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Hyeongwoo Kim and Deockhyun Ryu

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Measuring the Speed of Convergence of Stock Prices: A Nonparametric and Nonlinear Approach*

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Abstract

This paper evaluates the speed of convergence across national stock markets employing a nonlinear, nonparametric stochastic model of the relative stock price. To estimate the persistence of the relative stock price, we employ an operational algorithm that is based on two statistical notions: the short memory in mean (SMM) and the short memory in distribution (SMD). Using MSCI stock price indices of the G7 countries, we obtain strong empirical evidence of convergence of national stock prices in France, Germany, and the UK vis-à-vis the US index. Also, we obtain much faster convergence rates from our nonlinear models in comparison with those from linear alternatives. On the contrary, our results imply very limited evidence of convergence for Canada, Italy, and Japan. Similarly weak evidence of convergence was obtained from non-G7 developed countries. Our simulation exercise for portfolio switching strategies overall confirms the validity of empirical findings in the present paper.

JEL Classification: C14; C22; F36; G11; G14

Keywords: Persistence; Contrarian Strategy; Momentum Strategy; Short Memory in Mean; Short-Memory in Distribution; Max Half-Life; Portfolio Switching Strategies

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† Department of Economics, Auburn University, Auburn, AL 36849. Tel: +1.334.844.2928, Fax: +1.334.844.4615, Email: gmmkim@gmail.com.

‡ Contact Author: Deockhyun Ryu, Department of Economics, Chung-Ang University, 221 Heuksuk-Dong, Dongjak-Gu, Seoul 156-756, Korea. Tel: +82.2.820.5488, Email: dhryu@cau.ac.kr.

I Introduction

In the field of empirical financial economics, mean reversion properties of asset prices have been widely investigated to examine the usefulness of the contrarian investment strategy (DeBondt and Thaler, 1985) relative to the momentum investment strategy.

The contrarian strategy implicitly requires asset prices not to permanently deviate from its fundamental value path. When asset prices exhibit such properties, one may obtain excess returns by short-selling assets that have performed well and buying assets with relatively poor past performance. If asset prices are *not* mean-reverting, however, the momentum strategy may apply, that is, investors will need to buy better performing assets, selling assets that perform poorly to obtain excess returns, because deviations of asset prices from its fundamental value path are permanent.[1]

Mean reversion in the context of the international stock markets has been actively investigated; see Kasa (1992) and Richards (1995) for some early work, and Fung (2009), Mylonidis and Kollias (2010), Wälti (2011), Narayan, Mishra and Narayan (2011), Spierdijk et al. (2012), and Kim and Kim (2015) for more recent work on convergence in financial markets.[2] Balvers et al. (2000) employed a panel technique to deal with the mean-reverting hypothesis for 18 countries with well-

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[1] Empirical evidence on mean reversion in US stock prices is at best mixed. For instance, Fama and French (1988) and Poterba and Summers (1988) report some empirical evidence favoring the mean reversion hypothesis for the US stock returns. Many other researchers, however, question the validity/robustness of their findings. See among others, Richardson (1993), McQueen (1992), Kim et al. (1991), Richardson and Stock (1989).

[2] For example, Narayan et al. (2011) report strong evidence of convergence in international stock markets (market capitalization and trading volume) for 4 out of 11 panels using a growth model-type regression framework.

developed stock markets. They find strong evidence in favor of mean reversion.[3,4] Further, they report fairly short half-life estimates for stock index deviations from the fundamental, about 3 years, which supports the usefulness of the contrarian strategy because stock price reversals may occur in the short- or intermediate-term investment horizon.[5]

Kim (2009), however, reports a lot weaker evidence for the usefulness of the contrarian strategy when one controls for serial correlations and cross-section dependence. Further, he reports about 5 and 13 years half-life point estimates after correcting for bias when the MSCI (Morgan Stanley Capital International) World index and the US index are used for a reference index, respectively, which substantially weaken the practical usefulness of the contrarian strategy.[6]

On the other hand, extremely slow convergence rates of asset price deviations from fundamental values imply a superior performance of the momentum strategy over the contrarian strategy. However, Taylor (2001) demonstrates that half-life estimates from linear models are *upward* biased when the true data generating process (DGP) is nonlinear. Nonlinear models have been popularly employed for financial data, because the risks associated with contrarian trading/arbitrage coupled with the transaction costs naturally suggests nonlinear structures, reflecting

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[3] Bhojraj and Swaminathan (2006) find some evidence in favor of the long-run contrarian strategy using stock index data for 38 countries that include both developed and less developed economies.

[4] It should be noted that Balvers et al. (2000) use panel tests that require cross-section independence among the sample countries, which may suffer from severe size distortion problems when the assumption fails to hold (see for example, Phillips and Sul, 2003). Kim (2009) finds very weak evidence for mean-reversion when panel tests that allow cross-section dependence are used.

[5] The term half-life refers to the time period sufficient for the deviations to half-way adjust to its long-run equilibrium value.

[6] Narayan et al. (2011) report fast convergence rates from a regression model with exogenous covariates, which might help increase efficiency of their estimations. Narayan et al. (2015) reports some evidence that the efficient market hypothesis may be day-of-the-week dependent.

the fact that arbitrages occur only when deviations are large enough.[7] See among others, Boswijk et al. (2007), Kim et al. (2009), Chen and Kim (2011), Jawadi and Prat (2012), Kim and Kim (2015) that employ nonlinear models to analyze stock price adjustment dynamics.[8]

To study the persistence properties of nonlinear stochastic models of relative stock prices, we use more general time series concepts of the convergence toward the long-run equilibrium: short-memory-in-mean (SMM) and short-memory-in-distribution (SMD), which is closely related to the statistical notion of ϕ-mixing. SMM was proposed by Granger and Teräsvirta (1993) and Granger (1995) as an alternative to the *linear* notion of stationarity. Granger (1995) argued that SMM and SMD are better measures of persistence in more general models that nest linear models as a special case.

Our nonparametric approach has a number of advantages over other methods. First of all, our method does not require the knowledge on the parametric representation of transition functions nor any distributional assumptions. Hence, our method is less likely to generate mis-specification problems. For instance, since our method nests not only linear but also nonlinear models, our estimates are not subject to the upward bias that was suggested by Taylor (2001). Additionally, our approach provides more general notion for the long-run equilibrium implied by SMD in addition to SMM, which goes beyond the first moment (SMM). Note that

[7] See among others, Michael et al. (1997), Gallagher and Taylor (2001), Taylor et al. (2001), Peel and Taylor (2002), Chortareas et al.(2002), Sarno et al. (2004), McMillan (2008), and Kim and Moh (2010).

[8] These nonlinear models require parametric specifications for the transition function across regimes. Typical choices include the threshold autoregressive model, the exponential smooth transition model, and the logistic smooth transition model.

our method can still apply even when the first moment is not well-defined (e.g., Cauchy distribution).

Even though this approach is potentially very useful, it has been overlooked in the current empirical financial economics literature, because estimation and test methods using the concept of SMM/SMD are not yet well known to the profession. The present paper employs the operational algorithms developed by El-Gamal and Ryu (2006) to investigate general measures of persistence using the notions of SMM and SMD.

Using the MSCI stock indices for the G7 countries from December 1969 to February 2015, we report empirical findings that favor the contrarian investment strategy for France, Germany, and the U.K. relative to the U.S., while relative stock prices of Canada, Italy, and Japan show highly persistent dynamics that requires the momentum strategy. For the first group countries, we find a lot faster convergence rates toward the long-run equilibrium in comparison with those from linear models, which strengthens the usefulness of the contrarian investment strategy.

We also implement a sub-sample analysis for the pre- and the post-1990s periods, which overall confirms our empirical findings for the first group countries that show fast convergence rates in all sample periods. On the other hand, it turns out that very slow convergence rates for Canada and Italy are driven mainly by the post-1990s episode. We also estimate the persistence of relative stock prices for non-G7 developed countries. We obtain overall highly persistent dynamics from these countries. Idiosyncratic (country-specific) shocks in these small countries are likely to have limited or negligible effects on the US stock market. Since these shocks would create asymmetric effects, high persistence of relative stock prices may inherit the persistence of country-specific shocks from these small countries.

Lastly, we report the economic significance analysis via simulation exercise with an array of nonparametric portfolio switching strategies, which provides overall supporting evidence for our empirical results. For example, our exercise results in greater cumulative stock returns from the investment schemes that utilize short-horizon moving averages for the countries we obtain short half-lives such as France and the UK. On the contrary, greater returns were obtained from strategies with longer-horizon moving averages for Canada, Italy, and Japan vis-à-vis the US index, where we obtain weak evidence of convergence in relative stock prices.

The remainder of the present paper is organized as follows. Section 2 presents the baseline model and explains key statistical notions. We also describe our operational algorithms. In Section 3, we describe the data and provide major empirical findings. Section 4 reports an array of robustness check analyses and economic implications of our simulation exercise. Section 5 concludes.

II The Econometric Model

This section presents some useful definitions for our nonparametric model as an alternative to conventional linear models that are often employed in the current empirical financial economics literature. We also provide our nonparametric measures of persistence for general Markovian univariate time series models.

1 Linear Model

We first consider a linear model for the relative stock price as a benchmark model. Let p_t^i and f_t^i be the natural logarithm stock index in country i and its associated fundamental value, respectively. If p_t^i is mean-reverting around f_t^i, that is, if they are cointegrated with the cointegrating vector $[1,-1]$, its stochastic process has the following error correction representation.

$$\Delta\left(p_{t+1}^{i}-f_{t+1}^{i}\right)=a^{i}-\lambda^{i}\left(p_{t}^{i}-f_{t}^{i}\right)+\varepsilon_{t+1}^{i},$$

where $0<\lambda^{i}<1$ is the rate of convergence and ε_{t}^{i} is an idiosyncratic white noise shock. f_{t}^{i} is not directly observable, but it is assumed to have the following stochastic process.

$$f_{t}^{i}=c^{i}+p_{t}^{w}+v_{t}^{i},$$

where c^{i} is an idiosyncratic fixed effect constant, p_{t}^{w} is the reference stock index, and v_{t}^{i} is a white noise process. These two equations jointly imply the following stationary autoregressive process for the relative stock price, $x_{t}^{i}=p_{t}^{i}-p_{t}^{w}$.

$$x_{t+1}^{i}=\mu^{i}+\rho^{i}x_{t}^{i}+\eta_{t+1}^{i},$$

where $\mu^{i}=a^{i}+\lambda^{i}c^{i}$, $\rho^{i}=1-\lambda^{i}$, and $\eta_{t+1}^{i}=v_{t+1}^{i}-\left(1-\lambda^{i}\right)v_{t}^{i}+\varepsilon_{t+1}^{i}$. To put it differently, this equation implies that stock price deviations from the reference index are short-lived and eventually die out. Note that ρ^{i} is a measure of the persistence for x_{t}^{i} in this linear model representation.

