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## ABSTRACT

This paper examines the economic effects of forward guidance using a New Keynesian model with a zero lower bound (ZLB) constraint on the short-term nominal interest rate. Forward guidance is modeled with anticipated news shocks to the monetary policy rule. There are five key findings: (1) the stimulative effect of forward guidance falls as the economy deteriorates or as households expect a slower recovery because there is a smaller margin to lower expected policy rates; (2) longer forward guidance horizons do not generate increasingly larger impact effects on output when the total amount of news is fixed, unlike with an exogenous interest rate peg; (3) in steady state, an unanticipated shock has a larger impact effect on output than a news shock, but a news shock has a larger cumulative effect in every state of the economy; (4) at the ZLB, the cumulative effect on output from lengthening the forward guidance horizon increases over short horizons but decreases thereafter, which indicates the central bank faces limits on how far forward guidance can extend into the future and continue to add stimulus; and (5) forward guidance is stimulative in the absence of other shocks, but the observed effect on output is smaller or even negative if another shock simultaneously reduces demand.

*Keywords*: Monetary Policy; Forward Guidance; Zero Lower Bound; News Shocks
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# 1 INTRODUCTION

The global economic slowdown in 2008 led many central banks to sharply reduce their policy rates. When rates could not be reduced further, some central banks resorted to unconventional policies, such as forward guidance. Forward guidance refers to central bank communication about future policy, which has many forms including announcements about objectives, contingencies, policy actions, and speeches. This paper focuses on communication about the path of future policy rates.

We examine the effects of forward guidance using a New Keynesian model with a zero lower bound (ZLB) constraint on current and future policy rates. Forward guidance is modeled with anticipated news shocks to the monetary policy rule similar to Laséen and Svensson (2011), so news about future policy rates affects expectations in the same way as news about future technology.[1] The central bank implements forward guidance by communicating the news over a specific forward guidance horizon. The news is the difference between the expected policy rates before and after the central bank's announcement. News that the central bank intends to keep future policy rates lower than previously expected generates higher inflation, lowers real interest rates, and raises output.

We show how the ZLB constraint, the state of the economy, the size of the news shocks, the speed of the recovery, and the forward guidance horizon impact the efficacy of central bank forward guidance. Throughout our analysis, the total weight on the news shocks is held constant to isolate the effect of a longer horizon from a larger monetary policy shock. There are five key findings:

1. The stimulative effect of forward guidance falls as the economy deteriorates or as households expect a slower recovery because there is a smaller margin to lower expected policy rates.
2. Longer forward guidance horizons do not generate increasingly larger impact effects on output when the total amount of news is fixed, unlike with an exogenous interest rate peg.
3. In steady state, an unanticipated shock has a larger impact effect on output than a news shock, but a news shock has a larger cumulative effect in every state of the economy.
4. At the ZLB, the cumulative effect on output from lengthening the forward guidance horizon increases over short horizons but decreases thereafter, which indicates the central bank faces limits on how far forward guidance can extend into the future and continue to add stimulus.
5. Forward guidance is stimulative in the absence of other shocks, but the observed effect on output is smaller or even negative if another shock simultaneously reduces demand.

We use our results to interpret the effects of recent forward guidance by the Fed. Those policies likely had a limited effect on the economy for two main reasons. One, they only modestly flattened the yield curve because prior expectations of a weak economy gave policymakers a small margin to lower expected nominal interest rates. Two, the Fed's forward guidance was often accompanied by weak economic assessments, which caused consensus forecasts of output and inflation to decline.

To our knowledge, this paper is the first to study forward guidance with news shocks using a global solution method. This solution method enhances our analysis of forward guidance in several ways. One, it enables ZLB events to endogenously reoccur, which impacts households' expectations of future policy rates and the central bank's ability to provide economic stimulus. Two, we can assess the impact of forward guidance at the ZLB, near the ZLB, or at any other state of the economy. Three, it allows us to evaluate forward guidance in a setting where changes

[1] Gomes et al. (2013) and Milani and Treadwell (2012) estimate unconstrained New Keynesian models that include news shocks in the monetary policy rule. They find news shocks play an important role in matching data. Ben Zeev et al. (2015) and Campbell et al. (2012) develop methods to identify anticipated monetary policy shocks in the data.

in economic conditions affect both the probability and expected duration of a ZLB event. For example, a negative demand shock while the ZLB binds reduces a central bank's margin to lower expected policy rates by decreasing the probability of exiting the ZLB. Four, we are able to analyze forward guidance across all possible realizations of shocks, which nonlinearly impact the economy.

Campbell et al. (2012) introduce two terms to differentiate the types of forward guidance: Delphic and Odyssean. Delphic forward guidance is a central bank's forecast of its own policy, which is based on its projections for inflation and real GDP growth as well as an established policy rule. When combined with economic projections, Delphic forward guidance can help clarify the central bank's policy strategy.[2] Odyssean forward guidance is a commitment to deviate from the established policy rule at some point in the future by promising to set the policy rate lower than the policy rule recommends. News shocks are one way to model Odyssean forward guidance.

Central banks have recently used both date-based and threshold-based forward guidance. Date-based forward guidance provides information on the intended path of policy over a fixed period and is frequently modeled using an interest rate peg. To a modeler, an interest rate peg represents a promise by the central bank to fix the policy rate for a set number of periods. Researchers who use an interest rate peg to model forward guidance, such as Carlstrom et al. (2012), often assume the policy rate initially equals zero and stays there for a fixed period. Once that period ends, the policy rate rises and never returns to zero. Instead of restricting the model to one ZLB event with a predetermined length, we assume a set of stochastic shocks determines whether the ZLB binds.

With threshold-based forward guidance, the central bank agrees to maintain a certain policy rate until a particular event occurs. For example, the central bank might announce that it intends to keep its policy rate at zero until the unemployment rate falls below a specific value. Our news shock approach is similar to threshold-based forward guidance because it enables the policy rate to endogenously respond to changes in economic conditions, whereas an interest rate peg fixes the policy rate regardless of economic conditions. Specifically, expansionary news shocks push down expected policy rates, but those rates can still rise if the endogenous responses of output and inflation are strong enough to compensate for the news. Households also form expectations about the possibility the central bank will provide news that it intends to exit the ZLB in the near future.

Another advantage of using news shocks is that households' expectations incorporate the possibility that the central bank alters its previous forward guidance policy. For example, suppose the central bank announces a plan to keep its policy rate lower than its policy rule suggests for the next $q$ quarters. Households account for the possibility that the central bank might change its policy in the intervening periods. A strict interest rate peg, in contrast, does not allow for the possibility that future economic conditions may cause an unanticipated shift in the policy rate.[3] We compare the results from our news shock approach to an interest rate peg when there is an occasionally binding ZLB constraint. An interest rate peg generates much larger increases in output than is observed in the data, because it gives the central bank a stronger ability to influence expected interest rates.

Other papers examine the effects of forward guidance in an economy with a binding ZLB constraint through the perspective of optimal monetary policy under commitment (i.e., a promise to implement a specific policy regardless of changes in future economic conditions). Eggertsson and

[2] See the Bank of England (2013) for a discussion of how forward guidance helps the public form more accurate expectations about future central bank policy. See den Haan (2013) for a collection of essays about forward guidance and the International Monetary Fund (2013) for a detailed account of recent unconventional monetary policies.

[3] Walsh (2009) contends inflation-targeting central banks that promise expansionary future policies may lack the credibility to fulfill those promises. In our model, households account for this potential time inconsistency problem.

Woodford (2003) and Jung et al. (2005) solve for the optimal commitment policy assuming the policy rate initially equals zero and cannot return to its ZLB. They find the optimal policy is to maintain a policy rate equal to zero even after the natural real interest rate rises. Such a policy generates higher future inflation and lowers the real interest rate, which moderates the declines in output and inflation that occur at the ZLB.[4] Levin et al. (2010) show the optimal policy stabilizes the economy after small shocks but not after large and persistent shocks. In that situation, they argue that a central bank must employ other unconventional policies, such as large-scale asset purchases, to stabilize the economy. Adam and Billi (2006) relax the assumption that the policy rate initially equals zero by allowing the ZLB to occasionally bind. They find the optimal commitment policy is to react more aggressively to adverse shocks that cause output and inflation to decline.

There is also research on forward guidance outside the optimal policy literature.[5] Del Negro et al. (2012) use a log-linear New Keynesian model to show that extending the forward guidance horizon causes the model to overpredict the actual increases in output and inflation. They call that result the "forward guidance puzzle." Several papers offer explanations for the puzzle: McKay et al. (2015) introduce uninsurable income risk and borrowing constraints; Kiley (2014) considers a model with sticky information rather than sticky prices; De Graeve et al. (2014) and Haberis et al. (2014) account for imperfect credibility; and Caballero and Farhi (2014) develop a model where the ZLB binds due to a safety trap—a shortage of safe assets—as opposed to a demand-side shock.

We emphasize the ZLB constraint on current and future policy rates and the state of the economy as a way of explaining the forward guidance puzzle. In our model, demand shocks push the policy rate to its ZLB. The size of those shocks and whether news shocks occur determine how long the policy rate remains at zero. Households form expectations about whether the ZLB will bind in the future based on the entire distribution of demand shocks and news shocks. As demand falls, the ZLB constraint further limits the stimulative effect of forward guidance by preventing current and future policy rates from declining. Thus, there is a nonlinear relationship between the size of the news shocks, the forward guidance horizon, the state of the economy, and the stimulative effect.

The rest of the paper is organized as follows. Section 2 provides a post-financial crisis account of Federal Open Market Committee (FOMC) forward guidance in its policy statements. Section 3 describes our theoretical model. Sections 4 and 5 show the stimulative effects of forward guidance across horizons up to 10 quarters. Section 6 conducts case studies of recent FOMC forward guidance and uses our key findings to explain the effects of that communication. Section 7 concludes.

## 2 RECENT FEDERAL RESERVE FORWARD GUIDANCE

There are two primary ways the FOMC communicates information about the path of future policy rates. One, it releases the individual forecasts of its members four times per year, but that information can be diverse and reveal differences of opinions. Two, it provides forward guidance about

[4]There are several other optimal policy papers related to forward guidance. Krugman (1998) was one of the first to argue that the central bank can mitigate the effects of the ZLB by promising to allow prices to rise. Reifschneider and Williams (2000) develop the merits of that argument in a dynamic model. Werning (2011) shows it is also optimal to commit to higher future inflation when the ZLB binds in a continuous-time model. Adam and Billi (2007) find discretionary policy is unable to generate the higher inflation that is necessary to offset the adverse effects of the ZLB. English et al. (2015) show that introducing threshold-based forward guidance into the monetary policy rule generates outcomes that are closer to the optimal commitment policy. Coenen and Warne (2014) find that date-based forward guidance increases the risk of price instability, but introducing a threshold on inflation can help mitigate that risk.

[5]See Moessner et al. (2015) for a detailed summary and analysis of the recent research on forward guidance.

the future federal funds rate in its policy statements and has consistently done so since late 2008. We interpret those statements to determine the types of news households have recently received.[6]

At the December 16, 2008 meeting, the FOMC decided to target a range for the federal funds rate of $0\%$ to $0.25\%$ and announced it would likely remain at that low level for "some time." The FOMC continued to use similar language until its August 9, 2011 statement, which said that low range was likely warranted "at least through mid-2013." The announcement was the FOMC's first use of date-based forward guidance, and it had a modest effect on expected nominal interest rates.

The next change in forward guidance occurred in the statement release following the January 25, 2012 FOMC meeting. This policy statement was different in two ways. One, the time that the federal funds rate was expected to remain at zero was updated to read "at least through late 2014," which was an increase of six quarters. Two, the FOMC expressed a more pessimistic economic outlook and indicated the projected path for the federal funds rate was conditional on that outlook.

The forward guidance provided in the January 25, 2012 statement was likely viewed as Delphic for two reasons. One, the statement expressed more pessimism about the economy, which suggests the FOMC's policy rule was already projecting a much later date for raising the federal funds rate. Two, the FOMC never stated the new projected interest rate path was different from the path implied by its policy rule. By late summer 2012, the economy continued to disappoint policymakers, and the FOMC statement issued following the September 13, 2012 meeting was amended to read:

> To support continued progress toward maximum employment and price stability, ... *a highly accommodative stance of monetary policy will remain appropriate for a considerable time after the economic recovery strengthens*. ... the Committee also decided today to keep the target range for the federal funds rate at $0$ to $1/4$ percent and currently anticipates that exceptionally low levels for the federal funds rate are likely to be warranted *at least through mid-2015*.

The statement included a $2$-quarter extension to the time the FOMC promised to keep its policy rate at zero and a new pledge to add $85 billion to the Fed's balance sheet every month until the labor market significantly improved. The language "... for a considerable time after the economic recovery strengthens" conveys Odyssean forward guidance. Without that language, it suggests the FOMC would raise its policy rate as the recovery strengthens. On the other hand, the FOMC statement also included information about business spending that led to lower real GDP growth forecasts, which suggests the change in the FOMC's forward guidance was more Delphic in nature.

On December 12, 2012, the FOMC switched its forward guidance from the date-based language "at least through mid-2015" to threshold-based language. The policy statement read:

> ... this exceptionally low range for the federal funds rate will be appropriate *at least as long as the unemployment rate remains above* $6$-$1/2$ *percent*, inflation between one and two years ahead is projected to be no more than a half percentage point above the Committee's 2 percent longer-run goal, and longer-term inflation expectations continue to be well anchored.

FOMC participants' forecasts indicated the unemployment rate would likely hit $6.5\%$ in mid-2015. Therefore, the statement was not intended to change expectations about when the policy rate would rise, but rather to emphasize that any change in the policy rate is conditional on inflation expectations and labor market conditions. The phrase "at least as long as" suggests the unemployment

[6] Forward guidance has also been used by the Bank of Canada, Bank of England, European Central Bank, Bank of Japan, Reserve Bank of New Zealand, Norges Bank, and the Riksbank. See Andersson and Hofmann (2010), Filardo and Hofmann (2014), Kool and Thornton (2012), Moessner and Nelson (2008), Svensson (2014, 2011), and Swanson and Williams (2014) for an overview of the various policies and econometric analysis of their economic impacts.

rate threshold was not a trigger for when the FOMC would automatically raise its policy rate.

Over the next year, the labor market continued to improve, and it was evident the unemployment rate might cross the $6.5\%$ threshold. On December 18, 2013, the FOMC began tapering its monthly asset purchases and redrafted their forward guidance to explain how they intended to react to future economic conditions. The statement said "... it likely will be appropriate to maintain the current target range for the federal funds rate *well past the time that the unemployment rate declines below* 6-1/2 *percent*." The change in language from "at least as long as" to "well past" may have been viewed as Odyssean because it implied that the policy rate would remain near zero even though stronger economic conditions would normally cause the FOMC to raise its policy rate.

In 2014, the FOMC continued to reduce its asset purchases and communicate state-contingent forward guidance. For example, the July 18, 2014 statement said the Committee would likely target a low range for the federal funds rate for a "considerable time after the asset purchase program ends." On January 28, 2015, the FOMC changed its forward guidance to simply say "it can be patient in beginning to normalize" rates. By June 17, 2015, future rate increases appeared imminent as 15 of the 17 committee members were forecasting a rate increase in 2015. Those forecasts emphasize the importance of analyzing forward guidance not only at the ZLB, but also at states near the ZLB, especially since the FOMC has repeatedly said "economic conditions may, for some time, warrant keeping the target federal funds rate below levels the Committee views as normal."

## 3 ECONOMIC MODEL

We use a theoretical model to analyze the stimulative effect of forward guidance. The model has three sectors: a representative household that maximizes its utility, firms that produce intermediate inputs that are bundled together into a final good, and a central bank that sets the short-term nominal interest rate. Forward guidance enters our model through anticipated shocks to the policy rate.

**3.1 HOUSEHOLDS** A representative household chooses $\{c_t, n_t, b_t\}_{t=0}^{\infty}$ to maximize expected lifetime utility, $E_0 \sum_{t=0}^{\infty} \tilde{\beta}_t[\log c_t - \chi n_t^{1+\eta}/(1+\eta)]$, where $c$ is consumption, $n$ is labor hours, $b$ is the real value of a 1-period nominal bond, $1/\eta$ is the Frisch elasticity of labor supply, $E_0$ is an expectation operator conditional on information available in period 0, $\tilde{\beta}_0 \equiv 1$, and $\tilde{\beta}_t = \prod_{i=1}^{t>0} \beta_i$. Following Eggertsson and Woodford (2003), $\beta$ is a time-varying discount factor that follows $\beta_t = \bar{\beta}(\beta_{t-1}/\bar{\beta})^{\rho_\beta} \exp(\upsilon_t)$, where $\bar{\beta}$ is the steady-state discount factor, $0 \leq \rho_\beta < 1$, and $\upsilon_t \sim \mathbb{N}(0, \sigma_\upsilon^2)$.

