors	$j=1$	$j=3$	$j=6$	$j=9$	$j=12$
$\Delta\hat{F}1$	1.126^{*}	$1.296^{\dagger}$	$1.392^{\ddagger}$	$1.611^{\dagger}$	$1.612^{\dagger}$
$\Delta\hat{F}4$	1.088^{*}	$1.222^{\dagger}$	$1.372^{\ddagger}$	$1.611^{\dagger}$	$1.566^{\dagger}$
$\Delta\hat{F}7$	$1.095^{\dagger}$	$1.285^{\dagger}$	$1.305^{\ddagger}$	$1.688^{\dagger}$	$1.560^{\dagger}$
$\Delta\hat{F}1$, $\Delta\hat{F}4$	1.120^{*}	$1.232^{\dagger}$	$1.395^{\ddagger}$	$1.618^{\dagger}$	$1.598^{\dagger}$
$\Delta\hat{F}1$, $\Delta\hat{F}7$	$1.124^{\dagger}$	$1.289^{\dagger}$	$1.337^{\dagger}$	$1.673^{\dagger}$	$1.585^{\dagger}$
$\Delta\hat{F}1$, $\Delta\hat{F}4$, $\Delta\hat{F}7$	1.117^{*}	$1.222^{\dagger}$	$1.377^{\dagger}$	$1.679^{\dagger}$	$1.571^{\dagger}$

Recursive Method: RRMSPE vs. Autoregressive

Factors	$j=1$	$j=3$	$j=6$	$j=9$	$j=12$
$\Delta\hat{F}1$	$1.020^{\dagger}$	0.994	$1.028^{\ddagger}$	1.003^{*}	$1.017^{\dagger}$
$\Delta\hat{F}4$	0.997	0.951	$1.038^{\ddagger}$	0.982	0.990
$\Delta\hat{F}7$	0.991	$1.044^{\dagger}$	0.983	$1.033^{\dagger}$	0.995
$\Delta\hat{F}1$, $\Delta\hat{F}4$	$1.018^{\dagger}$	0.957	$1.069^{\ddagger}$	0.986	$1.014^{\dagger}$
$\Delta\hat{F}1$, $\Delta\hat{F}7$	1.010^{*}	1.037	1.001	1.038	1.016
$\Delta\hat{F}1$, $\Delta\hat{F}4$, $\Delta\hat{F}7$	1.008^{*}	0.998	1.051^{*}	1.031^{*}	$1.013^{\dagger}$

Rolling Window Method: RRMSPE vs. Autoregressive

Factors	$j=1$	$j=3$	$j=6$	$j=9$	$j=12$
$\Delta\hat{F}1$	$1.029^{\ddagger}$	0.999	$1.037^{\ddagger}$	0.992	$1.014^{\dagger}$
$\Delta\hat{F}4$	0.994	0.942	$1.022^{\ddagger}$	0.992	0.985
$\Delta\hat{F}7$	1.000	0.991	0.973	$1.040^{\ddagger}$	0.981
$\Delta\hat{F}1$, $\Delta\hat{F}4$	$1.023^{\ddagger}$	0.950	$1.039^{\dagger}$	0.996	1.005^{*}
$\Delta\hat{F}1$, $\Delta\hat{F}7$	$1.027^{\dagger}$	0.994	0.996	$1.030^{\dagger}$	0.997
$\Delta\hat{F}1$, $\Delta\hat{F}4$, $\Delta\hat{F}7$	$1.020^{\dagger}$	0.942	$1.026^{\dagger}$	$1.034^{\dagger}$	0.988

Note: *RRMSPE* denotes the mean square error from the random walk model relative to the mean square error from our factor model. Therefore, when *RRMSPE* is greater than one, our factor models perform better than the benchmark model.

