Auburn University
Department of Economics
Working Paper Series

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AUWP 2016-05

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March 28, 2016

Abstract

This study shows how the two alternative saving motives, life-cycle consumption smoothing and parental bequests, determine the relation between population growth and R&D-based economic growth, i.e. the sign of the "weak scale-effect". We take a textbook R&D-based growth model of infinitely living agents with no weak-scale effect, and analyze it in an Overlapping Generations framework - with and without bequest saving-motive. We show how the different saving motives determine the relation between population growth and per-capita income growth, which proves to be ambiguous in general, and may also be non-monotonic. Hence, we conclude that the counterfactual weak-scale effect that is present in the second and third generations of R&D-based growth models of infinitely-living agents depends on their specific demographic structure, and thus is not inherent to R&D-based growth theory itself.

JEL Classification: : O-31, O-40

Key-words: R&D-based Growth, Weak Scale Effect, Overlapping Generations

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†We thank Pietro Peretto for helpful comments.

1 Introduction

The second and third generations of R&D-based growth models were criticized for presenting a counterfactual weak scale-effect, which is a positive relation between population growth and economic prosperity[1]. A recent line of research, aimed to align the theory with empirical evidence, has proposed several modifications that yield ambiguous and non-monotonic weak-scale effect. A common element in these modified models is the introduction of human capital as a productive input in the R&D sector; See for example Dalgaard and Kreiner (2001), Strulik (2005), Bucci (2008, 2015), and Diwakar and Sorek (2016a,b).

Following the seminal works of Romer (1990), Grossman and Helpman (1991a,b) and Aghion and Howitt (1992), this literature has focused, almost exclusively, on the analysis of infinitely living homogenous agents[2]. Recent exceptions are the works by Strulik et al.(2013) and Prettner (2014), which studies the relation between population growth and innovation based prosperity in the Overlapping Generations (OLG) framework, with finitely living agents[3].

In this work we show that the different demographic structures of the two macroeconomic workhorse frameworks has, by itself, immediate implication to the presence of weak scale effect in R&D growth models. To this end, we take a standard textbook R&D-growth model, which was written for infinitely living agents with no human capital accumulation, and place it in the OLG framework to derive comparable results[4].

The two canonical demographic structures generate different incentives for saving. The infinitely living agents are assumed to share their assets (patent ownership in the current context and physical capital in the neoclassical models) with their offspring, and they fully internalize this into their saving decisions. Therefore, in this framework savings involve bequests, but they have no life-cycle consideration motive as labor supply is constant over life[5]. By contrast, in the standard OLG framework saving is aimed to smooth consumption through a finite lifetime which spans over working years and retirement period, and there are no intergenerational bequests. Hence, in this framework saving is motivated purely by life-cycle considerations.

We find that in the absence of bequest saving-motive, the effect of population growth on economic growth depends solely on the inter-temporal elasticity of substitution (IES): it is positive

[1] Seminal models of the second and third generations models are Jones 1995, Kortum 1997, and Segerstrom 1998, and Peretto 1998, Young 1998 and Howitt 1999, respectively. Jones 1999 provides a compact comparative summary of this literature. See Strulik et al. (2013) and Boikos et al.(2013) for recent summaries of the empirical literature.

[2] Which built on the neoclassical framework of Ramsey–Cass–Koopmans.

[3] Building on Diamond's (1965) neoclassical framework, Prettner (2014) shows that the relation between fertility rate and economic growth may depend on the provision of public education: teachers' productivity in the sector and per-student spending. Strulik et al.(2013) developed a unified growth model that incorporates endogenous fertility and transition from neoclassical technology to R&D-based growth.

[4] Earlier literature already showed that the different demographic structures has immediate implications to other key issues in neoclassical growth models - such as tax-policy, convergence patterns, and the feasibility of growth itself. Dalgaard and Jensen (2009, p.1639) summarize this literature. Sorek (2011) highlights the implications of the OLG demographic structure to patent policy.

[5] The infinitely living agents can be thought equivalently, and more realistically, as finitely living ones with strong altruism toward their offspring.