The household's choices are constrained by $c_t + b_t = w_t n_t + i_{t-1} b_{t-1}/\pi_t + d_t$, where $\pi_t$ is the gross inflation rate, $w_t$ is the real wage rate, $i_t$ is the gross nominal interest rate, and $d_t$ are the dividends from intermediate firms. The optimality conditions to the household's problem imply

$$w_t = \chi n_t^\eta c_t, \tag{1}$$
$$1 = i_t E_t[\beta_{t+1}(c_t/c_{t+1})/\pi_{t+1}]. \tag{2}$$

**3.2 FIRMS** The production sector consists of monopolistically competitive intermediate goods firms and a final goods firm. Intermediate firm $f \in [0,1]$ produces a differentiated good, $y_t(f)$, according to $y_t(f) = n_t(f)$, where $n_t(f)$ is the labor used by firm $f$. Each intermediate firm chooses its labor input to minimize operating costs, $w_t n_t(f)$, subject to its production function. The final goods firm purchases $y_t(f)$ from each intermediate firm to produce the final good, $y_t \equiv [\int_0^1 y_t(f)^{(\theta-1)/\theta} df]^{\theta/(\theta-1)}$, according to a Dixit and Stiglitz (1977) aggregator, where $\theta > 1$ is the elasticity of substitution between the intermediate goods. The demand function for intermediate inputs is $y_t(f) = (p_t(f)/p_t)^{-\theta} y_t$, where $p_t = [\int_0^1 p_t(f)^{1-\theta} df]^{1/(1-\theta)}$ is the price of the final good.

Following Rotemberg (1982), each firm faces a price adjustment cost, $adj_t(f)$. Using the functional form in Ireland (1997), $adj_t(f) = \varphi[p_t(f)/(\bar{\pi} p_{t-1}(f)) - 1]^2 y_t/2$, where $\varphi \geq 0$ scales the size of the adjustment costs and $\bar{\pi}$ is the steady-state gross inflation rate. Real dividends are then given by $d_t(f) = (p_t(f)/p_t)y_t(f) - w_t n_t(f) - adj_t(f)$. Firm $f$ chooses its price, $p_t(f)$, to maximize the expected discounted present value of real dividends $E_0 \sum_{t=0}^{\infty} \tilde{\beta}_t (c_0/c_t) d_t(f)$. In a symmetric equilibrium, all firms make identical decisions and the optimality condition implies

$$\varphi\left(\frac{\pi_t}{\bar{\pi}} - 1\right)\frac{\pi_t}{\bar{\pi}} = (1-\theta) + \theta w_t + \varphi E_t\left[\beta_{t+1}\frac{c_t}{c_{t+1}}\left(\frac{\pi_{t+1}}{\bar{\pi}} - 1\right)\frac{\pi_{t+1}}{\bar{\pi}}\frac{y_{t+1}}{y_t}\right]. \tag{3}$$

Without price adjustment costs, the gross markup of price over marginal cost equals $\theta/(\theta-1)$.

### 3.3 Central Bank and Forward Guidance

The policy rate is set according to

$$\begin{gathered} i_t = \max\{\underline{\imath}, i_t^*\}, \qquad i_t^* = \bar{\imath}(\pi_t/\bar{\pi})^{\phi_\pi}(y_t/\bar{y})^{\phi_y}\exp(x_t), \\ x_t \equiv \textstyle\sum_{j=0}^{q} \alpha_j \varepsilon_{t-j}, \quad \textstyle\sum_{j=0}^{q} \alpha_j = 1, \end{gathered} \tag{4}$$

where $\underline{\imath}$ is the effective lower bound on the nominal interest rate, $i_t^*$ is the notional interest rate (i.e., the policy rate the central bank would set if it were not constrained by the ZLB), $\bar{\pi}$ and $\bar{\imath}$ are the steady-state inflation and nominal interest rates, $\phi_\pi$ and $\phi_y$ are the policy responses to the inflation and output gaps, $\varepsilon_t \sim \mathbb{N}(0, \sigma_\varepsilon^2)$ is a monetary policy shock, $\alpha_j$ is the weight on the shock to the nominal interest rate $j$ periods ahead, and $q \geq 0$ is the forward guidance horizon. For example, when $(\alpha_0, \alpha_1, \ldots, \alpha_q) = (1, 0, \ldots, 0)$, the shock is unanticipated (no forward guidance) and when $(\alpha_0, \alpha_1, \ldots, \alpha_q) = (0, 0, \ldots, 1)$, the shock is anticipated $q$ periods in advance ($q$-period forward guidance). The restriction on the weights of the shocks ensures the total weight across all shocks is the same as the weight on the unanticipated shock without any forward guidance. That restriction allows us to isolate the effect of a longer forward guidance horizon from a larger policy shock.

### 3.4 Equilibrium

The resource constraint is $c_t = y_t - adj_t \equiv y_t^{gdp}$, where $y_t^{gdp}$ includes the value added by intermediate firms, which is their output minus price adjustment costs. Thus, $y_t^{gdp}$ represents real GDP in the model. A competitive equilibrium consists of sequences of quantities, $\{c_t, n_t, y_t, b_t\}_{t=0}^{\infty}$, prices, $\{w_t, i_t, \pi_t\}_{t=0}^{\infty}$, and discount factors, $\{\beta_t\}_{t=0}^{\infty}$, that satisfy the household's and firms' optimality conditions, (1)-(3), the monetary policy rule, (4), the production function, $y_t = n_t$, the bond market clearing condition, $b_t = 0$, the discount factor process, and the resource constraint, given initial conditions, $\beta_{-1}$ and $\{\varepsilon_{-j}\}_{j=1}^{q}$, and sequences of shocks, $\{\varepsilon_t, \upsilon_t\}_{t=0}^{\infty}$.

### 3.5 Calibration

We calibrate our model at a quarterly frequency to match moments in U.S. data from 1983Q1 to 2014Q4. The parameters are summarized in table 1. The steady-state discount factor, $\bar{\beta}$, is set to $0.9957$, which equals the average ratio of the quarter-over-quarter percentage change in the GDP implicit price deflator to the 3-month T-bill rate. The Frisch elasticity of labor supply, $1/\eta$, is set to 3, which is consistent with Peterman (2012). The leisure preference parameter, $\chi$, is calibrated so that steady-state labor equals $1/3$ of the available time. The elasticity of substitution between intermediate goods, $\theta$, is calibrated to 6, which corresponds to a 20% average markup of price over marginal cost. The price adjustment cost parameter, $\varphi$, is set to 160, which matches the estimate in Ireland (2003). The lower bound on the nominal interest rate, $\underline{\imath}$, is calibrated to $1.00022$, which equals the average $3$-month T-bill rate from 2009Q1 to 2014Q4.

| | | | | | |
|---|---|---|---|---|---|
| Steady-State Discount Factor | $\bar{\beta}$ | 0.9957 | Nominal Interest Rate Lower Bound | $\underline{\imath}$ | 1.00022 |
| Frisch Elasticity of Labor Supply | $1/\eta$ | 3 | Monetary Policy Response to Inflation | $\phi_\pi$ | 2 |
| Elasticity of Substitution between Goods | $\theta$ | 6 | Monetary Policy Response to Output | $\phi_y$ | 0.08 |
| Rotemberg Adjustment Cost Coefficient | $\varphi$ | 160 | Discount Factor Persistence | $\rho_\beta$ | 0.87 |
| Steady-State Labor | $\bar{n}$ | 0.33 | Discount Factor Standard Deviation | $\sigma_\varepsilon$ | 0.00225 |
| Steady-State Inflation Rate | $\bar{\pi}$ | 1.0057 | Monetary Policy Shock Standard Deviation | $\sigma_\nu$ | 0.003 |

Table 1: Calibrated parameters.

The steady-state inflation rate, $\bar{\pi}$, is calibrated to $1.0057$ to match the average GDP deflator inflation rate. Using the estimates in Smets and Wouters (2007), we set the monetary response to changes in inflation, $\phi_\pi$, equal to $2$ and the response to fluctuations in output, $\phi_y$, equal to $0.08$. We chose the parameters of the stochastic processes to match volatilities in the data. The persistence of the discount factor, $\rho_\beta$, equals $0.87$ and the standard deviation of the shock, $\sigma_\varepsilon$, equals $0.00225$, which are close to the estimates in Gust et al. (2013). The standard deviation of the monetary policy shock, $\sigma_\nu$, is set to $0.003$. In the data, the annualized standard deviations of quarter-over-quarter percent changes in real GDP, the GDP deflator inflation rate, and the 3-month T-bill rate are $2.58\%$, $0.99\%$, and $2.79\%$, respectively, per year. To compare our model to those values, we ran $10{,}000$ simulations that are each $128$ quarters long (i.e., the same length as our data). We then compute the median standard deviations of real GDP growth, the inflation rate, and the nominal interest rate. Those values and their $95\%$ credible intervals are $2.45\%$ $(1.92\%, 3.67\%)$, $1.07\%$ $(0.74\%, 1.63\%)$, and $2.29\%$ $(1.83\%, 2.90\%)$, respectively, per year. The median standard deviations in the model are near their historical averages, and all three credible intervals contain the values in the data.

The parameters of the discount factor processes are also chosen so our model is consistent with the length of time people *expected* the ZLB to bind, rather than the duration of the current ZLB episode. Prior to the FOMC's August 2011 date-based forward guidance, survey data indicated the 3-month T-bill rate was not expected to remain near zero for very long. Blue Chip consensus forecasts from 2009 and 2010 reveal that the 3-month T-bill rate was expected to exceed $0.5\%$ within three quarters. In our model, a ZLB event lasts an average of $2.12$ quarters when the economy is initialized at its steady state but rises to $3.10$ quarters when it is initialized at a notional interest rate that is consistent with estimates during and immediately after the Great Recession.[7] Therefore, our calibration produces ZLB events with a similar average duration to what was expected prior to the FOMC's forward guidance. The model can also generate much longer ZLB events. For example, $3.47\%$ $(0.93\%, 0.22\%)$ of the ZLB events are longer than $8$ quarters ($12$ quarters, $16$ quarters).

**3.6 SOLUTION METHOD** The model is solved using the policy function iteration algorithm described in Richter et al. (2014), which is based on the theoretical work on monotone operators in Coleman (1991). This method discretizes the state space and iteratively solves for updated policy functions until the tolerance criterion is met. We use linear interpolation to approximate future variables, since it accurately captures the kink in the policy functions, and Gauss-Hermite quadrature to numerically integrate. See appendix C for a formal description of the algorithm.[8]

[7]Bauer and Rudebusch (2014), Gust et al. (2013), Krippner (2013), and Wu and Xia (2014) all estimate the notional federal funds rate and find that it was well below zero during and immediately after the Great Recession.

[8]Benhabib et al. (2001) show that models with a ZLB constraint have two steady-state equilibria. See Gavin et al. (2015) for a discussion of the equilibrium that our algorithm converges to in both a deterministic and stochastic model.

# 4 ONE-QUARTER HORIZON RESULTS

This section uses decision rules and model simulations to quantify the stimulative effect of forward guidance over a 1-quarter horizon. We then show how the ZLB constraint, the initial state of the economy, the size of a monetary policy shock, and the speed of the recovery impacts those effects.

**4.1 EFFECTS OF FORWARD GUIDANCE** We begin by showing the economic effects of forward guidance over a 1-quarter horizon. Figure 1 plots the decision rules for real GDP, the inflation rate, and the current and expected future nominal interest rates as a function of the monetary policy shock, $\hat{\varepsilon}_t$.[9] The time subscript is the period households learn about the shock and not necessarily the period the shock impacts the economy. If the central bank provides no forward guidance, then $\hat{\varepsilon}_t$ is an unanticipated monetary policy shock that impacts the economy in period $t$. When the central bank provides 1-quarter forward guidance, $\hat{\varepsilon}_t$ is a news shock that households learn about in period $t$ but does not impact the economy until period $t+1$. Thus, a news shock creates an innovation in the expected nominal interest rate. We quantify the effects of 1-quarter forward guidance by comparing the differences in forecasts before and after the policy announcement. The vertical axis displays the marginal effect of a monetary policy shock relative to when there is no shock. For example, a 1-quarter news shock of $\hat{\varepsilon}_t = -0.25$ lowers the expected nominal interest rate by over $0.1$ percentage points and raises real GDP by about $0.1\%$ relative to when $\hat{\varepsilon}_t = 0$.

We focus on a cross section of the decision rules where the initial notional interest rate equals zero because it produces the largest stimulative effect of forward guidance when the central bank is constrained by the ZLB. The notional rate equals zero when the discount factor is $0.61\%$ above its steady state. The elevated discount factor signifies an increased desire by households to save, which lowers inflation and real GDP. Households, however, expect the discount factor to decline over time. If no forward guidance is provided, that belief raises the expected nominal interest rate.

When $(\alpha_0, \alpha_1) = (1, 0)$ (solid line), the central bank provides no forward guidance, so $\hat{\varepsilon}_t$ represents an unanticipated policy shock. If $\hat{\varepsilon}_t > 0$, then the shock contracts economic activity by raising the current nominal interest rate and lowering inflation and real GDP. The expected nominal interest rate is unaffected since the shock is serially uncorrelated. If, however, $\hat{\varepsilon}_t < 0$, then monetary policy has no impact on the nominal interest rate since it is already at its ZLB. Thus, the decision rules remain at zero when $\hat{\varepsilon}_t < 0$ since conventional monetary policy is ineffective.

When $(\alpha_0, \alpha_1) = (0, 1)$ (dashed line), the central bank provides households with 1-quarter forward guidance. The light-shaded regions represent the marginal effects of that policy. Without forward guidance, households expect the discount factor to decline, which causes the expected nominal interest rate to rise. If households receive news in period $t$ that an expansionary monetary policy shock will occur in period $t+1$, then the expected nominal interest rate is lower than it is without forward guidance. That expectational effect stimulates real GDP, which raises the inflation and nominal interest rates—what we refer to as *feedback effects*—even though the discount factor remains at the minimum value necessary for the ZLB to bind in this cross section. The maximum amount the expected nominal interest rate can decline is the difference between the expected rate in the absence of forward guidance and the ZLB, which is represented by a horizontal dashed line.

The feedback effect on the current nominal interest rate from 1-quarter forward guidance is counterfactual to recent FOMC forward guidance, and it would show up in expected nominal rates over longer horizons. In reality, the FOMC did not communicate an increase in either current or

[9] In our results, a hat denotes a percent change and a tilde denotes a percentage point difference between net rates.

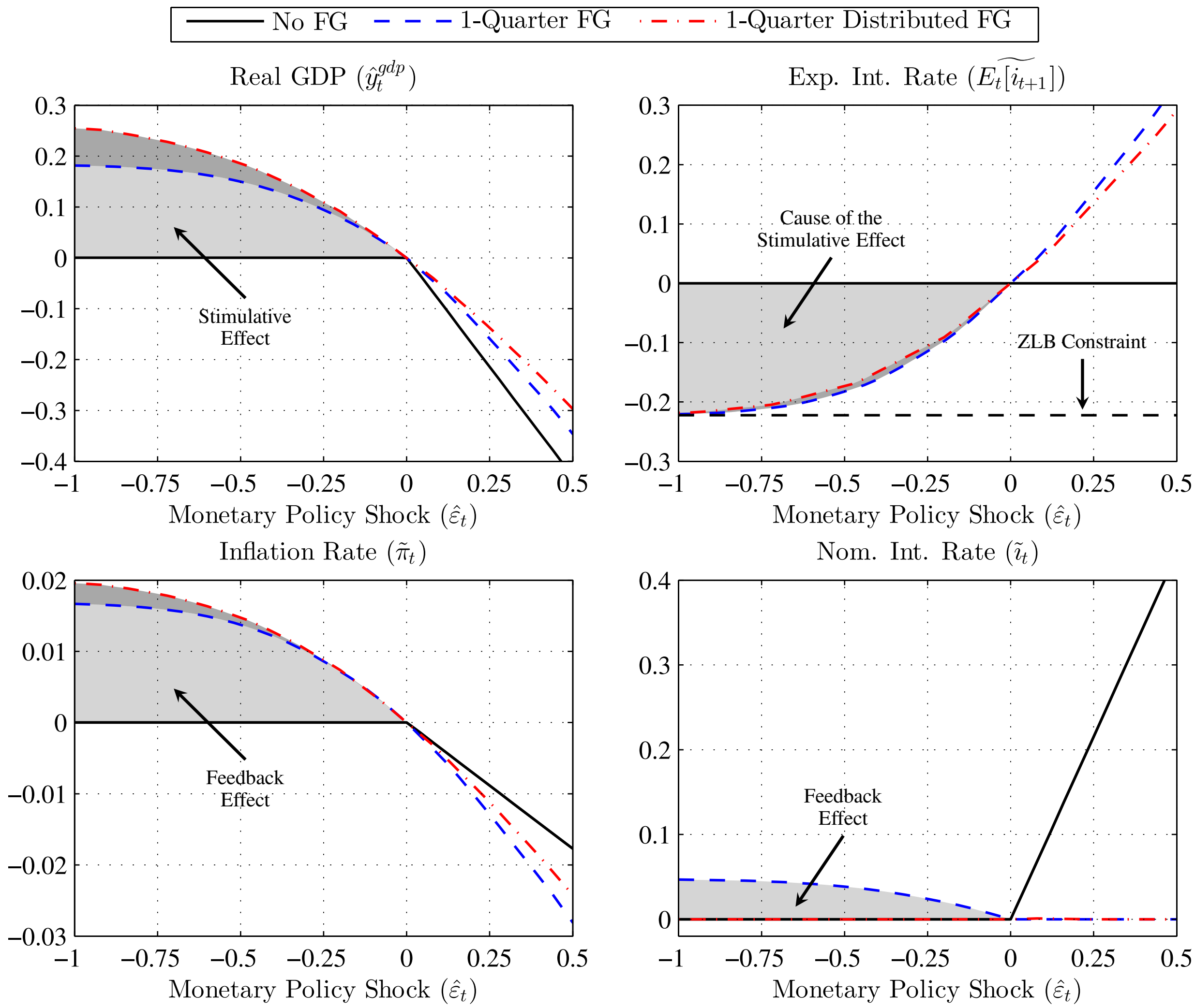


Figure 1: Decision rules as a function of the monetary policy shock with no forward guidance, $(\alpha_0, \alpha_1) = (1, 0)$ (solid line); 1-quarter forward guidance, $(\alpha_0, \alpha_1) = (0, 1)$ (dashed line); and 1-quarter distributed forward guidance, $(\alpha_0, \alpha_1) = (0.13, 0.87)$ (dash-dotted line). In this cross section, the initial notional interest rate equals zero.

future nominal interest rates. In our model, the central bank can eliminate the feedback effect by providing *additional* stimulus in the form of an unanticipated shock in the current period (e.g., set $(\alpha_0, \alpha_1) = (1, 1)$). That policy eliminates the feedback effect but increases the total weight on the policy shock, which generates counterfactually large increases in real GDP and inflation. Alternatively, the central bank can *redistribute* the weights on the policy shock, while holding the total weight fixed. An example of that policy is $(\alpha_0, \alpha_1) = (0.13, 0.87)$ (dash-dotted line), which we refer to as 1-quarter distributed forward guidance. In that case, just enough of the weight is taken from the 1-quarter ahead news shock, $\alpha_1$, and placed on the unanticipated shock, $\alpha_0$, so the current nominal interest rate remains at zero just like it did in the aftermath of the Great Recession.

