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Resource Depletion and Trade: Adding a Nonrenewable Resource to the Heckscher-Ohlin Model

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This paper develops the intertemporal equilibrium of a small open economy with a nonrenewable resource intensive export and a labor intensive import. Optimal depletion implies the resource price rises at the rate of the capital return. Capital grows with investment and labor at a steady rate, raising the issue of whether depletion necessarily diminishes. Effects of a depletion tax, import tariff, and export subsidy are examined. Simulations with Cobb-Douglas production functions illustrate model properties. The paper also considers a constant depletion rate, tragedy of the commons, and myopic resource owner.

Key words: resource depletion, trade, general equilibrium

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Resource Depletion and Trade:

Adding a Nonrenewable Resource to the Heckscher-Ohlin Model

Exports often rely on nonrenewable resources motivating the present model of a small open economy with a resource intensive export. Factors of production in the model include capital and labor as well as the nonrenewable resource. Capital grows with saving and investment while labor grows at a steady rate. Optimal depletion implies the resource price increases at the rate equal to the capital return, turning attention to analysis of the intertemporal equilibrium.

The model includes intemporal depletion, income distribution, and production of the export and the labor intensive import competing good. One issue is whether depletion necessarily diminishes given growth in capital and labor inputs and output adjustments. The intertemporal wage and capital return depend only on factor intensity while the two outputs depend on substitution, investment, labor growth, and the state of the economy as well. Effects of a depletion tax, import tariff, and export subsidy are examined. Simulations with Cobb-Douglas production functions illustrate the evolution of endogenous variables.

The first two sections introduce the basics of intertemporal dynamics and substitution in production. The third section presents the intertemporal equilibrium model followed by a section on a depletion tax, import tariff, and export subsidy. Simulations with Cobb-Douglas production in the fifth section illustrate the endogenous intertemporal variable paths. A final section considers depletion and trade assuming a constant depletion rate, tragedy of the commons, and myopic resource owner.

1. Dynamics of the nonrenewable resource, capital, and labor

Optimal depletion is developed by Dixit, Hammond, and Hoel (1980), Hamilton (1995), Withagen and Asheim (1998), and Sato and Kim (2002), and the growth model with optimal depletion but capital as the only other input by Stiglitz (1974), Dasgupta and Heal (1979), and Solow (1986) develop. Thompson (2013) extends this two factor growth model to include labor. The three factor model of production and trade developed by Ruffin (1981), Jones and Easton (1983), and Thompson (1985) relates to classical economics as discussed by Robinson (1980).

The present paper extends the three factor growth model with a nonrenewable resource to two traded goods. The present small open economy produces two traded goods with capital, labor, and the nonrenewable resource. Constant returns production functions are $x_{jt} = x_j(K_{jt}, L_{jt}, N_{jt})$ where x_{jt} is the output of good j = 1, 2 at time t, K_{jt} is capital input, L_{jt} is labor input, and N_{jt} is depletion or input of the nonrenewable resource. Inputs are fully employed, $K_t = \Sigma_j K_{jt}$ with similar conditions for labor L_t and the nonrenewable resource N_t.

The change in capital $K_t' \equiv dK_t/dt$ equals saving ζ_t assuming no depreciation or foreign investment. A share of output is transformed into capital at no cost. A constant saving rate σ out of income Y_t implies $\zeta_t = \sigma Y_t = K_t'$. The labor force L_t grows at the constant rate $\lambda \equiv L_t'/L_t$.

Income Y_t equals factor income $r_t K_t + w_t L_t + n_t N_t$ in the competitive economy where r_t, w_t, and n_t are the three factor prices. Equivalently, income is the value of output $\Sigma_j p_j x_{jt}$ with exogenous prices p_j of the two goods. Factors are paid values of marginal products with perfect mobility between the two sectors.

Depletion N_t diminishes the resource stock S_t according to $N_t = -S_t'$. Optimal depletion satisfies the Hotelling (1931) condition that equates the rate of return on the stock n_t'/n_t to the capital return r_t in a condition that binds the intertemporal equilibrium. This asset market clearing condition is

$$n_t' = r_t n_t. \tag{1}$$

2. Substitution between the three inputs

Allen (1938) and Takayama (1982) lay the foundation for the economics of substitution applied by Thompson (1985, 2006) to three factors. Resource depletion equals resource demand in the condition $N_t = \Sigma_j a_{Nj} x_{jt}$ where a_{Nj} is the cost minimizing input per unit of output j. Resource input changes according to $N_t' = \Sigma_j x_{jt} a_{Nj}' + \Sigma_j a_{Nj} x_{jt}'$. Unit inputs a_{Nj}' depend only on factor prices assuming homogeneous production functions. Expanding a_{Nj}' across factor price changes,

$$N_t' = S_{NK} r_t' + S_{NL} w_t' + S_{NN} n_t' + \Sigma_j a_{Nj} x_{jt}', \tag{2}$$

where $S_{NK} \equiv \Sigma_j x_{jt}(a_{Nj}'/r_t')$ represents cross price substitution of the resource relative to the capital return. Similarly S_{NL} represents substitution of the resource relative to the wage, and S_{NN} relative to its own price. Capital substitution S_{Ki} and labor substitution S_{Li} are similar where i = K, L, N. Substitution is positive between cross price substitutes but a pair of inputs may be technical complements.

