Auburn University
Department of Economics
Working Paper Series

The Zero Lower Bound: Frequency, Duration, and Determinacy

Alexander Richter[†] and Nathaniel Throckmorton[*]

[†]*Auburn University*, [*]*Indiana University*

AUWP 2013-16

The Zero Lower Bound: Frequency, Duration, and Determinacy

Alexander W. Richter* Nathaniel A. Throckmorton†

August 7, 2013

Abstract

When monetary policy faces a zero lower bound (ZLB) constraint on the nominal interest rate, determinacy is not guaranteed even if the Taylor principle is satisfied when the ZLB does not bind. This paper shows the boundary of the determinacy region imposes a clear tradeoff between the expected frequency and average duration of ZLB events. We show this tradeoff using a global solution to a nonlinear New Keynesian model with two alternative stochastic processes—one where monetary policy follows a 2-state Markov chain, which exogenously governs whether the ZLB binds, and the other where ZLB events arise endogenously due to technology shocks. In both cases, the household accounts for the possibility of going to and exiting the ZLB in expectation. We quantify the expectational effect of the ZLB and show it depends on the parameters of the stochastic process.

Keywords: Monetary policy; zero lower bound; determinacy; global solution method
JEL Classifications: E31; E42; E58

*Corresponding author, Department of Economics, Auburn University, 0332 Haley Center, Auburn, AL, 36849. Email: arichter@auburn.edu. Phone: +1(334)844-9638.

†Department of Economics, Indiana University, 100 S. Woodlawn, Wylie Hall 105, Bloomington, IN, 47401. Email: nathrock@indiana.edu. Phone: +1(765)658-4817.

1 INTRODUCTION

Since the beginning of the financial crisis in late 2007, many central banks around the world have targeted a policy rate near zero and promised to maintain a low rate until economic conditions improve. Despite this policy and numerous unconventional policies, most of these countries still face elevated unemployment levels and anemic output growth. This experience has ignited new research that studies the impacts of the zero lower bound (ZLB) on the nominal interest rate.

The ZLB constraint imposes a kink in the monetary policy rule. Most of the literature that studies the ZLB gets around this discontinuity by linearizing the equilibrium system, except the monetary policy rule. In these studies, ZLB events occur with certainty, they last for a predetermined duration, and there is no probability of returning [e.g., Braun and Körber (2011); Christiano et al. (2011); Eggertsson and Woodford (2003); Gertler and Karadi (2011)]. Recently global solution methods have been used to solve nonlinear models with a ZLB [e.g., Aruoba and Schorfheide (2013); Basu and Bundick (2012); Fernández-Villaverde et al. (2012); Gavin et al. (2013); Gust et al. (2012); Judd et al. (2011); Mertens and Ravn (2013)], but most of the work on determinacy uses a perfect foresight setup [e.g. Alstadheim and Henderson (2006); Benhabib et al. (2001a,b)].

When monetary policy faces a ZLB constraint, the Taylor principle—the principle that monetary policy pins down prices by adjusting the nominal interest rate more than one-for-one with inflation—does not guarantee determinacy even if it is satisfied when the ZLB does not bind. This paper shows the boundary of the determinacy region imposes a clear tradeoff between the expected frequency and average duration of ZLB events. We show this tradeoff using a global solution to a nonlinear New Keynesian model with two alternative stochastic processes—one where monetary policy follows a 2-state Markov chain, which exogenously governs whether the ZLB binds, and the other where ZLB events arise endogenously due to technology shocks. In both cases, the household accounts for the possibility of going to and exiting the ZLB in expectation. We quantify the expectational effect of the ZLB and show it depends on the parameters of the stochastic process.[1]

There are two main drawbacks with using a linearized equilibrium system to study the ZLB. First, the solution has large approximation errors that affect the qualitative properties of the model [Braun et al. (2012); Fernández-Villaverde et al. (2012)]. Second, it permits calibrations of the stochastic processes (e.g., highly persistent processes with large shocks) that do not yield a unique bounded equilibrium. Any monetary policy rule contains a ZLB, regardless of whether it is imposed by the modeler. This means calibrations of the stochastic processes, which affect the expected frequency and duration of ZLB events, must also affect determinacy in linear models. This fact is largely ignored in the literature, despite its important implications. Both of these drawbacks motivate using a nonlinear model to accurately account for the ZLB and study its consequences.

We solve the model using the policy function iteration algorithm described in Richter et al. (2013), which is based on the theoretical work on monotone operators in Coleman (1991). This solution method discretizes the state space and uses time iteration to solve for the updated policy functions until the tolerance criterion is met. We use piecewise linear interpolation to approximate future variables that show up in expectation, since this approach more accurately captures the kink in the policy functions than continuous functions, and Gauss-Hermite quadrature to numerically integrate. We set the bounds of the stochastic state variable so that they encompass 99.999 percent of the probability mass of its distribution. We specify $1{,}001$ grid points for each state variable and

[1] For a complete picture of the solution to New Keynesian models with and without capital see Gavin et al. (2013).

the maximum number of Gauss-Hermite weights (66) for each continuous shock. These techniques minimize extrapolation and ensure that the location of the kink is accurate.