Expansionary monetary policy shocks under both types of 1-quarter forward guidance have diminishing positive impacts on real GDP as the size of the shock increases. For example, a $-0.5\%$ news shock under 1-quarter forward guidance increases real GDP by $0.15$ percentage points, whereas a $-1\%$ news shock raises real GDP by $0.18$ percentage points. Thus, doubling the strength of the news shock only leads to a small additional increase in real GDP. The small marginal effect occurs because a larger expansionary policy shock increases the likelihood that next period's nominal interest rate will fall to its ZLB. That result is evident from the decision rule for the expected nominal interest rate. Consequently, the stimulative effect of forward guidance is limited by households' expectations about future policy rates, which cannot fall below the ZLB.

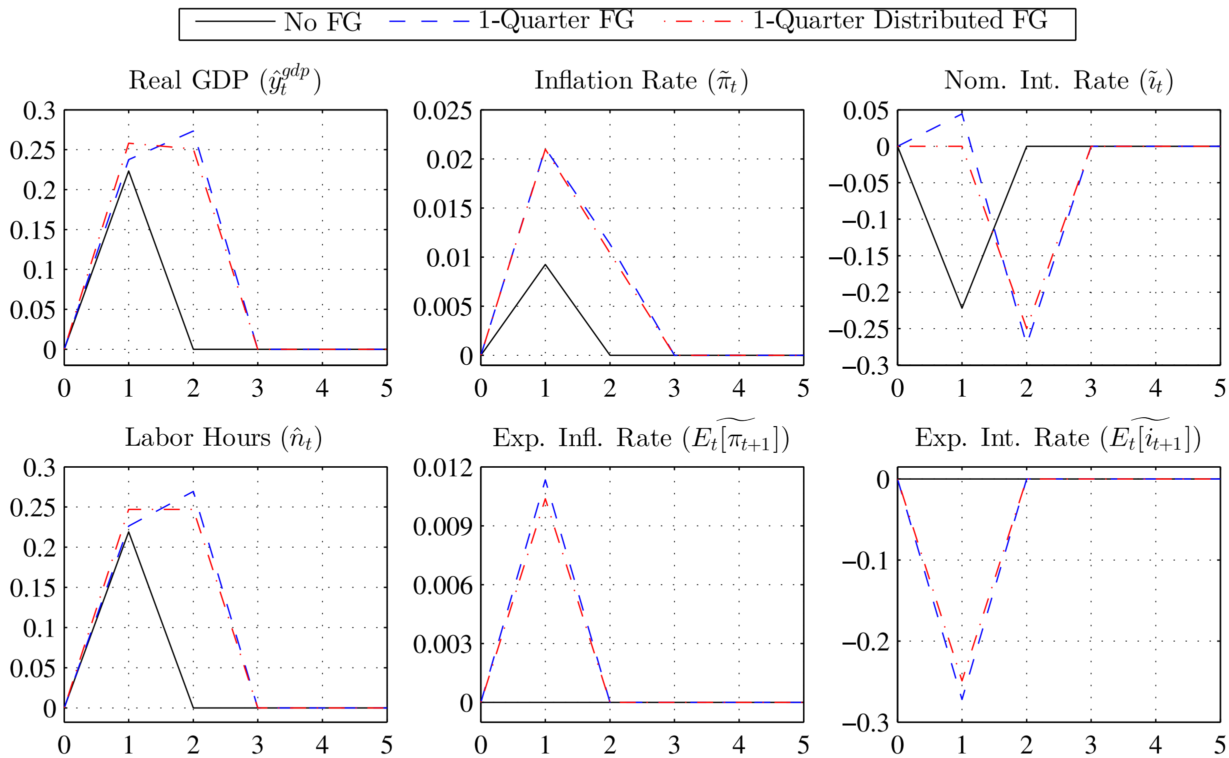


Figure 2: Generalized impulse responses to a $-0.5\%$ monetary policy shock with no forward guidance, $(\alpha_0, \alpha_1) = (1, 0)$ (solid line); 1-quarter forward guidance, $(\alpha_0, \alpha_1) = (0, 1)$ (dashed line); and 1-quarter distributed forward guidance, $(\alpha_0, \alpha_1) = (0.13, 0.87)$ (dash-dotted line). Each simulation is initialized at a notional rate equal to zero.

Another way to examine the effects of forward guidance is with generalized impulse response functions (GIRFs) following Koop et al. (1996). GIRFs are based on an average of model simulations where the realization of shocks is consistent with households' expectations. The advantage of GIRFs is that they show the dynamic effects of a policy shock, whereas the decision rules show the impact effects for a range of shocks. Figure 2 plots the responses to a $-0.5\%$ monetary policy shock at the ZLB with no forward guidance (solid line), 1-quarter forward guidance (dashed line), and 1-quarter distributed forward guidance (dash-dotted line). To compute the GIRFs, we calculate the mean of $10{,}000$ simulations conditional on random shocks. We then calculate a second mean from another set of $10{,}000$ simulations, but this time the random policy shock in the first quarter of each simulation is replaced with a $-0.5\%$ shock. The GIRFs are the percentage change (or the difference in rates) between the two means. Each simulation is initialized at the same notional interest rate as in figure 1. See appendix D for a detailed description of how the GIRFs are calculated.

In each simulation, households learn about the monetary policy shock in period 1. With no forward guidance, the shock is unanticipated and occurs in period 1. With 1-quarter forward guidance, households receive news in period 1 about a policy shock that will hit in period 2. The combination of a zero notional interest rate in period 0 and a mean reverting discount factor causes the period 1 nominal interest rate to rise above its ZLB in $59\%$ of the simulations without a monetary policy shock. Therefore, an unanticipated expansionary policy shock [$(\alpha_0, \alpha_1) = (1, 0)$, solid line] in period 1 reduces the nominal rate in most simulations, so the shock on average is stimulative.

A $-0.5\%$ 1-quarter forward guidance shock [$(\alpha_0, \alpha_1) = (0, 1)$, dashed line] lowers the expected nominal interest rate and raises expected consumption and inflation in period 2. That change causes households to increase their consumption and reduce their labor supply in period 1. It also pushes up the nominal interest rate in period 1 due to the feedback effect. Firms respond to the higher demand in period 1 by raising prices, real GDP, and labor demand. The higher labor demand dominates the decline in labor supply, so equilibrium hours and the real wage rate both increase. Therefore, $1$-quarter forward guidance stimulates the economy over the forward guidance horizon by lowering next period's real interest rate, just like it does under the optimal commitment policy.

The feedback effect of $1$-quarter forward guidance increases the nominal interest rate by $0.04\%$ in period 1. To offset that effect, the central bank could redistribute some of the weight on the news shock to the unanticipated shock. Our specification of $1$-quarter distributed forward guidance [$(\alpha_0, \alpha_1) = (0.13, 0.87)$, dash-dotted line] shifts just enough weight to the unanticipated shock to completely offset the feedback effect from the period 1 news shock, so the shock has no effect on the nominal interest rate in period 1. As a result, real GDP rises $0.02$ percentage points more on impact with distributed forward guidance, while the response in period 2 is only slightly smaller.

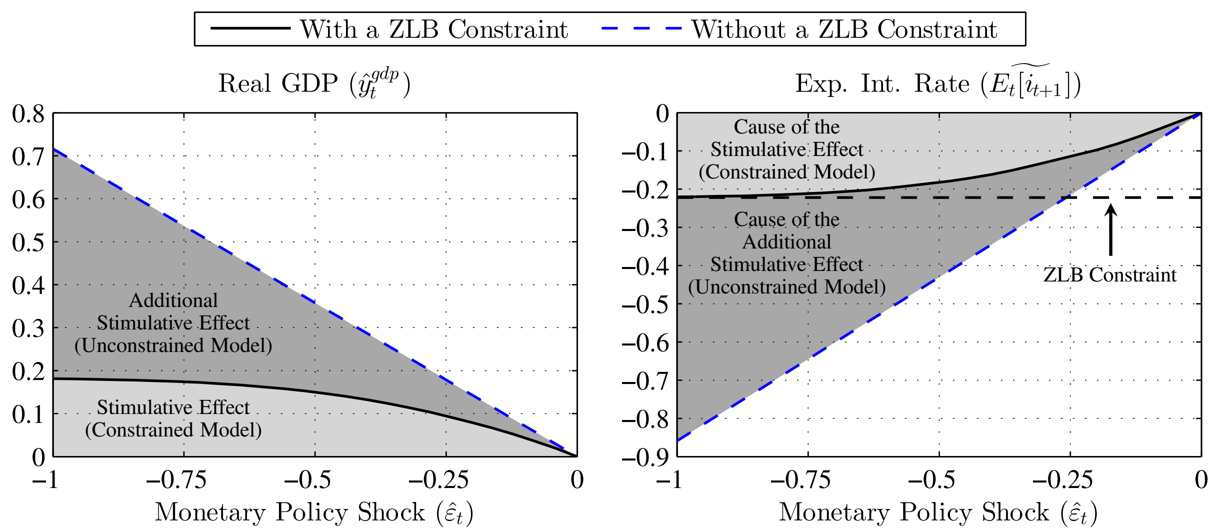


Figure 3: Comparison of decision rules with (solid line) and without (dashed line) a ZLB constraint given 1-quarter forward guidance, $(\alpha_0, \alpha_1) = (0, 1)$. In this cross section of the decision rules, the initial notional rate equals zero.

### 4.2 IMPORTANCE OF THE ZLB CONSTRAINT

The previous section shows forward guidance becomes progressively less stimulative as the expected nominal interest rate approaches zero. Essentially, the ZLB constraint truncates the distribution for the future nominal interest rate at zero, which limits the central bank's ability to lower the expected nominal rate. Figure 3 compares the effects of 1-quarter forward guidance with (solid line) and without (dashed line) a ZLB constraint under the assumption that the initial notional interest rate equals zero. That assumption enables us to analyze the effects of the ZLB constraint when the expected nominal interest rate is near zero.

We show the effects of 1-quarter forward guidance rather than distributed forward guidance, so the stimulative effect is only due to changes in the expected nominal interest rate. As in figure 1, the vertical axis measures the marginal effect of the news shock relative to when there is no shock.

Figure 3 reveals the stimulative effect of forward guidance is overstated when the model does not contain a ZLB constraint and the expected nominal interest rate is near or below zero. For example, a $-0.5\%$ ($-1\%$) news shock in the constrained model reduces the expected nominal interest rate by 18 (22) basis points and increases real GDP by $0.15$ ($0.18$) percentage points. The same shock in the unconstrained model pushes down the expected nominal rate by 43 (86) basis points and raises real GDP by $0.36$ ($0.72$) percentage points. In that example, the expected nominal rate is below its ZLB, but an overstatement of real GDP also transpires when the expected rate is positive but near zero because part of the distribution for the future nominal rate is negative. A similar overstatement would occur if the constraint is imposed in a simulation but not the solution.

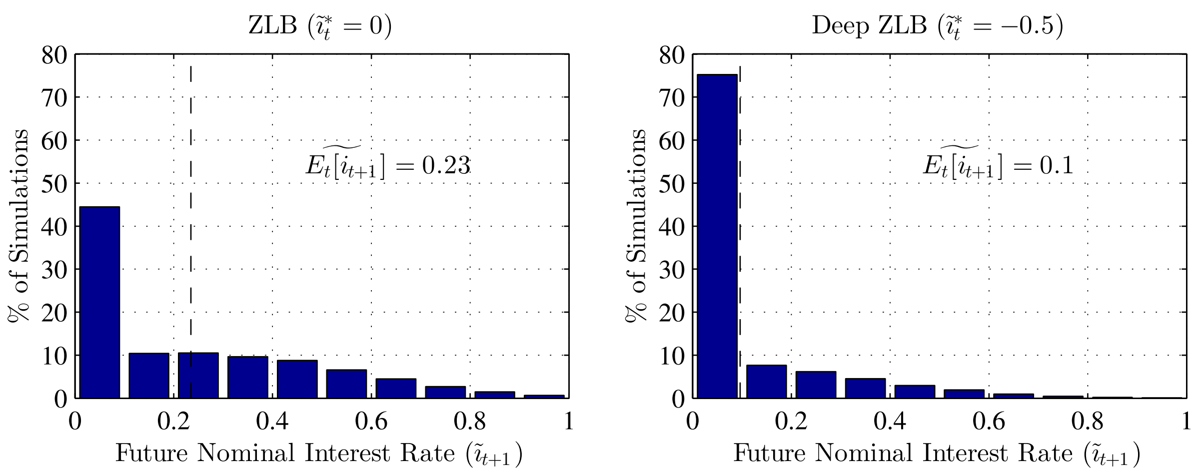


Figure 4: Histograms of the simulated values of next quarter's nominal interest rate without forward guidance. The simulations are initialized at two alternative notional interest rates: $\tilde{\imath}_t^* = 0$ (left panel) and $\tilde{\imath}_t^* = -0.5$ (right panel).

## 4.3 STATE OF THE ECONOMY

This section shows how a weak economy can render forward guidance less effective by examining different initial states of the economy. Figure 4 plots histograms of the simulated values of next quarter's nominal interest rate without forward guidance. The dashed lines represent the expected nominal interest rates. The simulations are initialized at two alternative notional interest rates: $\tilde{\imath}_t^* = 0$ (left panel) and $\tilde{\imath}_t^* = -0.5$ (right panel). These histograms reveal how much the initial notional rate skews the distribution for the future nominal interest rate. When $\tilde{\imath}_t^* = 0$, $\tilde{\imath}_{t+1}$ is between $0\%$ and $0.1\%$ in $45\%$ of the simulations, but that percentage rises to $75\%$ when $\tilde{\imath}_t^* = -0.5$. The expected nominal rates are $0.23\%$ and $0.10\%$ in those cases. Therefore, a weaker economy skews a larger fraction of the future nominal interest rate distribution towards the ZLB, which dampens the expected nominal rate. The lower expected nominal rate means forward guidance has a smaller margin to operate in order to stimulate the economy. Since estimates of the notional rate were below zero during and after the Great Recession, those results provide one reason why recent forward guidance likely had a limited effect on the economy.

