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Economic Development and Stage-Dependent IPR

Bharat Diwakar and Gilad Sorek

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Bharat Diwakar and Gilad Sorek*

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Abstract

We study growth-maximizing Intellectual Property Rights (IPR) policy for developing economy in a close Overlapping-Generations model. We first show that R&D-based growth in such economy is subject to threshold externalities and transitional dynamics. Then we show that the IPR policy that maximizes output growth rates is stage-dependent: in early phases of development weak IPR protection may be necessary to sustain and to fasten economic growth. This is because weaker IPR protection shifts income from the old to the young generation and thereby enhancing saving and investment, which otherwise are insufficient to initiate growth. However as the economy develops and growth sustains optimal IPR protection tightens.

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*Economics Depratment, Auburn University, Auburn Alabama.
Emails: bzd0013@tigermail.auburn.edu, gms0014@auburn.edu

1 Introduction

The strength of intellectual property rights (IPR) is positively correlated with economies' development-stage worldwide (See Eicher and Newiak 2013, and Chu et al. 2014). This empirical observation may suggest that developing countries fail to efficiently strengthen domestic IPR due to political (institutional) shortcomings, or that the optimal strength of IPR is positively related to the stage of economic development. In this work we present a normative case for tightening IPR over the course of economic development. We show that such stage dependent IPR policy maximizes growth along the transitional dynamics of a close Overlapping-Generations (OLG) economy. Moreover, weak IPR protection at early development phases may be necessary to initiate growth.

The demographic structure of the OLG economy highlights the role of IPR policy in shaping the inter-generational income distribution, thereby effecting innovation pace and output growth. Within each period weaker IPR over patented technologies, owned by the old, increase the income of the young thereby spurring saving and investment. This is a positive static effect of weak IPR on innovation. In the dynamic context, however, weak IPR shift the allocation of investment away from innovation activity toward the formation of physical capital. Hence weak IPR induce also a conventional negative dynamic effect on innovation and growth. We will show that during early development stages the static effect dominates the total impact of IPR on growth, but its relative importance diminishes as the economy develops and converges to its long run growth rate.

Jones and Manuelli (1992) showed that in Diamond's (1965) economy of finitely living agents growth is bounded by the ability of the young generation to purchase capital held by the old: saving out of labor income always falls short of the investment that is required to sustain long-run growth. One of the remedies they consider to support sustained growth in such economy is direct income transfers from old to young[1]. As explained above, in our work weak IPR serve the same purpose during early development stages - that is allocating income from the old patent holder to the young worker. In our model economy however the need for stimulating labor income to sustain growth vanishes as the economy takes off and converges to its long run growth rate.

Jones and Manuelli (1992) also showed that OLG model with capital externalities can present sustained growth, subject to threshold externalities in the spirit of Azariadis and Drazen (1990). That is a minimal level of initial capital stock is required to induce perpetual growth. In our endogenous R&D economy these threshold externalities are due to strategic complementarities in innovation, presented in the "Big-Push" theory by Murphy et al.(1989), or the knowledge spillover presented in Romer (1990). We will show that weak IPR in early development stage may be necessary for utilizing these externalities and initiating growth.

There is a large body of theoretical research on IPR policy, innovation and economic growth for close economies; See among others Li (2001), O'Donoghue and Zweimuller (2004), Furukawa (2007), Acemoglu and Akcigit (2012) Cysne and Turchick (2012) and Iwaisako and Futagami (2007,2013).

[1] Similarly, Uhlig and Yanagawa (1996) showed that reliance on capital-income taxation can enhance growth.

However this literature applies almost exclusively to economies with infinitely living agents, following the canonical frameworks of Romer (1990) Grossman and Helpman (1991) and Aghion and Howitt (1992). These close-economy models commonly lack transitional dynamics. Hence this literature has focused on the long-run IPR policy - along the balanced growth path, and so did the few works that studied IPR policy for OLG economies, as Chou and Shy (1992) and Sorek (2011)[2].

In a recent innovative study, Chu et al.(2014) present a model of stage-dependent optimal IPR policy for an open economy. The main message from their analysis is that weak IPR is optimal for open developing economies, because it enables imitation of foreign innovations. However as the developing economy catches up with the global technological frontier imitation opportunities are exhausted. Then stronger IPR become optimal as to support growth that is based on domestic innovation. Chu et al. (2013) provide evidence that China's IPR policy over the last few decades has followed such path. Our work proposes a complementary case for stage-dependent IPR policy - for a close OLG economy.

The remainder of the paper is organized as follows: Section 2 develops a the OLG model of product innovation. Section 3 presents the dynamics of innovation and growth in the model economy. Section 4 analyzes optimal IPR policy, and Section 5 concludes this study.

2 The Model

Our basic setup modifies Diamond's (1965) canonical OLG framework to incorporate variety expansion of specialized physical capital ("machines") in the fashion of Romer (1990). Population of constant size composes two overlapping generations in each and every period - "young" and "old". Each agent is endowed with one unit of labor to be supplied inelastically when young. Old agents retire to consume their saving. Up to Section 4 we assume complete IPR protection so innovators can charge the unconstrained optimal monopolistic price.

2.1 Production and Innovation

Final output that can be used for consumption and investment, is produced by perfectly competitive firms with labor and intermediate goods - i.e. "machines" - subject to the CRTS technology

$$Y_t = \left[\left(\int_0^{N_t} k_{i,t}^{\frac{1}{\varepsilon}} \, \mathrm{d}i \right)^{\varepsilon} \right]^{\alpha} L_t^{1-\alpha}, \quad \alpha \in (0,1) \tag{1}$$

where Y_t is the final good, L_t is the labor input, $k_{i,t}$ is the quantity of machine i used in the final good production, and N_t is the measure of machine varieties. We normalize the price of final output and each generation size to one. Hence aggregate measures coincide with per-worker measures.

[2] A related strand of literature studies the implications of IPR to international trade between economies at different development stages, in the "North-South" fashion of Helpman (1993). See Chu et al. (2013) for a recent compact summary of this literature.