(negative or zero) if the *IES* is greater than (lower than or equal to) one[6]. In comparison, the corresponding model with infinitely living agents yields no-relation between population growth and economic growth regardless of the *IES* value. In both models, population growth increases future demand for patented machines, thereby increasing the equilibrium interest rate . However, for the infinitely living agents population growth works also as a demographic discounting factor which discourages saving[7], and thus the two effects cancel out. In the OLG economy, population growth does not generate direct negative effect on saving and, due to the life-cycle structure of the OLG framework, the effect of the increased interest rate on saving depends on the *IES*.

Once we introduce bequest-motive for saving, the effect of population growth on economic growth becomes complex, and possibly non-monotonic. We show how it is determined by both the *IES* and the specification of parental utility from bequest-giving - namely, the interaction between the level of bequest per-child and the number of kids in parents' utility. Parents may care only about their per-child bequest giving, in line with Millian specification. However, their utility from the level of per-child bequest may also increase or decrease with the number of children, consistently with Benthamite specification and the formulation emphasized by Barro and Becker (1989) and Becker et al.(1990), respectively. The latter specification forms a trade-off between the number of kids and per-child bequest level in parent's total utility from giving. We will show that this specification is crucial to the effect of population growth on economic prosperity.

Previous works showed that in R&D-based models with human capital accumulation and infinitely living agents, dynastic altruism affects the sign and strength of weak scale effect, based on the Millian and Benthamite specifications; See Dalgaard and Kreiner (2001), Strulik (2005), and Bucci (2013). In Diwakar and Sorek (2016b) we generalize these results by allowing non-linear altruism factor in the Barro-Becker (1989) fashion to establish non-monotonic weak scale effect. There however, the mechanism at work is different from the one studied here: it involves the tension between a positive effect of population growth on saving in the presence of dynastic altruism and its negative (diluting) effect on the accumulation of human capital.

Our study is closely related to the work by Dalgaard and Jensen (2009), hereafter "*DJ*", on the effect of alternative saving motives on the presence of strong scale effect - that is the effect of population *size* on economic growth. They showed that population size has positive effect on growth when the bequest motive is dominant but it may turn negative when the life-cycle motive dominates, and may be non-monotonic as well. Their work adds bequest saving-motive to an otherwise standard OLG model with capital externalities, and derives comparative statics with respect to the values of bequest motive parameter.

Our analysis follows a similar methodological approach to study the effect of alternative saving motives on presence of weak scale-effect, yet departing from *DJ* along two lines. First, we study a full-fledged textbook model of R&D-based growth, and analyze it in the OLG framework, to derive comparable results with the infinitely-living-agents framework.

[6] The empirical literature suggests that the IES is lower than one; See Hall (1988), Beaudry and Wincoop (1996), Ogaki and Reinhart (1998), Engelhardt and Kumar (2009).

[7] Following the standard Euler condition $\frac{\dot{c}}{c} = \frac{1}{\theta}(r - \rho - n)$

Second, and most importantly, as population in our analysis is constantly growing, we are required to address the interaction between the level of bequest per-child and the number of kids in parents' utility. This specification, which is not relevant for *DJ*'s analysis of strong scale effect, proves to be crucial to the relation between population growth and economic growth. Nonetheless, our results confirm that the different demographic structures of the two macroeconomic workhorse models have immediate implications to the role of population in modern growth theory.

The remainder of the paper is organized as follows. Section 2 presents the detailed model. Section 3 studies weak-scale effects with life-cycle saving only. Section 4 introduces the bequest motive for saving, and Section 5 concludes this study.

2 The Model

We take the variety-expansion growth model presented in the textbook of Barro and Sala-I-Martin (2004, ch.6)[8], hereafter "*BS*", and accommodate it to the OLG framework. Hence, preferences and technologies presented below, and the implied static optimization problems of the firms are identical to those presented in *BS*. However, unlike *BS* who study the infinitely living agents, we analyze the OLG demographic setup: each consumer lives for two periods. In the first period she supplies one unit of labor and in the second period she retires. Cohort (generation) size is increasing at an exogenous constant rate n, which is also the growth rate of the labor force and overall population.