GIRFs are a practical tool to show how the stimulative effect of forward guidance is influenced by the state of the economy. Figure 5 displays generalized impulse responses to two different types

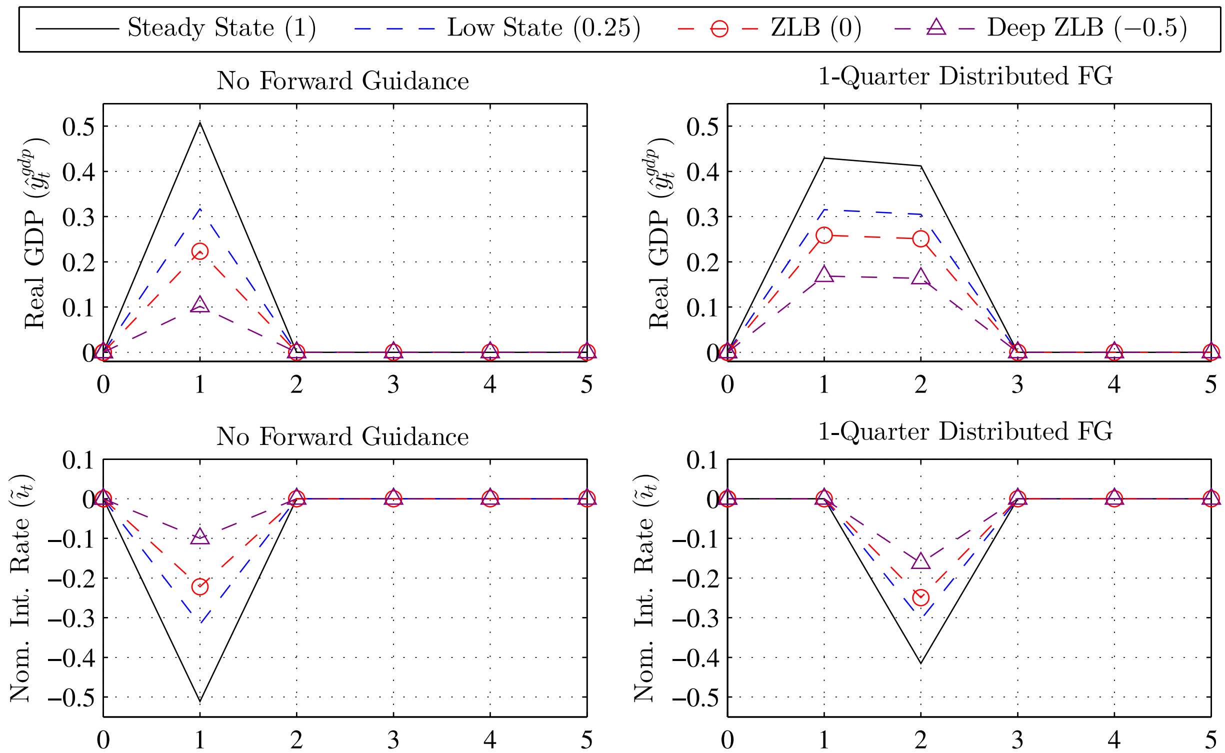


Figure 5: Generalized impulse responses to a $-0.5\%$ monetary policy shock. Two types of monetary policy are examined: No forward guidance, $(\alpha_0, \alpha_1) = (1, 0)$, (left panels) and 1-quarter distributed forward guidance (right panels). Each line represents a simulation initialized at a specific notional interest rate. In each case, the weights on the 1-quarter distributed forward guidance shock are set to eliminate any feedback effects on the nominal interest rate.

of $-0.5\%$ monetary policy shocks: an unanticipated shock (left panels) and a 1-quarter distributed forward guidance shock (right panels). The effect of each shock is examined given four alternative initial notional interest rates: (1) $\tilde{\imath}_0^* = 1$ (solid line) represents an economy at its steady state; (2) $\tilde{\imath}_0^* = 0.25$ (dashed line) is a low policy rate that is consistent with the FOMC's June 2015 forecast for 2016; (3) $\tilde{\imath}_0^* = 0$ (circle markers) denotes an economy that is just weak enough, so the ZLB binds (i.e., the same value used in earlier figures); and (4) $\tilde{\imath}_0^* = -0.5$ (triangle markers) represents an economy in a severe recession where the policy rate is constrained by the ZLB, which is based on its estimated value during the Great Recession. In each case, the weights on the 1-quarter distributed forward guidance shock (i.e., $\alpha_0$ and $\alpha_1$) are set so that monetary policy does not affect the nominal interest rate in period 1 (i.e., the feedback effect is eliminated). A policy that does not generate feedback effects on the nominal rate is consistent with recent FOMC forward guidance.

There are two key takeaways from our simulations. One, monetary policy shocks are less stimulative when the initial notional interest rate is near or below zero. In steady state ($\tilde{\imath}_0^* = 1$), a $-0.5\%$ shock (unanticipated or anticipated) generates the largest decline in the nominal interest rate and has the greatest stimulative effect on real GDP because the policy rate rarely falls by enough to hit its ZLB. As the initial notional interest rate declines, the effects of the policy shock become more limited. For example, a $-0.5\%$ shock has a smaller effect on real GDP when $\tilde{\imath}_0^* = 0.25$ because the current and expected nominal interest rates are closer to zero and, as a result, have less room to fall after the shock. Those effects are further reduced when $\tilde{\imath}_0^*$ equals $0\%$ and $-0.5\%$.

Two, an unanticipated shock is more stimulative on impact than a news shock when the economy is at steady state, while a news shock becomes relatively more stimulative as the policy rate approaches its ZLB. At steady state ($\tilde{\imath}_0^* = 1$), a $-0.5\%$ unanticipated shock initially increases real GDP by $0.51\%$, whereas a 1-quarter distributed forward guidance shock pushes up real GDP by $0.43\%$. That same shock raises real GDP by only $0.10\%$ in a severe recession ($\tilde{\imath}_0^* = -0.5$), while the distributed shock increases real GDP by $0.17\%$. The relative effectiveness of unanticipated shocks versus news shocks depends on how far the current and expected nominal interest rates are from the ZLB. When $\tilde{\imath}_0^* = 1$, the initial notional rate is high enough that the ZLB binds only $1\%$ of the time. The low probability enables the entire unanticipated shock to stimulate the economy most of the time. That result changes when $\tilde{\imath}_0^* = -0.5$. At that state, the ZLB binds $67\%$ of the time, so unanticipated shocks hardly have any effect. The stimulative effect of the distributed shock also declines as the policy rate approaches zero. Its economic effects, however, depend on how close the expected nominal rate, as opposed to the current nominal rate, is to the ZLB. Therefore, if the economy is expected to improve, then the expected nominal rate will be higher than the current rate, which gives news shocks a larger margin to stimulate the economy than unanticipated shocks.

4.4 SIZE OF THE SHOCK The size of the monetary policy shock is another factor that determines whether an unanticipated or distributed news shock is more stimulative on impact. Figure 6 plots the decision rules as a function of the entire distribution of policy shocks with no forward guidance (left panels) and 1-quarter distributed forward guidance (right panels) for the same four initial notional interest rates examined in figure 5. In each cross section, the distributed forward guidance weights ($\alpha_0$ and $\alpha_1$) are set so the news shock has no feedback effects on the nominal interest rate.

A comparison of the right and left panels of figure 6 enables us to determine whether an unanticipated shock or news shock is more stimulative in each state without having the analysis distorted by the feedback effect. When the economy is at steady state ($\tilde{\imath}_t^* = 1$), an unanticipated shock (solid line, left panel) always raises real GDP more on impact than a 1-quarter distributed forward guidance shock (solid line, right panel). The economic effects of an unanticipated shock, however, are more limited when the initial notional interest rate is low enough that the shock causes the ZLB to bind. If the economy is expected to improve, situations exist in which a promise to lower future nominal interest rates generates a larger increase in real GDP than an equivalent shock to the current nominal rate, which cannot fall below the ZLB. Consider the case where $\tilde{\imath}_t^* = 0.25$. A small unanticipated shock, $\hat{\varepsilon}_t > -0.26$, does not drive the nominal interest rate to its ZLB, so the jump in real GDP is higher than with a 1-quarter distributed shock. A moderate-sized unanticipated shock, $-0.42 < \hat{\varepsilon}_t < -0.26$, reduces the nominal interest rate to zero, but the initial stimulative effect is still stronger than the effect of distributed forward guidance. A large unanticipated shock, $\hat{\varepsilon}_t < -0.42$, causes an increasingly smaller rise in real GDP than the same distributed news shock.

When a recession is severe enough to cause the ZLB to bind ($\tilde{\imath}_t^* = 0$), distributed forward guidance is always more stimulative because an unanticipated policy shock cannot reduce the nominal interest rate. In a deeper recession ($\tilde{\imath}_t^* = -0.5$), the probability of exiting the ZLB next period becomes smaller, which reduces the expected nominal interest rate and limits the stimulative effect of forward guidance. Those results reinforce our finding from figure 5 that the stimulative effect of forward guidance is much more limited in a severely depressed economy. In fact, forward guidance will not have any stimulative effect if the initial notional interest rate is sufficiently low.

In figures 5 and 6, we focused on the impact effects of forward guidance in different states of the economy and for different-sized shocks. Later in the paper, we will examine longer forward guidance horizons, which allow us to compare the dynamic effects of distributed news shocks.

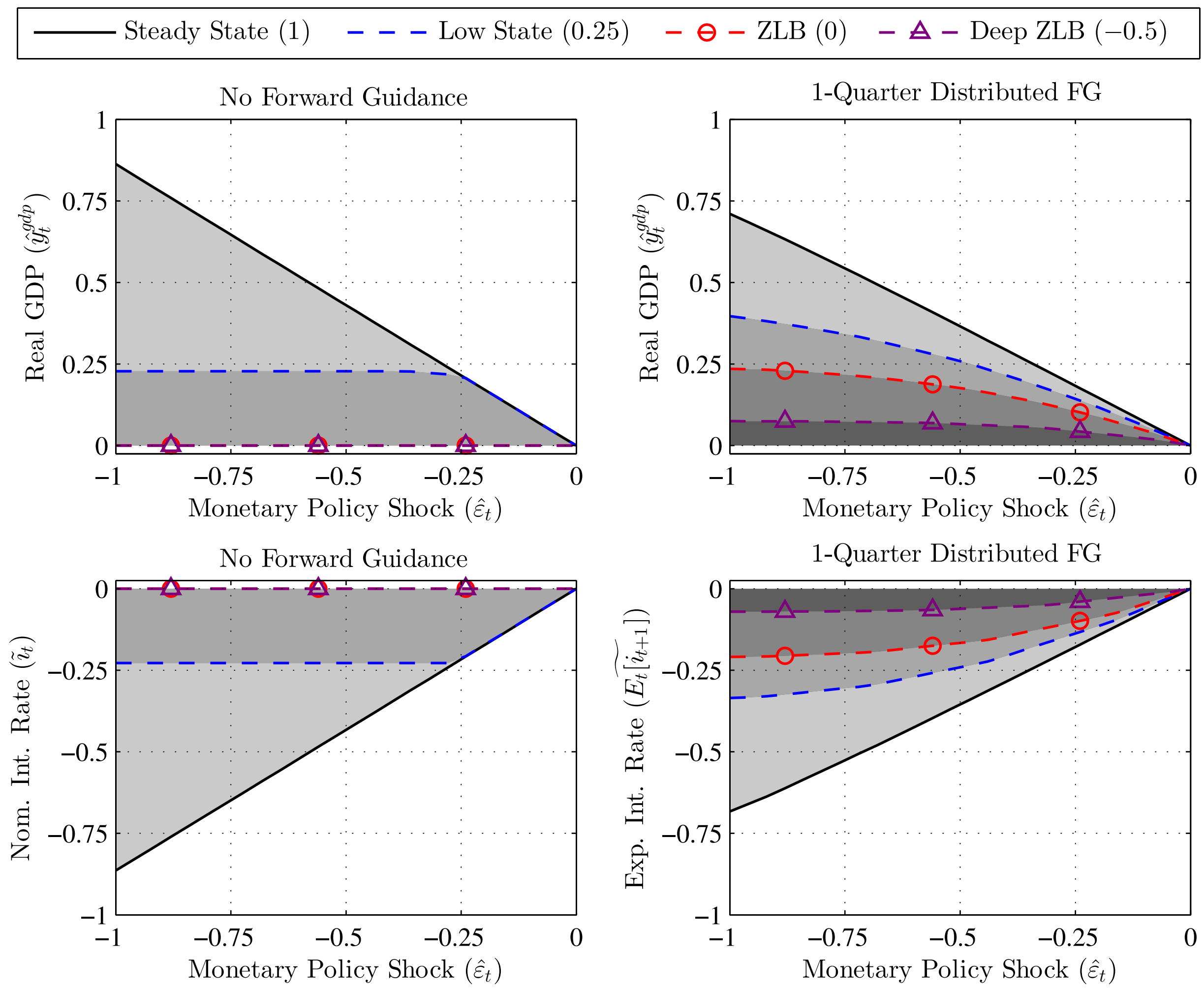


Figure 6: Decision rules with no forward guidance, $(\alpha_0, \alpha_1) = (1, 0)$, (left panels) and distributed forward guidance (right panels). Each line represents a cross section of the decision rules. In each cross section, the weights on the distributed forward guidance shock ($\alpha_0$ and $\alpha_1$) are set to eliminate any feedback effects on the nominal interest rate.

**4.5 SPEED OF THE RECOVERY** Another important determinant of the stimulative effect of forward guidance is how quickly households expect the economy to recover from a recession where the ZLB binds. Unfortunately, the continuous process for the discount factor makes it impossible to change the probability of leaving the ZLB (i.e., the speed of the recovery) without simultaneously changing the probability of going to the ZLB (i.e., the likelihood of a recession). To avoid that problem, we follow Eggertsson and Woodford (2003) and assume the discount factor follows a 2-state Markov chain with transition matrix $\Pr\{s_t = j | s_{t-1} = i\} = p_{ij}$ for $i, j \in \{1, 2\}$. The discount factor is at its steady state in state 1, whereas the discount factor is high enough for the ZLB to bind in state 2. The probability that the discount factor switches from state 1 to state 2, $p_{12}$, is $1\%$, which implies the nominal rate enters the ZLB state once every $25$ years on average. The probability of switching from state 2 to state 1, $p_{21}$, determines the expected speed of the recovery.

Figure 7 shows decision rules with 1-quarter forward guidance as a function of the monetary policy shock, given a slow recovery ($p_{21} = 0.19$, solid line) and a fast recovery ($p_{21} = 0.21$, dashed line). The light-shaded region represents the stimulative effect of forward guidance when

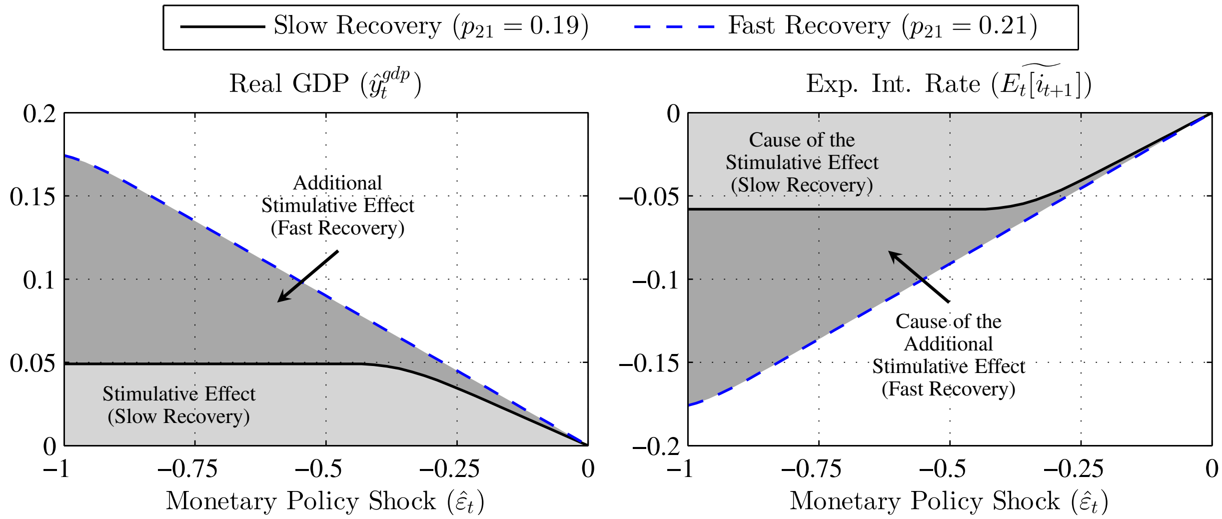


Figure 7: The stimulative effect of 1-quarter forward guidance, $(\alpha_0, \alpha_1) = (0, 1)$, given a slow recovery (solid line) and a fast recovery (dashed line). In this cross section of the decision rules, the initial notional interest rate equals zero.

the economy recovers slowly and the dark-shaded region is the marginal effect of a faster recovery. As in figure 1, the initial notional interest rate equals zero in this cross section of the decision rules.