In both the exogenous and endogenous setups, we classify the algorithm as *non-convergent* whenever the iteration step, defined as the maximum distance between policy function values on successive iterations, increases at an increasing rate for more than 50 iterations or when all of the values in any policy function consistently drift from their steady-state value. Additionally, when ZLB events are endogenous, we require that the ZLB binds on fewer than 50 percent of the nodes in the state space since it is infeasible for the ZLB to bind at the stochastic steady state. We classify the algorithm as *convergent* whenever the iteration step is less than 10^{-13} (the tolerance criterion) for 10 successive iterations, which prevents the algorithm from jumping to the tolerance criterion.

Davig and Leeper (2007) contains two monetary policy rules—one that aggressively responds to inflation and one that reacts less aggressively to inflation—governed by a 2-state Markov chain. Their setup is similar to a model with a ZLB constraint, since a pegged zero interest rate regime is a special case of passive monetary policy. Thus, we use their regime switching setup as a benchmark for our algorithm. When we adopt the models they use (log-linear Fisherian economy, log-linear New Keynesian economy), our algorithm yields the same determinacy regions they analytically derive. This means our algorithm is non-convergent whenever the monetary policy parameters are outside their analytical determinacy region and convergent whenever the Long-run Taylor Principle is met. Our numerical solutions to these models also equal the minimum state variable (MSV) solutions Davig and Leeper (2007) derive. Thus, we define any convergent solution as a determinate rational expectations equilibrium and the set of convergent solutions as the determinacy region (i.e. the set of parameters that deliver a unique stable MSV solution).[2]

Within the class of Markov-switching rational expectations models, Farmer et al. (2009, 2010), Barthélemy and Marx (2013), and Cho (2013) prove that non-MSV solutions with fundamental or non-fundamental components may still exist even when the MSV solution is determinate. To be clear, our numerical algorithm rules out many indeterminate equilibria that are subject to sunspot fluctuations (e.g., in models that do not contain a ZLB constraint, our algorithm only converges when the Taylor principle is satisfied), but it cannot capture the types of non-MSV solutions that may exist when a determinate MSV solution exists.[3] Studying these types of solutions in models with a ZLB constraint is an important topic for future research, but we believe locating regions of the parameter space that deliver a determinate MSV solution while accurately capturing the ZLB is significant since most macroeconomic research (including estimation) is based on MSV solutions.

Models that include a ZLB constraint contain two deterministic steady states [Benhabib et al. (2001a,b)]. Specifically, there are two nominal interest rate/inflation rate pairs consistent with the steady state equilibrium system. In one case the monetary authority meets its positive inflation target, while in the other, deflation occurs. Similar to the sunspot shocks in Aruoba and Schorfheide (2013) and the confidence shocks in Mertens and Ravn (2013), exogenous switches in the monetary policy state that occur in our model cause the economy to switch between these two equilibria. However, the model only simulates to a deflationary steady state when the expected duration of ZLB events is sufficiently long. In our other setup where ZLB events are endogenous, the model never simulates to the deflationary steady state due to mean reversion in the stochastic process.

[2]The code used to produce our results and replicate Davig and Leeper (2007) is available upon request.

[3]Barthélemy and Marx (2013) refer to unique bounded MSV solutions as bounded Markovian solutions (i.e., dependent on a finite number of past regimes). Our numerical algorithm rules out the possibility of multiple bounded Markovian solutions, but there may exist other bounded non-Markovian solutions. We call these non-MSV solutions.

The paper is organized as follows. Section 2 describes the model economy. Sections 3 and 4 lay out the two alternative stochastic processes that drive the economy to the ZLB and show the tradeoff between the expected frequency and average duration of ZLB events. Section 5 concludes.

2 MODEL ECONOMY AND BASELINE CALIBRATION

A representative household chooses $\{c_t, n_t, b_t\}_{t=0}^{\infty}$ to maximize expected lifetime utility, given by, $E_0 \sum_{t=0}^{\infty} \beta^t \{c_t^{1-\sigma}/(1-\sigma) - \chi n_t^{1+\eta}/(1+\eta)\}$, where $1/\sigma$ is the intertemporal elasticity of substitution, $1/\eta$ is the Frisch elasticity of labor supply, c_t is consumption of the final good, n_t is labor hours, and β is the discount factor. The choices are constrained by $c_t + b_t = w_t n_t + r_{t-1} b_{t-1}/\pi_t + \tau_t + d_t$, where $\pi_t = p_t/p_{t-1}$ is the gross inflation rate, w_t is the real wage rate, τ_t is a lump-sum tax, b_t is a one-period real bond, r_t is the gross nominal interest rate, and d_t are profits from intermediate firms. The optimality conditions to the household's utility maximization problem imply