Omitting the constant term and superscript i, we may consider the following representation for x_{t} which nests the previous linear model as a special case.

$$x_{t+1}=m(x_{t})+\varepsilon_{t+1}$$

Note that $m(x_{t})$ is the conditional expectation of x_{t+1} at time t given information set. In what follows, we extend this nonlinear representation into a framework that go beyond the first moment.

2 Non-linear and nonparametric model

This section introduces nonparametric measures of persistence for general non-linear models, which are based on the framework proposed by El-Gamal and Ryu (2006) for a first-order Markovian univariate time series $\{x_t\}$. Abandoning linearity in the time domain, we pursue nonlinearity in the density domain instead. From the Chapman-Kolmogorov equation, we define the transition probability kernel and the Markov operator, which can be approximated by a finite transition matrix. We also employ the consistent tests of ergodicity and mixing proposed by Domowitz and El-Gamal (1993, 1996, 2001) to our relative stock index data.

We define the short memory in distribution (SMD) and the short memory in mean (SMM) as stated in El-Gamal and Ryu (2006).

Definition 1. The time series is said to have *Short Memory in Distribution* (SMD) if $F_s(x) \Rightarrow \overline{F}(x)$, as $s \uparrow \infty$ where $F_s(x) = \Pr(x_{t+s} \le x \mid \mathrm{A}_t)$ is the cumulative distribution function of x_{t+s} conditional on the past information set $\mathrm{A}_t = \sigma(x_{t-j}; j \ge 0)$, and $\overline{F}$ is a fixed (unconditional) distribution function.

Definition 2. The time series is said to have the *Short Memory in Mean* (SMM) property if $\| E[x_{t+s} \mid \mathrm{A}_t] - E[x_{t+s}] \| < c_s;\ c_s \xrightarrow{s\uparrow\infty} 0$.

Note that SMM is equivalent to *mixing in mean* or *mixingales* as discussed in McLeish (1978) and Gallant and White (1988), while SMD shares a property of mixing.

We use the asymptotic independence notion of uniform or ϕ-mixing to study SMD and SMM. As shown by El-Gamal and Ryu (2006), we can calculate the SMD and SMM numerically. That is, we can get the finite grid analog $\phi_n(s)$ which converges to $\phi(s)$ as the grid size $n \uparrow \infty$. Similarly, we can also get the *grid* $MDM_n(s)$ which converges to the Maximum Distance in Mean, $MDM(s)$, the measure of SMM, as the grid size $n \uparrow \infty$. We provide detailed explanations on the numerical algorithms to compute our persistence measures in Appendix.[9]

The notion of half-life can now be replaced by the value of s at which $MDM_n(s) = 0.5 \times MDM_n(0)$, that is, the number of periods needed for the worst possible transitory shock from the unconditional mean to be cut in half. This notion may then be extended beyond half-life to consider *Max m-life* as the number of time periods before the worst possible shock would shrink to (1-*m*) of its original magnitude. For instance, we may define *Max quarter-life* by the number of time periods before the worst possible shock would shrink to 0.25, i.e., *m* = 0.75 of its initial one unit shock.[10]

For non-parametric estimations of $P_{T,n}$ using a kernel estimator, we begin with the estimated $\phi(s)$ and *Max m-life* using Silverman's rule of thumb: $h_T = \sigma_T \; T^{-1/5}$, where σ_T is the standard deviation of our series. The estimated *Max m-life* with this bandwidth selection rule typically implies substantially less persistent dynamics. However, as El-Gamal and Ryu (2006) show, such results may

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[9] See El-Gamal and Ryu (2006) for more detailed description about the numerical calculation and convergence arguments of finite grids of SMD and SMM.

[10] This metric is an extension of the quarter-life that is introduced by Steinsson (2008), which is based on linear regression models. This additional measure of persistence can be used to see if the convergence takes place monotonically.

not be reliable because this selection rule tends to produce over-smoothed estimates of the transition density, which may generate downward bias in the estimates of $\phi(s)$ and *Max m-life*. Therefore, the rule of thumb tends to generate empirical support in favor of the contrarian strategy, which may not be robust to the bandwidth choice.

Realizing this issue, we implement estimations for an array of the choice of the level of under-smoothing, k. That is, we modify Silverman's rule of thumb as follows.

$$h_T = \left(\frac{\sigma_T}{k}\right) T^{-1/5}$$

And we report our estimation results for k ranging from 1 to 10. We note that our estimates for $\phi(s)$ (or *Max m-life*) often converge each other as k approaches to 10. We interpret such results as strong empirical evidence of SMM/SMD, which support the contrarian investment strategy.

III Empirical Results

1 Data and Summary Statistics

We use monthly frequency data from Morgan Stanley Capital International (MSCI) for stock market indices for the G7 countries: Canada, France, Germany, Italy, Japan, the UK, and the US.[11] We also obtain stock indices for 11 non-G7 developed

[11] Sampling frequency may influence statistical inferences. See, for example, Narayan and Sharma (2015) and Boswijk and Klaasseen (2012). Empirical findings with annual frequency data overall produced qualitatively similar results.

countries from the same source.[12] Observations span from December 1969 to February 2015. We use end-of-period observations rather than average data to avoid a time aggregation bias (Taylor, 2001). All stock prices are value-weighted index prices in the US dollar that include gross dividends. See Figure 1 for the log transformed stock indices.

Table 1 provides basic summary statistics of national stock index deviations from the US index as the reference index (e.g., Balvers et al., 2000; Kim, 2009). The mean values of the national index deviations relative to the US index range from −1.013 for Italy to 0.742 for Japan, and the standard deviations vary from 0.215 for the UK to 0.706 for Japan. Half of the deviation series have negative skewness, while for the rest the right tail is more pronounced than the left tail. All deviations have a leptokurtic (high peak) distribution. The Jarque-Bera statistic implies a non-normal distribution for all series at the 95% significance level.[13] Overall, these results support our nonparametric approach to study the relative stock index dynamics.

Figure 1 and Table I around here

2 Linear Model Estimates

As a benchmark, we report persistence estimates from a linear model, the augmented Dickey-Fuller regression model, in Table II. Since the least squares estimate for the persistence parameter is downward biased, we correct for the

[12] Among MSCI's 23 developed markets (DM) countries, we excluded Finland, Ireland, Israel, New Zealand, and Portugal due to lack of observations.

[13] We use the critical values from Deb and Sefton (1996) to deal with a size distortion problem using an asymptotic chi-square distribution with two degrees of freedom.

median bias by grid bootstrap method (Hansen, 1999) with 10,000 nonparametric simulations on each of 51 fine grid points in the vicinity of the least squares point estimate.[14]

Our estimates are roughly consistent with relative stock price graphs shown in Figure 2. For instance, the national stock index of Canada and Japan relative to the US index exhibit more persistent movements compared with other relative stock indices that show more frequent price reversals/adjustments. Such eyeball metric is confirmed by much longer half-life estimates for Canada and Japan than those for France, Germany, and the UK. We noticed that dynamics of Italy's relative stock price becomes more persistent when we include recent observations after the financial crisis. Recall that the US stock index continued to outperform Italy's, as Italy shows not much of recovery since the late 2000s.

Overall, these half-life estimates support the momentum investment strategy rather than the contrarian strategy, because most point estimates imply very sluggish adjustments toward the long-run equilibrium under the linear stochastic model framework. The shortest half-life point estimate is over 2 years for Germany vis-à-vis the US index. The longest one is the case of Japan (and practically Canada as well) relative to the US, where the half-life point estimate is positive infinity. Further, median bias-corrected 95% confidence intervals are very wide and extend to positive infinity for all relative stock prices.

We note, however, that this seemingly slow rate of adjustment does not necessarily imply strong support against the use of the contrarian investment strategy. As Taylor (2001) points out, if the true DGP is nonlinear, statistical

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[14] Kim (2009) employs a similar framework for general autoregressive processes, AR(p), whereas Balvers et al. (2000) use Andrew's (1993) parametric bootstrap method for the AR(1) process.

inferences on the persistence of the stochastic process via linear models may become invalid because the persistence parameter estimate is upward biased.

Figure 2 around here

Table II around here

Our persistence measure estimates are not subject to this type of bias because our framework allows any type of Markovian transition functions, nesting linear models as a special case. In what follows, we report substantially shorter half-life estimates for 3 out of 6 relative stock prices in comparison with those from linear models, which favors the contrarian investment strategy, while we find empirical support for the momentum strategy for the other three relative stock prices.

3 Nonparametric Model Estimates

This section provides nonlinear statistical test results of mean reversion in terms of ergodicity and mixing via the methods proposed by Domowitz and El-Gamal (2001) for our relative stock index variables. Results are reported in Table III. We account for nonlinearity in the process describing the evolution of relative stock market returns through nonparametric estimations of the transition densities that underlie our test statistics. This circumvents the problem of taking a stand with respect to exact parametric specifications of the model, and eliminates the need for extra computations of critical values.

The mean reversion property is reformulated in terms of general ergodic failure, as opposed to the specific case of a unit root process. This is more appropriate in nonlinear settings, given the essential linearity of unit root analyses, as discussed in Granger (1995). The test rejects the null hypothesis of ergodicity if the p-value of a single randomized test is smaller than a pre-specified value. We then determine rejections of the ergodicity null by the percentiles of the density of p-values which are less than or close to a pre-specified number, e.g., 5% or 10%.