The cost of producing each machine is one final output unit, and machines fully depreciate after one period. The parameter ε measures of substitutability of different machine varieties, where the elasticity of substitution is $s = \frac{\varepsilon}{\varepsilon-1} > 1$. When ε approaches 1 all machines are perfect substitutes and (1) falls back to the neoclassical form of homogenous capital $Y_t = K_t^{\alpha} L_t^{1-\alpha}$.

Following Romer (1987, 1990), the current literature confined attention to the case $\varepsilon = \frac{1}{\alpha}$, for which (1) simplifies to $Y_t = \left(\int_0^{N_t} k_{i,t}^{\alpha} \, \mathrm{d}i \right) L_t^{1-\alpha}$.[3] This assumption implies that different intermediate varieties are not direct substitutes nor complements in production, whereas for $\varepsilon > \frac{1}{\alpha}$ ($\varepsilon < \frac{1}{\alpha}$) different machines are complements (substitutes)[4]. Under symmetric equilibrium $k_{i,t} = k_t \; \forall i$ and thus

$$Y_t = N_t^{\varepsilon\alpha} {k_t}^{\alpha} \tag{1a}$$

We assume perfectly competitive labor market hence the wage equals the marginal product of labor

$$w_t = (1-\alpha) N_t^{\varepsilon\alpha} {k_t}^{\alpha} \tag{2}$$

Final good producers set demand for each machine variety as to maximize profit

$$\max_{k_{i,t}} : \pi_{i,t} = \left(\int_{i=1}^{N_t} k_{i,t}^{\frac{1}{\varepsilon}} \, \mathrm{d}i \right)^{\varepsilon\alpha} - \int_{i=1}^{N_t} p_{i,t} k_{i,t} \, \mathrm{d}i$$

where $p_{i,t}$ is the price of machine i. The first order condition for maximal profit yields the following demand for each machine variety, denoted k^d

$$k_{i,t}^d = \left[\frac{\alpha}{p_{i,t}} \left(\int_{i=1}^{N_t} k_{i,t}^{\frac{1}{\varepsilon}} \, \mathrm{d}i \right)^{\varepsilon\alpha - 1} \right]^{\frac{\varepsilon}{\varepsilon-1}} \tag{3}$$

Under symmetric equilibrium (3) simplifies to

$$k_t^d = \left(\frac{\alpha N_t^{\varepsilon\alpha-1}}{p_t} \right)^{\frac{1}{1-\alpha}} \tag{3a}$$

Equations (3)-(3a) imply that for the commonly assumed case $\varepsilon = \frac{1}{\alpha}$ demand for each variety is independent of the variety span N, whereas for $\varepsilon > \frac{1}{\alpha}$ demand for each variety increase with variety span. These variety complementarities are equivalent to strategic complementarities presented in Murphy et al.(1989) and are particular example of the capital externalities discussed by Jones and Manuelli (1992).

[3] Barro and Sala-I-Martin (2004) and Aghion and Howitt (2009) employ this exact specification for infinitely living agents, in their textbook chapters on variety expansion models - Chapters 6.1 and 3.4, respectively.

[4] Romer (1990) considered the possibility of complementarity as potential extension to his analysis, but left it for future research (p.81).

The services of specialized machines are rented under patent protection. Given the demand (3) the price of each machine is set to maximize the profit function

$$\max_{p_{i,t}} \Pi_{i,t} = k^d_{i,t}(p_{i,t} - 1) = \left[\frac{\alpha}{p_{i,t}} \left(\int_{i=1}^{N_t} k_{i,t}^{\frac{1}{\varepsilon}} \, \mathrm{d}i \right)^{\varepsilon\alpha - 1} \right]^{\frac{\varepsilon}{\varepsilon - 1}} (p_{i,t} - 1)$$

The first order condition for this maximization problem implies the standard monopolistic pricing $p_{i,t} = \varepsilon$. Plugging this price back in (3a) and then substituting (3a) into (1a) we obtain the following expression for total output, showing that output growth is determined by the rate of variety expansion

$$Y_t = \left(\frac{\alpha}{\varepsilon}\right)^{\frac{\alpha}{1-\alpha}} N_t^{\frac{\alpha(\varepsilon-1)}{1-\alpha}} \tag{4}$$

The innovation technology follows the conventional specification

$$\Delta N_t = (N_{t+1} - N_t) = \delta R_t N_t^{\gamma} \tag{5}$$

where R is R&D investment in new varieties. Each new variety is immediately granted with eternal patent. Equation (5) implies that the cost of a new variety, denoted ϕ_t is

$$\phi_t \equiv \frac{R_t}{(N_{t+1} - N_t)} = \frac{1}{\delta N_t^{\gamma}} \tag{5a}$$

Where $\gamma \in [0, 1]$. For $\gamma = 0$ this cost is time invariant, and for positive (negative) γ the per-variety innovation cost is declining (increasing) with economic development level, implying dynamic knowledge spillover ("fishing out" effect)[5]. We will show that such knowledge spillover induce the same dynamic properties as the strategic complementarities (i.e. $\varepsilon > \frac{1}{\alpha}$), and that both imply stage-dependent IPR.

2.2 Preferences

Lifetime utility from consumption over two periods follows the tractable logarithmic formulation[6]

$$u(c_1, c_2) = \ln c_1 + \beta \ln c_2 \tag{6}$$

where $\beta \in (0, 1)$ is the subjective discount factor. Young agents allocate their labor income between consumption and saving, denoted s. The solution for optimal saving in this standard problem is $s_t = \frac{\beta}{1+\beta} w_t$. Substituting the explicit expression for w_t from (2) we obtain

$$S_t = \frac{\beta}{1+\beta}(1-\alpha) N_t^{\frac{\alpha(\varepsilon-1)}{1-\alpha}} \left(\frac{\alpha}{\varepsilon}\right)^{\frac{\alpha}{1-\alpha}} \tag{7}$$

[5] See Jones (1999) for discussion on the alternative fomulations of the innovation cost function in the literature.
[6] In Appendix C we consider the more general CRRA instantaneous utility function.