2.1 Production and Innovation

The final good Y is produced by perfectly competitive firms with labor and differentiated intermediate goods, to which we refer as "machines"

$$Y_t = AL_t^{1-\alpha} \int_0^{N_t} K_{i,t}^{\alpha}\,\mathrm{d}i \quad \alpha \in (0,1) \tag{1}$$

where A is a productivity factor, L_t and $K_{i,t}$ are labor input and the utilization level of machine i in period t, respectively, and N_t measures the number of available machine varieties[9]. Once invented, machines variety is eternally patented. Machines fully depreciate after one usage period, and the final good price is normalized to one. Under symmetric equilibrium, utilization level for all machines is the same, i.e. $K_{i,t} = K_t \ \forall\ i$, and thus total output is

$$Y_t = AN_t {K_t}^{\alpha} L_t^{1-\alpha} \tag{1a}$$

The labor market is perfectly competitive, and therefore the equilibrium wage and aggregate labor income are $w_t = A(1-\alpha)N_t {K_t}^{\alpha} L_t^{-\alpha}$ and $w_t L_t = A(1-\alpha)N_t {K_t}^{\alpha} L_t^{1-\alpha}$, respectively. The profit for

[8] Aghion and Howitt (2009) use the same model in Chapter 3.4 of their textbook.
[9] The elasticity of substitution between different varieties is $\frac{1}{\alpha}$.

the final good producer is $\pi_{i,t} = AL_t^{1-\alpha} \int_0^{N_t} K_{i,t}^{\alpha} \, \mathrm{d}i - \int_{i=1}^{N_t} p_{i,t} K_{i,t} \, \mathrm{d}i - w_t L_t$, where $p_{i,t}$ is the price of intermediate good i. Profit maximization yields the demand for each machine: $K_{i,t}^d = A^{\frac{1}{1-\alpha}} L_t \left(\frac{\alpha}{p_{i,t}}\right)^{\frac{1}{1-\alpha}}$. Given this demand, the periodic surplus for the patented machine i is $PS_{i,t} = [p_{i,t} - (1+r_t)] K_{i,t}^d$, which is maximized by the standard monopolistic price $p_{i,t} = \frac{1+r_t}{\alpha} \forall i, t$[10]. Plugging this price in $K_{i,t}^d$ and then back in (1a) yields the following expression for total output

$$Y_t = A^{\frac{1}{1-\alpha}} \left(\frac{\alpha^2}{1+r_t}\right)^{\frac{\alpha}{1-\alpha}} N_t L_t \tag{1b}$$

where $y_t \equiv \frac{Y_t}{L_t}$. Innovation technology follows the specification of BS in the analysis of scale effect[11], where the cost of innovating a new variety, denoted η_t, is

$$\eta_t = \eta A^{\frac{1}{1-\alpha}} \left(\frac{\alpha^2}{1+r_t}\right)^{\frac{\alpha}{1-\alpha}} L_t \tag{2}$$

Where $\eta > 0$ is a cost parameter. New and old varieties play equivalent role in production as, reflected in their symmetric presentation in (1). Therefore the market value of old varieties equals the cost of inventing a new one - η_t. As we assume machine-varieties are patented forever, patents are being traded inter-generationally - young buy patents from old. Hence the return on patent ownership - over old and new technologies is $1 + r_{t+1} = \frac{PS_{i,t+1} + \eta_{t+1}}{\eta_t}$. Plugging the explicit expressions for the surplus and the innovation cost and imposing stationary interest rate, we obtain[12]

$$1 + r = (1+n)\left[\frac{\alpha(1-\alpha)}{\eta} + 1\right], \forall t \tag{3}$$

Following (1b), output growth rate and the per-capita output growth[13], denoted g_Y and g_y respectively, depend on the expansion rate of machine-varieties range, denoted g_N

$$\begin{aligned} 1 + g_{Y,t+1} &\equiv \frac{Y_{t+1}}{Y_t} = (1+n)(1+g_{N,t+1}) \\ 1 + g_{y,t+1} &\equiv \frac{y_{t+1}}{y_t} = 1 + g_{N,t+1} \end{aligned} \tag{4}$$

[10] BS abstract from the timing of investment, setting the cost of each machines (in terms of output units) to one, and therefore having the optimal monopolistic price $p = \frac{1}{\alpha}$ (equations 6.9-6.10 on pp. 291-292 there). In their continuous time framework this abstraction has no effect on any of the results (in our framework this abstraction would have a quantitative effect on our results).