Applying those tests, we reject the null hypothesis of ergodicity for Japan as shown in Table III, since the percentiles of the density of p-values are substantially above 5% and 10%. Also, we have weak support for the null when the test applies to France, which may reflect a size distortion problem (Domowitz and El-Gamal, 2001). For Canada, Germany, Italy, and the UK, the percentiles are close to 5% and 10%, respectively, so we conclude that the test fails to reject the null hypotheses of ergodicity. These test results overall show that the mean reversion took place for cases of Canada, Germany, Italy, and the UK, which imply relative usefulness of the contrarian strategy, but not for the remaining two countries. Although the results are somewhat consistent with other research like Kim (2009) under the linear model framework, we note that this progression does not presume linearity of the underlying process at the outset, which may prove useful in nonlinear settings.

In contrast, our mixing test supports the null hypothesis for all countries, because the estimated percentiles are close to 5% and 10% for all cases. That is, we conclude that the randomized test fails to reject the null hypotheses of mixing. However, we know from Monte Carlo results of Domowitz and El-Gamal (2001) that for a sample size as small as $T = 499$ our estimates of P_n^t might be over-smoothed, thus these tests may suffer from low power. Therefore, we need to employ additional approaches to get a complete argument of mean reversion properties in stock market returns.[15]

Table III around here

Next, we report our Max half-life (MHL) estimates as well as Max quarter-life (MQL) for the smoothing parameter (k) ranging from 1 to 10 to see how robust our estimates are.

We first report our estimates for the SMM (mixingale) property in Table IV. We note that MHL and MQL estimates tend to increase as k goes up. However, MHL estimates from France, Germany, and the UK exhibit convergence as k approaches to 10. That is, as we can see in Figures 3 through 8, the MHL with $k = 10$ becomes an upper limit for these countries, while the MHL is not well-defined for Canada, Italy, and Japan even when $k = 10$.

Similarly, the MHL is not well-defined for those three countries when we investigate the persistence based on the SMD property (Table V), while we obtain well-defined MHL for the other three countries. We again observe convergence for all normalized *MDM(s)* and $\phi(s)$ as k approaches to 10. Therefore, we do not think it is necessary to try estimations with more values for k.

These findings suggest very strong support for the contrarian investment strategy for France, Germany, and the UK against the US, while the momentum

[15] Linear unit root tests such as the augmented Dickey Fuller test failed to reject the null of nonstationarity, which might reflect the fact that linear tests have low power when the true DGP is nonlinear. Test results are available upon request. Alternatively, one may use nonlinear tests such as Lee and Strazicich (2003) and Narayan and Popp (2010), if one believe that the DGP is nonlinear in the time domain. For tests with state-dependent nonlinearity, the inf-t test by Park and Shintani (forthcoming) might be employed.

strategy is supported for Canada, Italy, and Japan vis-à-vis the US stock index. It should be noted that unlike the results from linear models that often suggest very sluggish adjustment rates, our nonparametric measures imply quite fast speeds of adjustment which support the contrarian strategy whenever MHLs are well-defined. For instance, the MHL for the UK vs. the US ranges from 4 to 16 months for both SMM and SMD, which sharply contrasts with estimates from our linear model that extends to positive infinity (Table II).

We also investigate whether adjustments exhibit a non-monotonic pattern, employing a metric proposed by Steinsson (2008) for linear models. Note that MHL should equal to MQL – MHL if the adjustment takes place monotonically. This idea can be formulated by 2MHL minus MQL. We report estimates for this metric in Tables IV and V. We obtain mostly negative values when convergence is made, which implies a slower adjustment in the second half than in the first half.[16]

Tables IV and V around here

Figures 3 through 8 around here

IV Robustness Check and the Economic Significance Analysis

1 Persistence Estimations: Non-G7 Developed Markets Countries

This sub-section presents our nonparametric estimation results of the persistence in relative stock prices for Non-G7 MSCI developed markets (DM) countries relative to the US. We obtain monthly frequency national stock price data from MSCI for 11

[16] Steinsson (2008) reports mostly positive estimates using the US real exchange rate data, which may be consistent with hump-shape dynamics.

non-G7 countries from December 1969 to February 2015. Among MSCI's 23 DM countries, Finland, Ireland, Israel, New Zealand, and Portugal are excluded due to lack of observations. Results are reported in Table VI. To save space, we report MHL estimates only with the SMM property as we obtained similar results with the SMD notion.

We note that most MHL estimates are substantially longer than those from the G7 countries reported in previous section. The shortest estimate was around 2-year for Switzerland when $k = 10$. MHL estimates were longer than 4 years for 8 out of 11 countries. And among those countries, we fail to see meaningful convergence of half-lives when k increases to 10. That is, relative stock prices of these smaller economies vis-à-vis the US stock index seem to exhibit overall high degree persistence.

Such high persistence might inherit that of the idiosyncratic component in these small economies. Unlike the G7 countries, the rest of the world is not likely to respond much to perturbations/shocks originated from these small countries, meaning that idiosyncratic shocks from these small economies would be somewhat isolated within the countries without creating similar effects in the rest of the world. Highly persistent idiosyncratic shocks, therefore, will create long-lasting asymmetric effects to the national stock index relative to the US stock price, and will generate sluggish adjustments towards the new equilibrium, if any.

Table VI around here

2 Sub-Sample Analysis

This sub-section reports a robustness analysis with respect to alternative sample periods. Since we do not have mixing-based econometric techniques that detect the existence of potential structural breaks, and because our empirical framework requires sufficiently large number of observations, we take an ad hoc approach by splitting the entire sample period into two sub-sample periods: the pre-1990s and the post-1990s periods.[17] Results are reported in Table VII. Again, we report MHL estimates with SMM only in order to save space.

Overall results are quite similar for France, Germany, Japan, and the UK. One caveat is that these results may not be reliable due to small samples because we are using only 50% of entire observations. For example, we obtain a little shorter MHL estimates from Japan in each of these sub-samples (and convergence) than the one from the full sample. This may reflect a small sample bias that is reported by El Gamal and Ryu (2006). However, we still obtain a lot shorter MHL estimates from France, Germany, and the UK than those from Canada, Italy, and Japan especially in the post-1990s period, which confirms our overall findings.

Another interesting finding is that high degree persistence of relative national stock prices in Canada and Italy might have been driven mainly by the post-1990s episode. For Italy, recent episodes of the European sovereign debt crisis in the late 2000s (PIGS, that is, Portugal, Italy, Greece, and Spain) and accompanying financial market meltdown might explain substantially more persistent deviations of the national stock price relative to the US stock index during the post-1990s sub-sample period. In Canada, we note a big downward swing of the national stock index since the 1990s (see Figure 2), which surely contributes to high persistence of the relative

[17] We thank an anonymous referee and the editor who suggested this robustness check.

stock price in Canada. One may also point out a big swing in the Canadian dollar exchange rate relative to the US dollar observed in the post-1990s. This also contributes greatly to high persistence of the relative stock prices because all stock prices in our study are denominated in the US dollar.

Table VII around here

3 Portfolio Switching Simulations and the Economic Significance Analysis

This section presents portfolio switching simulation exercise to see whether our proposed models yield economically useful implications on portfolio choice strategies.

We consider the following nonparametric investment strategy which is based on an m-month backward moving average. Starting with a fixed amount investment, say \$1, we buy the national stock whenever the following condition holds.

$$x_t \leq \mu_t = \frac{1}{m}\sum_{j=t-m}^{t-1} x_j,$$

where $x_s = p_s^i - p_s^{US}$ is the log stock price differential relative to the US stock price. At time t, if the current national stock price relative to the US price (x_t) is less than its conditional expectation (first moment), proxied by an m-month backward moving average, we expect a price reversal to occur toward the national stock, so we buy the national stock selling all existing stocks. We buy the US stock otherwise. At time $t+1$, we repeat the same strategy to construct a new portfolio by selling all currently holding assets and buying new assets following this strategy. Note that this investment strategy resembles the nonparametric contrarian strategy proposed by Debondt and Thaler (1985) who used a 3-year moving average. We report and compare cumulative returns from competing strategies in Figures 9 through 11.

We consider 5 alternative investment strategies for $m = 12, 36, 60, 120, 240$, that is, portfolio choice strategies with 1- through 20-year backward moving averages. Note that shorter half-lives (less persistent dynamics) imply a better performance when portfolio choice strategies are combined with shorter-period backward moving averages. On the contrary, more conservative buy and hold type investment strategies should work better when the relative stock price shows a very persistent movement, because price reversals are less likely to occur.

Our half-life estimates from linear models (Table II) are about 3 years for France, Germany, and the UK, while we obtain a lot longer half-lives for Canada, Italy, and Japan. On the other hand, half-lives from our nonlinear and nonparametric models (Tables IV and V) range from about a half-year to one and a half-year for the first group countries, while a lot longer half-lives were observed from the rest of the G7 countries. Therefore, we conjecture the portfolio switching schemes with a 1-year ($m = 12$) or 3-year ($m = 36$) would be consistent with the contrarian strategy based on our nonparametric models for the first group countries, while the portfolio strategies with longer-horizon moving averages, say, $m = 120, 240$, would work better for Canada, Italy, and Japan.[18]

In Figure 9, we report cumulative stock returns from the portfolio switching strategies with a 1-year moving average (solid line) and with a 10-year moving average (dashed line). Since we implement simulations with up to a 20-year moving average, we compare the portfolio performances starting in January 1990 when the

[18] Half-life estimates from linear models would suggest portfolio choice schemes with long-horizon moving averages for all countries.

first 20-year moving average becomes available. Our simulations demonstrate that the 10-year portfolio switching strategy performs better for Canada, Italy, and Japan, whereas the 1-year strategy performs better for France and the UK.

Note that these results are consistent with our empirical findings in Tables IV and V that provide supporting evidence of the momentum strategy for Canada, Italy, and Japan. Recall that the empirical evidence of reasonably short half-life estimates for France and the UK, which suggest a more frequent portfolio switching strategy such as the one with a 1-year moving average. Germany is at odds with this prediction based on our empirical findings because the 10-year strategy outperforms the 1-strategy, even though we obtained reasonably short half-life estimates.