3 Equilibrium and Growth Dynamics

The savings from labor income in (7) are allocated to three types of investment: buying patents over old technologies, inventing new varieties and physical capital (i.e. specialized machines). As new and old varieties play equivalent role in production the market value of old variety equals the cost of inventing a new one - ϕ_t. The market for specialized machines clears as the supply of each variety equals the demand in (3a). Hence, investments in each period -denoted I - satisfies

$$I_t = (N_t + \Delta N_t)\,\phi_t + (N_t + \Delta N_t)k^d_{t+1} = N_{t+1}\left[\phi_t + \left(\frac{\alpha N_{t+1}^{\varepsilon\alpha-1}}{\varepsilon}\right)^{\frac{1}{1-\alpha}}\right] \tag{8}$$

Equalizing (8) to (7) we impose the resources-uses equilibrium condition $I_t = S_t$, to obtain the dynamic equation which governs the economies' growth rate

$$N_{t+1}\left[\phi_t + \left(\frac{\alpha N_{t+1}^{\varepsilon\alpha-1}}{\varepsilon}\right)^{\frac{1}{1-\alpha}}\right] = \frac{\beta}{1+\beta}(1-\alpha)N_t^{\frac{\alpha(\varepsilon-1)}{1-\alpha}}\left(\frac{\alpha}{\varepsilon}\right)^{\frac{\alpha}{1-\alpha}} \tag{9}$$

To enhance tractability, from this point on we will present the effects of strategic complementarities and knowledge spillover separately, focusing on knowledge spillover first.

3.1 Knowledge Spillover

Focusing on knowledge spillover first we assume $\gamma > 0$ and $\varepsilon = \frac{1}{\alpha}$. Applying these assumption to (9) and rearranging yields the following expression for the rate of variety expansion, which defines the output growth rate g

$$1 + g_{t+1} = \frac{N_{t+1}}{N_t} = \frac{\frac{\beta}{1+\beta}(1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}}}{\frac{1}{\delta N_t^{\gamma}} + \alpha^{\frac{2}{1-\alpha}}} \tag{10}$$

Let us also define the long run growth rate $\widetilde{g} \equiv \lim\limits_{t\to\infty} g_t$

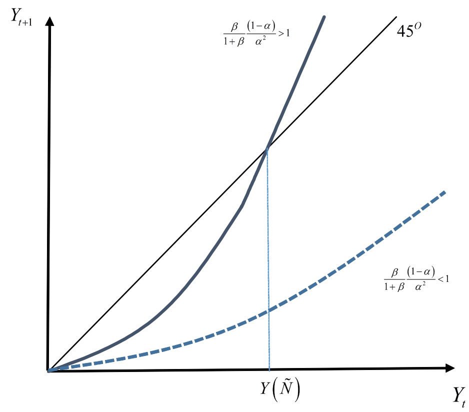
Proposition 1 *For* $\varepsilon = \frac{1}{\alpha}$, $\gamma > 0$ *and* $\frac{\beta}{1+\beta}\frac{(1-\alpha)}{\alpha^2} > 1$, $\exists N_0$ *such that* $\forall t > 0$:$g_{t+1} > 0$ *and* $\widetilde{g} \equiv \lim\limits_{t\to\infty} g_t > 0$ *, If an initial variety threshold level is reached there exist transitional dynamics to a sustainable finite long run growth rate.*

Proof. For $\varepsilon = \frac{1}{\alpha}$ and $\gamma > 0$, under sustained growth the right side of (10) is monotonically decreasing with N_t and $\widetilde{g} = \frac{\beta}{1+\beta}\frac{(1-\alpha)}{\alpha^2}$. Long term growth cannot sustain if $\gamma < 0$, and is constant for $\gamma = 0$: $(1+\widetilde{g}) = \frac{\frac{\beta}{1+\beta}(1-\alpha)(\alpha)^{\frac{2\alpha}{1-\alpha}}}{\frac{1}{\delta}+(\alpha)^{\frac{2}{1-\alpha}}}$, implying no transitional dynamics and development stages.

(10) implies also that $\frac{N_{t+1}}{N_t} > 1 \Rightarrow g_{t+1} > 0$ iff $N_t^{\gamma} > \frac{1}{\delta\alpha^{\frac{2}{1-\alpha}}\left[\frac{\beta}{1+\beta}\frac{(1-\alpha)}{\alpha^2}-1\right]} \equiv \widetilde{N}^{\gamma}$ ■

In Diwakar and Sorek (2015) we study the implications of population growth on output growth abstracting from IPR policy. We show there that as long as population grows faster than innovation cost long run growth can be sustained. Diagram 1 illustrates the threshold output level that is required to initiate perpetual growth.

Diagram 1: Threshold level for sustained growth



For $\frac{\beta}{1+\beta}\frac{(1-\alpha)}{\alpha^2} < 1$ the economy is subject to the dynamic path of the dashed curve and cannot initiate positive output growth. For $\frac{\beta}{1+\beta}\frac{(1-\alpha)}{\alpha^2} > 1$ the economy follows the dynamic path of the solid curve. If $N_0 < \widetilde{N}$ the economy is to the left of $Y\left(\widetilde{N}\right)$ and is degenerating under a negative growth rate. For $\widetilde{N}_0^{\gamma} = \widetilde{N}$ the economy is stagnated, and for $\widetilde{N}_0^{\gamma} > \widetilde{N}$ the economy is on a sustained growth path that is converging asymptotically to $1+\widetilde{g} = \frac{\beta}{1+\beta}\frac{(1-\alpha)}{\alpha^2}$.