The stimulative effect of forward guidance is dampened when households expect a slower economic recovery. A less rapid return to steady state reduces demand and lowers the expected nominal interest rate. The smaller jump in the expected nominal rate implies that a promise to maintain a low policy rate in the future will have a weaker effect on real GDP because there is a smaller margin for policy to push down the expected nominal rate in order to stimulate real GDP.[10]

The decision rules under the slow recovery exhibit a kink due to the lower expected nominal interest rate. For small news shocks, $\hat{\varepsilon}_t > -0.25\%$, the expected nominal rate decreases linearly since expectations are a convex combination of the future interest rates across the two states. For large news shocks, $\hat{\varepsilon}_t < -0.50\%$, the expected nominal rate is at the ZLB in both states, so its decision rule is flat. With a fast recovery, however, large news shocks do not push the expected nominal rate to its ZLB, so they generate a larger increase in real GDP that grows with the size of the news shock. For example, news this period that the policy rate will be cut by $0.5\%$ ($1\%$) next period causes real GDP to rise by $0.05\%$ ($0.05\%$) when $p_{21} = 0.19$ and by $0.09\%$ ($0.17\%$) when $p_{21} = 0.21$. Those results demonstrate that forward guidance has a more limited stimulative effect if the policy causes households to revise their expectations about the future economy or it is communicated at the same time they learn about a weaker economic outlook from other sources.

## 5 Longer Horizon Results

This section first compares the stimulative effects of forward guidance over horizons up to 10 quarters. We then show how a decrease in demand can obscure the impact of forward guidance. It concludes by showing how an exogenous interest rate peg overstates the stimulative effect of forward guidance and argues that news shocks better reflect actual policy and its observed effects.

[10] Levin et al. (2010) also show a slower expected recovery hinders forward guidance. They assume a real rate shock hits the economy, decays at a constant rate for four periods, and then switches to a slower rate of decay. Eggertsson and Mehrotra (2014) argue that forward guidance is less effective when the economy is in a near-permanent slump.

5.1 METHODOLOGY Our results in section 4 use Gauss-Hermite quadrature to evaluate expectations. That approach allows us to obtain an accurate approximation of the decision rules and to quantify the stimulative effect of forward guidance for a continuous range of monetary policy shocks, which is important because the responses of key economic variables do not move one-for-one with the size of the shock. Using that technique, appendix A presents the economic effects of 2-quarter forward guidance across all policy shocks. That solution method, however, is numerically infeasible with longer forward guidance horizons because the state space grows exponentially.

We reduce the dimensionality of the problem when analyzing horizons beyond $2$ quarters by discretizing the continuous distribution of the shock process using the method in Tauchen (1986). Specifically, we assign three values for each monetary policy shock, $(-0.6, 0, 0.6)$, and calculate the probabilities of each transitional event. Tauchen's (1986) method is particularly useful for examining longer forward guidance horizons because it enables us to analyze the effects of specific shocks to the news process without solving the model across the complete distribution of shocks. See appendix E for more details on how this solution procedure differs from the previous method.

5.2 COMPARISON ACROSS FORWARD GUIDANCE HORIZONS Figure 8 shows generalized impulse responses to a $-0.6\%$ monetary policy shock distributed over $1$-, $4$-, $8$-, and $10$-quarter forward guidance horizons. For each horizon, we set the weights on the shocks so they sum to one, and there are no feedback effects on the nominal interest rate. In the top, middle, and bottom panels, the simulations are initialized at steady state ($\tilde{\imath}_0^* = 1$), the ZLB ($\tilde{\imath}_0^* = 0$), and a severe recession ($\tilde{\imath}_0^* = -0.5$), respectively. In each case, households learn about the policy shock in period $1$.

When the economy is initialized at steady state ($\tilde{\imath}_0^* = 1$, top panels), the unanticipated monetary policy shock raises real GDP more on impact than the distributed news shock, regardless of the forward guidance horizon. Unlike the effects of an unanticipated shock, which disappear after period $1$, the impact of a $q$-quarter distributed forward guidance shock persists for $q$ more quarters. Distributed forward guidance shifts some of the impact of a policy shock from period $q + 1$ to periods $1$ to $q$ to eliminate the feedback effect on the nominal interest rate. The result is that the size of the shock in period $q + 1$ becomes smaller as $q$ increases (i.e., $\alpha_q$ declines as $q$ rises). The smaller shock dampens the initial rise in output, but the increase persists over the longer forward guidance horizon. Beyond period $q + 1$, forward guidance has no effect on the economy.[11] There are two key takeaways from those results. One, longer forward guidance horizons do not generate increasingly larger impact effects on real GDP when the total amount of news is fixed, unlike with an interest rate peg. Two, a central bank's ability to lower future policy rates, while maintaining the current policy rate in the intervening periods, declines as the forward guidance horizon lengthens.

When the economy begins in a recession that is just severe enough for the ZLB to bind ($\tilde{\imath}_0^* = 0$, middle panels), the initial rise in real GDP is similar across all forward guidance horizons. The increase, however, is smaller in every period over the forward guidance horizon than occurs when the economy is initialized at steady state. The reduced stimulative effect is due to the smaller margin that the central bank has to lower expected nominal interest rates over the next few periods.

In an economic downturn similar to the Great Recession ($\tilde{\imath}_0^* = -0.5$, bottom panels), the stimulative effect of forward guidance is even more limited, especially over shorter horizons. At longer horizons, the response of real GDP is relatively unaffected by the initial state of the economy. Those results show forward guidance can have a larger cumulative effect on real GDP without increasing the total weight on the shocks when the news is distributed across a longer horizon.

[11] De Graeve et al. (2014) show that if the model contains backward-looking endogenous state variables, such as habit formation or inflation indexation, then the effects of the policy will persist beyond the forward guidance horizon.

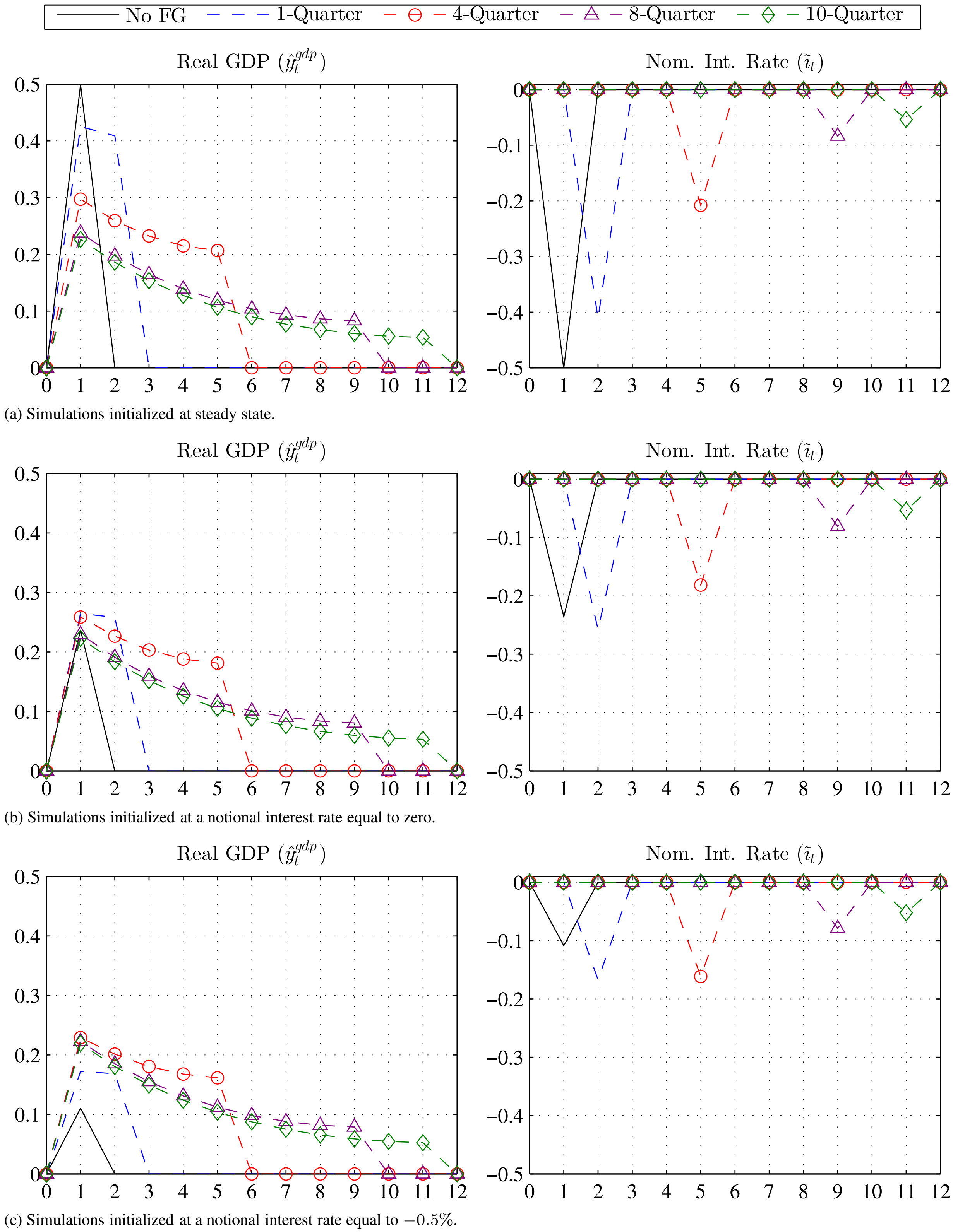


Figure 8: Generalized impulse responses to a $-0.6\%$ monetary policy shock with no forward guidance, $(\alpha_0, \alpha_1) = (1, 0)$, (left panels) and distributed forward guidance (right panels). In each simulation, the weights on the distributed forward guidance shock ($\alpha_j$, $j = 0, 1, \ldots, q$) are set to eliminate any feedback effects on the nominal interest rate.

The difference between the GIRFs for two of the forward guidance horizons provides some intuition for the effects of extending the horizon, which is a policy the FOMC instituted in 2012. Suppose the central bank previously provided four quarters of forward guidance and decides to extend it by four quarters. The difference between the $4$-quarter and $8$-quarter distributed forward guidance GIRFs suggests that extending the horizon by four quarters will push up real GDP in the second year but have little effect in the first year if the size of the policy shock remains unchanged.

To quantify the stimulative effect of the forward guidance policies shown in figure 8 across the entire horizon, we calculate the present value of the percent change in real GDP in every period:

$$\text{Cumulative Effect } \hat{y}(q) = \frac{1}{N}\sum_{j=1}^{N}\sum_{t=1}^{q+1}\frac{100(y_{j,t}^{\varepsilon}/y_{j,t}^{no\ \varepsilon}-1)}{\prod_{k=2}^{t} r_{j,k}},$$

where $y_{j,t}^{no\ \varepsilon}$ is real GDP conditional on draw $j$ of the shocks, $y_{j,t}^{\varepsilon}$ is real GDP conditional on the same draw of shocks except $\hat{\varepsilon}_1 = -0.6\%$, $r_{j,t}$ is the gross real interest rate from draw $j$, and $N$ is the number of simulations. Table 2 shows the present value of the cumulative percent change in real GDP over various forward guidance horizons in response to a $-0.6\%$ monetary policy shock.

| | Forward Guidance Horizon | | | | |
|---|---|---|---|---|---|
| Initial State of the Economy | 0 | 1 | 4 | 8 | 10 |
| Steady State ($\tilde{\imath}_0^* = 1$) | 0.50 | 0.83 | 1.19 | 1.20 | 1.17 |
| Recession ($\tilde{\imath}_0^* = 0$) | 0.23 | 0.51 | 1.00 | 1.09 | 1.09 |
| Deep Recession ($\tilde{\imath}_0^* = -0.5$) | 0.11 | 0.33 | 0.87 | 1.03 | 1.04 |

Table 2: Present value of the cumulative percent change in real GDP in response to a $-0.6\%$ monetary policy shock.

For all states of the economy, $q$-quarter distributed forward guidance always has a larger cumulative effect on real GDP than an unanticipated shock. The size of the cumulative effect, however, depends on both the state of the economy and the forward guidance horizon. In steady state ($\tilde{\imath}_0^* = 1$), extending the forward guidance horizon to $4$ quarters increases the cumulative effect on real GDP, but provides little effect thereafter. At the ZLB ($\tilde{\imath}_0^* = 0$), increasing the horizon from $4$ to $8$ quarters only raises the present value of real GDP by $0.09\%$, while increasing the horizon beyond $8$ quarters has no additional effect. In a deep recession ($\tilde{\imath}_0^* = -0.5$), an increase in the horizon from $4$ to $8$ quarters increases the present value of real GDP by $0.16\%$, but no meaningful increase beyond $8$ quarters. Those results indicate it is more beneficial to extend the horizon when the economy is facing worse economic conditions, but in any state of the economy the central bank faces limits on how far forward guidance can extend into the future and continue to add stimulus.

Carlstrom et al. (2012) and De Graeve et al. (2014) show that endogenous state variables can affect the dynamics generated by forward guidance. To test the robustness of our results, appendix B extends our model in section 3 to include habit formation. That feature dampens, delays, and extends the stimulative effect of forward guidance, but all of our key findings continue to hold.[12]

[12]Carlstrom et al. (2012) find the impact effect on output is positive when the policy rate is pegged at zero for a short duration, but over longer horizons that effect can suddenly turn negative if the model includes endogenous state variables. In our model with habit formation, we find no evidence of reversals. Instead, the impact effect on output declines over longer horizons. Similar to De Graeve et al. (2014), we separately examined inflation indexation, but found that feature had a much smaller effect. Combining both features exacerbated the effects of habit formation.

**5.3 FORWARD GUIDANCE AND LOWER DEMAND** Despite using forward guidance and other unconventional policy measures since late 2008, economic growth has been anemic, leading many to claim that unconventional monetary policy has had little effect on real GDP. The Fed's response is that interest rates have fallen across the term structure and that the economy would be worse off without unconventional policy. This section shows our model is consistent with those arguments.[13]

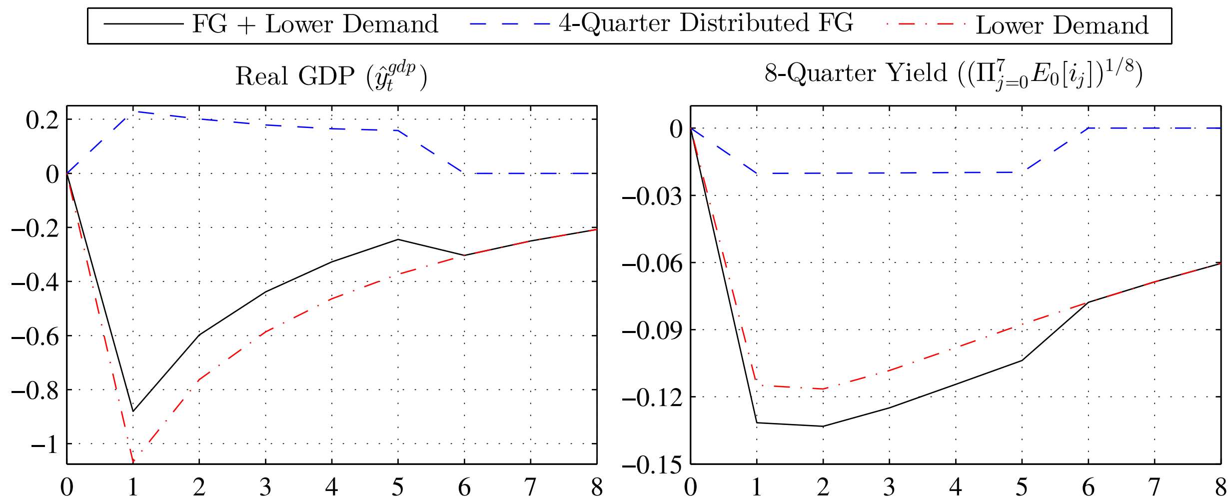


Figure 9: Generalized impulse responses to a 1 standard deviation positive discount factor shock and a $-0.6\%$ monetary policy shock with 4-quarter distributed forward guidance (solid line). The combined responses are compared to responses with only the monetary policy shock (dashed line) and only the discount factor shock (dash-dotted line). Each simulation is initialized at a notional rate equal to $-0.5\%$ to reflect the environment of recent forward guidance.

Figure 9 compares the economic effects of a decline in demand and an announcement of 4-quarter distributed forward guidance. To assess their combined effects, we compute generalized impulse responses to a simultaneous 1 standard deviation positive discount factor shock that reduces demand and a $-0.6\%$ forward guidance shock distributed over 4 quarters (solid line). Those responses are then compared to the responses with only the forward guidance shock (dashed line) and the responses with only the discount factor shock (dash-dotted line). The simulations are initialized at a notional interest rate equal to $-0.5\%$. The distance between the dashed line and the solid line measures the effect of the negative demand shock, whereas the distance between the dash-dotted line and the solid line is the marginal benefit of 4-quarter distributed forward guidance.

An announcement in period 1 of 4-quarter distributed forward guidance reduces the 8-quarter yield and raises real GDP, whereas a negative demand shock in period 1 also pushes down the yield curve but at the expense of lower real GDP. When the two shocks simultaneously hit the economy, the yield curve shifts down, and the real GDP response depends on which of the two shocks dominate. In figure 9, the discount factor shock dominates the forward guidance announcement, so real GDP falls. Those findings illustrate two important points. One, identifying the source of empirically-observed changes in the yield curve is challenging because households often receive forward guidance and information about current and future economic conditions at the same time.