Figures 9 around here

Figure 10 reports cumulative returns from the 10-year strategy (dashed) and from the 3-year strategy (solid), a little longer period portfolio choice scheme than the 1-year strategy. We obtained similar supporting evidence in favor of our empirical findings for Canada, Italy, Japan as well as France and the UK. We note that cumulative returns for Germany are similar each other in this exercise as we use an investment strategy with a little bit longer-horizon moving average, 3-year instead of 1-year. This result seems to be consistent with our work that show longer half-lives for Germany in comparison with those for the UK and France as *k* becomes close to 10. We also compare the returns from investment strategies with the 5-year (solid) and the 20-year (dashed) moving averages. See Figure 11 for results, which again confirm our empirical findings. We obtain the portfolio switching scheme with a 20-year moving average outperforms the one with a shorter 5-year moving average for Canada, Italy, and Japan, whereas opposite results were observed for France and the UK. Again, we obtain a little mixed evidence for Germany.

Figures 10 and 11 around here

V Concluding Remarks

We revisit the usefulness of the contrarian investment strategy relative to the momentum strategy in international stock markets. Previous studies that employ linear stochastic models often provide fairly weak empirical support for the contrarian strategy, finding very persistent dynamics of relative stock prices.[19]

This paper employs a nonlinear, nonparametric stochastic model of relative international stock prices that utilizes two statistical notions: the short memory in mean (SMM) and the short memory in distribution (SMD). This allow us to use very general measures of persistence that avoid potential upward bias that arises when linear models are used even though the true DGP is nonlinear (Taylor, 2001).

Using monthly frequency stock prices from G7 countries, we obtain favorable empirical evidence supporting the contrarian strategy for France, Germany, and the UK. For these countries, we find reasonably short MHL estimates for both SMM and SMD that are robust to the choice of bandwidth. For the rest of G7 countries, Canada, Italy, and Japan, relative to the US, we report empirical results that favor the momentum strategy as we find weak evidence of SMM and SMD.

[19] This may occur when the stochastic process exhibits a (near) unit-root process, when linear models are employed.

The robustness check analysis confirms our empirical findings for the first group countries while highly persistent relative stock price dynamics in Canada and Italy mainly come from the post-1990s episodes when we often observe wide swings in the national stock markets and foreign exchange markets. We observe highly persistent MHL estimates for non-G7 developed countries vis-à-vis the US index. Idiosyncratic (country-specific) perturbations from these countries tend to have limited or negligible effects on the rest of the world including the US. If country-specific shocks are highly persistent, relative stock prices, therefore, will inherit such characteristics, because idiosyncratic shocks are likely to generate asymmetric effects across countries.

Our simulation exercises for portfolio switching schemes show overall supporting evidence for our empirical findings. For example, short MHL estimates from our empirical findings imply more frequent price reversals in investment opportunities between the US and France, while price reversals are less likely to occur for Japan and the US as we see virtually no evidence of convergence. Our simulation exercises imply greater stock price returns when one employs investment strategies according to such expectations. To put it differently, our economic significance analysis overall confirms the practical usefulness of our empirical models.

Appendix: Numerical algorithms for computing *Max m-life* and $\phi(s)$ functions

We present numerical algorithms used in this paper for computing our measures of SMD and SMM for known models and transition density estimates. We begin with the assumption of having a known transition matrix $P_n(.,.)$ on an $n \times n$ grid.

Algorithm A: $MDM_n(s)$

1. Fix the grid $\boldsymbol{x} = (x_1, ..., x_n)$.
2. Compute the invariant measure f_n^* by iterating on $P_n^s f$ for any initial f, and $s = 1, 2, ...$, until convergence (in the sup norm) is made.
3. Compute the unconditional expectation $\mu = E_{f_n^*}[x_{t+s}] = \boldsymbol{x}' f_n^*$.
4. Define $MDM_n(0) = \max(\mu, 1-\mu)$.
5. For each s, and each point on the grid $\{x_i\}_{i=1}^n$, compute the conditional expectation $\mu_i(s) = E_{P_n^s \delta\{x_i\}}[x_{t+s} \mid x_t = x_i] = \boldsymbol{x}' P_n^s \delta_{\{x_i\}}$. Then, compute $MDM_n(s) = \max_i(|\mu_i(s) - \mu|)$.
6. Normalize $MDM_n(s)$ by defining $1 - m_n(s) = MDM_n(s) / MDM_n(0)$.
7. Plot *Max m-life* as against $(1 - m_n(s))$.

Algorithm B: $\phi_n(s)$

1. Perform steps 1-2 of Algorithm A.
2. For each s, and each point on the grid $\{x_i\}_{i=1}^n$, compute $\phi_{n,i}(s) = \| f_n^* - P_n^s \delta_{\{x_i\}} \|$.
3. Set $\phi_n(s) = \max_i(\phi_{n,i}(s))$.
4. Plot $\phi_n(s)$ against *s*.

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Figure 1. MSCI National Stock Prices vs. US Stock Price (dashed line)