3.2 Strategic Complementarities

Focusing on strategic complementarities in innovation we turn here to the case $\gamma = 0$ and $\varepsilon > \frac{1}{\alpha}$, under which the growth rate equation (10) modifies to

$$(1+g)^{\frac{1-\alpha}{\alpha(\varepsilon-1)}} = \frac{N_{t+1}}{N_t} = \frac{\frac{\beta}{1+\beta}(1-\alpha)\left(\frac{\alpha}{\varepsilon}\right)^{\frac{\alpha}{1-\alpha}}}{\phi N_{t+1}^{\frac{1-\alpha\varepsilon}{1-\alpha}} + \left(\frac{\alpha}{\varepsilon}\right)^{\frac{1}{1-\alpha}}} \tag{11}$$

Proposition 2 *For $\gamma = 0$ there exist transitional dynamics to sustainable long run growth only if $\varepsilon > \frac{1}{\alpha}$ and $\frac{\beta}{1+\beta}\frac{(1-\alpha)}{\alpha}\varepsilon > 1$.*

Proof. For $\varepsilon < \frac{1}{\alpha}$ the right side in (11) is decreasing monotonically with N_{t+1} and approaching zero contradicting with $1+g > 1$. For $\varepsilon > \frac{1}{\alpha}$ the right side of (11) is increasing monotonically with N_{t+1} and approaching $\frac{\beta}{1+\beta}\frac{(1-\alpha)}{\alpha}\varepsilon$. In the limit case $\varepsilon = \frac{1}{\alpha}$ the growth rate is constant and equals $\frac{\frac{\beta}{1+\beta}(1-\alpha)(\alpha^2)^{\frac{\alpha}{1-\alpha}}}{\phi+(\alpha^2)^{\frac{1}{1-\alpha}}}$ ■

Equation (11) implies also that for $\varepsilon > \frac{1}{\alpha}$ the threshold variety -level, denoted $\widetilde{N}$, that is required to initiate sustained growth is

$$\widetilde{N} > \left[\frac{\phi}{\left(\frac{\alpha}{\varepsilon}\right)^{\frac{1}{1-\alpha}} \left[\frac{\beta}{1+\beta}(1-\alpha)\frac{\varepsilon}{\alpha} - 1\right]} \right]^{\frac{1-\alpha}{\alpha\varepsilon-1}} \tag{12}$$

If investment cannot support the threshold variety-level (12) the economy will fail to induce growth and will degenerate. However, Iwaisako (2002) shows that reaching this investment capacity does not guarantee that private investors will choose to adopt the new progressive technology. Appendix A illustrates the nature of threshold externalities in the broader context of transforming from neoclassical technology of capital accumulation and long-term stagnation to innovation activity and R&D-based growth. Next we turn to study growth maximizing IPR policies.

4 Intellectual Property Rights

We model IPR policy with the parameter λ, which defines the ability of patent holders to charge the unconstraint optimal monopolistic price: $p(\lambda) = \lambda p^* = \lambda\varepsilon$ where $\lambda \in \left(\frac{1}{\varepsilon}, 1\right)$ hence $p(\lambda) \in (1, \varepsilon)$. One can think of $p(\lambda)$ as the maximal price a patent holder can set and still deter competition by imitators. Weaker IPR protection lowers the cost of imitation, thereby imposing a lower deterrence price on patent holders. This is equivalent to regulating the price of the patentee monopolist. When $\lambda = 1$ IPR are perfectly enforced and innovators can charge the unconstraint optimal price $p = \varepsilon$. This case was analyzed in Section 3. With zero protection $\lambda = \frac{1}{\varepsilon}$ patent holders are losing their market power completely, therefore selling at marginal cost price.

Plugging $p(\lambda) = \lambda\varepsilon$ in (3a) shows how demand for each variety is decreasing with IPR due to higher price: $k_t = \left[\frac{\alpha N_t^{\varepsilon\alpha-1}}{\lambda\varepsilon}\right]^{\frac{1}{1-\alpha}}$. Modifying (7) for the IPR-dependent pricing shows that weaker IPR increase labor income and saving for a given variety span

$$s_t = \frac{\beta}{1+\beta}(1-\alpha)N_t^{\frac{\alpha(\varepsilon-1)}{1-\alpha}} \left(\frac{\alpha}{\lambda_t\varepsilon}\right)^{\frac{\alpha}{1-\alpha}} \tag{13}$$

Modifying (8) correspondingly shows that weaker IPR also shifts investment from patents to physical capital

$$I_t = N_{t+1}\phi_t + N_{t+1}k_{t+1}^d = \phi_t N_{t+1} + N_{t+1}^{\frac{\alpha(\varepsilon-1)}{1-\alpha}} \left(\frac{\alpha}{\lambda_{t+1}\varepsilon}\right)^{\frac{1}{1-\alpha}} \tag{13a}$$

The complete analysis of optimal IPR policy that follows, focuses on the case of knowledge spillover. Equalizing (13) and (13a) with $\varepsilon = \frac{1}{\alpha}$ and $\gamma > 0$ yields the output growth rate

$$(1+g_{t+1}) = \frac{\frac{\beta}{1+\beta}(1-\alpha)\left(\frac{\alpha^2}{\lambda_t}\right)^{\frac{\alpha}{1-\alpha}}}{\frac{1}{\delta N_t^{\gamma}} + \left(\frac{\alpha^2}{\lambda_{t+1}}\right)^{\frac{1}{1-\alpha}}} \tag{14}$$

Equation (14) presents a form of the conventional trade-off between static and dynamic efficiency faced by the IPR policy maker. The expression in the numerator is decreasing with IPR protection, through boosting aggregate saving by (13) for a given variety level N_t. This is the negative static effect of IPR protection on growth rate. It implies that in a one shot game current IPR protection should be zero, $\lambda_t = 0$. However the denominator in (14) is also decreasing with λ_{t+1} through the positive effect of IPR protection on investment in variety expansion, by (13a). This is the positive dynamic effect of IPR on growth. Let us denote λ^*_{LR} the stationary IPR policy which maximizes growth rate(14). Observe that the relative effect of λ_{t+1} in the denominator on the growth rate, is increasing with variety span (compared with the effect of λ_t in the numerator).