Table 6. j-Period Ahead Out-of-Sample Forecast: FSI-Stock

Recursive Method: RRMSPE vs. Random Walk

Factors	$j=1$	$j=3$	$j=6$	$j=9$	$j=12$
$\Delta\hat{F}1$	1.016	0.999	1.062	1.090	$1.238^{\dagger}$
$\Delta\hat{F}4$	1.022	1.055	1.082	1.088	$1.253^{\dagger}$
$\Delta\hat{F}8$	1.033	1.056	1.114	1.103	1.239^{*}
$\Delta\hat{F}1$, $\Delta\hat{F}4$	1.028	1.000	1.063	1.075	$1.261^{\dagger}$
$\Delta\hat{F}1$, $\Delta\hat{F}8$	1.008	0.995	1.087	1.106	$1.268^{\dagger}$
$\Delta\hat{F}1$, $\Delta\hat{F}4$, $\Delta\hat{F}8$	1.013	0.995	1.086	1.093	$1.298^{\dagger}$

Rolling Window Method: RRMSPE vs. Random Walk

Factors	$j=1$	$j=3$	$j=6$	$j=9$	$j=12$
$\Delta\hat{F}1$	1.051	1.022	1.070	1.104	$1.300^{\dagger}$
$\Delta\hat{F}4$	1.010	1.076	1.129	1.143	$1.340^{\dagger}$
$\Delta\hat{F}8$	1.038	1.074	1.161	1.152	$1.302^{\dagger}$
$\Delta\hat{F}1$, $\Delta\hat{F}4$	1.035	1.020	1.076	1.096	$1.324^{\dagger}$
$\Delta\hat{F}1$, $\Delta\hat{F}8$	1.053	1.007	1.096	1.122	$1.296^{\ddagger}$
$\Delta\hat{F}1$, $\Delta\hat{F}4$, $\Delta\hat{F}8$	1.036	1.004	1.105	1.122	$1.330^{\ddagger}$

Recursive Method: RRMSPE vs. Autoregressive

Factors	$j=1$	$j=3$	$j=6$	$j=9$	$j=12$
$\Delta\hat{F}1$	1.000	0.945	0.975	$1.004^{\ddagger}$	$1.024^{\ddagger}$
$\Delta\hat{F}4$	$1.006^{\dagger}$	0.998	0.993	1.002	$1.037^{\ddagger}$
$\Delta\hat{F}8$	$1.016^{\dagger}$	0.999	$1.022^{\ddagger}$	1.016^{*}	$1.025^{\ddagger}$
$\Delta\hat{F}1$, $\Delta\hat{F}4$	1.011	0.945	0.975	0.991	$1.043^{\ddagger}$
$\Delta\hat{F}1$, $\Delta\hat{F}8$	0.992	0.941	$0.997^{\ddagger}$	$1.019^{\ddagger}$	$1.049^{\ddagger}$
$\Delta\hat{F}1$, $\Delta\hat{F}4$, $\Delta\hat{F}8$	0.997	0.941	0.996	1.007^{*}	$1.074^{\ddagger}$

Rolling Window Method: RRMSPE vs. Autoregressive

Factors	$j=1$	$j=3$	$j=6$	$j=9$	$j=12$
$\Delta\hat{F}1$	1.034^{*}	0.949	0.944	0.966	0.992
$\Delta\hat{F}4$	0.993	0.999	0.996	1.001	$1.023^{\dagger}$
$\Delta\hat{F}8$	$1.022^{\ddagger}$	0.997	$1.024^{\ddagger}$	1.009^{*}	0.994
$\Delta\hat{F}1$, $\Delta\hat{F}4$	1.018^{*}	0.947	0.949	0.960	1.011^{*}
$\Delta\hat{F}1$, $\Delta\hat{F}8$	$1.036^{\dagger}$	0.935	0.967	0.982	0.989
$\Delta\hat{F}1$, $\Delta\hat{F}4$, $\Delta\hat{F}8$	1.019^{*}	0.933	0.975	0.983	$1.015^{\dagger}$

Note: *RRMSPE* denotes the mean square error from the random walk model relative to the mean square error from our factor model. Therefore, when *RRMSPE* is greater than one, our factor models perform better than the benchmark model.