$$w_t = \chi n_t^{\eta} c_t^{\sigma}, \tag{1}$$

$$1 = \beta r_t E_t\{(c_t/c_{t+1})^{\sigma}/\pi_{t+1}\}. \tag{2}$$

The production sector consists of a continuum of monopolistically competitive intermediate goods firms and a perfectly competitive final goods firm. Each firm $i \in [0, 1]$ in the intermediate goods sector produces a differentiated good, $y_t(i)$, according to $y_t(i) = z_t n_t(i)$, where z_t is technology and $n_t(i)$ is the level of employment used by firm i. The final goods firm purchases $y_t(i)$ units from each intermediate firm to produce the final good, $y_t \equiv [\int_0^1 y_t(i)^{(\theta-1)/\theta} di]^{\theta/(\theta-1)}$, according to a Dixit and Stiglitz (1977) aggregator, where $\theta > 1$ is the price elasticity of demand between intermediate goods. Profit maximization yields the demand function for good i, $y_t(i) = (p_t(i)/p_t)^{-\theta} y_t$, where $p_t = [\int_0^1 p_t(i)^{1-\theta} di]^{1/(1-\theta)}$ is the final good price. Each intermediate firm chooses its price level, $p_t(i)$, to maximize the expected present value of real profits, $E_t \sum_{k=t}^{\infty} q_{t,k} d_k(i)$, where $q_{t,t} \equiv 1$, $q_{t,t+1} = \beta(c_t/c_{t+1})^{\sigma}$ is the pricing kernel between periods t and $t+1$, and $q_{t,k} \equiv \prod_{j=t+1}^{k} q_{j-1,j}$. Following Rotemberg (1982), each firm faces a cost to adjusting its price, which emphasizes the potentially negative effect that price changes can have on customer-firm relationships. Using the functional form in Ireland (1997), firm i's real profits are

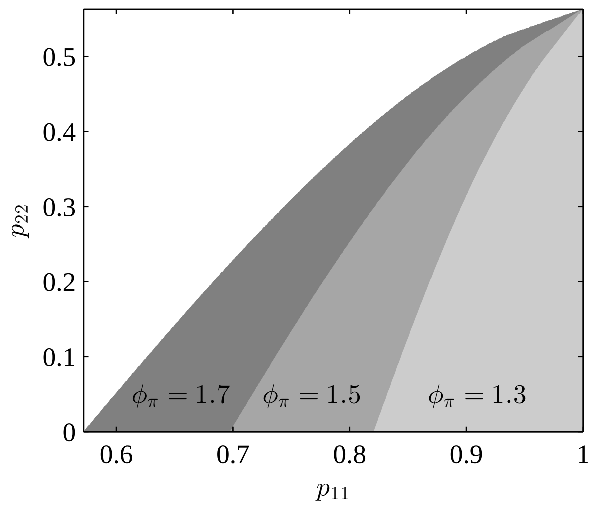
$$d_t(i) = \left[\left(\frac{p_t(i)}{p_t}\right)^{1-\theta} - \Psi_t \left(\frac{p_t(i)}{p_t}\right)^{-\theta} - \frac{\varphi}{2}\left(\frac{p_t(i)}{\bar{\pi} p_{t-1}(i)} - 1\right)^2\right] y_t,$$

where $\varphi \geq 0$ determines the magnitude of the adjustment cost, Ψ_t is the real marginal cost of producing a unit of output, and $\bar{\pi}$ is the steady-state gross inflation rate. In a symmetric equilibrium, all intermediate goods firms make the same decisions and the optimality condition reduces to

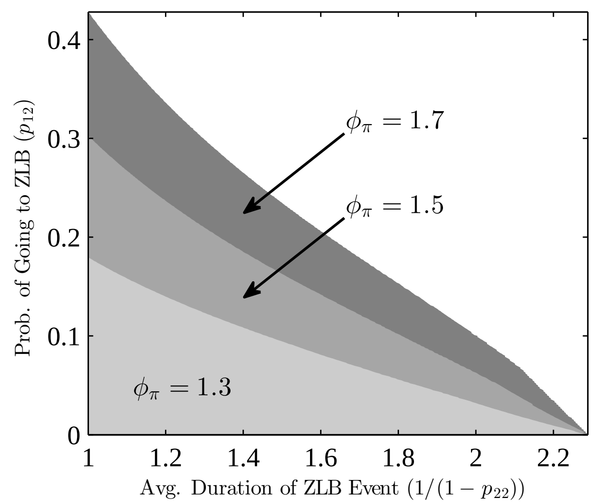
$$\varphi\left(\frac{\pi_t}{\bar{\pi}} - 1\right)\frac{\pi_t}{\bar{\pi}} = (1-\theta) + \theta\Psi_t + \varphi E_t\left[q_{t,t+1}\left(\frac{\pi_{t+1}}{\bar{\pi}} - 1\right)\frac{\pi_{t+1}}{\bar{\pi}}\frac{y_{t+1}}{y_t}\right]. \tag{3}$$

In the absence of price adjustment costs (i.e., $\varphi = 0$), the real marginal cost equals $(\theta - 1)/\theta$, which is the inverse of the firm's markup of price over marginal cost.