Proposition 3 $\lambda^*_{LR} = 1$, *long run growth is maximized with full IPR protection.*

Proof. The long run growth rate (14) approaches $\frac{\beta}{1+\beta}\frac{1-\alpha}{\alpha^2}\lambda$, that it is increasing with λ, hence $\lambda^* = 1$. ■

Proposition 4 *For* $\frac{\beta}{1+\beta}\frac{(1-\alpha)}{\alpha} > 1$ *and* $g_0 = 0$ *, Incomplete IPR protection is necessary to initiate growth.*

Proof. Suppose the economy is initially stagnated under complete IPR protection. Hence $\forall t$: $\lambda = 1$ and $g = 0$, which by (14) implies that (a) $\frac{\beta}{1+\beta}(1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}} = \frac{1}{\delta \tilde{N}^{\gamma}} + \alpha^{\frac{2}{1-\alpha}}$, and (b) $\frac{\partial(1+g)}{\partial \lambda} < 0$ if $\frac{1}{\delta \tilde{N}^{\gamma}} + \alpha^{\frac{2}{1-\alpha}} > \alpha^{\frac{2}{1-\alpha}-1}$. Conditions (a) and (b) both hold if $\frac{\beta}{1+\beta}(1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}} > \alpha^{\frac{2}{1-\alpha}-1} \Rightarrow \frac{\beta}{1+\beta}\frac{(1-\alpha)}{\alpha} > 1$ ■

Next we turn to explore IPR policy that maximizes growth along the transitional dynamics, denoted λ^*_t. It is easy to verify that there is no stationary optimal IPR policy. To see that assume $\lambda^*_t = \lambda^*_{t+1} = \lambda^*$, and differentiate (14) to obtain $\lambda^* = \left[\left(\alpha^{\frac{1+\alpha}{1-\alpha}} - \alpha^{\frac{2}{1-\alpha}}\right)\delta N_t^{\gamma}\right]^{1-\alpha}$. As the right side of the latter expression depends on N_t the optimal IPR policy cannot be stationary and independent of the stage of economic development N_t. Consider now instead a policy maker that maximizes output over time with perfect foresight and full commitment, by setting periodic IPR policy. The objective function $\underset{\lambda_t}{Max} : \sum_{t=0}^{\infty} Y_t$ is subject to the initial condition : $Y_0 = \left(\frac{\alpha^2}{\lambda}\right)^{\frac{\alpha}{1-\alpha}} N_0^{\frac{\alpha(\varepsilon-1)}{1-\alpha}}$, and the dynamic constraint (14) : $N_{t+1} = N_t \frac{\frac{\beta}{1+\beta}(1-\alpha)\left(\frac{\alpha}{\lambda_t \varepsilon}\right)^{\frac{\alpha}{1-\alpha}}}{\frac{1}{\delta N_t^{\gamma}} + \left(\frac{\alpha}{\lambda_{t+1}\varepsilon}\right)^{\frac{1}{1-\alpha}}}$.

Appendix B shows that first order condition for this maximization problem yields the following rule for optimal IPR policy denoted ψ

$$\psi_{t+1} = \lambda^*_{t+1} = \alpha \frac{\left(\frac{\alpha^2}{\lambda^*_{t+1}}\right)^{\frac{\alpha}{1-\alpha}}}{\frac{1}{\delta N_t^{\gamma}} + \left(\frac{\alpha^2}{\lambda^*_{t+1}}\right)^{\frac{1}{1-\alpha}}} \tag{15}$$

Equation (15) is an implicit function of the optimal IPR policy λ^*_{t+1}.

Proposition 5 *$\exists \lambda^*_{t+1} \in (\alpha, 1)$ where $\frac{\partial \lambda^*_{t+1}}{\partial N_t} > 0$. There exists an economic development phase along which optimal IPR protection is increasing over time.*

Proof. The right side of (15), $\psi_{t+1} = \alpha \frac{\left(\frac{\alpha^2}{\lambda^*_{t+1}}\right)^{\frac{\alpha}{1-\alpha}}}{\frac{1}{\delta N_t^{\gamma}} + \left(\frac{\alpha^2}{\lambda^*_{t+1}}\right)^{\frac{1}{1-\alpha}}}$, is a continuous function of λ^*_{t+1} that is increasing (decreasing) for $\frac{1}{\delta N_t^{\gamma}} < \frac{\alpha^{\frac{1+\alpha}{1-\alpha}} - \alpha^{\frac{2}{1-\alpha}}}{\lambda_{t+1}^{\frac{1}{1-\alpha}}}$ ($\frac{1}{\delta N_t^{\gamma}} > \frac{\alpha^{\frac{1+\alpha}{1-\alpha}} - \alpha^{\frac{2}{1-\alpha}}}{\lambda_{t+1}^{\frac{1}{1-\alpha}}}$). Define $\bar{\epsilon} \equiv (\alpha)^{\frac{\alpha}{1-\alpha}} (1-\alpha)$. For $\frac{1}{\delta N_t^{\gamma}} \in \left(\alpha^{\frac{1}{1-\alpha}} \bar{\epsilon}, \bar{\epsilon}\right)$ we get $\psi_{t+1}\left(\lambda^*_{t+1} = \alpha\right) > \alpha$ and $\frac{\partial \psi_{t+1}}{\partial \lambda^*_{t+1}}|_{\lambda^*_{t+1}=\alpha} > 0$, and $\psi\left(\lambda_{t+1} = 1\right) < 1$ and $\frac{\partial \psi_{t+1}}{\partial \lambda^*_{t+1}}|_{\lambda^*_{t+1}=1} < 0$. Hence by the fix point theorem $\frac{1}{\delta N_t^{\gamma}} \in \left(\alpha^{\frac{1}{1-\alpha}} \bar{\epsilon}, \bar{\epsilon}\right)$: $\exists \lambda^*_{t+1} \in (\alpha, 1)$. Hence there is a corresponding range of variety span for which optimal policy is incomplete IPR. As $\frac{\partial \psi_{t+1}}{\partial N_t} > 0$, within this range optimal IPR is increasing over time - with variety expansion ■