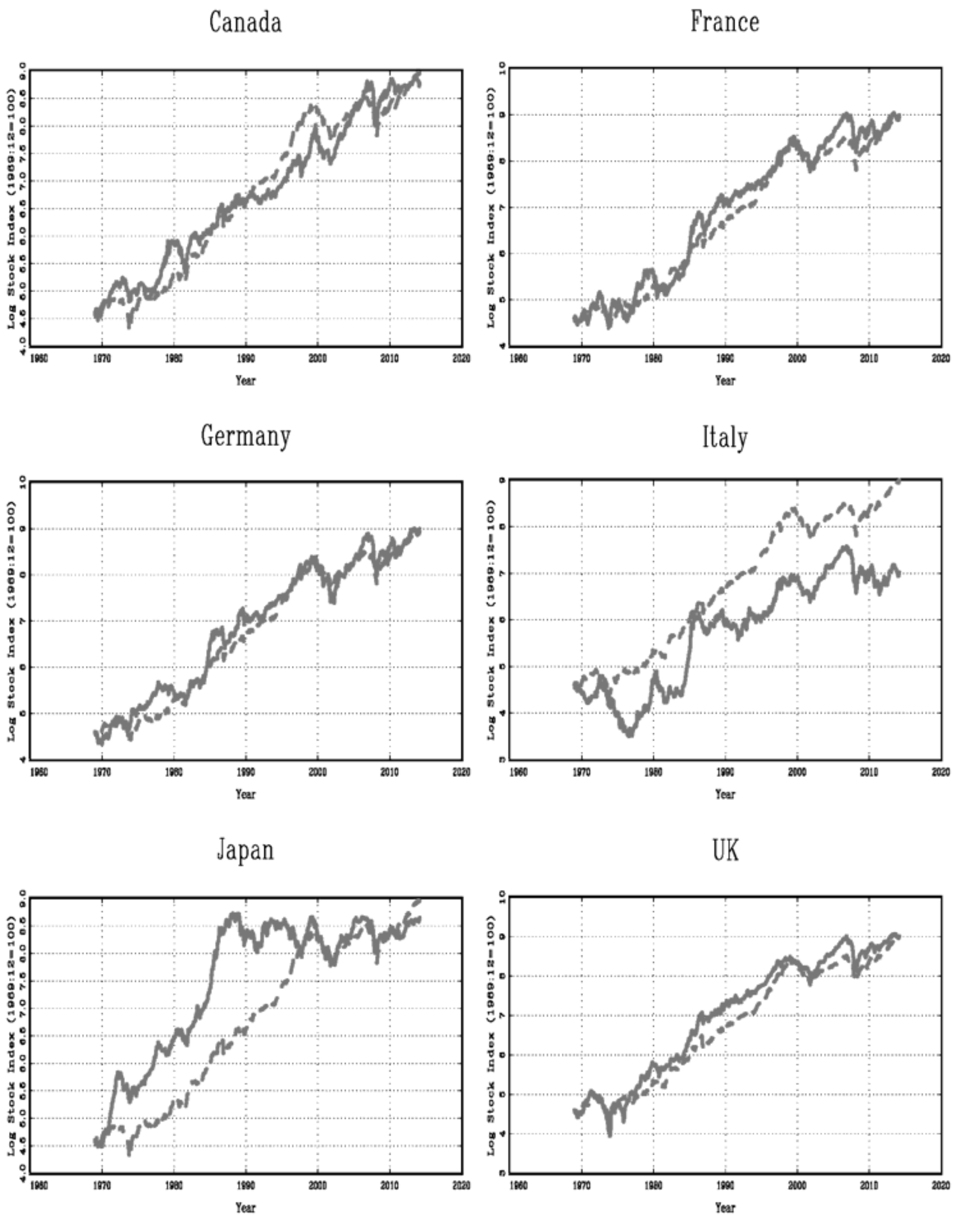


Figure 2. Log National Stock Prices relative to US Stock Price

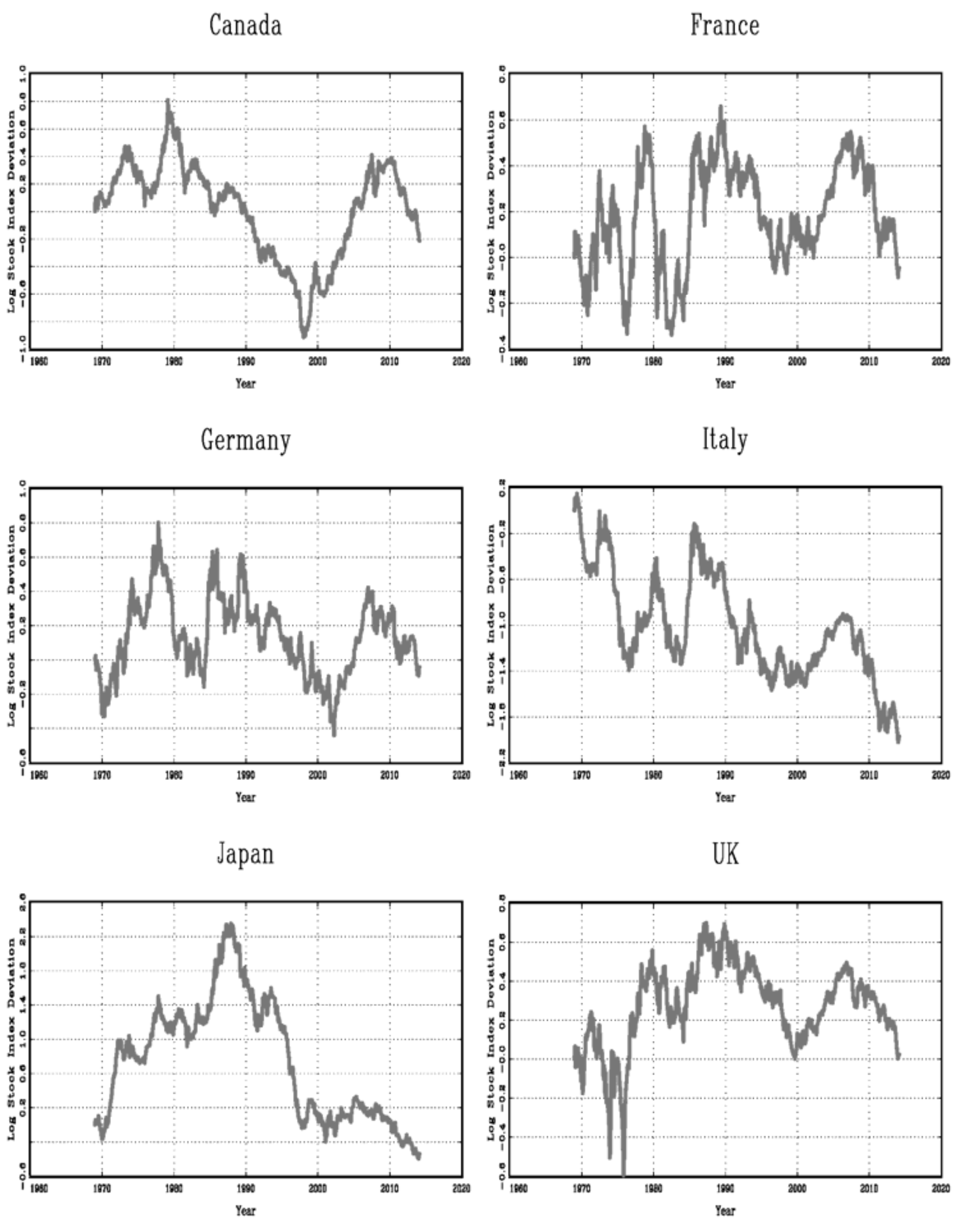
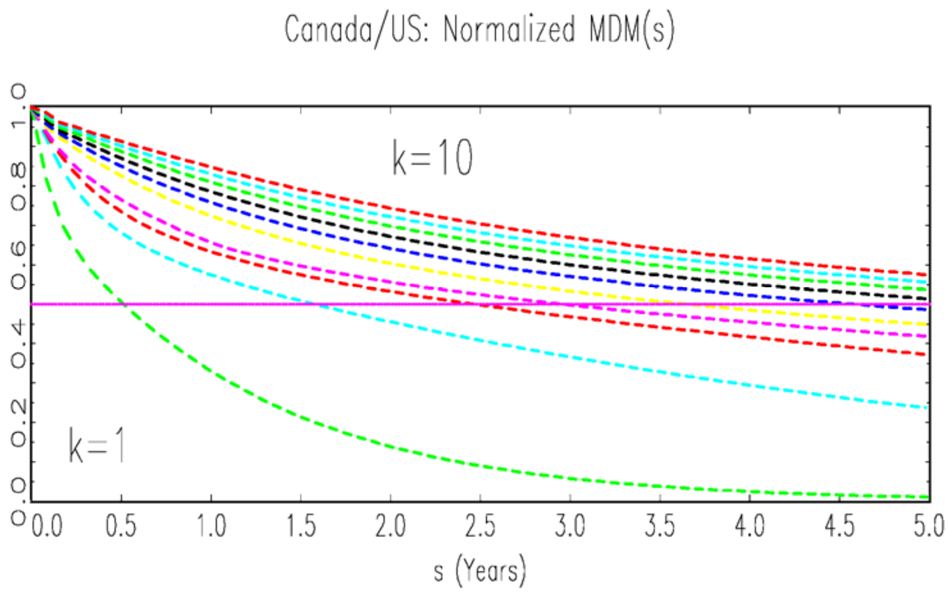


Figure 3. SMM and SMD Properties: Canada vs. US



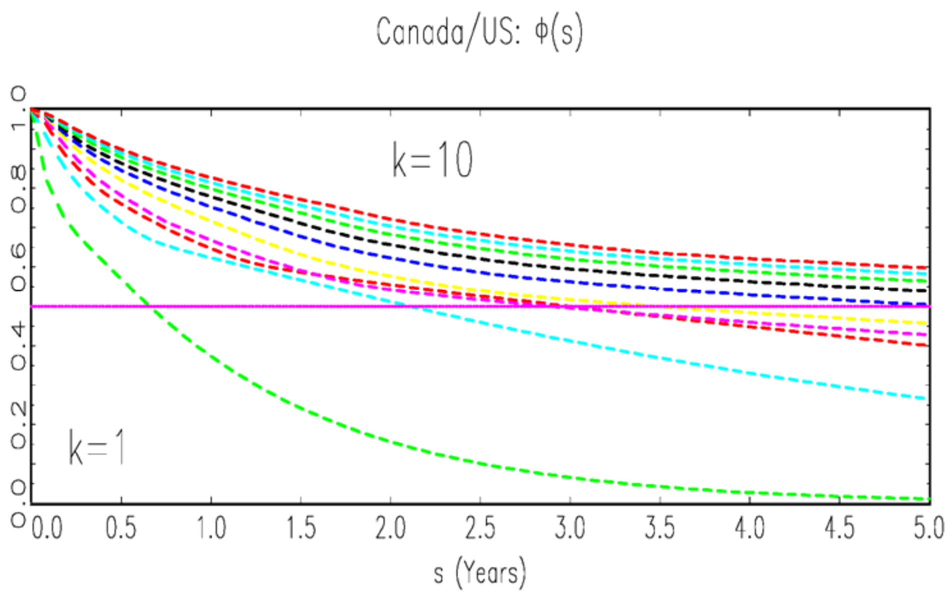
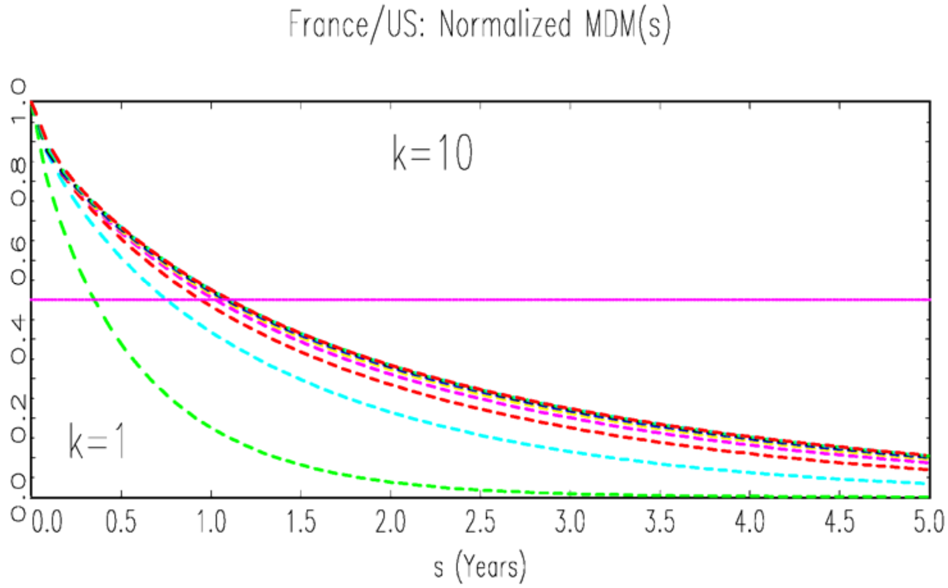


Figure 4. SMM and SMD Properties: France vs. US



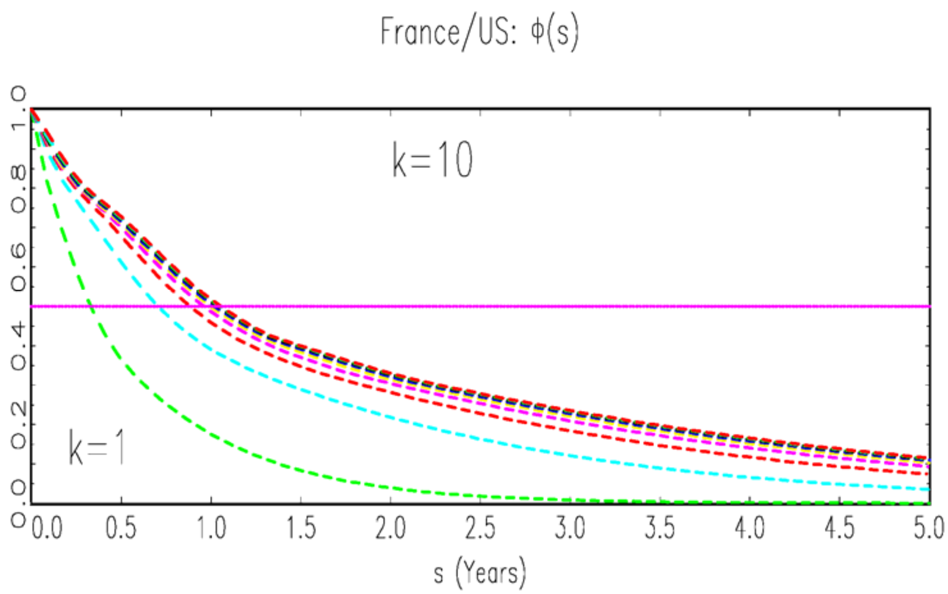
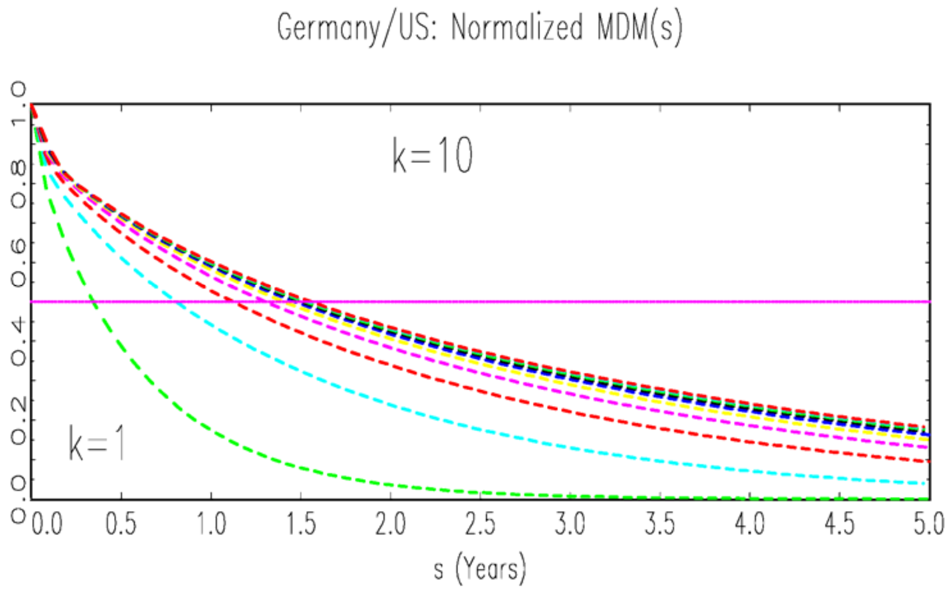