Each period the fiscal authority finances its spending, $\bar{g}$, by levying lump-sum taxes ($\tau_t = \bar{g}$). The resource constraint is $c_t + \bar{g} = [1 - \varphi(\pi_t/\bar{\pi} - 1)^2/2] y_t$. The household's and firm's optimality conditions, the government's budget constraint, the monetary policy rule (defined below), the bond market clearing condition ($b_t = 0$), and the resource constraint form the equilibrium system.



(a) Determinacy (shaded) regions in (p_{11},p_{22})-space



(b) Expected frequency/average duration of ZLB events tradeoff

Figure 1: Determinacy regions across alternative monetary policy responses to inflation.

The model is calibrated at a quarterly frequency using values that are common in the literature. We set $\beta = 0.99$ and $\sigma = 1$, implying log utility in consumption. The Frisch elasticity of labor supply, $1/\eta$, is set to 1 and the leisure preference parameter, χ, is set so that steady-state labor equals $1/3$ of the available time. The price elasticity of demand between intermediate goods, θ, is calibrated to 6, which corresponds to an average markup of price over marginal cost equal to 20 percent. The costly price adjustment parameter, φ, is set to 58.25, which is consistent with a Calvo (1983) price-setting specification where prices change on average once every four quarters. Steady-state technology, $\bar{z}$, is normalized to 1. In the policy sector, the steady-state gross inflation rate, $\bar{\pi}$, is calibrated to 1.005, which implies an annual (net) inflation rate target of 2 percent. The steady-state ratio of government spending to output is calibrated to 20 percent.

3 EXOGENOUS ZLB EVENTS: MONETARY POLICY SWITCHING

In this section, the monetary authority sets the gross nominal interest rate according to

$$r_t = \begin{cases} \bar{r}(\pi_t/\bar{\pi})^{\phi_\pi}\exp(\varepsilon_t) & \text{for} \quad s_t = 1 \\ 1 & \text{for} \quad s_t = 2 \end{cases}, \tag{4}$$

where ϕ_π is the policy response to deviations of inflation from its steady state and $\varepsilon_t \sim \mathbb{N}(0, \sigma_\varepsilon^2)$ is a discretionary monetary policy shock. The monetary policy state evolves according to a 2-state Markov chain with transition matrix $\Pr\{s_t = j|s_{t-1} = i\} = p_{ij}$, for $i, j \in \{1, 2\}$. When $s_t = 1$, the monetary authority follows the Taylor principle ($\phi_\pi > 1$) and when $s_t = 2$, the monetary authority exogenously pegs the (net) nominal interest rate at zero. We set $z_t = \bar{z}$ and $\sigma_\varepsilon = 0.003$, which is small enough that the ZLB never binds due to a discretionary monetary policy shock. Thus, all ZLB events in this section are due to exogenous changes in the monetary policy state, s_t.

As long as the household places sufficient expectation on the monetary authority pinning down prices in the future, a unique bounded MSV solution still exists even though the Taylor principle is violated when the ZLB binds. Figure 1a plots the determinacy (shaded) regions in (p_{11}, p_{22})-space for $\phi_\pi \in \{1.3, 1.5, 1.7\}$. Within this region, the dynamics of the endogenous variables are standard; as the household places greater weight on going to the ZLB, expected deflation raises the real interest rate and reduces consumption and output.

The boundary of the shaded region for each ϕ_π represents the largest p_{22} value that yields a determinate solution for each p_{11} value. These results show a clear tradeoff between p_{11} and p_{22}. When there is a low probability of going to the ZLB (i.e., a higher p_{11}), it is possible for the household to expect longer ZLB events (i.e., a higher p_{22}) and still guarantee a determinate solution. To see this more clearly, figure 1b plots the probability of going to the ZLB (i.e., p_{12}) as a function of the average duration of each ZLB event (i.e., $1/(1-p_{22})$) for each value of ϕ_π. When the average duration of ZLB events is short, the economy can support a higher expected frequency of ZLB events. However, as the average duration of ZLB events increases, the maximum expected frequency of ZLB events must fall to guarantee a determinate solution. An implication of our results is that stochastic processes that are commonly embedded in dynamic models do not generate ZLB events that are consistent with observed ZLB events, which is similar to the points made in Chung et al. (2012) and Fernández-Villaverde et al. (2012).