Figure 2: Optimal incomplete IPR

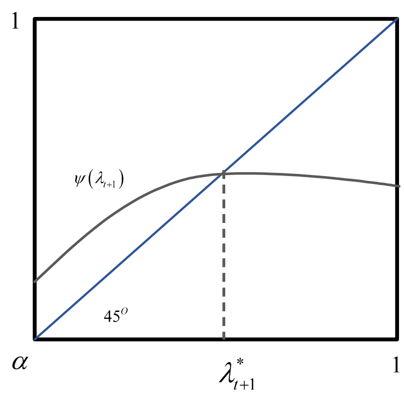


Figure 2 illustrates the optimal policy $\lambda^* \in (\alpha, 1)$ for $\frac{1}{\delta N_t^{\gamma}} \in \left(\alpha^{\frac{1}{1-\alpha}} \bar{\epsilon}, \bar{\epsilon}\right)$, defined by the fix point that solves (15). The 45^o line marks the left side of (15) and the curved one, denoted $\psi\left(\lambda_{t+1}\right)$, marks the right side of (15). For $\frac{1}{\delta N_t^{\gamma}} \in \left(\alpha^{\frac{1}{1-\alpha}} \bar{\epsilon}, \bar{\epsilon}\right)$ we have $\psi\left(\lambda_{t+1} = \alpha\right) > \alpha$ and $\frac{\partial \psi_{t+1}}{\partial \lambda^*_{t+1}}|_{\lambda^*_{t+1}=\alpha} > 0$, and $\psi\left(\lambda_{t+1} = 1\right) < 1$ and $\frac{\partial \psi_{t+1}}{\partial \lambda^*_{t+1}}|_{\lambda^*_{t+1}=\alpha} < 0$. Within the range $\left(\alpha^{\frac{1}{1-\alpha}} \bar{\epsilon}, \bar{\epsilon}\right)$ the $\psi\left(\lambda_{t+1}\right)$ curve is shifting upward as variety span N increases, thereby shifting the intersection point that defines λ^* to the right. For $\frac{1}{\delta N_t^{\gamma}} < \alpha^{\frac{1}{1-\alpha}} \bar{\epsilon}$ ($\frac{1}{\delta N_t^{\gamma}} > \bar{\epsilon}$) the $\psi\left(\lambda_{t+1}\right)$ curve is entirely above (below) the 45^o line, implying corner solutions.

Under strategic complementarities, with $\varepsilon > \frac{1}{\alpha}$ and $\gamma = 0$, the periodic growth rate as a function of IPR policy is given by

$$(1+g)^{\frac{1-\alpha}{\alpha(\varepsilon-1)}} = \frac{N_{t+1}}{N_t} = \frac{\frac{\beta}{1+\beta}(1-\alpha)\left(\frac{\alpha}{\lambda_t \varepsilon}\right)^{\frac{\alpha}{1-\alpha}}}{\phi N_{t+1}^{\frac{1-\varepsilon\alpha}{1-\alpha}} + \left(\frac{\alpha}{\lambda_{t+1}\varepsilon}\right)^{\frac{1}{1-\alpha}}} \tag{16}$$

The growth rate under strategic externalities in (16) is very similar to the one obtained in (14) under knowledge externalities. The main difference is that on the right side of (16) we have N_{t+1} whereas on the right side of (14) we have N_t. Hence the explicit analysis of optimal patent policy becomes less tractable.

5 Concluding remarks

In this work we studied optimal IPR policy for a close OLG economy of R&D-based growth. We have shown that this economy presents threshold externalities, which are conventional to the literature on economic growth and development. The threshold externalities studied here were due to dynamic knowledge spillover or strategic complementarities in innovation. Once the economy surpasses the threshold level of investment needed to initiate sustainable growth, it follows transitional dynamics that resemble economic-development process that converges to a steady long run growth rate.

We have characterized a stage-dependent IPR policy that maximizes output growth, as it tightens IPR protection along the transitional dynamics of economic development. Moreover, we have shown that weak IPR protection may be necessary for the economy to initiate growth.

The demographic structure of the model (OLG) economy highlights the differential effect of IPR policy across different age groups, generating a unique form of static-dynamic trade-off faced by the policy-maker. As patent ownership is concentrated by the old, weaker IPR protection shifts income from old to young saving workers thereby boosting aggregate investment.

However, at the same time it also shifts the allocation of investment away from R&D activity toward physical capital. We have shown that the relative impact of these two contradicting effects changes along the course of economic development in favor of tighten IPR protection.

Our theoretical results provide a novel normative explanation for the observed positive correlation between IPR and economic development, across states and over time. This explanation is consistent with the inter-generational transfers proposed by Jones and Manuelli (1992), and with the actual IPR policy implemented in developing economies as China over the last few decades. It is complementary to the one proposed by Chu et al. (2014) for open developing economy.

Note that the interests of young and old generations are conflicting with respect to optimal patent policy in each period. For a given N_t the old generation in period t gains higher returns on investment the higher λ_t is. The young generation of period t gains a higher labor income the lower λ_t is. Hence if the policy reflects some electoral political-equilibrium economies with relatively young population are likely to implement weaker patent policy and demographic ageing will lead to implementing stronger IPR protection.

Appendix A: Threshold extenalites and technological transition

Suppose the economy first uses the neoclassical-"traditional" - technology with homogeneous capital $Y_t = AK_t^{\alpha} L_t^{1-\alpha}$. While using this technology the economy accumulates physical capital subject to the standard saving condition $s_t = (1-\alpha)\frac{\beta}{1+\beta} AK_t^{\alpha}$.