Figure 5. SMM and SMD Properties: Germany vs. US



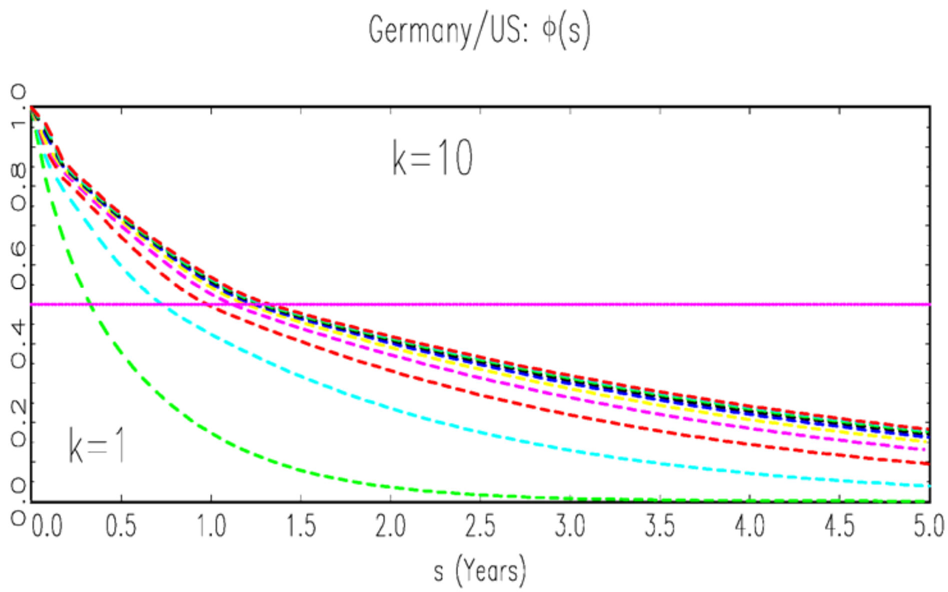


Figure 6. SMM and SMD Properties: Italy vs. US

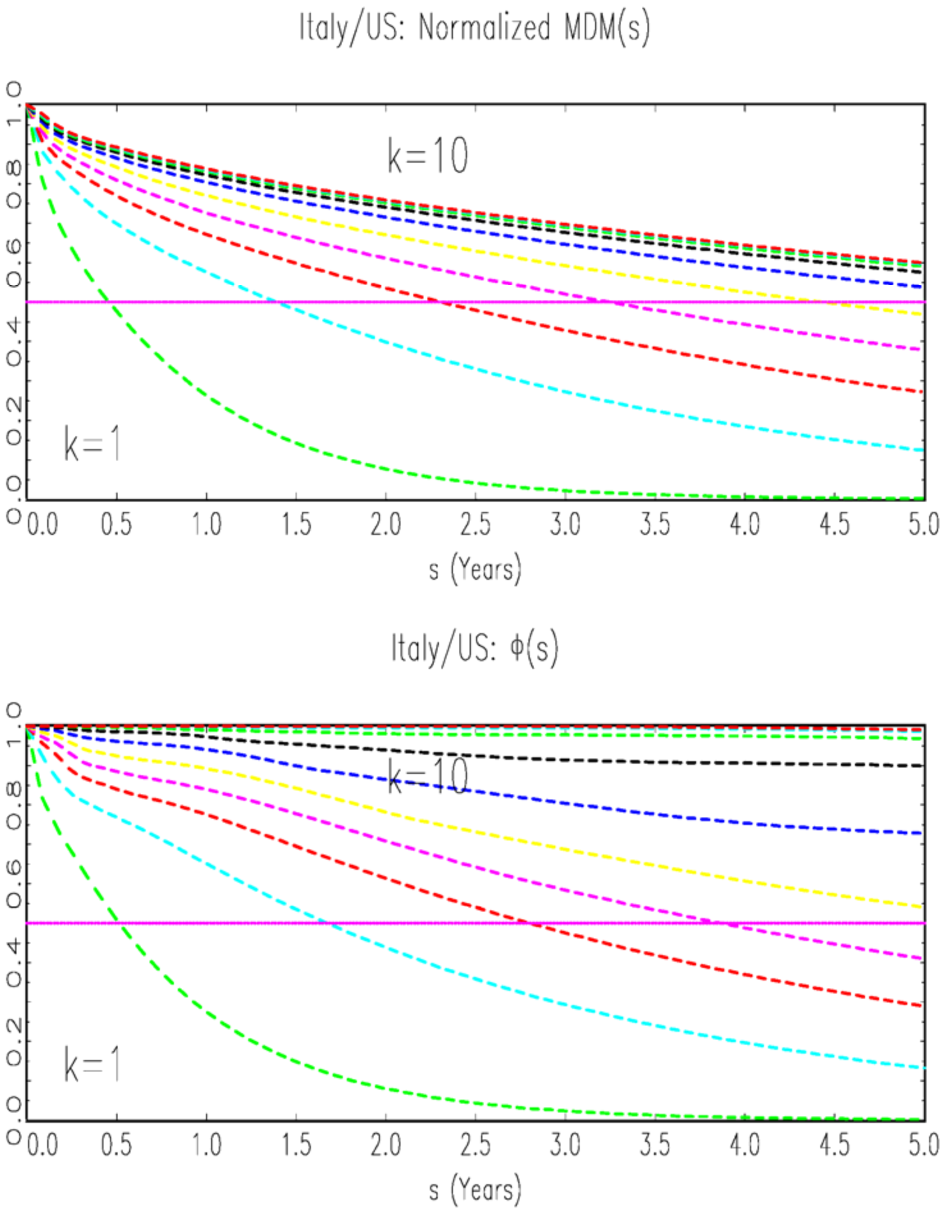
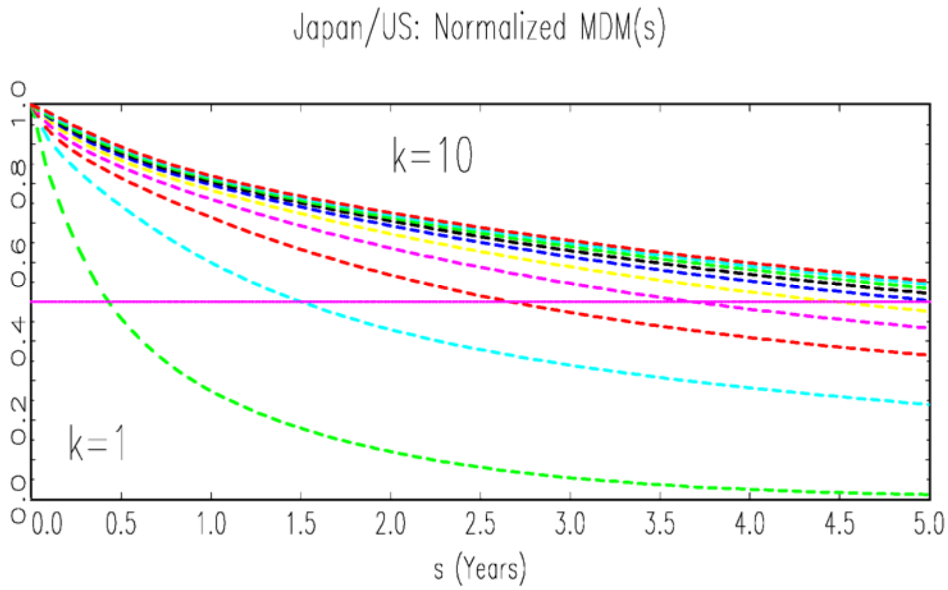