The determinacy region also depends critically on how strongly the monetary authority responds to inflation when the ZLB does not bind. If the monetary authority responds more aggressively to inflation when $s_t = 1$ (i.e., a higher ϕ_π) and $p_{11} < 1$, the determinacy region widens, since the household expects the monetary authority to return to a regime where prices are more stable across the alternative realizations of the monetary policy shock. This means the economy can support longer and/or more frequent trips to the ZLB. However, it is interesting that regardless of the value of ϕ_π, the longest average ZLB event inside the determinacy region is the same. As p_{11} rises, the likelihood of recurring ZLB events declines and the model approaches a fixed-regime setup where increases in ϕ_π beyond a minimum threshold have no effect on determinacy. As long as the ZLB state exists, it is always possible for the ZLB to bind if the state exogenously switches to $s_t = 2$, but these switches are unexpected by the household if $p_{11} = 1$.

The deep parameters in the model (e.g., σ, η, β, φ) also affect the expected frequency and average duration of ZLB events and therefore the size of the determinacy region, given monetary policy. A larger price adjustment cost adds price stability to the model and expands the determinacy regions. When the degree of risk aversion, σ is higher, the household is less willing to intertemporally substitute consumption. When the Frisch elasticity of labor supply, $1/\eta$, is larger, the household's willingness to supply labor is more sensitive to changes in the real wage rate. Both of these effects make hours worked, consumption, and the inflation rate less volatile when the ZLB binds, which expands the determinacy region. When the household is more patient (i.e., a higher β), the steady state nominal interest rate is lower, which increases the frequency of ZLB events.

Our finding that there exists a tradeoff between the expected frequency and average duration of ZLB events and that it depends on the monetary authority's response to inflation is similar to the main finding in Davig and Leeper (2007). Using two different log-linearized models, they prove that when there are distinct monetary policy regimes, the existence of a unique bounded MSV solution does not require the Taylor principle to hold in both regimes. As long as one of the regimes satisfies the Taylor principle, the monetary authority can passively respond to inflation (i.e., adjust the nominal interest rate less than one-for-one with inflation) in the other regime and still deliver a determinate solution. They also find that if the monetary authority visits the passive monetary policy regime infrequently (i.e., a higher p_{11}), the model can support longer trips to the passive monetary regime (i.e., higher p_{22}) and still achieve determinacy. However, there are two key differences. First, an occasionally binding ZLB constraint truncates the distribution of interest rates, which significantly affects the household's expectations. Second, the calibration of the stochastic process impacts the size of the determinacy region, which we quantify in section 4.

4 ENDOGENOUS ZLB EVENTS: STOCHASTIC TECHNOLOGY

This section replaces the exogenous Markov-switching process with a continuous technology process that determines the frequency and duration of ZLB events. Technology follows

$$z_t = \bar{z}(z_{t-1}/\bar{z})^{\rho_z} \exp(\varepsilon_t), \tag{5}$$

where $0 \leq \rho_z < 1$ and $\varepsilon_t \sim \mathbb{N}(0, \sigma_\varepsilon^2)$. We define $\sigma_z = \sigma_\varepsilon/\sqrt{1-\rho_\varepsilon^2}$ as the standard deviation of (5). We remove the monetary policy shock, so the gross nominal interest rate is set according to

$$r_t = \max\{1, \bar{r}(\pi_t/\bar{\pi})^{\phi_\pi}\}. \tag{6}$$

Positive realizations of technology shocks act as positive aggregate supply shocks. At high levels of technology, firms' per unit marginal cost of production is low. Firms react by lowering their prices and raising their production. This eventually causes deflation and the (net) nominal interest rate to fall to zero, given the Taylor rule in (6). Thus, ZLB events are endogenous in this section.

Section 3 makes clear that when episodes at the ZLB are exogenous, the boundary of the determinacy region imposes a clear tradeoff between the expected frequency and average duration of ZLB events. This same tradeoff is present when technology follows (5) and ZLB events are endogenous. We discretize the technology state into N elements such that $z_{t-1} \in \{z_1, \ldots, z_N\}$. Let $s_t \in \{1, 2\}$ indicate that the ZLB is either not binding or binding, respectively. Let n denote the index corresponding to the minimum level of technology where the ZLB binds, which partitions the state-space into two subsets. Denote the corresponding sets of indices as $\mathcal{I}_{1,t-1} = \{1, \ldots, n-1\}$ and $\mathcal{I}_{2,t-1} = \{n, \ldots, N\}$. Since technology follows (5), the probability of going to the ZLB (the analog of $p_{12} = 1 - p_{11}$ in the transition matrix defined in section 3) is given by