Cost minimization and Shephard's lemma imply unit inputs a_{ij} are partial derivatives of the unit cost function $c_j(r_t, w_t, n_t)$. Substitution terms are symmetric by Young's theorem. Assuming these unit cost functions are homogeneous of degree one in the factor prices, the unit input terms $a_{ij}(.)$ are homogeneous of degree zero. Concave cost functions imply negative own price substitution.

3. The intertemporal equilibrium

The first equation in the intertemporal system (3) below is capital employment similar to resource employment in (2) with $K_t' = \sigma Y_t$ as the change in the capital stock. The change n_t' in the resource price in (3) is replaced by $r_t n_t$ as in (1). The second equation in (3) is the change in labor employment where $L_t' = \lambda L_t$. The third equation is the resource market clearing condition (2).

The last two equations in (3) represent competitive pricing of the two goods. Price equals cost, $p_{jt} = a_{Kj}r_t + a_{Lj}w_t + a_{Nj}n_t$ for j = 1, 2. Differentiating, the envelope condition of cost minimization implies $p_{jt}' = a_{Kj}r_t' + a_{Lj}w_t' + a_{Nj}n_t'$.

The economy is in a temporary equilibrium defined by their employment and pricing conditions. The model solves for the intertemporal equilibrium adjustments r_t', w_t', N_t', and x_{jt}' in the system

$$\begin{pmatrix} S_{KK} & S_{KL} & 0 & a_{K1} & a_{K2} \\ S_{KL} & S_{LL} & 0 & a_{L1} & a_{L2} \\ S_{KN} & S_{LN} & -1 & a_{N1} & a_{N2} \\ a_{K1} & a_{L1} & 0 & 0 & 0 \\ a_{K2} & a_{L2} & 0 & 0 & 0 \end{pmatrix} \begin{bmatrix} r_t' \\ w_t' \\ N_t' \\ x_{1t}' \\ x_{2t}' \end{bmatrix} = \begin{pmatrix} \sigma Y_t - S_{KN}r_t n_t \\ \lambda L_t - S_{LN}r_t n_t \\ -S_{NN}r_t n_t \\ p_1' - a_{N1}r_t n_t \\ p_2' - a_{N2}r_t n_t \end{pmatrix}. \quad (3)$$

Intertemporal capital K_t' and income Y_t' are derived separately from the endogenous adjustments in (3). Assume constant exogenous world prices imply $p_1' = p_2' = 0$ with import tariffs and export subsidies analyzed in Section 4.

The negative determinant of the system in (3) is $\Delta = -a_{KL}^2$ where the term $a_{KL} \equiv a_{K1}a_{L2} - a_{L1}a_{K2}$ describes capital/labor intensity. Assume export production is capital intensive relative to labor, $a_{KL} > 0$. The similar resource/capital and resource/labor intensity terms are $a_{NK} \equiv a_{N1}a_{K2} - a_{K1}a_{N2}$ and $a_{NL} \equiv a_{N1}a_{L2} - a_{L1}a_{N2}$.

Solving (3) for the intertemporal equilibrium with Cramer's rule, the changes in the capital return and wage are

$$r_t' = -a_{NL} r_t n_t / a_{KL} \qquad (4)$$

$$w_t' = a_{NK} r_t n_t / a_{KL}.$$

Intertemporal factor prices are independent of growth in capital and labor due to the factor price equalization property. Smaller capital/labor intensity leverages the factor price adjustments as a_{KL} approaches zero with capital and labor effectively becoming a single input.

If the resource is extreme in the import competing sector in the ranking $a_{K1}/a_{K2} > a_{L1}/a_{L2} > a_{N1}/a_{N2}$ then the intensity terms $A \equiv (a_{KL} \;\; a_{NL} \;\; a_{NK})$ have signs $(+ \;\; - \;\; -)$ implying $(r_t' \;\; w_t')$ is $(+ \;\; -)$. The rising resource price increases the price of extreme capital and lowers payment to middle factor labor. If the resource is the middle factor $a_{K1}/a_{K2} > a_{N1}/a_{N2} > a_{L1}/a_{L2}$ the signs of A are $(+ \;\; + \;\; -)$ and of $(r' \;\; w')$ are $(- \;\; -)$. Prices of both extreme factors fall with the rising price of the middle factor.