[11] See Chapter 6.1.7 on the analysis of scale effect and population growth (p.302 there). Equation (2) implies that variety expansion rate, which defines productivity growth in this model, depends positively on the share of output devoted to R&D. This specification aligns the model with the empirical data summarized in that chapter, which were originally pointed out by Jones (1995).

[12] Our results would hold if we assume that patents ownership is transferred freely from parents to off-spring, like in the model with infinitely living agents. Then, however, the interest rate would be $1 + r = \frac{(1+n)\alpha(1-\alpha)}{\eta}$, which corresponds to the one presented in BS (adjusted for continuous time).

[13] Notice that total population and the labor force grow at the same rate, implying equal growth rates for per-worker output and per-capita output.

2.2 Preferences

Lifetime utility is derived from consumption over two periods, based on the *CRRA* instantaneous-utility specification

$$U = \frac{c_t^{1-\theta}}{1-\theta} + \rho \frac{c_{t+1}^{1-\theta}}{1-\theta} \tag{5}$$

where $\rho \in (0,1)$ is the subjective discount factor, and $\frac{1}{\theta}$ is the elasticity of inter-temporal substitution. Young agents allocate their labor income between consumption and saving, denoted s. The solution for the standard optimal saving problem is $s_t = \frac{w_t}{1+\rho^{-\frac{1}{\theta}}(1+r)^{\frac{\theta-1}{\theta}}}$. Hence, aggregate saving is $S_t = \frac{w_t L_t}{1+\rho^{-\frac{1}{\theta}}(1+r)^{\frac{\theta-1}{\theta}}}$, which after substituting the explicit expressions for w_t becomes

$$S_t = \frac{N_t(1-\alpha)A^{\frac{1}{1-\alpha}}\left(\frac{\alpha^2}{1+r}\right)^{\frac{\alpha}{1-\alpha}} L_t}{1+\rho^{-\frac{1}{\theta}}(1+r)^{1-\frac{1}{\theta}}} \tag{6}$$

3 Life-Cycle Saving

The saving from labor income in (6) are allocated to three types of investment: buying patents over old technologies, inventing new varieties, and forming specialized machines. Hence aggregate investment in each period, I_t , satisfies

$$I_t = N_{t+1}\left[\eta_t + A^{\frac{1}{1-\alpha}} L_{t+1}\left(\frac{\alpha^2}{1+r}\right)^{\frac{1}{1-\alpha}}\right] \tag{7}$$

Notice that higher population growth rate between period t and $t+1$, has direct positive effect on the demand for each machine variety - due to the increase in L. However, following (3), a higher population growth rate also increases the interest rate, which thereby increases machines price and therefore decreases the demand for each machine variety. By equalizing (6) and (7), we impose the equilibrium condition $I_t = S_t$, to obtain the dynamic equation that governs variety expansion rate:

$$\frac{N_{t+1}}{N_t} \equiv 1 + g_N = 1 + g_y = \frac{(1-\alpha)A^{\frac{1}{1-\alpha}}\left(\frac{\alpha^2}{1+r}\right)^{\frac{\alpha}{1-\alpha}} L_t}{\left[\eta_t + A^{\frac{1}{1-\alpha}} L_{t+1}\left(\frac{\alpha^2}{1+r}\right)^{\frac{1}{1-\alpha}}\right]\left[1+\rho^{-\frac{1}{\theta}}(1+r)^{1-\frac{1}{\theta}}\right]} \tag{8}$$

Plugging (2) and (3) in (8) yields

$$1 + g_y = \frac{\left(\frac{\alpha(1-\alpha)}{\eta} + 1\right)(1-\alpha)}{(\alpha+\eta)\left[1+\rho^{-\frac{1}{\theta}}\left[\frac{\alpha(1-\alpha)(1+n)}{\eta}\right]^{1-\frac{1}{\theta}}\right]} \tag{8a}$$