Figure 7. SMM and SMD Properties: Japan vs. US



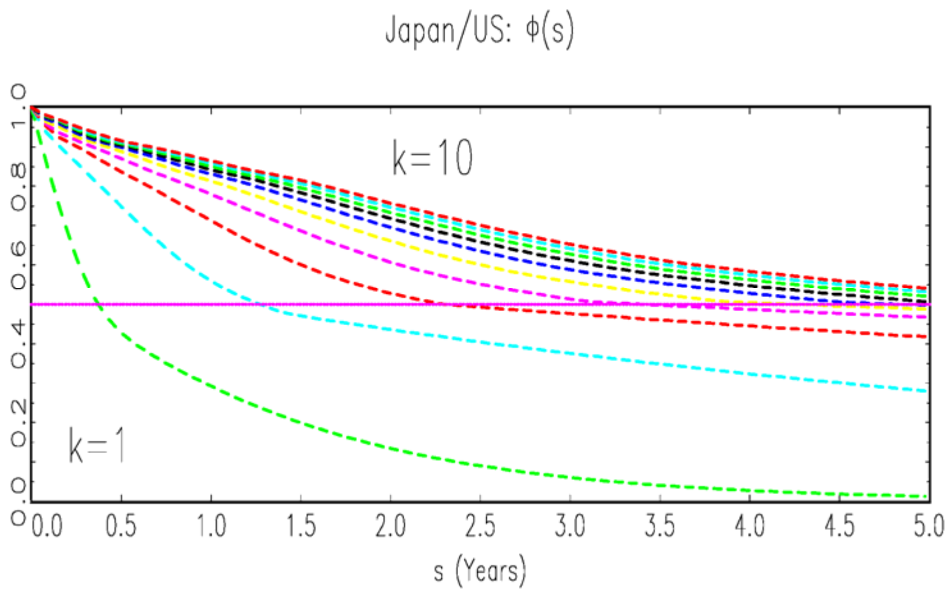
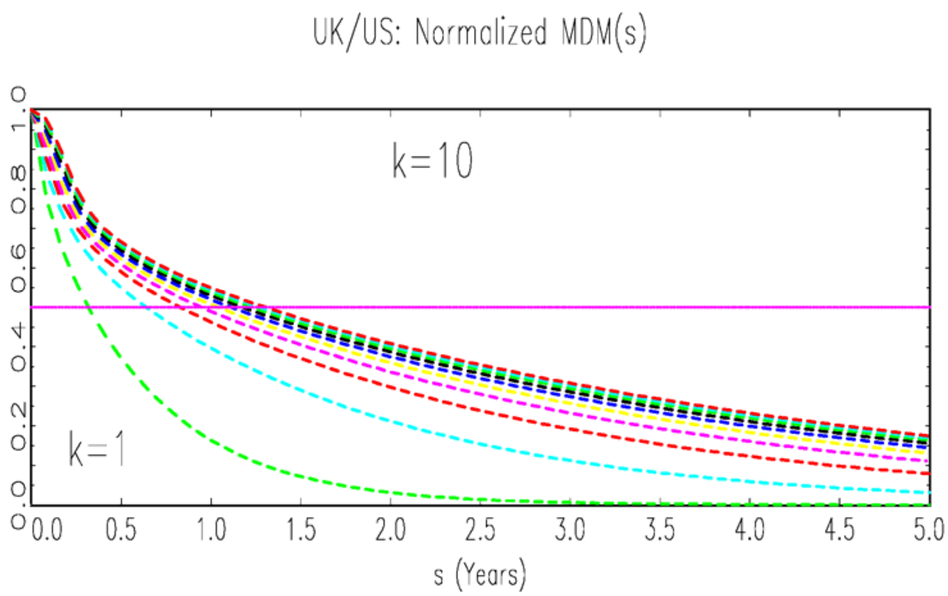