For the present paper assume the resource is extreme in export production, $a_{N1}/a_{N2} > a_{K1}/a_{K2} > a_{L1}/a_{L2}$. The signs of A are all positive implying $(r_t' \;\; w_t')$ is $(- \;\; +)$. The falling capital return and rising wage are consistent with capital deepening in growth theory even though capital/labor ratio may not rise. As the resource price rises, the price of the other extreme factor labor also rises as the price of the middle factor capital falls.

Solving (3) for intertemporal depletion,

$$N_t' = -r_t n_t \Delta_{32}/\Delta < 0, \qquad (5)$$

where Δ_{32} is the negative determinant of the model with three factors and two goods. *Mutatis mutandis* resource demand is downward sloping in the general equilibrium, not an obvious property given the output adjustments and growth in capital and labor.

The intertemporal equilibrium output of the resource intensive export in (3) is

$$x_{1t}' = [-r_t n_t(a_{NK}S_1 + a_{NL}S_2 + a_{KL}S_3) + a_{KL}\kappa]/a_{KL}^2, \quad (6)$$

where $S_1 \equiv a_{L2}S_{KL} - a_{K2}S_{LL} > 0$, $S_2 \equiv a_{K2}S_{KL} - a_{L2}S_{KK} > 0$, $S_3 \equiv a_{L2}S_{KN} - a_{K2}S_{LN}$, and $\kappa \equiv a_{L2}\sigma Y_t - a_{K2}\lambda L_t$. Signs of S_3 and κ are uncertain. There is a presumption that $x_{1t}' < 0$ but the direction of the effect depends on factor intensity, substitution, investment relative to labor growth, and the state of the economy. Increased export production would be favored by weak substitution, low prices r_t and n_t of the intensive factors, high investment, low labor growth, and strong substitution of capital K combined with weak substitution of labor L with respect to the rising resource price n_t.

The presumption, however, is that $x_{1t}' < 0$. The mirror image presumption is that import competing production rises, $x_{2t}' > 0$. Both outputs may rise, however, based on investment and labor growth as shown in the simulations. Both outputs may fall as well based on declining depletion. Finally $x_{1t}' > 0$ and $x_{2t}' < 0$ is possible as well.

The output adjustments are reflected in intertemporal income, $Y_t' = K_t r_t' + L_t w_t' + N_t n_t' + n_t N_t'$. Substituting from (4) and (5),

$$Y_t' = r_t n_t[a_{KL}(a_{NK}L_t - a_{NL}K_t) + a_{KL}^2 N_t + n_t\Delta_{32})]/a_{KL}^2. \quad (7)$$

Rising income is favored by high levels of labor and resource inputs, a low level of capital input, a low resource price n_t, and weak substitution with Δ_{32} close to zero. Income per capita y_t would rise only if $Y_t'/Y_t > \lambda$.

Given the presumption of falling export production and rising import competing production, intertemporal income in (7) reflects the potential gains from trade. Income rises if the increased import competing output outweighs decreased export production as the economy moves to a higher terms of trade line where $-p_1 x_{1t}' < p_2 x_{2t}'$. The economy may, however, move to a lower terms of trade line reflected by lower income in (7). Regardless of the change in income, the level of trade falls assuming homothetic utility.

Figure 1 illustrates intertemporal trade with a homothetic utility function implying a constant consumption ratio c_1/c_2 given exogenous prices in the small open economy. Temporary equilibrium production at point P_0 is on the production frontier with utility maximizing consumption at C_0 as illustrated. The terms of trade line tt that connects P_0 and C_0 reflects the level of income. As export production falls and import competing production rises, the intertemporal production point shifts northeast. Income increases if the new production point is above the tt line as the higher tt line reflects higher income. The trade triangle shrinks with reduced trade. If the new production point were below the tt line, then income would fall. The level of trade would increase if both outputs rise with the increase in x_1/x_2 greater than C_1/C_2.

4. A depletion tax, import tariff, and export subsidy

A depletion tax t_N raises the price of the resource to $(1 + t_N)n_t$ amplifying the intertemporal dynamics. Depletion N′ in (5) becomes more negative as the higher resource price reduces quantity demanded. Adjustments in the capital return and wage in (4) are amplified with labor benefiting as the economy shifts toward production of the import competing good. Substitution S_{LK} between labor and capital in (6) favors this shift toward import competing production as do low saving σ and high labor growth λ. The depletion tax amplifies the intertemporal income adjustment in (7).

Import tariffs and export subsidies add their effects to the intertemporal equilibrium through p_j' in (3). An import tariff reinforces the two factor prices according to

$$r_t'/p_2' = -a_{L1}/a_{KL} + r_t' < 0 \qquad (8)$$

$$w_t'/p_2' = a_{K1}/a_{KL} + w_t' > 0,$$

where r_t' and w_t' are the underlying adjustments in (4). The tariff favors intensive labor in the import competing industry. Larger difference in capital/labor intensity diminishes these factor price effects.