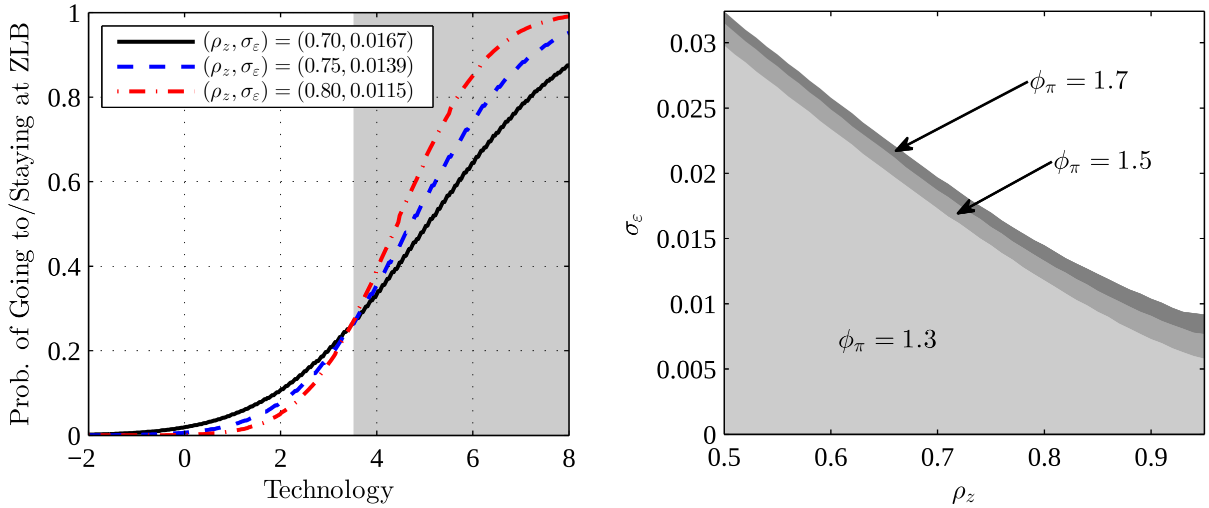
$$\Pr\{s_t = 2 | s_{t-1} = 1\} = \frac{\sum_{i \in \mathcal{I}_{1,t-1}} \Pr\{s_t = 2 | z_{t-1} = z_i\} \phi(z_i | \bar{z}, \sigma_z)}{\sum_{i \in \mathcal{I}_{1,t-1}} \phi(z_i | \bar{z}, \sigma_z)},$$

where

$$\Pr\{s_t = 2 | z_{t-1} = z_i\} = \frac{\sum_{j \in \mathcal{I}_{2,t}} \phi(\varepsilon_j | 0, \sigma_\varepsilon)}{\sum_{j \in \mathcal{I}_{1,t} \cup \mathcal{I}_{2,t}} \phi(\varepsilon_j | 0, \sigma_\varepsilon)}, \tag{7}$$

$\phi(x|\mu, \sigma)$ is the normal probability density function, given mean μ and standard deviation σ. For each z_{t-1}, there is a vector of realizations of z_t, where each realization corresponds to a Gauss-Hermite quadrature node, ε_j (the roots of the Hermite polynomial).

Figure 2a plots (7) as a function of the technology state for three alternative parameterizations of (5). The shaded region corresponds to technology states where the ZLB binds, which begins when technology is 3.5 percent above its steady-state value. The three combinations of $(\rho_z, \sigma_\varepsilon)$ are chosen to keep the boundary of the ZLB region unchanged. In technology states below the boundary, the probability on the vertical axis is the probability of going to the ZLB in the next quarter. In technology states above the boundary, it is the probability of staying at the ZLB. This figure demonstrates the tradeoff between the probability of hitting the ZLB and the average duration of



(a) Probabilities of going to the ZLB, given the technology state and $\phi_\pi = 1.5$. Technology is in percent deviations from steady state. The shaded region corresponds to the states where the ZLB binds.

(b) Tradeoff between the technology process persistence and shock standard deviation across alternative monetary policy responses to inflation. The shaded regions correspond to the determinacy regions.

Figure 2: Properties of the model where ZLB events arise endogenously due to technology shocks.

ZLB events. As ρ_z increases and σ_ε decreases, it is less likely the ZLB will bind in technology states below the boundary and more likely the ZLB will continue to bind once the ZLB is hit.

The combinations of $(\rho_z, \sigma_\varepsilon)$ shown in figure 2a are *not* on the boundary of the determinacy region in $(\rho_z, \sigma_\varepsilon)$-space. The boundary of the ZLB region is a function of $(\rho_z, \sigma_\varepsilon)$, which affects the probabilities of going to and staying at the ZLB. Since ZLB events are endogenous due to (5), there is no way to map $(\rho_z, \sigma_\varepsilon)$ into equivalent (p_{11}, p_{22}) values and generate a picture equivalent to figure 1 (i.e., we cannot increase p_{22} by changing $(\rho_z, \sigma_\varepsilon)$ without altering p_{11}). Thus, fixing the boundary of the ZLB region offers the closest comparison to the Markov chain process in section 3.