Proposition 1 *With no bequest motive the effect of population growth on per-capita output growth depends on the sign of* $1-\theta$*.*

Proof. Proof is in equation (8a). ■

In the corresponding model of infinitely living agents, presented in *BS*, aggregate consumption growth follows the standard Euler equation[14]: $\frac{\dot{C}}{C} = \frac{1}{\theta}(r-\rho)$, and per-capita consumption follows $\frac{\dot{c}}{c} = \frac{1}{\theta}(r-\rho-n)$ where the interest rate is given by[15] $r = n + \frac{\alpha(1-\alpha)}{\eta}$. Hence, the stationary growth rates for aggregate and per-capita consumption, which apply also for aggregate and per-capita income are

$$g_{C,Y} = \frac{1}{\theta}\left[n + \frac{\alpha(1-\alpha)}{\eta} - \rho\right], \quad g_{c,y} = \frac{1}{\theta}\left[\frac{\alpha(1-\alpha)}{\eta} - \rho\right]$$

Hence, in the corresponding model of infinitely-living agents population growth has no effect on per-capita output growth. *DJ* showed that, in the absence of bequest, the sign of the strong scale effect in their model depends not only on the *IES* but also on technological parameters (see Theorem 1 there). For the corresponding technological parameters we are employing here - unit elasticity of substitution between machines and labor - their model economy presents strong scale effect for $IES = 1$ (see discussion of Corollary 1 on p.1643 there), whereas we find no weak-scale effect for our model.

4 Bequests

We introduce bequest motive for saving that resembles a joy-of-giving in consumers' preferences, similar to *DJ*

$$u(c_t, c_{t+1}, b_t) = \frac{\left(w_t + \frac{b_{t-1}}{1+n} - s_t\right)^{1-\theta}}{1-\theta} + \rho\left[\frac{[s_t(1+r) - b_t]^{1-\theta}}{1-\theta} + \kappa(1+\varphi(n))\frac{\left(\frac{b_t}{1+n}\right)^{1-\theta}}{1-\theta}\right] \tag{9}$$

where b_t is the total bequest left by a representative parent in period t. The parameter $\kappa \geq 0$ measures the weight placed on utility from bequest. Our formulation departs from *DJ* by the term $\varphi(n)$, which captures potential interaction between the number of kids and the utility from per-child bequest-giving level. At this point we do not define exact specification for $\varphi(n)$, but we will further discuss below, following Proposition 1.

Differentiating (9) with respect to s and b we obtain the following first order conditions

$$s_t = \frac{w_t + \frac{b_{t-1}}{1+n}}{\frac{\rho^{-\frac{1}{\theta}}(1+r)^{\frac{\theta-1}{\theta}}}{1+(1+n)^{\frac{\theta-1}{\theta}}[\kappa(1+\varphi(n))]^{\frac{1}{\theta}}} + 1}, \quad b_t = s_t\frac{1+r}{\frac{(1+n)^{\frac{1-\theta}{\theta}}}{[\kappa(1+\varphi(n))]^{\frac{1}{\theta}}} + 1} \tag{10}$$

[14] Equation (6.22) on p.295 there.
[15] Equation (6.35) on p. 302 there.

The first condition in (10) implies that aggregate savings is given by

$$S_t = \frac{(1-\alpha)A^{\frac{1}{1-\alpha}}N_t\left(\frac{\alpha^2}{1+r}\right)^{\frac{\alpha}{1-\alpha}}L_t + B_{t-1}}{\frac{\rho^{-\frac{1}{\theta}}(1+r)^{\frac{\theta-1}{\theta}}}{1+(1+n)^{\frac{\theta-1}{\theta}}[\kappa(1+\varphi(n))]^{\frac{1}{\theta}}}+1} \tag{11}$$

Where $B_{t-1} = \frac{L_t b_{t-1}}{1+n}$, is aggregate bequests given to workers who were born in period t. Notice that for $\kappa = 0$ the aggregate saving level defined in (11) falls back to the one presented in (6). The denominator of (11) reveals the way bequest motive interacts with the weak scale effect in determining aggregate saving and thereby economic growth: the term $(1+n)^{\frac{\theta-1}{\theta}}$ marks the diluting effect of n on per-child bequest level. The sign of this effect on saving, just like the interest rate, depends on θ, i.e. the *IES*. The term $[\kappa(1+\varphi(n))]^{\frac{1}{\theta}}$ marks the strength of the saving motive for per-child bequest-giving. The impact of n on this term depends on the sign of $\varphi'(n)$.