The tariff affects depletion according to

$$N_t'/p_2' = -(S_4a_{NK} + S_5a_{NL} + S_6a_{KL})/a_{KL}^2 + N_t', \qquad (9)$$

where $S_4 \equiv a_{L1}S_{KL} - a_{K1}S_{LL} > 0$, $S_5 \equiv a_{K1}S_{KL} - a_{L1}S_{KK} > 0$, $S_6 \equiv a_{L1}S_{KN} - a_{K1}S_{LN}$, and $N_t' < 0$ is the intertemporal decline in (5). The two terms S_4 and S_5 reinforce declining depletion but a negative S_6 with capital a complement for the resource would weaken depletion.

An export subsidy has a positive effect on the intertemporal capital return,

$$r_t'/p_1' = a_{L2}/a_{KL} + r_t', \qquad (10)$$

where $r_t' < 0$ is the intertemporal decrease in (4). An export subsidy could result in a net increase in the capital return. A small resource price n_t and small labor/resource intensity in (4) under the condition $r_t < a_{L2}/a_{NL}n_t$.

The effect of the export subsidy on the wage is

$$w_t'/p_1' = -a_{K2}/a_{KL} + w_t', \qquad (11)$$

where $w_t' > 0$ is the intertemporal adjustment in (4). An export subsidy offsets the rising wage. A wage decrease occurs if $a_{K2}/a_{NK} > r_tn_t$ favored by weak capital/resource intensity and low capital and resource prices. An export subsidy would raise N_t in an expression similar to (9).

Output changes due to an import tariff or export subsidy involve substitution in production. The effect of a subsidy on export production is

$$x_{1t}'/p_1' = (2a_{K2}a_{L2}S_{KL} - a_{K2}^2S_{LN} - a_{L2}^2S_{KK}) + x_{1t}', \qquad (12)$$

where x_{1t}' is the intertemporal adjustment in (6). Stronger substitution implies a less concave production frontier and favors increased x_{1t}. The export subsidy unambiguously lowers x_{2t} in a similar expression.

5. Simulations of depletion and production

These simulations illustrate the intertemporal model with Cobb-Douglas production functions. The resource intensive export is produced according to $x_{1t} = K_{1t}^{0.6}L_{1t}^{0.1}N_{1t}^{0.3}$ and the labor intensive import competing $x_{2t} = K_{2t}^{0.4}L_{2t}^{0.5}N_{2t}^{0.1}$. The model is simulated over 10 time periods starting at initial values $K_1 = 100{,}000$ and $L_1 = 100$. The positive factor intensity terms A = (a_{KL} a_{NL} a_{NK}) imply a rising wage and falling capital return.

Exogenous world prices are set equal to 1. The saving rate is $\sigma = 0.25$ and labor growth $\lambda = 0.01$ with sensitivity examined. The initial equilibrium is determined assuming $n_1 = 24$ implying $N_1 = 9.5$. The resource price increases according to $n_{t+1} = (1 + r_t)n_t$ determining depletion N_{t+1} and the other intertemporal adjustments in system (3). Variables are rescaled for the Figures.

This economy in Figure 2 strongly trends toward production of import competing good 2 as depletion N decreases at a decreasing rate. Income Y falls but only slightly as the rising wage w and resource price n nearly offset the declining capital return r. The level of trade falls with income and consumption of both goods.

* Figure 2 *

Figures 3 pictures an otherwise identical economy with higher saving at $\sigma = 0.40$ and lower labor growth $\lambda = 0.005$. Output of exported good 1 rises as capital growth more than offsets the slower decline in depletion N. Import competing production x_2 expands more slowly than in Figure 2. Factor price paths of w and r are identical to Figure 2 due to factor price equalization.

Income Y increases with the wage w and resource price n. Consumption of both goods increases as could the level of trade.

* Figure 3 *

Figure 4 shows an economy with the original saving rate $\sigma = 0.25$ but slightly negative labor growth at $\lambda = -0.05$. Both outputs fall along with income Y as capital growth does not support production with the declining depletion and labor force. Consumption of both goods falls even though the level of trade could increase with a sharper reduction in import competing production.

* Figure 4 *

The depletion tax of $t_N = 10\%$ at t = 5 in Figure 5 reduces depletion N as the tax raises the input price to $(1 + t_N)n_t$ along a higher optimal intertemporal path. Resource intensive x_1 and the capital return r both fall before resuming negative trends. Output of labor intensive x_2 increases as labor and capital are released from producing exported good 1.

* Figure 5 *

Figure 6 shows the effects of a 10% import tariff at t = 5 with the original saving $\sigma = 0.25$ and labor growth $\lambda = 0.01$ in Figure 2. The tariff reinforces the underlying factor price paths of the wage w and capital return r in (8). Depletion N and export production x_1 both fall to lower transition paths as production of the import competing good x_2 jumps to a higher trend in (9).