Figure 8. SMM and SMD Properties: UK vs. US



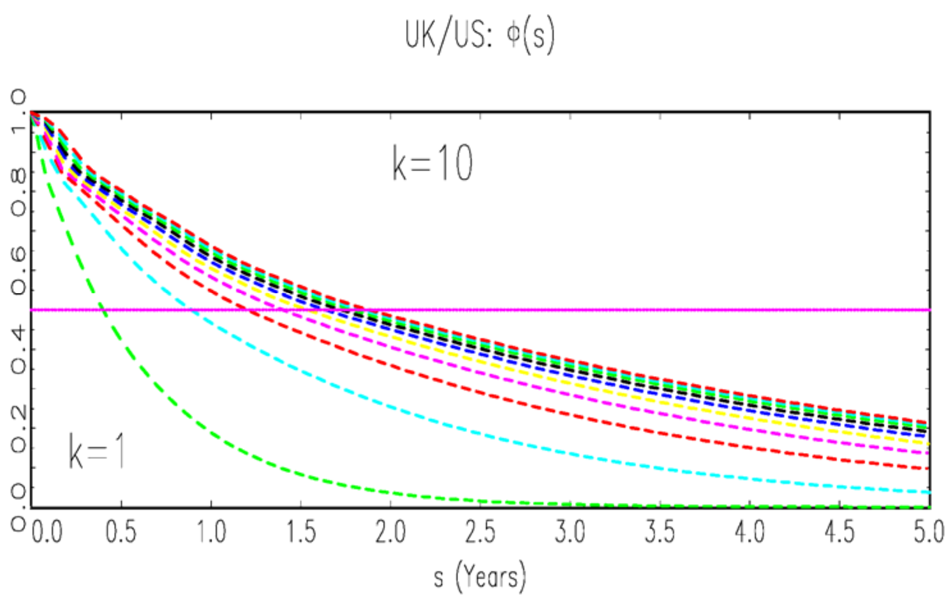


Figure 9. Simulation Exercises: 1-Year vs. 10-Year Rules

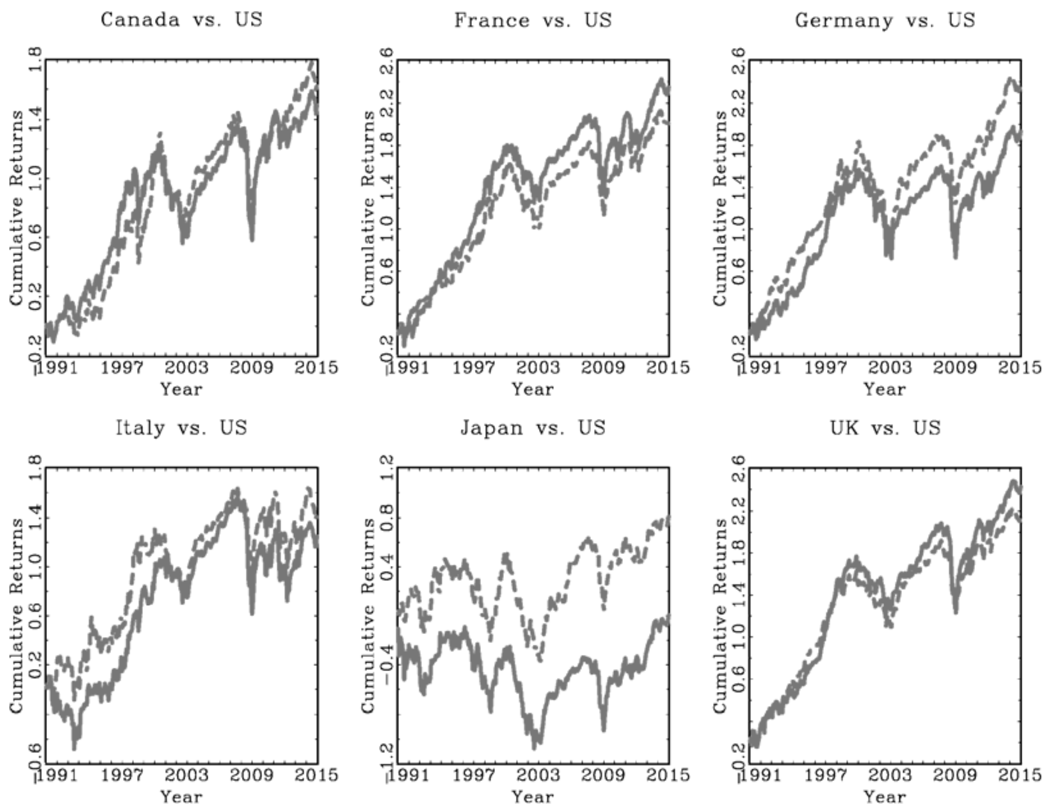


Figure 10. Simulation Exercises: 3-Year vs. 10-Year Rules

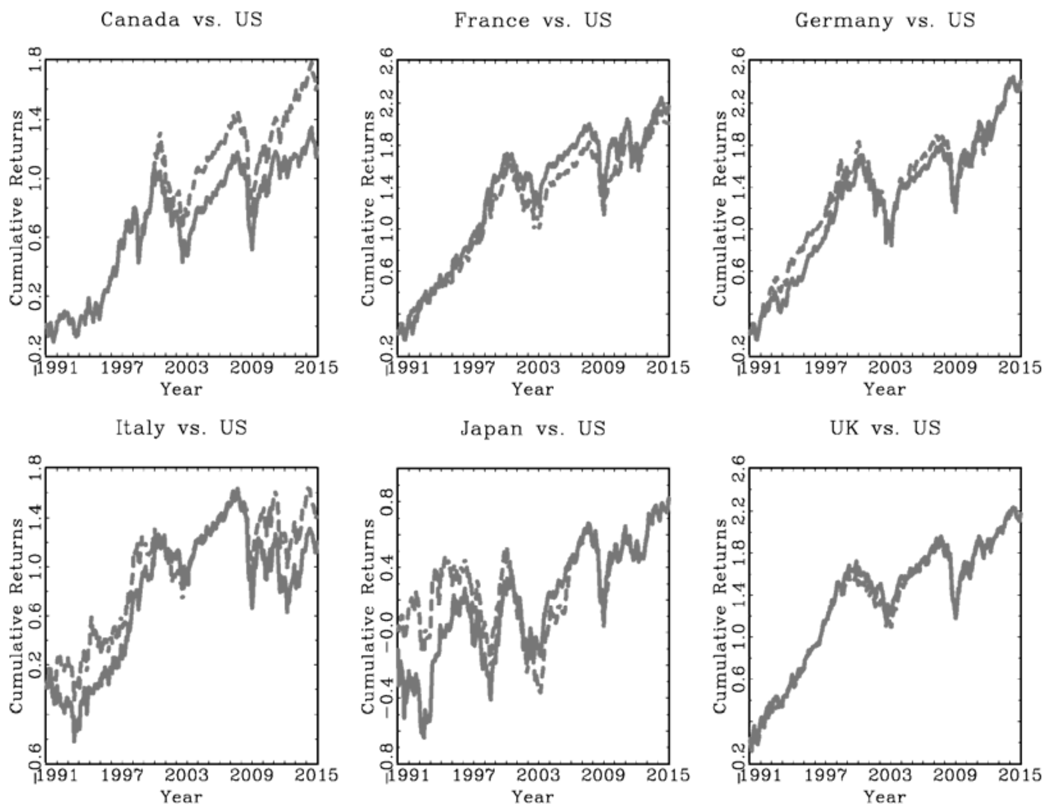


Figure 11. Simulation Exercises: 5-Year vs. 20-Year Rules

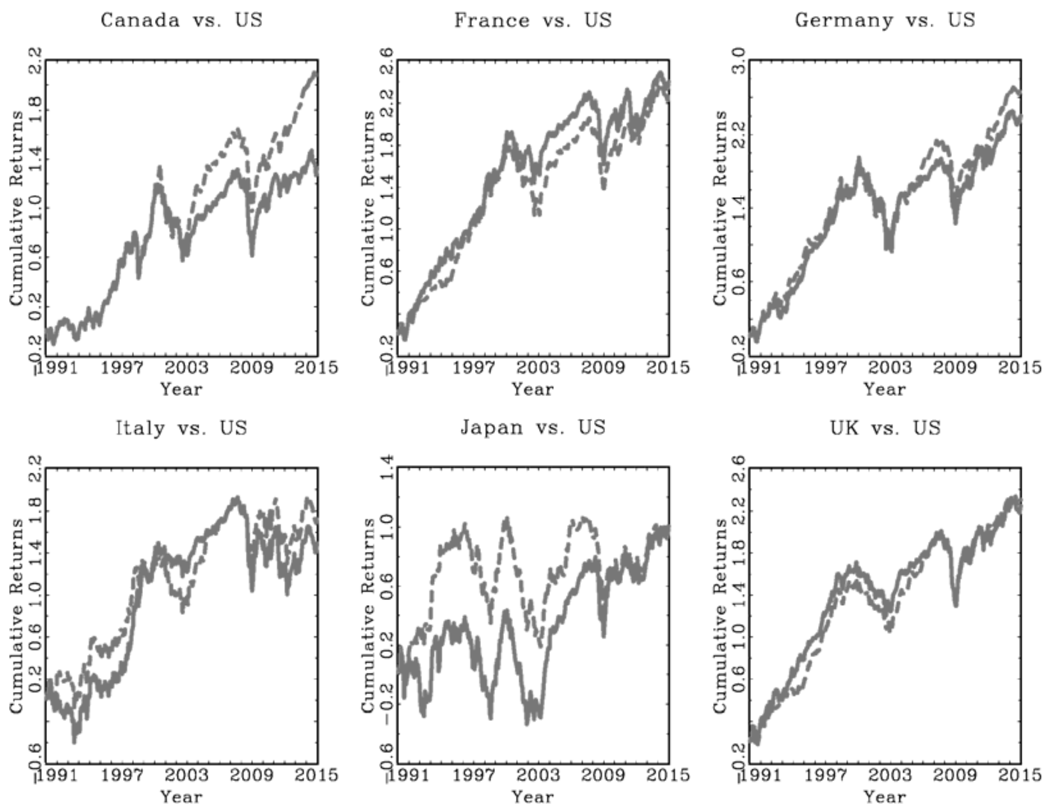


Table I. Summary Statistics of Relative Stock Prices

	Mean	SD	Min	Max	Skew	Kurt	JB
Canada	0.014	0.349	-0.915	0.811	-0.124	4.229	35.49
France	0.183	0.225	-0.339	0.659	-0.119	4.896	84.73
Germany	0.140	0.227	-0.441	0.803	-0.174	4.185	34.44
Italy	-1.013	0.454	-2.024	0.151	0.087	4.200	33.21
Japan	0.742	0.706	-0.395	2.356	0.013	4.549	54.18
UK	0.274	0.215	-0.595	0.699	0.203	6.847	338.0

Note: i) Relative stock prices are defined as the log national stock index minus the log US stock index. ii) JB denotes the Jarque-Bera statistics. We obtained the statistics for the residual of each series after filtering out with an AR(1) specification.

Table II. Half-Life Estimation from a Linear Model

	ρ	Conf. Interval	HL (month)	Conf. Interval
Canada	0.997	[0.986, 1.008]	262.5	[47.56, ∞)
France	0.978	[0.957, 1.004]	31.10	[15.59, ∞)
Germany	0.974	[0.953, 1.002]	26.44	[14.36, ∞)
Italy	0.990	[0.974, 1.012]	66.53	[25.99, ∞)
Japan	1.000	[0.990, 1.007]	∞	[69.38, ∞)
UK	0.979	[0.958, 1.005]	32.87	[16.35, ∞)

Note: i) ρ denotes the persistence parameter from a linear augmented Dickey-Fuller regression equation, that is, $x_t = c + \rho x_{t-1} + \sum_{j=1}^{k} \beta_j \Delta x_{t-j} + e_t$. ii) The lag parameter (k) is chosen by the general-to-specific rule with 12 maximum number of lags. iii) The point estimate and the 95% confidence interval was constructed by Hansen's (1999) grid bootstrap method to correct for median bias. For this, 10,000 bootstrap simulations on each of 51 grid points were implemented.

Table III. Ergodicity and Mixing Test Results

(a) Ergodicity Test

Country	% p-values < 0.05	% p-values < 0.10
Canada	4	9
France	10	16
Germany	4	8
Italy	6	13
Japan	22	30
UK	4	9

(b) Mixing Test

Country	% p-values < 0.05	% p-values < 0.10
Canada	3	7
France	3	6
Germany	3	6
Italy	3	6
Japan	5	10
UK	3	7

Note: i) These are randomized tests proposed by Domowitz and El-Gamal (2001). ii) The numbers in the table are the percentage of rejections at the 5% and the 10% significance level from 1,000 independent randomized runs.

Table IV. Max Half-Life and Max Quarter-Life: Short Memory in Mean

(a) Max Half-Life (month)			
Country	$k = 1$	$k = 10$	Convergence
Canada	7	>60	No
France	5	14	Yes
Germany	4	19	Yes
Italy	6	>60	No
Japan	6	>60	No
UK	4	16	Yes

(b) Max Quarter-Life (month)			
Country	$k = 1$	$k = 10$	Convergence
Canada	16	>60	No
France	10	33	Yes
Germany	10	47	Yes/No
Italy	14	>60	No
Japan	14	>60	No
UK	9	45	No

(c) 2MHL – MQL (month)			
Country	$k = 1$	$k = 10$	Convergence
Canada	-2	n.a.	No
France	0	-5	Yes
Germany	-2	-9	Yes/No
Italy	-2	n.a.	No
Japan	-2	n.a.	No
UK	-1	-13	No

Note: i) We estimate Max Half-Life (MHL) and Max Quarter-Life (MQL) for the smoothing parameter k raning 1 through 10. ii) We denote "Yes" in the last column when the m-life estimates converge as k approaches to 10, that is, when greater values for k produces no substantial difference in MHL and MQL estimates of the normalized Maximal Distance Measure (MDM). iii) $2MHL - MQL$ metric is suggested by Steinsson (2008). Zero values for $2MHL - MQL$ imply monotonic adjustment process towards the long-run equilibrium. Negative values occur when $MHL < MQL - MHL$.

Table V. Max Half-Life and Max Quarter-Life: Short Memory in Distribution

(a) Max Half-Life (month)			
Country	$k = 1$	$k = 10$	Convergence
Canada	8	>60	No
France	5	14	Yes
Germany	4	16	Yes
Italy	7	>60	No
Japan	6	>60	No
UK	4	16	Yes

(b) Max Quarter-Life (month)			
Country	$k = 1$	$k = 10$	Convergence
Canada	18	>60	No
France	10	33	Yes
Germany	10	47	Yes/No
Italy	13	>60	No
Japan	14	>60	No
UK	9	45	Yes/No

(c) 2MHL - MQL (month)			
Country	$k = 1$	$k = 10$	Convergence
Canada	-2	n.a.	No
France	0	-5	Yes
Germany	-2	-15	Yes/No
Italy	1	n.a.	Yes
Japan	-2	n.a.	No
UK	-1	-13	Yes/No

Note: i) We estimate Max Half-Life (MHL) and Max Quarter-Life (MQL) for the smoothing parameter k raning 1 through 10. ii) We denote "Yes" in the last column when the m-life estimates converge as k approaches to 10, that is, when greater values for k produces no substantial difference in MHL and MQL estimates of the normalized Maximal Distance Measure (MDM). iii) $2MHL - MQL$ metric is suggested by Steinsson (2008). Zero values for $2MHL - MQL$ imply monotonic adjustment process towards the long-run equilibrium. Negative values occur when $MHL < MQL - MHL$.

Table VI. Max Half-Life: Non-G7 Developed Countries

Country	*Max half-life (month)* $k = 1$	$k = 10$	Convergence
Australia	10	>60	No
Austria	7	>60	No
Belgium	7	48	Yes
Denmark	6	41	Yes
Hong Kong	6	35	Yes
Netherlands	8	>60	No
Norway	7	56	No
Singapore	7	>60	No
Spain	6	52	No
Sweden	6	>60	No
Switzerland	5	23	Yes

Note: i) We estimate Max Half-Life (MHL) for the smoothing parameter k raning 1 through 10. ii) We denote "Yes" in the last column when the m-life estimates converge as k approaches to 10, that is, when greater values for k produces no substantial difference in MHL estimates.

Table VII. Max Half-Life: Sub-Sample Analysis

(a) Pre-1990s Period (1969-1990)

Country	$k = 1$	$k = 10$	Convergence
Canada	5	28	Yes
France	3	17	Yes
Germany	4	22	Yes
Italy	4	16	Yes
Japan	5	45	Yes
UK	3	21	Yes

(b) Post-1990s Period (1991-2015)

Country	$k = 1$	$k = 10$	Convergence
Canada	4	>60	No
France	4	29	Yes
Germany	4	16	Yes
Italy	6	>60	No
Japan	10	48	Yes
UK	4	27	Yes

Note: i) We estimate Max Half-Life (MHL) for the smoothing parameter k raning 1 through 10. ii) We denote "Yes" in the last column when the m-life estimates converge as k approaches to 10, that is, when greater values for k produces no substantial difference in MHL estimates.