Table 7. j-Period Ahead Out-of-Sample Forecast: FSI-Financial Industry

Factors	$j=1$	$j=3$	$j=6$	$j=9$	$j=12$
Recursive Method: RRMSPE vs. Random Walk					
$\Delta\hat{F}1$	0.961	0.992	1.117	$1.246^{\ddagger}$	$1.428^{\ddagger}$
$\Delta\hat{F}4$	1.027	$1.123^{\ddagger}$	$1.218^{\dagger}$	$1.315^{\ddagger}$	$1.405^{\ddagger}$
$\Delta\hat{F}5$	1.023	1.068	1.177^{*}	$1.267^{\ddagger}$	$1.409^{\ddagger}$
$\Delta\hat{F}1$, $\Delta\hat{F}4$	0.946	1.015	1.172^{*}	$1.294^{\ddagger}$	$1.423^{\ddagger}$
$\Delta\hat{F}1$, $\Delta\hat{F}5$	0.953	0.970	1.132	$1.245^{\ddagger}$	$1.426^{\ddagger}$
$\Delta\hat{F}1$, $\Delta\hat{F}4$, $\Delta\hat{F}5$	0.937	0.996	1.213^{*}	$1.308^{\ddagger}$	$1.423^{\ddagger}$

Factors	$j=1$	$j=3$	$j=6$	$j=9$	$j=12$
Rolling Window Method: RRMSPE vs. Random Walk					
$\Delta\hat{F}1$	0.953	0.979	1.117	$1.256^{\ddagger}$	$1.424^{\ddagger}$
$\Delta\hat{F}4$	1.017	$1.130^{\dagger}$	$1.193^{\dagger}$	$1.386^{\ddagger}$	$1.400^{\ddagger}$
$\Delta\hat{F}5$	1.024	1.044	$1.217^{\dagger}$	$1.236^{\ddagger}$	$1.388^{\ddagger}$
$\Delta\hat{F}1$, $\Delta\hat{F}4$	0.923	0.997	1.140	$1.356^{\ddagger}$	$1.427^{\ddagger}$
$\Delta\hat{F}1$, $\Delta\hat{F}5$	0.951	0.954	1.153^{*}	$1.223^{\ddagger}$	$1.407^{\ddagger}$
$\Delta\hat{F}1$, $\Delta\hat{F}4$, $\Delta\hat{F}5$	0.920	0.979	1.203^{*}	$1.350^{\ddagger}$	$1.417^{\ddagger}$

Factors	$j=1$	$j=3$	$j=6$	$j=9$	$j=12$
Recursive Method: RRMSPE vs. Autoregressive					
$\Delta\hat{F}1$	0.934	0.891	0.947	0.974	$1.010^{\ddagger}$
$\Delta\hat{F}4$	0.998	1.007	$1.032^{\dagger}$	1.029	0.994
$\Delta\hat{F}5$	0.995	0.959	0.998	0.991	0.997
$\Delta\hat{F}1$, $\Delta\hat{F}4$	0.920	0.911	0.993	1.012	$1.007^{\dagger}$
$\Delta\hat{F}1$, $\Delta\hat{F}5$	0.927	0.871	0.959	0.974	$1.009^{\dagger}$
$\Delta\hat{F}1$, $\Delta\hat{F}4$, $\Delta\hat{F}5$	0.911	0.894	1.028	1.023	1.007

Factors	$j=1$	$j=3$	$j=6$	$j=9$	$j=12$
Rolling Window Method: RRMSPE vs. Autoregressive					
$\Delta\hat{F}1$	0.926	0.875	0.931	0.975	$1.014^{\ddagger}$
$\Delta\hat{F}4$	0.988	1.009^{*}	0.995	$1.076^{\ddagger}$	0.996
$\Delta\hat{F}5$	0.995	0.932	1.015^{*}	0.960	0.987
$\Delta\hat{F}1$, $\Delta\hat{F}4$	0.896	0.891	0.950	$1.053^{\ddagger}$	$1.015^{\dagger}$
$\Delta\hat{F}1$, $\Delta\hat{F}5$	0.923	0.852	0.961	0.950	$1.001^{\dagger}$
$\Delta\hat{F}1$, $\Delta\hat{F}4$, $\Delta\hat{F}5$	0.894	0.874	1.003^{*}	$1.048^{\ddagger}$	$1.008^{\dagger}$

Note: *RRMSPE* denotes the mean square error from the random walk model relative to the mean square error from our factor model. Therefore, when *RRMSPE* is greater than one, our factor models perform better than the benchmark model.