[13] Christiano et al. (2015) find that if the FOMC had not provided any forward guidance after 2011, then the policy rate would have started to rise in 2014 and output would have been 2% lower. Bernanke (2014) reiterated this viewpoint by saying, "Skeptics have pointed out that the pace of recovery has been disappointingly slow...[h]owever...economic growth might well have been considerably weaker, or even negative, without substantial monetary policy support."

Two, forward guidance is stimulative in the absence of any other shocks, but the observed effect on real GDP is smaller or even negative if another shock simultaneously reduces current demand.

While figure 9 focuses on one particular discount factor shock, we can simulate the model over a selected range of shocks in order to mimic the outcomes possible under certain economic conditions. That approach is suitable to analyze the effectiveness of forward guidance in the immediate aftermath of the Great Recession because it was a period when the economy recovered much slower than households expected. Specifically, we restrict our sample of shocks to those values of the discount factor that keep the policy rate at its ZLB in the absence of forward guidance. That set of shocks then is used to generate the distribution of real GDP outcomes with and without forward guidance. The differences between those two distributions provide some indication of the effectiveness of forward guidance in an economic environment similar to the Great Recession.

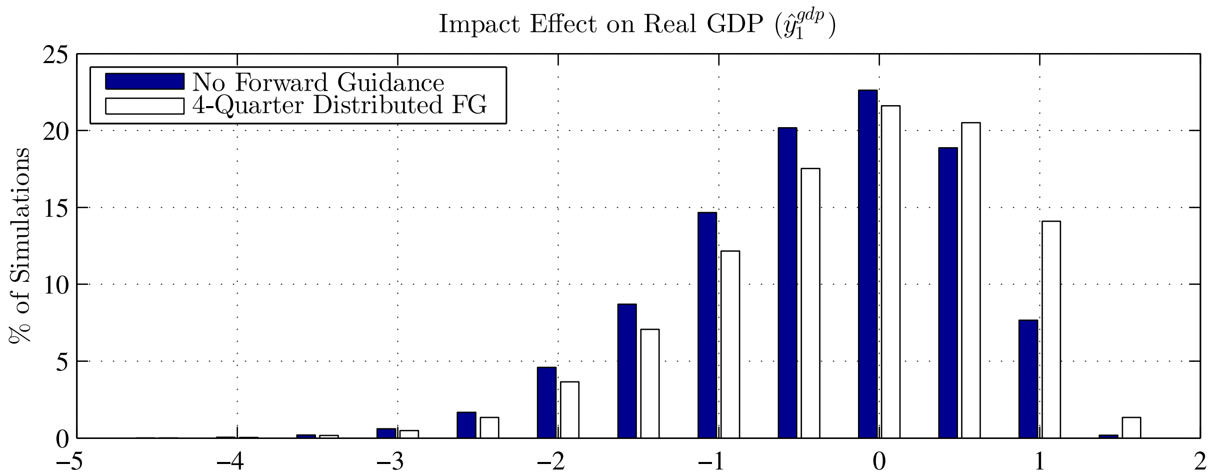


Figure 10: Impact effect on real GDP from generalized impulse responses with no forward guidance (dark bars) and 4-quarter distributed forward guidance (light bars). Each simulation is initialized at a notional rate equal to $-0.5\%$.

Figure 10 shows the distribution of the impact effect on real GDP from generalized impulse responses with no forward guidance (dark bars) and a $-0.6\%$ 4-quarter distributed forward guidance shock (light bars). The simulations used to produce both distributions are initialized at the deep ZLB state ($\tilde{\imath}_t^* = -0.5$) and are based on sequences of discount factor shocks that keep the nominal interest rate at zero for a minimum of five periods, so the economy does not recover as fast as households expect. Those expectations, however, are strong enough to provide the central bank with a small margin to lower expected nominal interest rates even though the actual nominal rate remains at its ZLB over the entire forward guidance horizon. A comparison of both distributions for real GDP reveals that forward guidance shifts the distribution to the right, so real GDP falls less often on impact. That is, the impact effect on real GDP is negative in $62\%$ of the simulations without forward guidance and drops to $53\%$ with forward guidance. In both cases, the declines in real GDP are caused by a negative demand shock, but the forward guidance shock is strong enough in some cases to prevent real GDP from falling. In the remaining portion of the distribution where real GDP declines with forward guidance, the demand shocks are large enough to mask the stimulative effect of the news shock. These findings reinforce our contention that forward guidance boosts real GDP, even though the evidence from recent forward guidance might suggest otherwise.

We do not take a position on why demand shifts in our model. In reality, there are several reasons why the discount factor may change when forward guidance is announced. One, households may interpret news of lower future policy rates as a signal of a weaker economy or a slower economic recovery. Two, policy statements may also provide a forecast of economic conditions that is worse than private forecasts, which leads households to revise their forecasts downward. Three, other sources may provide information that the economy is not performing as well as previously expected at the same time as the forward guidance announcement. Any of those scenarios could decrease real GDP, even if forward guidance is successful at reducing expected policy rates.

**5.4 COMPARISON WITH AN INTEREST RATE PEG** An alternative to using news shocks to model forward guidance is an exogenous interest rate peg. This section uses the same model as section 3, but modifies (4) with a Markov process that governs the central bank's forward guidance policy. Specifically, the central bank sets the gross nominal interest rate according to

$$i_t = \begin{cases} \max\{\underline{\imath}, i_t^*\} & \text{for } e_t = 0 \\ \underline{\imath} & \text{for } e_t = 1 \end{cases}.$$

In period $t$, the gross nominal interest rate is determined endogenously when $e_t = 0$ and is exogenously pegged at its ZLB when $e_t = 1$. Central bank forward guidance is characterized by a vector of nominal interest rate policies, $[e_t, e_{t+1}, \ldots, e_{t+q}]$, communicated to households in period $t$ over horizon $q$. The state of forward guidance is $s_t$ and a particular forward guidance policy is given by $\mathbf{f}(s_t, q) = [s_t \bmod 2, \lfloor s_t/2 \rfloor \bmod 2, \ldots, \lfloor s_t/2^q \rfloor \bmod 2]$, where $s_t \in \{0, \ldots, 2^{q+1}-1\}$. The matrix of all forward guidance policies is defined by $F(q) \equiv [\mathbf{f}(s_t, q)]$. The forward guidance state, $s_t$, evolves according to a $2^{q+1}$-state Markov chain with a transition matrix given by

$$P(q) \equiv [\underbrace{I_{2^q} \otimes [p, p]'}_{2^{q+1} \times 2^q}, \underbrace{I_{2^q} \otimes [1-p, 1-p]'}_{2^{q+1} \times 2^q}].$$

Our approach to modeling an interest rate peg is unique because it permits a stochastic exit and entrance.[14] For example, if forward guidance is communicated over a 1-quarter horizon, then

$$F(1) = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix}, \quad P(1) = \begin{bmatrix} p & 0 & 1-p & 0 \\ p & 0 & 1-p & 0 \\ 0 & p & 0 & 1-p \\ 0 & p & 0 & 1-p \end{bmatrix},$$

where the first (second) column of $F$ corresponds to the realization of $e_t$ ($e_{t+1}$). In state 0, the nominal interest rate is endogenous in periods $t$ and $t+1$. Thus, economic conditions and not an exogenous interest rate peg determine whether the ZLB binds. The probability that forward guidance remains in state 0 is $p$, whereas $1-p$ is the probability that forward guidance will enter state 2. In state 2, the period $t$ nominal interest rate is still set endogenously, but the central bank credibly announces the period $t+1$ nominal rate will be pegged to $\underline{\imath}$ regardless of economic conditions. That promise exogenously sets $i_{t+1}$ unlike news shocks, which allow for the possibility that $i_{t+1} > \underline{\imath}$. Forward guidance then transitions from state 2 to state 1 with probability $p$ such that the nominal interest rate is pegged in period $t$ but is endogenously set in period $t+1$. Alternatively,

[14] Blake (2012) examines alternate ways to peg the policy rate in a model with an endogenous monetary policy rule.

a $1-p$ probability exists that forward guidance moves from state 2 to state 3, which extends the interest rate peg by one quarter. In that case, households only know with certainty that the peg will last one quarter, although it may actually last for several quarters. With a longer forward guidance horizon, households expect the central bank may peg the nominal interest rate for more periods.

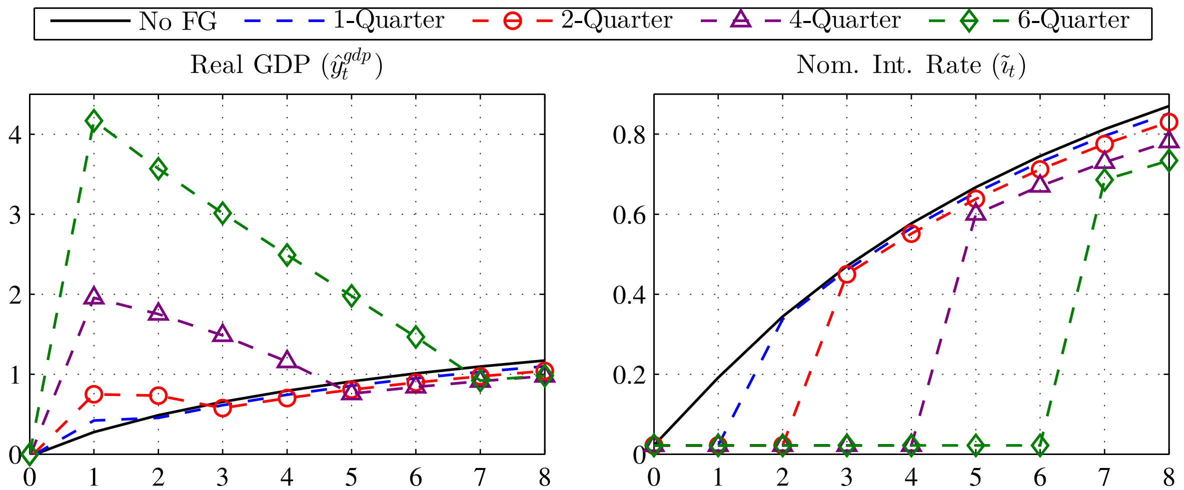


Figure 11: Economic effects of alternative interest rate pegs when the initial notional interest rate equals zero.

Figure 11 shows the economic effects of alternative interest rate pegs when the initial notional interest rate equals zero. With a 1-quarter peg, forward guidance begins in state 3, where the central bank promises to keep the nominal interest rate at zero until next period. Forward guidance then transitions to state 1 in the first period, where the central bank holds the current nominal interest rate at its ZLB but allows the peg to lapse in period 2. Without an interest rate peg in period 2 and beyond, households expect the nominal interest rate to rise as the economy recovers. With a 2-quarter peg, forward guidance begins in state 7, which guarantees the nominal rate will remain at zero for two periods. Forward guidance then transitions to states that reflect the number of periods that remain in the interest rate peg. In period 1, there is one period remaining in the peg, so forward guidance is in state 3. In period 2, forward guidance is in state 1 because households know the interest rate peg will lapse in the next period. The 4- and 6-quarter pegs evolve in a similar way.

With an exogenous interest rate peg, longer forward guidance horizons generate increasingly larger impact effects on real GDP because every additional quarter the nominal interest rate is pegged at zero is equivalent to a news shock that is large enough to drive the expected nominal rate to its ZLB. Therefore, an interest rate peg gives the central bank a much stronger ability to affect expected nominal interest rates than is observed in the data. In our model with news shocks, the impact effects on real GDP diminish with longer horizons, since we fix the total amount of news.

There are four main reasons why we model forward guidance using news shocks instead of an exogenous interest rate peg. One, an interest rate peg cannot respond to changes in economic conditions, which is inconsistent with the threshold-based nature of recent forward guidance. Two, news shocks are more flexible since a 1-quarter interest rate peg corresponds to a large anticipated shock that pushes the expected nominal interest rate to zero. Three, households never expect the central bank to modify previously announced forward guidance policies with an interest rate peg. Four, an interest rate peg does not allow the effects of additional news to be separated from a longer horizon because the longer horizon is analogous to providing increasingly large news shocks.

## 6 CASE STUDIES OF FEDERAL RESERVE FORWARD GUIDANCE

This section uses the qualitative predictions of our theoretical model to help explain the economic effects of three recent FOMC policy statements that communicated date-based forward guidance.[15]

**6.1 2011 POLICY STATEMENT** On August 9, 2011, the FOMC announced it "anticipates that economic conditions. . . are likely to warrant exceptionally low levels for the federal funds rate at least through mid-2013," which was the Committee's first use of date-based forward guidance. It also said, "The Committee now expects a somewhat slower pace of recovery over coming quarters," but the Fed's quantitative easing policy was unchanged, which makes this statement ideal to study.

Blue Chip forecasts of interest rates and real GDP changed following the August 9th FOMC statement. Assessing the effect of that announcement on economic forecasts is complicated by a downward revision of GDP on July 29, 2011. The GDP revision reduced real GDP growth in most quarters since the beginning of the Great Recession. For example, real GDP growth in 2011Q1 (2008Q4) declined from $1.9\%$ ($-7.0\%$) before the revision to $0.4\%$ ($-9.2\%$) after the revision. To separate the impact of the two events, we follow Crump et al. (2013) and use the consensus 3-month T-bill rate forecasts from the July and August 2011 Blue Chip Financial Forecasts (BCFF) survey and the same forecasts from the August 2011 Blue Chip Economic Indicators (BCEI) survey.

| Date | 2011Q4 | 2012Q1 | 2012Q2 | 2012Q3 | 2012Q4 |
|---|---|---|---|---|---|
| BCFF (7/20-21) | 0.14 | 0.26 | 0.43 | 0.75 | 1.08 |
| BCEI (8/4-5) | 0.13 | 0.19 | 0.29 | 0.50 | 0.77 |
| BCFF (8/24-25) | 0.07 | 0.09 | 0.12 | 0.14 | 0.20 |
| Total Change | $-0.07$ | $-0.17$ | $-0.31$ | $-0.61$ | $-0.88$ |
| Change following GDP | $-0.01$ | $-0.07$ | $-0.13$ | $-0.25$ | $-0.31$ |
| Change following FOMC | $-0.06$ | $-0.10$ | $-0.18$ | $-0.36$ | $-0.57$ |

Table 3: Blue Chip consensus forecasts of the 3-month T-Bill rate. All values are annualized net rates.

Table 3 shows the consensus BCFF and BCEI forecasts of the 3-month T-bill rate from 2011Q4 to 2012Q4. The BCFF forecasts were made on July 20-21 before the GDP revision was released on July 29th, while the BCEI forecasts were made on August 4-5 just prior to the August 9th FOMC statement. The difference between the late-July and early-August forecasts is an implicit measure of the impact that the GDP revision had on the forecasts. The next BCFF forecasts were made on August 24-25. The difference between the BCEI's August 4-5 forecasts and the BCFF's August 24-25 forecasts indirectly measures the effect of the FOMC statement, which communicated date-based forward guidance and provided an assessment of current and expected economic conditions.

Data indicate the July 29th GDP revision led to a $13$ basis point decline in the consensus forecast of the 2012Q2 3-month T-bill rate and a 31 basis point decline in the 2012Q4 rate. After the FOMC statement, there were even larger decreases in expected interest rates, with the 2012Q2 and 2012Q4 rates falling by an additional $18$ and $57$ basis points, respectively. In fact, 3-month T-bill forecasts for all of 2012 declined more after the FOMC statement than after the GDP revision. Following both events, the 2012Q4 rate was only $13$ basis points higher than the 2011Q4 rate, which means forecasters believed the policy rate would remain unchanged at least until 2013. Moessner

[15] See Campbell et al. (2012) for a thorough analysis of FOMC forward guidance before and after the financial crisis.

(2013) and Raskin (2013) find the FOMC statement had similar effects on future interest rates. For example, the 2-year ahead Eurodollar rate fell by about 25 basis points after the statement.

Table 4 displays consensus forecasts of real GDP growth from 2011Q4 to 2012Q4. The forecast for 2011Q4 dropped 0.56 percentage points after the GDP revision but only 0.36 percentage points after the FOMC statement. The 2012Q4 forecast of real GDP growth declined by almost 0.3 percentage points after the GDP revision but slightly increased after the FOMC statement. A comparison of all forecast horizons through 2012 reveals that the decline in the forecasts of real GDP after the GDP revision is larger than the change that was observed after the FOMC statement.

| Date | 2011Q4 | 2012Q1 | 2012Q2 | 2012Q3 | 2012Q4 |
|---|---|---|---|---|---|
| BCFF (7/20-21) | 3.09 | 2.75 | 2.97 | 3.07 | 3.17 |
| BCEI (8/4-5) | 2.53 | 2.38 | 2.59 | 2.81 | 2.88 |
| BCFF (8/24-25) | 2.17 | 2.13 | 2.44 | 2.69 | 2.90 |
| Total Change | −0.92 | −0.62 | −0.53 | −0.38 | −0.27 |
| Change following GDP | −0.56 | −0.37 | −0.38 | −0.26 | −0.29 |
| Change following FOMC | −0.36 | −0.25 | −0.15 | −0.12 | 0.02 |

Table 4: Blue Chip consensus forecasts of quarter-over-quarter real GDP growth. All values are annualized net rates.