Figure 2b shows that along the boundary of the determinacy (shaded) region, there is a clear tradeoff between the persistence of the technology process, ρ_z, and the standard deviation of the shock, σ_ε. As the persistence of the process increases, the standard deviation of the shock must decline to avoid an indeterminacy region. This tradeoff reflects that ρ_z and σ_ε both impact the expected frequency and average duration of ZLB events, as figure 2a shows. Once again, the monetary policy response to inflation, ϕ_π, affects the size of the determinacy region. For a given ρ_z, an increase in ϕ_π permits a larger σ_ε, as prices are more stable when the ZLB does not bind.[4]

The fact that the parameters of the stochastic process impact the determinacy region is significant, because these parameters do not affect determinacy in linearized models, regardless of whether the ZLB is imposed. In models that impose a ZLB, it is common to linearize every equation in the equilibrium system, except for the Taylor rule, and assume ZLB events last for a predetermined duration with no probability of recurrence. This approach does not account for the expectational effects of going to and exiting the ZLB, which are critical for determinacy.

Figure 3 compares the inflation rate policy functions across two parameterizations of (5), both of which are on the boundary of the determinacy region in $(\rho_z, \sigma_\varepsilon)$-space. The horizontal dashed line is the steady-state inflation rate ($\bar{\pi} = 1.005$). When the technology state equals the steady-

[4] Although these results are based on a technology shock, a similar tradeoff would exist with a preference shock.

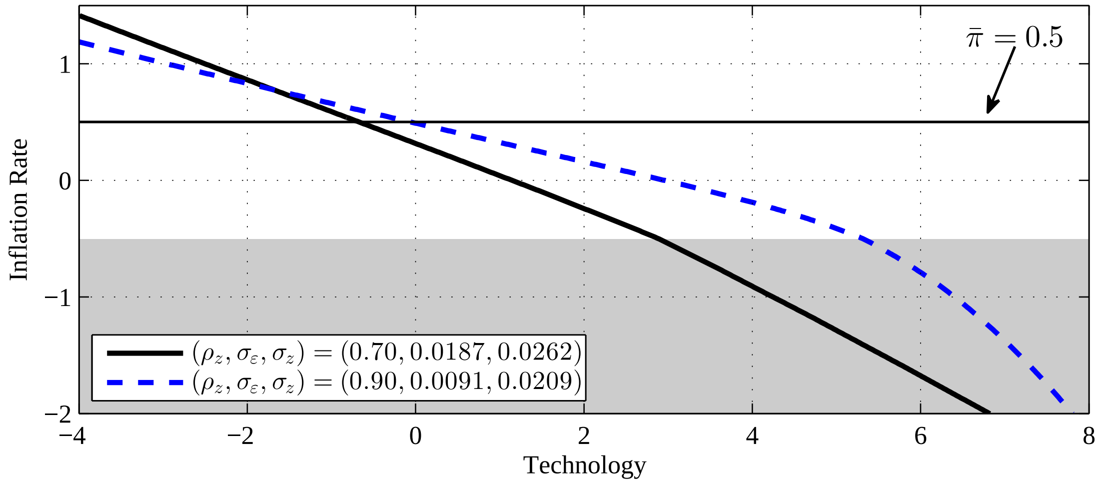


Figure 3: Comparison of the policy functions for $(\rho_z, \sigma_\varepsilon)$ combinations on the boundary of the determinacy region ($\phi_\pi = 1.5$). Technology is in percent deviations from steady state. The solid horizontal line represents the steady-state gross inflation rate. The shaded region corresponds to the technology states where the ZLB binds.

state technology level ($\bar{z} = 1$), the deviations of the inflation rate from its steady-state value provide a measure of the expectational effect of hitting the ZLB. The shaded region represents values of the inflation rate where the ZLB binds. When σ_ε is relatively small (dashed line), the expectational effect is small because the likelihood of hitting the ZLB in expectation is also small. As σ_ε increases, and σ_z increases with it, the expectational effect of hitting the ZLB also increases.

When the ZLB binds, higher real interest rates reduce consumption and put downward pressure on inflation as firms respond to the lower demand. Thus, when there is a higher probability of going to the ZLB (solid line), the slope of the inflation rate policy function is steeper. Since the downward pressure on inflation happens across the entire state space, it also influences where the ZLB first binds in the state space. For smaller standard deviations of (5), the probability of hitting the ZLB in expectation is smaller and the boundary of the ZLB region lies at a higher technology state. Unlike linear models, where the calibration of the stochastic process has a much smaller effect on the policy functions, these results imply that changes in the calibration of the stochastic process can significantly impact the quantitative properties of the model.

5 Conclusion

This paper demonstrates that the boundary of the determinacy region imposes a clear tradeoff between the expected frequency and average duration of episodes at the ZLB, regardless of whether ZLB events arise exogenously or endogenously. This tradeoff is critical for at least three reasons. First, even though the Taylor principle is violated at the ZLB, it shows that central banks can still pin down prices when the nominal interest rate is pegged at its ZLB, so long as households have a strong enough expectation of returning to a monetary policy regime where the central bank aggressively responds to inflation. Second, it imposes an important constraint on the parameter space that the econometrician must account for when estimating the nonlinear model. Third, it implies that small changes in the parameters of stochastic processes significantly impact the policy functions and the state at which the ZLB first binds.

Acknowledgements We thank Eric Leeper and Todd Walker for helpful comments.