* Figure 6 *

An export subsidy of 5% at t = 5 in Figure 7 leads to adjustments based on (7), (8), and (9). Export production x_1 jumps with a sharp increase in depletion N before both resume downward trends, the effects of the subsidy only temporary. Production of import competing x_2 falls before adjusting and resuming its upward trend, similar to the wage w. The capital return r rises temporarily but then returns to its intertemporal decline. The subsidy has only transitory effects as the economy adjusts and returns to its transition path. An export subsidy of 10% collapses the import competing sector.

* Figure 7 *

An export tax of 10% leads to the mirror image adjustments in Figure 8. The wage w and import competing production x_2 rise substantially and assume higher trends. Depletion N_t falls sharply as does export production x_1 and the capital return r as they adjust to new trends. Reaction to the export tax is similar but not identical to the import tariff in Figure 6 as the differences in factor prices and depletion relax the Lerner symmetry theorem.

* Figure 8 *

6. Alternative assumptions on depletion

A constant depletion rate implies the same fraction α of the resource stock S_t is depleted each time period according to $N_t = \alpha S_t$. The intertemporal change $(N_t/S_t)'$ implies $N_t'S_t = N_tS_t'$ or $N_t' = -\alpha N_t$ in a relationship added to (3) with the endogenous n_t'. The resulting system is similar to the static model with three factors and two goods. A higher depletion rate α implies higher N_t', n_t', and r_t' but lower w_t'. A higher saving rate σ raises capital growth and w_t' but lowers r_t' and n_t' as capital replaces the resource in export production. Higher labor growth λ would lower w_t' but raise r_t' and n_t'. Other properties of the model depend on substitution as well. An import tariff and an export subsidy have the expected output effects but their factor price effects depend on the state of the economy.

In a tragedy of the commons, the resource is priced at marginal physical extraction cost E_t. Constant E_t implies $n_t' = 0$ eliminating r_tn_t in the exogenous vector of (3). The resulting constant resource price implies a constant wage w_t and capital return r_t as well. Production of the export evolves according to $x_{1t}' = (a_{L2}\sigma Y_t - a_{K2}\lambda L_t)a_{KL}^{-1}$ in the first term of (6). Higher capital growth and lower labor growth both favor increased x_{1t}. The change in import competing output $x_{2t}' = (a_{K1}\lambda L_t - a_{L2}\sigma Y_t)a_{KL}^{-1}$ would be negative with higher capital growth or lower labor growth. Depletion rises according to $N_r' = \Sigma_j a_{Nr} x_j' = (a_{NL}\sigma Y_t + a_{NK}\lambda L_t)a_{KL}^{-1} > 0$. Income rises due to the gains from competition, $Y_t' = w_t L_t' + r_t K_t' + n_t N_t' = [(w_t a_{KL} + a_{NK})\lambda L_t + (r_t a_{KL} + a_{NL})\sigma Y_t]a_{KL}^{-1} > 0$. Rising marginal extraction cost would lead to more complex properties following the model of optimal depletion in Section 3.

A myopic monopolistic resource owner maximizes immediate profit disregarding the asset value of the resource stock and setting marginal revenue R_t equal to marginal extraction cost E_t. Total resource revenue $n_t N_t$ implies $R_t = (n_t N_t)'/N_t' = n_t + N_t n_t'/N_t' = E_t$ or $n_t' = N_t'(E_t - n_t)/N_t$. The resource price n_t would have to be greater than E_t implying opposite signs for n_t' and N_t'. The myopic resource owner suffers a falling income share.

7. Conclusion

The present small open economy produces a nonrenewable resource intensive export and a labor intensive import competing good. Optimal depletion implies a rising resource price. The wage rises and capital return falls consistent with capital deepening in growth theory. Depletion diminishes regardless of adjustments in the two outputs. Trade falls with declining export production but income may rise with production of the import competing good. A depletion tax and an import tariff amplify these trends while an export subsidy has a dampening effect.

This model of depletion and trade can be modified in various ways. A renewable or backstop resource can be analyzed. The implications of optimal saving and endogenous labor growth can be analyzed. Utility maximization would lead to the intertemporal trade levels for a small open economy, or to the terms of trade between two such large open economies.