The second condition in (10) implies that $B_{t-1} = \frac{1+r}{\left(\frac{(1+n)^{\frac{1-\theta}{\theta}}}{[\kappa(1+\varphi(n))]^{\frac{1}{\theta}}}+1\right)}S_{t-1}$, and the equilibrium condition $S_{t-1} = I_{t-1}$ requires

$$B_{t-1} = \frac{1+r}{\left(\frac{(1+n)^{\frac{1-\theta}{\theta}}}{[\kappa(1+\varphi(n))]^{\frac{1}{\theta}}}+1\right)}N_t\left(\eta_{t-1} + A^{\frac{1}{1-\alpha}}L_t\alpha^{\frac{2}{1-\alpha}}\right)$$

Substituting the latter expression along with (3) back into (11) and equalizing to (7), i.e. setting $S_t = I_t$, we obtain

$$\frac{N_{t+1}}{N_t} = \frac{\left[\frac{(1-\alpha)}{\alpha+\eta}\left[(1+n)^{\frac{1-\theta}{\theta}} + [\kappa(1+\varphi(n))]^{\frac{1}{\theta}}\right] + [\kappa(1+\varphi(n))]^{\frac{1}{\theta}}\right]\left[\frac{\alpha(1-\alpha)}{\eta}+1\right]}{\rho^{-\frac{1}{\theta}}\left[\frac{\alpha(1-\alpha)}{\eta}+1\right]^{\frac{\theta-1}{\theta}} + (1+n)^{\frac{1-\theta}{\theta}} + [\kappa(1+\varphi(n))]^{\frac{1}{\theta}}} \tag{12}$$

Rearranging (12) yields

$$\frac{N_{t+1}}{N_t} = \frac{\left[\frac{(1-\alpha)}{\alpha+\eta} + \frac{1}{1+\psi}\right]\left[\frac{\alpha(1-\alpha)}{\eta}+1\right]}{\frac{\rho^{-\frac{1}{\theta}}(1+n)^{\frac{\theta-1}{\theta}}\left[\frac{\alpha(1-\alpha)}{\eta}+1\right]^{\frac{\theta-1}{\theta}}}{1+\psi^{-1}}+1} \tag{12a}$$

Where $\psi \equiv \frac{(1+n)^{\frac{1-\theta}{\theta}}}{[\kappa(1+\varphi(n))]^{\frac{1}{\theta}}}$. The growth rate defined in (12a) presents complex impact of the population growth rate, which works through the bequest motive that is captured in ψ and the interest rate effect presented in the denominator of (12a). The sign of the interest-rate effect depends solely on the IES, i.e. θ, as defined in Proposition 1. The sign of the bequest motive effect, i.e. the sign of $\frac{\partial\psi}{\partial n}$, depends on the sign of $(1-\theta)(1+n)^{-1} - \varphi'(n)(1+\varphi(n))^{-1}$, which is a function of n. Hence (12a) implies ambiguous effect of population growth on per-capita output growth that may be non-monotonic. To further characterize the relation between population growth and output growth, we focus first on the case $\theta = 1$ for which (12) becomes

$$\frac{N_{t+1}}{N_t} = \frac{1}{\frac{1}{\rho[1+\kappa(1+\varphi(n))]}+1}\left[\frac{(1-\alpha)}{\alpha+\eta}+\frac{1}{\frac{1}{\kappa(1+\varphi(n))}+1}\right]\left[\frac{\alpha(1-\alpha)}{\eta}+1\right] \tag{13}$$

Proposition 2 *In the presence of bequest saving-motive, for $\theta = 1$ the effect of population growth on per-capita income growth, $\frac{\partial g_y}{\partial n}$, depends solely on the sign of $\varphi'(n)$.*

Proof. Differentiating (13) for n reveals that sign of $\frac{\partial g_y}{\partial n}$ is given by the sign of $-\varphi'(n)(1+\varphi(n))^{-1}$. ■