Our model predicts forward guidance will reduce expected interest rates and push up real GDP when it is communicated without conflicting information. Data following the FOMC statement, however, indicate that near-term real GDP forecasts declined. Our theory provides two potential factors that contributed to the decline in real GDP. One, the GDP revision just before the FOMC statement lowered expected interest rates and limited the central bank's ability to stimulate the economy. Two, the forward guidance was communicated at the same time households received information about a much weaker economic outlook. A comparison of the forecasts following the GDP revision and the FOMC statement reveals that there was a larger decline in expected interest rates and a smaller decline in forecasts of real GDP after the FOMC statement. Figure 9 shows a positive forward guidance announcement accompanied by a negative demand shock lowers expected interest rates and may cause real GDP to fall. In fact, the effects of the FOMC statement on real GDP are well within the distribution of outcomes shown in figure 10. Furthermore, real GDP is higher and expected nominal rates are lower than without forward guidance. Those theoretical results are consistent with the changes in Blue Chip forecasts following the FOMC statement.[16]

### 6.2 2012 POLICY STATEMENTS

The January 25, 2012 and September 13, 2012 statements lengthened the forward guidance horizon for the federal funds rate. The January statement extended the horizon by six quarters (from mid-2013 to late-2014), but it was announced five quarters before the end of the August 2011 horizon. The September statement extended the horizon by two quarters (from late-2014 to mid-2015), six quarters before the January forward guidance ended.

[16]Walsh (2009) cautions that aggressively reducing the policy rate in response to adverse shocks may cause a downward revision in people's economic outlook when their information set differs from the central bank. Campbell et al. (2012) suggest that real GDP declined in response to recent forward guidance because forecasters believed the Fed's communication was based on information about future economic conditions that was not available to the public. Bullard (2012) and Woodford (2012) argue date-based forward guidance may cause people to expect worse economic conditions over its horizon, whereas threshold-based forward guidance alleviates that problem by linking policy rate changes to economic conditions. Yellen (2013, 2014) refers to that type of communication as an "automatic stabilizer."

| Statement | 1 | 2 | 3 | 4 | 6 | 8 | 10 |
|---|---|---|---|---|---|---|---|
| 08/09/2011 | $-0.09$ | $-0.18$ | $-0.25$ | $-0.29$ | $-0.28$ | $-0.20$ | $-0.12$ |
| 01/25/2012 | $-0.04$ | $-0.10$ | $-0.13$ | $-0.14$ | $-0.11$ | $-0.05$ | $0.01$ |
| 09/13/2012 | $-0.01$ | $-0.03$ | $-0.05$ | $-0.07$ | $-0.06$ | $-0.01$ | $0.05$ |

Table 5: Expected changes in forward rates $j$-years ahead on the date of the statement. Values are annualized net rates.

The January and September 2012 statements only contained news lowering expected nominal rates beyond five quarters because previous forward guidance announcements already conveyed the policy rate was expected to remain at its ZLB for at least the next five quarters. Blue Chip forecasts, however, do not extend that far into the future. Thus, we use daily term structure data from Gürkaynak et al. (2007), which is regularly updated by the Board of Governors. Table 5 shows changes in instantaneous forward rates $j$-years ahead on the date of the FOMC statements. Following the January 2012 statement, the decline in the forward rates at $1$ to $4$ years ahead was about half the decline that occurred after the August 2011 statement. At longer horizons, the response is smaller and at $10$ years ahead it is near zero. The September 2012 statement had an even smaller effect. Similarly, Raskin (2013) argues the August 2011 and January 2012 statements had different effects since the market was surprised by the first FOMC statement but not the second.

| Date | 2012Q2 | 2012Q3 | 2012Q4 | 2013Q1 | 2013Q2 |
|---|---|---|---|---|---|
| BCFF (1/4-5) | 2.14 | 2.30 | 2.66 | 2.59 | 2.72 |
| BCEI (2/6-7) | 2.21 | 2.40 | 2.59 | 2.51 | 2.68 |
| Change | 0.06 | 0.10 | $-0.07$ | $-0.08$ | $-0.04$ |

(a) January 25, 2012 FOMC statement release.

| Date | 2012Q4 | 2013Q1 | 2013Q2 | 2013Q3 | 2013Q4 |
|---|---|---|---|---|---|
| BCFF (9/5-6) | 1.90 | 1.78 | 2.34 | 2.66 | 2.83 |
| BCEI (9/24-25) | 1.84 | 1.91 | 2.23 | 2.61 | 2.80 |
| Change | $-0.06$ | 0.12 | $-0.12$ | $-0.06$ | $-0.03$ |

(b) September 13, 2012 FOMC statement release.

Table 6: Blue Chip consensus forecasts of quarter-over-quarter real GDP growth. All values are annualized net rates.

Table 6 displays survey data analogous to what is shown in table 4. The data indicate that the January and September FOMC statements had little effect on real GDP forecasts. The small marginal effect is consistent with our theory for two reasons. One, the August 2011 policy change reduced expected interest rates so much that the modest extension of the forward guidance horizon had a smaller margin to lower expected rates in order to stimulate real GDP. Two, the FOMC's extension of its existing forward guidance horizon was more likely to be anticipated and have smaller economic effects than the initial August 2011 announcement of a date-based horizon. Our results in figure 8 show that modest extensions to the forward guidance horizon can lead to a larger cumulative effect on real GDP, but most of that increase occurs at the very end of the horizon. It is also possible that concurrent information about a weak economy dampened real GDP forecasts, just like in figures 9 and 10. Interestingly, a headline in the *New York Times* on the day of the January statement read, "Fed Signals That a Full Recovery Is Years Away." Such a reaction illustrates the challenge central banks face in achieving the desired effect of their forward guidance policy.

## 7 CONCLUSION

This paper examines the stimulative effects of forward guidance at and away from the ZLB. The central bank conducts forward guidance by promising to keep future nominal interest rates lower than its policy rule suggests. That policy can stimulate economic activity if households believe the economy will recover and exit the ZLB. If, however, households do not expect any meaningful recovery, then future nominal interest rates will remain near zero and forward guidance will have little effect on the economy. Therefore, the ability of forward guidance to stimulate demand is limited when the economy is in a deep recession or households expect a slow economic recovery.

Distributed forward guidance has a smaller impact effect than a conventional monetary policy shock at steady state, but it has a larger cumulative effect in every state of the economy. At the ZLB, the cumulative effect on real GDP from lengthening the forward guidance horizon increases until the horizon reaches eight quarters but not thereafter, which suggests that the central bank faces limits on how far forward guidance can extend into the future and continue to add stimulus. More intense news in the current period or news in future periods may generate a larger cumulative effect, but that effect is limited by the central bank's influence over expected future policy rates.

Empirical estimates indicate recent FOMC forward guidance reduced expected interest rates. It is unclear, however, how much of that decline was due to forward guidance and how much was due to changes in current and expected economic conditions. We find that if there is a decline in demand when communicating forward guidance, then the policy will have a more limited effect.

This paper demonstrates that the forward guidance horizon, the state of the economy, the speed of the recovery, and the size of monetary policy shocks all nonlinearly impact the economic effects of forward guidance due to the ZLB constraint on current and future policy rates. Future research could generalize our model to examine other features that might influence the effects of forward guidance. For example, households could learn about the policy rule over time, instead of knowing the rule with certainty, in order to examine how much forward guidance increases welfare by reducing uncertainty. Another possibility is to assume the news shocks depend on the discount factor shocks. In that case, central bank communication would depend directly on the future state of the economy, which would provide a new way to model threshold-based forward guidance. It would also be interesting to examine various forms of communication about exiting the ZLB. Overall, we believe our findings provide a solid foundation for future research on forward guidance.

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# A TWO-QUARTER HORIZON RESULTS

This section examines how the results in figure 1 change when the forward guidance horizon is extended by one quarter. Figure 12 plots the 2-quarter forward guidance [$(\alpha_0, \alpha_1, \alpha_2) = (0, 0, 1)$, dashed line] decision rules for real GDP and the current and expected nominal interest rates as a function of the monetary policy shock, $\hat{\varepsilon}_t$. With 2-quarter forward guidance, households receive news about a policy shock two periods before the shock hits the economy. As a reference, we also show the decision rules without forward guidance [$(\alpha_0, \alpha_1, \alpha_2) = (1, 0, 0)$, solid line]. Once again, we focus on a cross section of the decision rules where the initial notional interest rate equals zero.

When households receive news in period $t$ about an expansionary monetary policy shock that will occur in period $t + 2$, the impact on real GDP is similar to the impact with 1-quarter forward guidance. Given households prefer a smooth consumption path, the expectation of monetary stimulus in period $t + 2$ encourages households to raise their consumption not only in period $t + 2$, but also in periods $t$ and $t + 1$. The higher consumption in those periods stimulates current real GDP.

Central banks, in practice, offset the feedback effects on current and expected future nominal interest rates by promising to keep the nominal rate at zero over the entire forward guidance horizon. Thus, figure 12 also shows the decision rules when households receive 2-quarter distributed

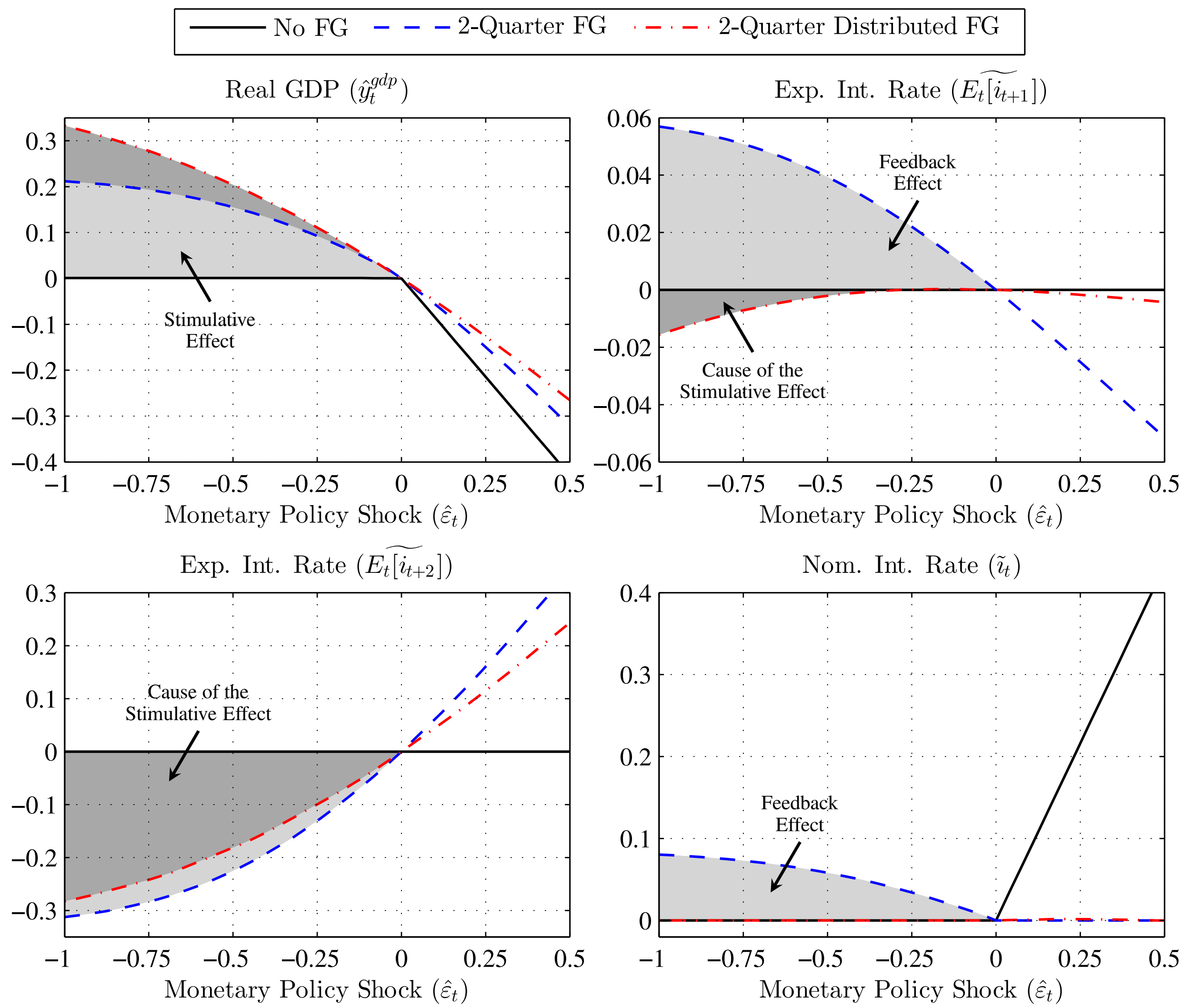



Figure 12: Decision rules as a function of the policy shock with no forward guidance, $(\alpha_0, \alpha_1, \alpha_2) = (1, 0, 0)$ (solid line); 2-quarter forward guidance, $(\alpha_0, \alpha_1, \alpha_2) = (0, 0, 1)$ (dashed line); and 2-quarter distributed forward guidance, $(\alpha_0, \alpha_1, \alpha_2) = (0.16, 0.125, 0.715)$ (dash-dotted line). In this cross section, the initial notional rate equals zero.

forward guidance [$(\alpha_0, \alpha_1, \alpha_2) = (0.16, 0.125, 0.715)$, dash-dotted line]. Substantial differences exist between the two types of 2-quarter forward guidance. With 2-quarter distributed forward guidance, the central bank announces in period $t$ that an expansionary monetary policy shock will occur in periods $t$, $t+1$, and $t+2$. The shocks in periods $t$ and $t+1$, which are not present with 2-quarter forward guidance, hold the current nominal interest rate at zero and lower the expected rate in period $t+1$. Those two additional policy shocks more than compensate for the smaller weight on the period $t+2$ news shock, so 2-quarter distributed forward guidance produces a slightly larger stimulative effect than the more heavily weighted news shock that occurs in period $t+2$. For example, a $-0.5\%$ ($-1\%$) shock announced in period $t$ raises real GDP by $0.05$ ($0.12$) percentage points more with 2-quarter distributed forward guidance than with 2-quarter forward guidance.

Extending the horizon from 1 to 2 quarters less than doubles the stimulative effect on real GDP. For example, a $-0.5\%$ ($-1\%$) policy shock increases real GDP by $0.18$ ($0.25$) percentage points with 1-quarter distributed forward guidance and by 0.20 (0.33) percentage points with 2-quarter distributed forward guidance. The extra quarter of news only raises real GDP by an additional 0.02 (0.08) percentage points, since the total amount of news is held constant across the two horizons.

# B MODEL WITH HABIT FORMATION

This section shows how the effects of forward guidance change when we extend the model in section 3 to allow for habit formation in the household's preferences—a feature many economists argue improves the model's empirical fit [e.g., Christiano et al. (2005) and Smets and Wouters (2007)]. A representative household chooses $\{c_t, n_t, b_t\}_{t=0}^{\infty}$ to maximize $E_0 \sum_{t=0}^{\infty} \tilde{\beta}_t[\log(c_t - hc^a_{t-1}) - \chi n_t^{1+\eta}/(1+\eta)]$, where $c^a$ is aggregate consumption, which is taken as given by the household, and $h$ is the degree of external habit formation. The household's choices are constrained by $c_t + b_t = w_t n_t + i_{t-1}b_{t-1}/\pi_t + d_t$. The optimality conditions to the household's problem imply

$$w_t = \chi n_t^{\eta}(c_t - hc^a_{t-1}), \tag{5}$$

$$1 = i_t E_t[q_{t,t+1}/\pi_{t+1}], \tag{6}$$

where $q_{t,t+1} \equiv \beta_{t+1}(c_t - hc^a_{t-1})/(c_{t+1} - hc^a_t)$ is the pricing kernel between periods $t$ and $t+1$ and $c_t = c^a_t$ in equilibrium. The production sector is unchanged, except firms now discount future dividends by $q_{t,k} \equiv \prod_{j=t+1}^{k>t} q_{j-1,j}$. When $h = 0$, the model is identical to the one in section 3. Gust et al. (2013) use a particle filter to estimate a constrained nonlinear model similar to this model. Thus, we set the habit formation parameter, $h$, to their mean posterior estimate of 0.46629.