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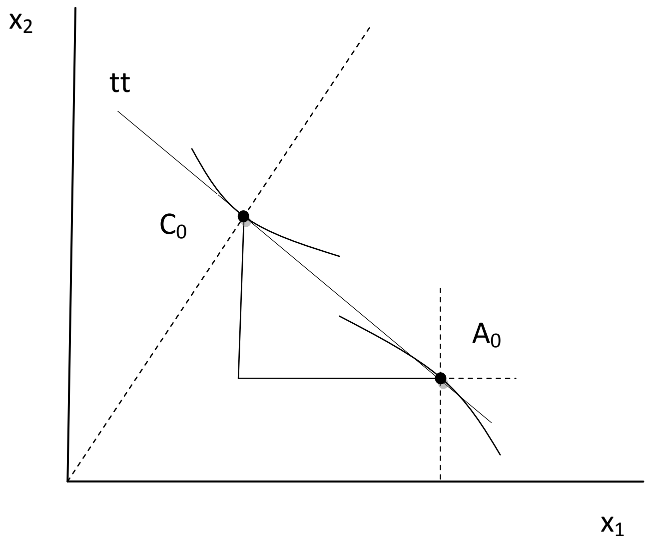

Figure 1. Production and Trade

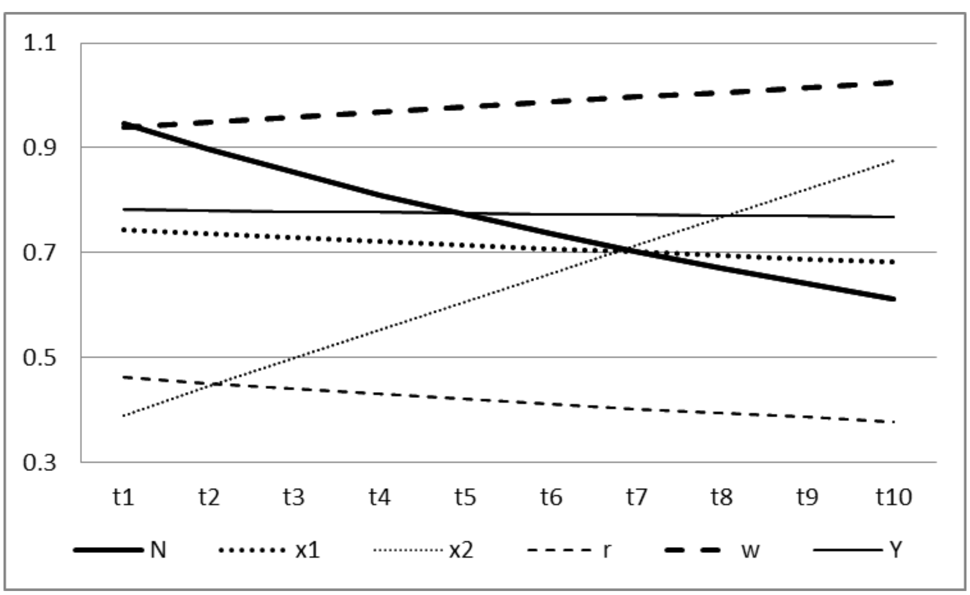

Figure 2. Cobb-Douglas production

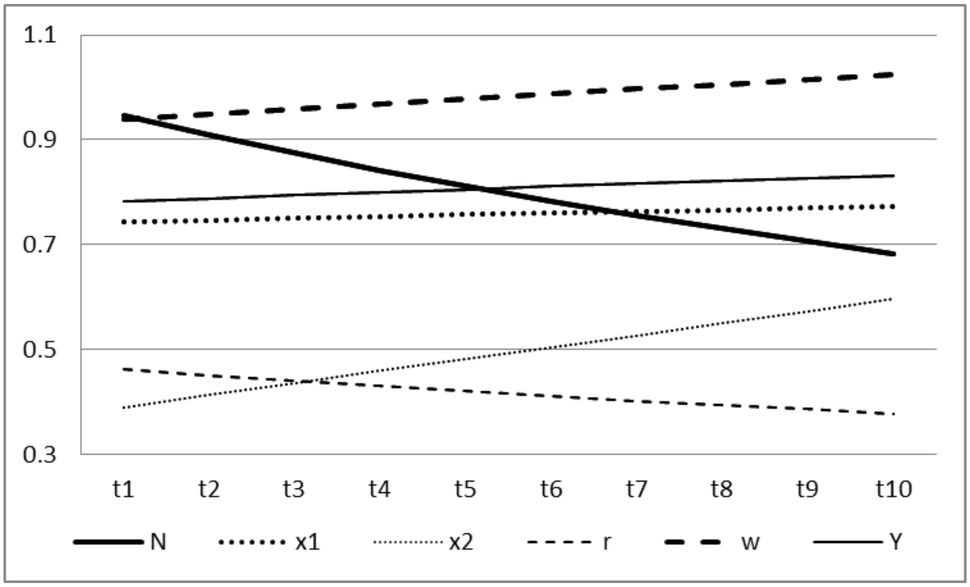