Before the economy can initiate R&D-based growth by adopting the progressive technology (1), it must be able to support an investment level that satisfies the minimal variety span (12). Equalizing the latter saving expression to (8) yields the following necessary condition that is required for the transition from the traditional technology to the progressive one and initiate sustainable growth

$$K_t^{\alpha} \geq \frac{\widetilde{N}\left[\phi + \left(\frac{\alpha \widetilde{N}^{\varepsilon\alpha - 1}}{\varepsilon}\right)^{\frac{1}{1-\alpha}}\right]}{(1-\alpha)\frac{\beta}{1+\beta} A}$$

If long-run (stationary) capital level with the traditional technology, given by $\widetilde{K} = \left[(1-\alpha)\frac{\beta}{1+\beta} A\right]^{\frac{1}{1-\alpha}}$, is lower than the right side of (12), the economy will be locked in a poverty trap of long-run stagnation.

Diagram A: Conditional transition from traditional technology to R&D-based growth

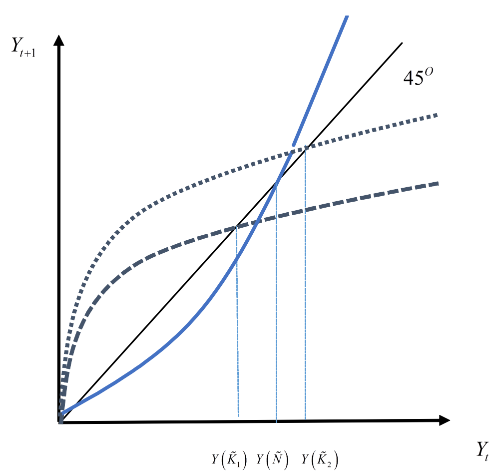


Figure A illustrates the conditional transition from the neoclassical homogenous capital technology to the R&D based technology of specialized capital. The two concave curves mark neoclassical convergence paths for two traditional economies that may differ in their productivity parameter $A_2 > A_1$. The (weakly) convex curve marks the R&D-based growth dynamics presented in Diagram 1. Only with the traditional technology that is marked by the higher concave curve will reach the threshold level required for transition to R&D-based perpetual growth. This is a necessary condition for transition. However, Iwaisako (2002) shows that reaching the investment capacity does not guarantee that private investors will choose to adopt the new progressive technology.

Appendix B: Optimal IPR policy

The policy maker discounts future output by the factor $\rho \in [0,1]$. For non-negative growth rates the expression to be maximized $\underset{\lambda_t}{Max} : \sum_{t=0}^{\infty} \rho^t Y_t$ is infinite. As the long run growth rate is finite and constant the expression approaches a geometric sequence. Equation (10) implies that output maximization coincides with the variety maximization $\underset{\lambda_t}{Max} : \sum_{t=0}^{\infty} \rho^t N_t$. To illustrate the properties of this expression we apply the dynamic constraint (14) to look at its the first three addends following period t, which enables focusing on optimizing λ_{t+1} given N_t and λ_t

$$N_{t+1} = \rho N_t \frac{\frac{\beta}{1+\beta}(1-\alpha)\left(\frac{\alpha^2}{\lambda_t}\right)^{\frac{\alpha}{1-\alpha}}}{\frac{1}{\delta N_t^{\gamma}}+\left(\frac{\alpha^2}{\lambda_{t+1}}\right)^{\frac{1}{1-\alpha}}}$$

$$N_{t+2} = \rho^2 N_t \frac{\frac{\beta}{1+\beta}(1-\alpha)\left(\frac{\alpha^2}{\lambda_t}\right)^{\frac{\alpha}{1-\alpha}}}{\frac{1}{\delta N_t^{\gamma}}+\left(\frac{\alpha^2}{\lambda_{t+1}}\right)^{\frac{1}{1-\alpha}}} \frac{\frac{\beta}{1+\beta}(1-\alpha)\left(\frac{\alpha^2}{\lambda_{t+1}}\right)^{\frac{\alpha}{1-\alpha}}}{\frac{1}{\delta N_{t+1}^{\gamma}}+\left(\frac{\alpha^2}{\lambda_{t+2}}\right)^{\frac{1}{1-\alpha}}}$$

$$N_{t+3} = \rho^3 N_t \frac{\frac{\beta}{1+\beta}(1-\alpha)\left(\frac{\alpha^2}{\lambda_t}\right)^{\frac{\alpha}{1-\alpha}}}{\frac{1}{\delta N_t^{\gamma}}+\left(\frac{\alpha^2}{\lambda_{t+1}}\right)^{\frac{1}{1-\alpha}}} \frac{\frac{\beta}{1+\beta}(1-\alpha)\left(\frac{\alpha^2}{\lambda_{t+1}}\right)^{\frac{\alpha}{1-\alpha}}}{\frac{1}{\delta N_{t+1}^{\gamma}}+\left(\frac{\alpha^2}{\lambda_{t+2}}\right)^{\frac{1}{1-\alpha}}} \frac{\frac{\beta}{1+\beta}(1-\alpha)\left(\frac{\alpha^2}{\lambda_{t+2}}\right)^{\frac{\alpha}{1-\alpha}}}{\frac{1}{\delta N_{t+2}^{\gamma}}+\left(\frac{\alpha^2}{\lambda_{t+3}}\right)^{\frac{1}{1-\alpha}}}$$