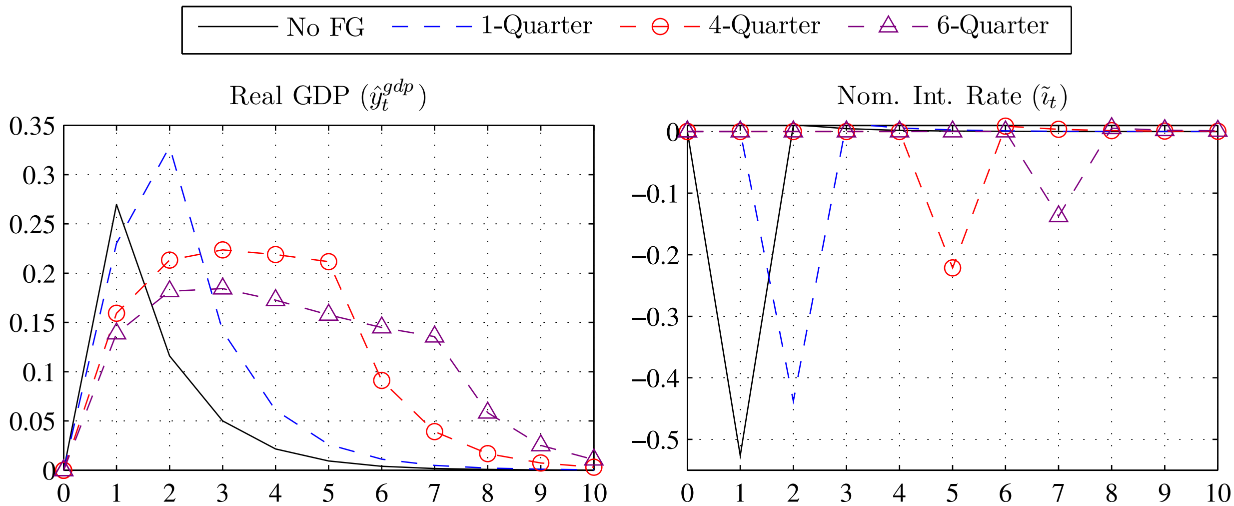
| | Forward Guidance Horizon | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | Without Habit Formation | | | | With Habit Formation | | | |
| Initial State of the Economy | 0 | 1 | 4 | 6 | 0 | 1 | 4 | 6 |
| Steady State ($\tilde{\imath}^*_0 = 1$) | 0.50 | 0.83 | 1.19 | 1.21 | 0.47 | 0.80 | 1.17 | 1.19 |
| Recession ($\tilde{\imath}^*_0 = 0$) | 0.23 | 0.51 | 1.00 | 1.08 | 0.22 | 0.48 | 0.97 | 1.06 |
| Deep Recession ($\tilde{\imath}^*_0 = -0.5$) | 0.11 | 0.33 | 0.87 | 0.99 | 0.09 | 0.29 | 0.84 | 0.97 |

Table 7: Present value of the cumulative percent change in real GDP in response to a $-0.6\%$ monetary policy shock.

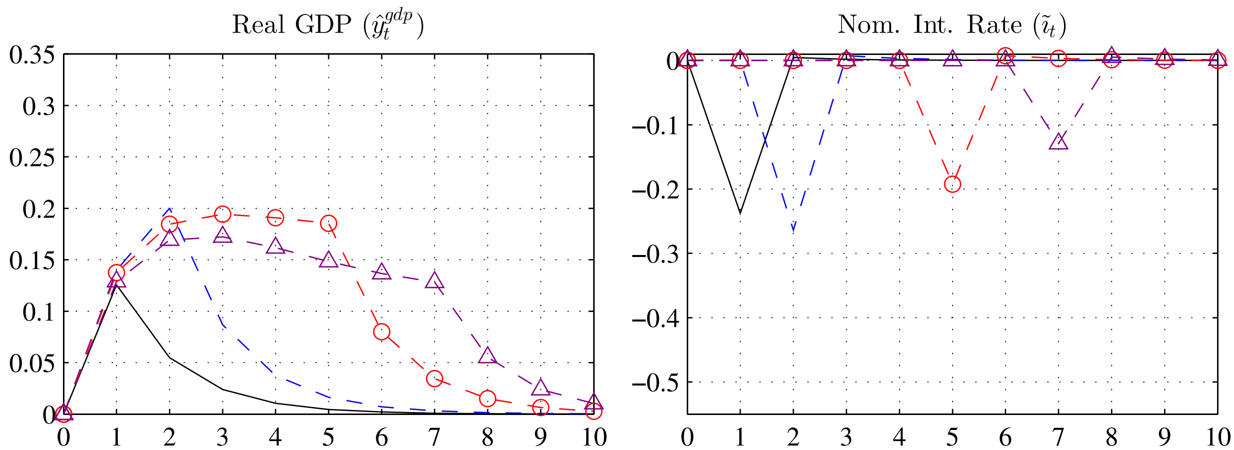
Habit formation in consumption influences both the impact effect and duration of real GDP's response to forward guidance. Nevertheless, this paper's five key findings are unaffected by habit formation. Table 7 shows generalized impulse responses to a $-0.6\%$ monetary policy shock distributed over 1-, 4-, and 6-quarter forward guidance horizons in the model with habit formation. The assumptions underlying the GIRFs are identical to figure 8 and are thus directly comparable.

There are four important differences in the responses compared to the model without habit formation. One, the impact effect of forward guidance is much smaller. Two, the peak response of real GDP is delayed such that real GDP increases gradually until about half way through the forward guidance horizon. Three, the stimulative effect of forward guidance lasts beyond the forward guidance horizon, although its post-horizon effect is small relative to real GDP's response in each quarter over the horizon. Four, despite being more persistent, table 7 shows the cumulative effect of forward guidance at each horizon and each initial notional interest rate is slightly smaller.

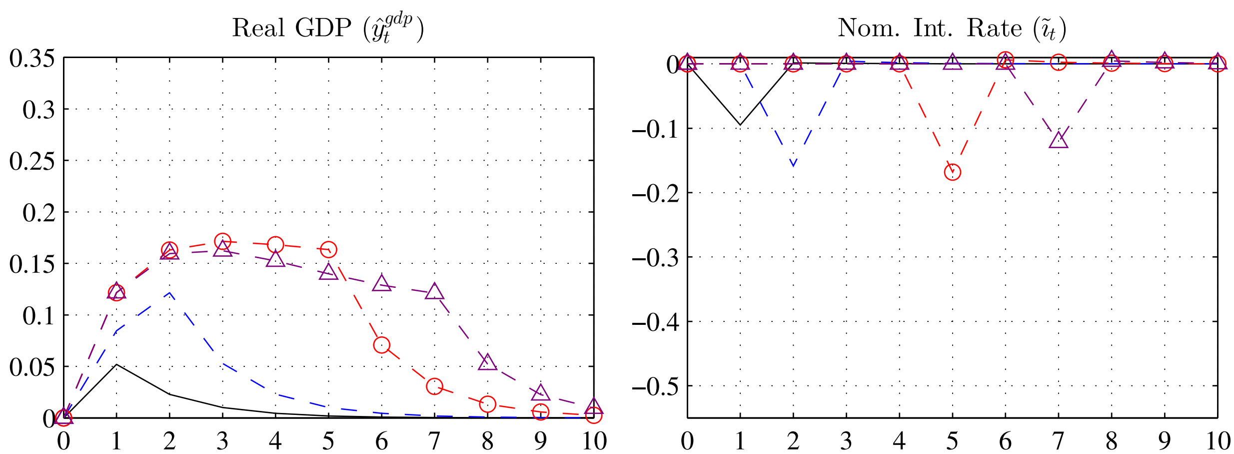
Essentially, the presence of habit formation breaks the link between consumption growth and the real interest rate, so current real GDP is less sensitive to changes in current and expected




(a) Simulations initialized at steady state.




(b) Simulations initialized at a notional interest rate equal to zero.




(c) Simulations initialized at a notional interest rate equal to $-0.5\%$.

Figure 13: Generalized impulse responses to a $-0.6\%$ monetary policy shock with no forward guidance, $(\alpha_0, \alpha_1) = (1, 0)$, (left panels) and distributed forward guidance (right panels). In each simulation, the weights on the distributed forward guidance shock ($\alpha_j$, $j = 0, 1, \ldots, q$) are set to eliminate any feedback effects on the nominal interest rate.

future real interest rates. Therefore, a distributed news shock, which simultaneously eliminates the feedback effect on the nominal interest rate and pushes up inflation, causes real GDP to peak on impact in our model without habit formation but is delayed in our model with habit formation.

## C Numerical Algorithm

A formal description of the numerical algorithm begins by writing the model compactly as

$$\mathbb{E}[f(\mathbf{v}_{t+1}, \mathbf{w}_{t+1}, \mathbf{v}_t, \mathbf{w}_t)|\Omega_t] = 0,$$

where $f$ is a vector-valued function that contains the equilibrium system, $\mathbf{v} = \beta$ is a vector of exogenous variables, $\mathbf{w} = (c, m, y, w, \pi, i)$ is a vector of endogenous variables, and $\Omega_t = \{S, P, \mathbf{z}_t\}$ is the household's information set in period $t$, which contains the structural model, $S$, its parameters, $P$, and the state vector, $\mathbf{z}$. For example, with 1-quarter distributed forward guidance, $\mathbf{z}_t = (\varepsilon_{t-1}, \varepsilon_t, \beta_t)$. Each state variable is discretized into $61$ points, so the state space contains $226{,}981$ nodes. The bounds of each state variable are $\pm 4$ standard deviations of their processes.

The following steps outline our policy function iteration algorithm:

1. Obtain initial conjectures for the approximating functions, $\hat{c}_0$ and $\hat{\pi}_0$, on each node from the log-linear model without the ZLB imposed. We use `gensys.m` to obtain those conjectures.
2. For iteration $i \in \{1, \ldots, I\}$ and node $n \in \{1, \ldots, N\}$, implement the following steps:
   (a) On each node, solve for $\{y_t, i_t, w_t\}$ given $\hat{c}_{i-1}(\mathbf{z}_t^n)$ and $\hat{\pi}_{i-1}(\mathbf{z}_t^n)$ with the ZLB imposed.
   (b) Linearly interpolate $\{c_{t+1}, \pi_{t+1}\}$ given the state, $\{\varepsilon_t, \varepsilon_{t+1}^m, \beta_{t+1}^m\}_{m=1}^M$ (1-quarter forward guidance). Each of the $M$ pairs of $\{\varepsilon_{t+1}^m, \beta_{t+1}^m\}$ are Gauss-Hermite quadrature nodes. In a constrained model, the accuracy of expectations is crucial, so we use $31$ nodes on each shock ($M = 31^2$). We use Gauss-Hermite quadrature, since it is accurate for normally distributed shocks. We use piecewise linear interpolation to approximate future variables that show up in expectation, since that approach more accurately captures the kink in the decision rules than continuous functions such as Chebyshev polynomials.
   (c) On each node, solve for time $t+1$ variables, $\{y_{t+1}^m, c_{t+1}^m\}_{m=1}^M$, that enter the expectation operators. Then, numerically integrate to approximate the expectations by computing

$$\mathbb{E}\left[f(\mathbf{x}_{t+1}^m, \mathbf{x}_t^n)|\Omega_t\right] \approx \frac{1}{\pi}\sum_{m=1}^{M} f(\mathbf{x}_{t+1}^m, \mathbf{x}_t^n)\phi(\varepsilon_{t+1}^m, \beta_{t+1}^m),$$

   where $\mathbf{x} \equiv (\mathbf{z}, \mathbf{w})$, and $\phi$ are the respective Gauss-Hermite weights. The superscripts on $\mathbf{x}$ indicate which realizations of the state variables are used to compute expectations. Finally, use the nonlinear solver, `csolve.m`, to minimize the Euler equation errors.
3. Define $\mathrm{maxdist}_i \equiv \max\{|\hat{c}_i - \hat{c}_{i-1}|, |\hat{\pi}_i - \hat{\pi}_{i-1}|\}$. Repeat step 2 until $\mathrm{maxdist}_i < 10^{-9}$ on every node for $10$ consecutive iterations. At that point, the algorithm converged to a solution.

Richter et al. (2014) demonstrate the accuracy of this algorithm in a model with a ZLB constraint.

## D Generalized Impulse Response Functions

The general procedure for calculating GIRFs is described in Koop et al. (1996). The GIRFs are based on the average path from repeated simulations of our model and generated by following:

1. Initialize each simulation by solving for the constant discount factor shock that yields the desired notional interest rate. Define the corresponding state vector as $\mathbf{z}_0$.
2. Draw random monetary policy and discount factor shocks, $\{\varepsilon_t, \upsilon_t\}_{t=0}^N$, for each simulation, where $N$ is the number of quarters in the simulation. Beginning at the initial state vector, $\mathbf{z}_0$, simulate $R$ equilibrium paths, $\{\mathbf{x}_t^j(\mathbf{z}_0)\}_{t=0}^N$, where $j \in \{1, 2, \ldots, R\}$ and $R = 100{,}000$.
3. Using the same $R$ draws of shocks from step 2, replace the policy rate shock in period one with a $-0.5\%$ shock (i.e., set $\varepsilon_1 = -0.5$ for all $j \in \{1, 2, \ldots, R\}$). Simulate the model with these alternate sequences of shocks. This yields $R$ equilibrium paths, $\{\mathbf{x}_t^j(\mathbf{z}_0, \varepsilon_{z,1})\}_{t=0}^N$.
4. Average across the $R$ simulations from step 2 and step 3 to obtain average paths given by
$$\bar{\mathbf{x}}_t(\mathbf{z}_0) = R^{-1} \textstyle\sum_{j=1}^R \mathbf{x}_t^j(\mathbf{z}_0), \qquad \bar{\mathbf{x}}_t(\mathbf{z}_0, \varepsilon_{z,1}) = R^{-1} \textstyle\sum_{j=1}^R \mathbf{x}_t^j(\mathbf{z}_0, \varepsilon_{z,1}).$$
5. The difference between the two paths is a GIRF. In our figures, a variable with a hat equals $100(\bar{\mathbf{x}}_t(\mathbf{z}_0, \varepsilon_{z,1})/\bar{\mathbf{x}}_t(\mathbf{z}_0) - 1)$, and a variable with a tilde is $100(\bar{\mathbf{x}}_t(\mathbf{z}_0, \varepsilon_{z,1}) - \bar{\mathbf{x}}_t(\mathbf{z}_0))$.

## E COMPUTING LONGER HORIZONS

To make our numerical algorithm tractable across forward guidance horizons up to 10 quarters, we discretize each monetary policy shock with 3 points by following the procedure in Tauchen (1986). The state vector, $\mathbf{z}_t = (\beta_t, s_{0,t}, s_{1,t})$, is independent of the horizon. The monetary policy state, $s_{0,t} \in \{0, 1, 2\}$, determines the realization of the monetary policy shock, $\varepsilon_t$, according to

$$\varepsilon_t = \begin{cases} -0.006 & \text{for} \quad s_{0,t} = 0 \\ 0 & \text{for} \quad s_{0,t} = 1 \\ 0.006 & \text{for} \quad s_{0,t} = 2 \end{cases}.$$

A particular realization of the lagged monetary policy states in the news process is given by $\mathbf{e}(s_{1,t}, q) = [s_{1,t} \bmod 3, \lfloor s_{1,t}/3 \rfloor \bmod 3, \ldots, \lfloor s_{1,t}/3^{q-1} \rfloor \bmod 3]$, where $s_1 \in \{0, 1, \ldots, 3^q - 1\}$. The matrix of all realizations of lagged states is $E(q) \equiv [\mathbf{e}(s_1, q)]$. For example, when $q = 2$

$$E(2) = \begin{bmatrix} 0 & 1 & 2 & 0 & 1 & 2 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 & 1 & 1 & 2 & 2 & 2 \end{bmatrix}',$$

where the first (second) column of $E$ corresponds to the state underlying the realization of $\varepsilon_{t-1}$ ($\varepsilon_{t-2}$). The evolution of the state of lagged policy shocks is given by $s_{1,t+1} = s_{0,t} + 3(s_{1,t} \bmod 3^{q-1})$. If we further suppose $s_{0,t} = 2$ and $s_{1,t} = 3$, then $(\varepsilon_t, \varepsilon_{t-1}, \varepsilon_{t-2}) = (0.006, -0.006, 0)$. In order for $s_{1,t+1}$ to be consistent with the history of shocks, it must equal 2, which is given by $2 + 3(3 \bmod 3)$, so $(\varepsilon_t, \varepsilon_{t-1}) = (0.006, -0.006)$ (i.e., the third row of $E$).

The transition matrix for $s_{0,t}$ is ergodic and is characterized by a single vector of probabilities,

$$P = (\lambda_1, \lambda_2, \lambda_3) = (0.1587, 0.6827, 0.1587),$$

where $\lambda_k = \Pr(s_{0,t+1} = k)$. We discretize the initial discount factor, $\beta_t$, into 61 points, so the state space contains $N = 61 \times 3 \times 3^q$ nodes. We approximate the expectation operators by computing

$$\mathbb{E}\left[f(\mathbf{x}_{t+1}^k, \mathbf{x}_t^n)|\Omega_t\right] \approx \frac{1}{\pi} \sum_{k=1}^{3} \lambda_k \sum_{m=1}^{M} f(\mathbf{x}_{t+1}^{m,k}, \mathbf{x}_t^n)\phi(\beta_{t+1}^m),$$

where $k$ is the realization of $s_{0,t+1}$. In all other aspects, the algorithm is the same as in appendix C.