Figure 3. High saving and low labor growth

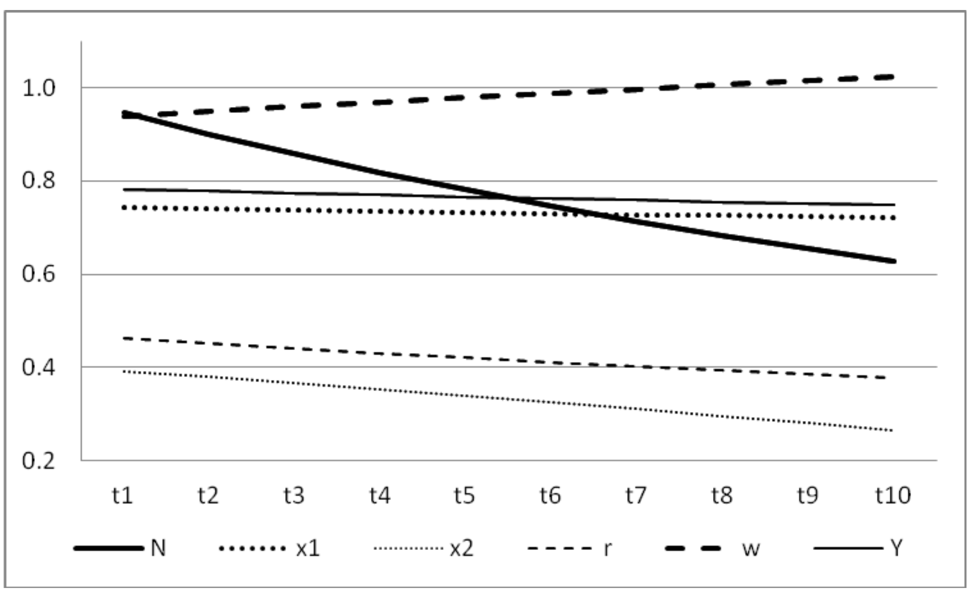

Figure 4. Negative labor growth

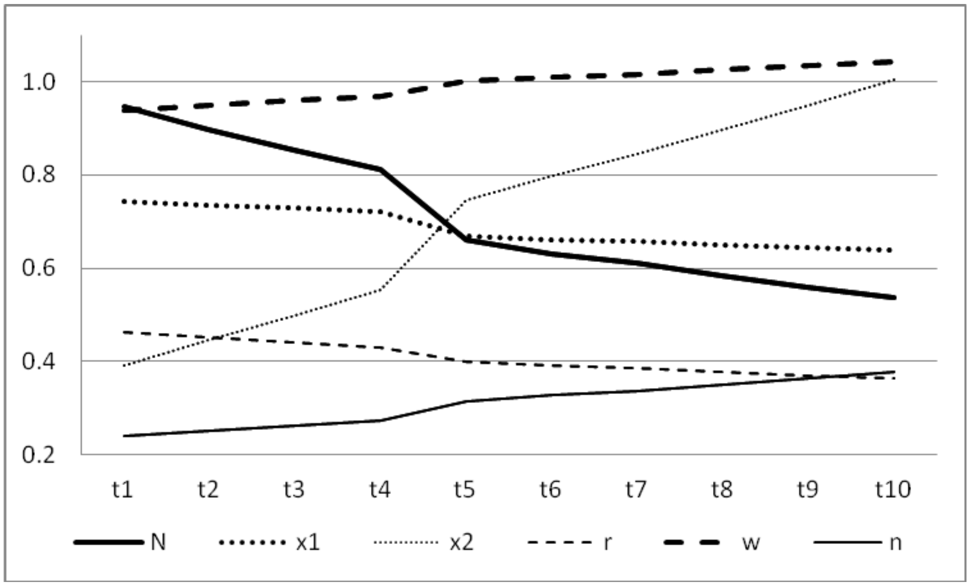

Figure 5. 10% depletion tax at t = 5