Hence, if $\theta = 1$ and parent care about per-child bequest only, i.e. $\varphi(n) = 0$, there is no weak scale effect that is $\frac{\partial g_y}{\partial n} = 0$. These parental preferences are in line with the Millian preference specification employed in *BS* and *DJ*. By comparison, in *BS* there is no weak scale effect for any value of θ, and *DJ* find that for $\theta = 1$ there is strong scale effect for the technological parameters used in our model[16]. For the case $\varphi'(n) > 0$ (< 0), which is in line with the Benthamite ("Beckerian"[17]) specification, Proposition 2 implies positive (negative) weak scale effect.

We turn now to further explore the case where parents care about per-child bequest giving $\varphi(n) = 0$, for which equation (12) becomes

$$\frac{N_{t+1}}{N_t} = \frac{\left[\frac{1-\alpha}{\alpha+\eta}\left(\kappa^{-\frac{1}{\theta}}(1+n)^{\frac{1-\theta}{\theta}}+1\right)+1\right]\left[\frac{\alpha(1-\alpha)}{\eta}+1\right]}{\rho^{-\frac{1}{\theta}}\kappa^{-\frac{1}{\theta}}\left[\frac{\alpha(1-\alpha)}{\eta}+1\right]^{\frac{\theta-1}{\theta}}+\kappa^{-\frac{1}{\theta}}(1+n)^{\frac{1-\theta}{\theta}}+1} \tag{14}$$

Proposition 3 *For $\varphi(n) = 0$, the effect of population growth on per-capita output growth, $\frac{\partial g_y}{\partial n}$, is positive (negative) for sufficiently weak (strong) bequest motive.*

Proof. Differentiating (14) for n reveals that, for $\theta < 1$ ($\theta > 1$), $\frac{\partial g_y}{\partial n} > 0$ if $\rho^{-1}\left(\frac{1-\alpha}{\alpha+\eta}\right)^{\theta}\left[\frac{\alpha(1-\alpha)}{\eta}+1\right]^{\theta-1} > \kappa$ ($\rho^{-1}\left(\frac{1-\alpha}{\alpha+\eta}\right)^{\theta}\left[\frac{\alpha(1-\alpha)}{\eta}+1\right]^{\theta-1} < \kappa$). for $\theta = 1$, the sign of $\frac{\partial g_y}{\partial n}$ is independent of κ as already stated in Proposition 2. ■

By comparison, *DJ* find that sufficiently *strong* bequest motive is necessary for the presence of strong scale effect in their model, where the bequest motive is defined solely by the parameter κ, as here.

5 Conclusions

This study shows how the two alternative saving motives - life-cycle considerations and intergenerational bequests, determine the relation between population growth and economic prosperity. First,

[16]See Corollary 2 on p. 1643 there for $\sigma = 1$ (by their notation), which is the elasticity of substitution between labor and capital in our model.
[17]Presented in Barro and Becker (1988) and Becker et al.(1990).

we showed that in the standard OLG economy, where life-cycle considerations are the sole saving motive, the effect of population growth on economic growth depends on the *IES*, whereas in our reference textbook model of infinitely living agents there is no weak scale effect for any value of the *IES*.

Acknowledging that both modelling approaches are unrealistically extreme, we also analyzed a hybrid model with both bequest and life-cycle saving motives. In this case the relation between population growth and economic growth is complex in general and may be non-monotonic, as it depends on the exact specification of the bequest motive.

In particular, for the more empirically valid case, of low inter-temporal elasticity of substitution, we find that the effect of population growth on economic growth depends on how parent utility from per-child bequest level changes with the number of children. This property of parental utility has already been identified as central in other contexts of the literature on economic growth.

Our results are in line with Dalgaard Jensen (2009) who showed that strong scale effect depend on the saving motive in a model where growth is driven by capital externalities. We conclude that the role of population in modern growth theory interacts with the assumed intergenerational links and the demographic structure of the model economy. Hence, the counterfactual relation between population and output growth rates that present in models of infinitely living agents depends on their specific demographic structure, and thus is not inherent to R&D-based growth theory itself.

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