Observe that all addends are products of $N_t \frac{\frac{\beta}{1+\beta}(1-\alpha)\left(\frac{\alpha^{\alpha}}{\lambda_t \varepsilon}\right)^{\frac{\alpha}{1-\alpha}}}{\frac{1}{\delta N_t^{\gamma}}+\left(\frac{\alpha}{\lambda_{t+1}\varepsilon}\right)^{\frac{1}{1-\alpha}}}$ and all addends from the second one and on are also products of $\left(\frac{\alpha}{\lambda_{t+1}\varepsilon}\right)^{\frac{\alpha}{1-\alpha}}$. Hence $\sum_{t}^{\infty} \rho^t N_t$ can be written as

$$\rho N_t \frac{\beta(1-\alpha)}{1+\beta}\left(\frac{\alpha^2}{\lambda_t}\right)^{\frac{\alpha}{1-\alpha}} \left\{ \frac{1}{\frac{1}{\delta N_t^{\gamma}}+\left(\frac{\alpha^2}{\lambda_{t+1}}\right)^{\frac{1}{1-\alpha}}} \left[1 + \left(\frac{\alpha^2}{\lambda_{t+1}}\right)^{\frac{\alpha}{1-\alpha}} \underbrace{\left[\begin{array}{c} \frac{\rho\frac{\beta}{1+\beta}(1-\alpha)}{\frac{1}{\delta N_{t+1}^{\gamma}}+\left(\frac{\alpha^2}{\lambda_{t+2}}\right)^{\frac{1}{1-\alpha}}} + \\ \frac{\rho^2\left(\frac{\beta}{1+\beta}\right)^2(1-\alpha)^2\left(\frac{\alpha^2}{\lambda_{t+1}}\right)^{\frac{\alpha}{1-\alpha}}}{\frac{1}{\delta N_{t+1}^{\gamma}}+\left(\frac{\alpha^2}{\lambda_{t+2}}\right)^{\frac{1}{1-\alpha}}} \frac{\left(\frac{\alpha^2}{\lambda_{t+2}}\right)^{\frac{\alpha}{1-\alpha}}}{\frac{1}{\delta N_{t+2}^{\gamma}}+\left(\frac{\alpha^2}{\lambda_{t+3}}\right)^{\frac{1}{1-\alpha}}} + \\ +.... \end{array} \right]}_{\equiv W} \right] \right\}$$

Note that the element W has infinite number of addends, but none of them is a function of λ_{t+1}. Maximizing the above expression for λ_{t+1} yields the following first order condition

$$\lambda_{t+1}^{*} = \frac{\alpha\left[\frac{1}{W}+\left(\frac{\alpha^2}{\lambda_{t+1}^{*}}\right)^{\frac{\alpha}{1-\alpha}}\right]}{\frac{1}{\delta N_t^{\gamma}}+\left(\frac{\alpha^2}{\lambda_{t+1}^{*}}\right)^{\frac{1}{1-\alpha}}}$$

Having $\rho \cdot (1+\widetilde{g}) = \rho\frac{\beta}{1+\beta}\frac{1-\alpha}{\alpha^2} > 1$ implies $W \to \infty$ hence the above expression approaches (15). This is surely the case for $\rho = 1$ assumed for (15).

Appendix C: CRRA utility function

Here we consider a deviation from logarithmic utility function to general CRRA formulation

$$U(c_t, c_{t+1}) = \frac{c_t^{1-\theta}}{1-\theta} + \beta \frac{c_{t+1}^{1-\theta}}{1-\theta}$$

where $\frac{1}{\theta}$ is the inter-temporal elasticity of substitution ($\theta > 0$). Hence the indirect lifetime utility function is

$$U(c_t, c_{t+1}) = \frac{(w_t - s_t)^{1-\theta}}{1-\theta} + \beta \frac{[s_t(1+r_{t+1})]^{1-\theta}}{1-\theta}$$

and optimal saving is given by

$$s_t = \frac{w_t}{1 + \beta^{-\frac{1}{\theta}} (1+r_{t+1})^{\frac{\theta-1}{\theta}}}$$

Free entry to innovation activity implies the following expression for the interest rate

$$(1+r_{t+1}) = \frac{(\lambda_{t+1}\varepsilon - 1)\, N_{t+1}^{\frac{(\varepsilon-1)}{1-\alpha}} \left(\frac{\alpha}{\lambda_{t+1}\varepsilon}\right)^{\frac{1}{1-\alpha}} + \phi_{t+1}}{\phi_t + N_{t+1}^{\frac{(\varepsilon-1)}{1-\alpha}} \left(\frac{\alpha}{\lambda_{t+1}\varepsilon}\right)^{\frac{1}{1-\alpha}}}$$

Hence under either strategic externalities or knowledge externalities (or both) the long term interest rate approaches the mark-up term $(\lambda\varepsilon - 1)$. Equalizing aggregate saving to aggregate investment yields the growth rate

$$(1+g_{t+1})^{\frac{\alpha(\varepsilon-1)}{1-\alpha}} = \frac{(1-\alpha)(\frac{\alpha}{\lambda_t \varepsilon})^{\frac{\alpha}{1-\alpha}}}{\left[\phi_t N_{t+1}^{\frac{1-\varepsilon\alpha}{1-\alpha}} + (\frac{\alpha}{\lambda_{t+1}\varepsilon})^{\frac{1}{1-\alpha}}\right] \left[1 + \beta^{-\frac{1}{\theta}} \left(\frac{(\lambda_{t+1}\varepsilon-1) N_{t+1}^{\frac{(\varepsilon-1)}{1-\alpha}} \left(\frac{\alpha}{\lambda_{t+1}\varepsilon}\right)^{\frac{1}{1-\alpha}} + \phi_{t+1}}{\phi_t + N_{t+1}^{\frac{(\varepsilon-1)}{1-\alpha}} \left(\frac{\alpha}{\lambda_{t+1}\varepsilon}\right)^{\frac{1}{1-\alpha}}}\right)^{\frac{\theta-1}{\theta}}\right]}$$

The analysis of the transitional dynamics is clearly more complicated here, as it should account for the effect IPR on the interest rate and thereby on growth rate. The effect of the interest rate on growth rate however depends on the preference parameter θ. The long term growth is

$$(1+g)^{\frac{\alpha(\varepsilon-1)}{1-\alpha}} = \frac{(1-\alpha)\frac{\lambda\varepsilon}{\alpha}}{1 + \beta^{-\frac{1}{\theta}} (\lambda\varepsilon - 1)^{\frac{\theta-1}{\theta}}}$$

For $\theta \leq 1$ complete IPR protection is the optimal policy for the long run. However for $\theta > 1$, the IPR protection parameter λ has contradicting effect on the above expression, due to the negative effect of the interest rate on saving (as the inter-temporal elasticity of substitution in this case is lower than one).

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