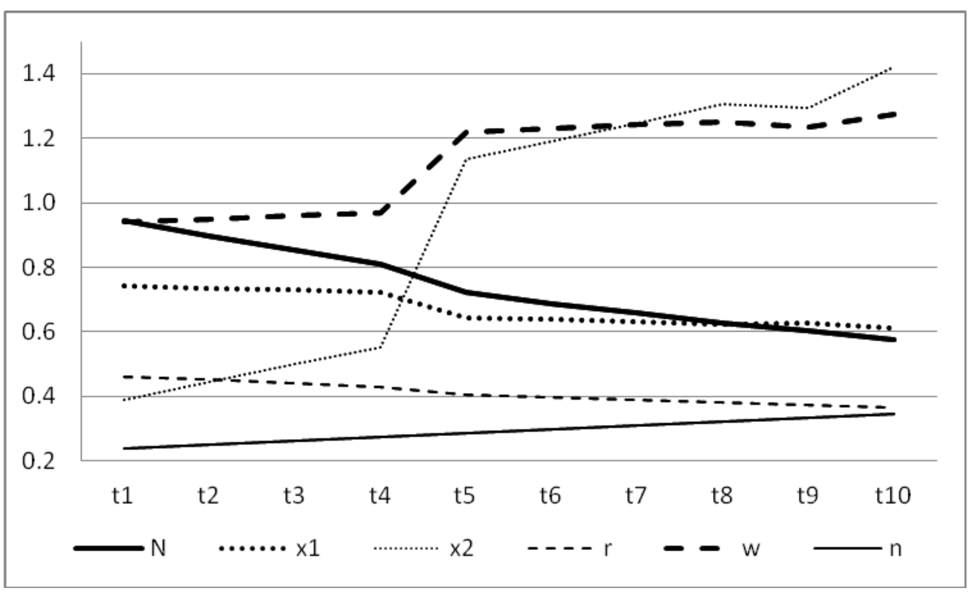

Figure 6. 10% import tariff at t = 5

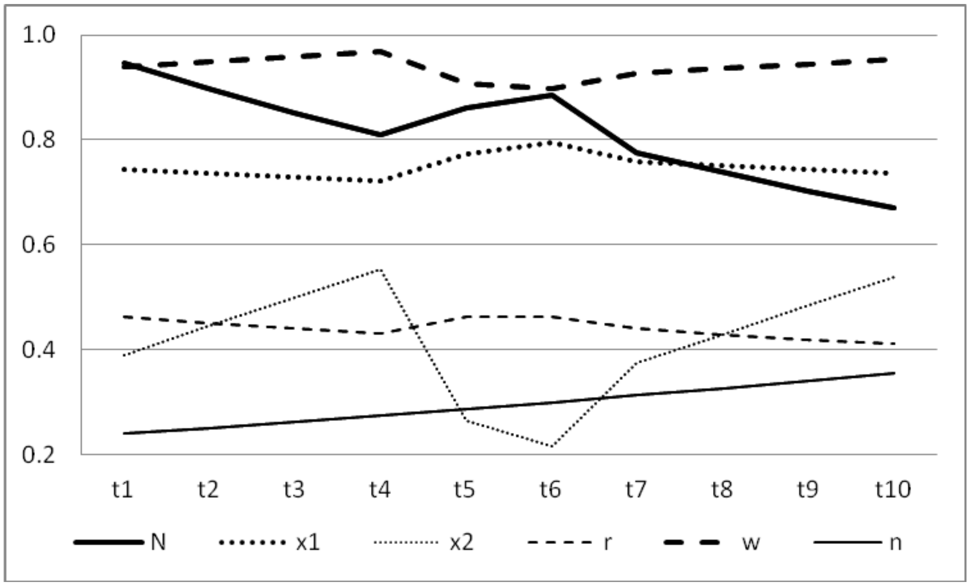

Figure 7. 5% export subsidy at t = 5

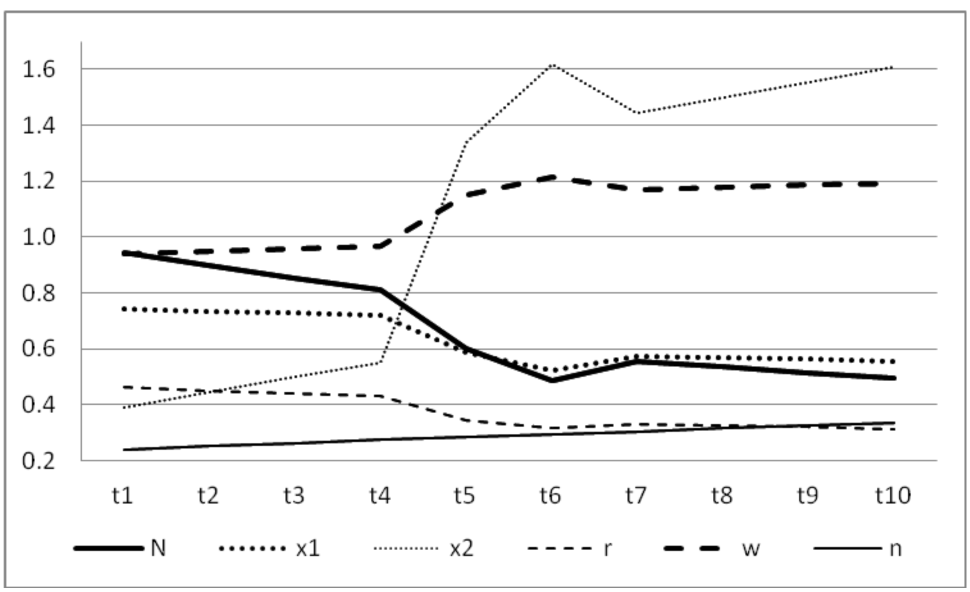

Figure 8. 10% export tax at t = 5