REFERENCES

ALSTADHEIM, R. AND D. W. HENDERSON (2006): "Price-Level Determinacy, Lower Bounds on the Nominal Interest Rate, and Liquidity Traps," *The B.E. Journal of Macroeconomics*, 6, 1–27.

ARUOBA, S. AND F. SCHORFHEIDE (2013): "Macroeconomic Dynamics Near the ZLB: A Tale of Two Equilibria," Manuscript, University of Pennsylvania.

BARTHÉLEMY, J. AND M. MARX (2013): "Determinacy Conditions for Markov Switching Rational Expectations Models," Manuscript, Banque de France.

BASU, S. AND B. BUNDICK (2012): "Uncertainty Shocks in a Model of Effective Demand," NBER Working Paper 18420.

BENHABIB, J., S. SCHMITT-GROHE, AND M. URIBE (2001a): "Monetary Policy and Multiple Equilibria," *American Economic Review*, 91, 167–186.

——— (2001b): "The Perils of Taylor Rules," *Journal of Economic Theory*, 96, 40–69.

BRAUN, R. A. AND L. M. KÖRBER (2011): "New Keynesian Dynamics in a Low Interest Rate Environment," *Journal of Economic Dynamics and Control*, 35, 2213–2227.

BRAUN, R. A., L. M. KÖRBER, AND Y. WAKI (2012): "Some Unpleasant Properties of Log-Linearized Solutions When the Nominal Rate is Zero," FRB Atlanta Working Paper 2012-5a.

CALVO, G. A. (1983): "Staggered Prices in a Utility-Maximizing Framework," *Journal of Monetary Economics*, 12, 383–398.

CHO, S. (2013): "Characterizing Markov-Switching Rational Expectations Models," Manuscript, Yonsei University.

CHRISTIANO, L., M. EICHENBAUM, AND S. REBELO (2011): "When Is the Government Spending Multiplier Large?" *Journal of Political Economy*, 119, 78–121.

CHUNG, H., J.-P. LAFORTE, D. REIFSCHNEIDER, AND J. C. WILLIAMS (2012): "Have We Underestimated the Likelihood and Severity of Zero Lower Bound Events?" *Journal of Money, Credit and Banking*, 44(S1), 47–87.

COLEMAN, II, W. J. (1991): "Equilibrium in a Production Economy with an Income Tax," *Econometrica*, 59, 1091–1104.

DAVIG, T. AND E. M. LEEPER (2007): "Generalizing the Taylor Principle," *American Economic Review*, 97, 607–635.

DIXIT, A. K. AND J. E. STIGLITZ (1977): "Monopolistic Competition and Optimum Product Diversity," *American Economic Review*, 67, 297–308.

EGGERTSSON, G. AND M. WOODFORD (2003): "The Zero Bound on Interest Rates and Optimal Monetary Policy," *Brookings Papers on Economic Activity*, 34(1), 139–235.

FARMER, R. E., D. F. WAGGONER, AND T. ZHA (2009): "Understanding Markov-switching rational expectations models," *Journal of Economic Theory*, 144, 1849 – 1867.

FARMER, R. E. A., D. F. WAGGONER, AND T. ZHA (2010): "Generalizing the Taylor Principle: Comment," *American Economic Review*, 100, 608–17.

FERNÁNDEZ-VILLAVERDE, J., G. GORDON, P. A. GUERRÓN-QUINTANA, AND J. RUBIO-RAMÍREZ (2012): "Nonlinear Adventures at the Zero Lower Bound," NBER Working Paper 18058.

GAVIN, W. T., B. D. KEEN, A. W. RICHTER, AND N. A. THROCKMORTON (2013): "Global Dynamics at the Zero Lower Bound," FRB St. Louis working paper 2013-007B.

GERTLER, M. AND P. KARADI (2011): "A Model of Unconventional Monetary Policy," *Journal of Monetary Economics*, 58, 17–34.

GUST, C., D. LÓPEZ-SALIDO, AND M. E. SMITH (2012): "The Empirical Implications of the Interest-Rate Lower Bound," CEPR Discussion Paper 9214.

IRELAND, P. N. (1997): "A Small, Structural, Quarterly Model for Monetary Policy Evaluation," *Carnegie-Rochester Conference Series on Public Policy*, 47, 83–108.

JUDD, K. L., L. MALIAR, AND S. MALIAR (2011): "A Cluster-Grid Algorithm: Solving Problems with High Dimensionality," Manuscript, Stanford University.

MERTENS, K. AND M. O. RAVN (2013): "Fiscal Policy in an Expectations Driven Liquidity Trap," Manuscript, Cornell university.

RICHTER, A. W., N. A. THROCKMORTON, AND T. B. WALKER (2013): "Accuracy, Speed and Robustness of Policy Function Iteration," Manuscript, Indiana University.

ROTEMBERG, J. J. (1982): "Sticky Prices in the United States," *Journal of Political Economy*, 90, 1187–1211.