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Incentive to reduce cost under incomplete information

Aditi Sengupta

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Abstract

I examine how *ex ante* symmetric firms that compete in prices strategically decide to invest in research and development of cost-reducing technology when the rival firm and the consumers are not aware of the actual outcome of the investment. I also compare the strategic incentive to invest and market outcomes under incomplete information with that of the full information. I find that equilibrium investment under incomplete information with unobservable investment is same as that of (symmetric) full information equilibrium and is also socially optimal.

Key-words: Cost-reducing technology; Duopoly; Incomplete information; Price competition; Strategic investment.

JEL Classification: D43, D82, L13.

1 Introduction

Rent-seeking firms often invest in process innovation i.e., research and development (R&D) to invent and adopt cost-reducing production technology. However, the success of cost reducing R&D is not always guaranteed for sure. If a firm invests in an uncertain innovation process it eventually gets to know the final outcome of its own investment i.e., whether it has successfully achieved to adopt the low-cost technology. However, the information structure in the market depends on voluntary disclosure by firms or anti-trust regulations on information sharing among firms. In this paper, I try to examine the strategic incentive of a firm to invest in cost-reducing technology when the rival and the consumers are unaware of the actual outcome of its investment. In particular, I consider two specific incomplete information structures i.e., either a firm observes rival's investment decision or not. Lack of information among strategically competing firms, in turn, determines firms' market power, profitability, and incentive to invest. I also compare the strategic incentive to invest, market power, and profitability of firms under incomplete information with that of the full information and socially optimal outcomes. One may consider this paper as an attempt to study the effect of disclosure of information among strategically competing firms on their investment behavior. Surprisingly, I find that the least possible information sharing among firms not only yields similar investment outcome as that of the full information which is also socially optimal but it also happens to generate strictly positive expected profit unlike the latter. Moreover, not only both firms find it profitable to invest under incomplete information but also the aggregate investment in the industry under incomplete information with observable investment can be higher compared to the full information.

To the best of my knowledge, this is the first attempt to critically examine the strategic investment behavior of *ex ante* symmetric firms that compete in prices when the actual investment outcomes of the cost-reducing investment remain as private knowledge. Thomas (1997) considers *ex ante* asymmetric firms (i.e., firms could be high-cost or low-cost types with different probabilities) and allows only one firm to make a stochastic investment in freely available cost-reducing technology. Unlike Thomas (1997), I find that both firms have strictly positive strategic incentive to invest in cost-reducing technology under incomplete information even when the cost-reducing technology is not free. Jansen (2010) investigates the strategic incentive of a firm to disclose the cost (parameter) of investment when in the next stage both firms simultaneously choose their R&D investments where the probability of success of the innovation depends on the investment. Unlike Jansen, I

consider a framework where the incidence of information sharing among firms is exogenously given. In other words, this paper is silent about the strategic incentive of information sharing among firms and primarily focuses on the incentive to invest in (production) cost reducing technology.

The remainder of the paper is organized as follows. Section 2 describes the model. In section 3, I discuss the price and investment equilibriums under full and incomplete information. Section 4 briefly compares the outcomes under incomplete information with that of the full information and also with respect to socially optimal outcomes.

2 The model

I consider an oligopolistic market with two identical firms that compete in prices and produce homogenous product. The production technology of each firm can be of two potential types: high-cost (H) and low-cost (L). Each firm produces at constant unit cost. The unit production cost of a high-cost type (defined by c_H) is greater than that of a low-cost type (defined by c_L) i.e., $0 < c_H < c_L$. There is a unit mass of risk neutral consumers in the market. Consumers have unit demand i.e., each consumer buys at most one unit of the good. Each consumer is willing to pay V for a unit produced by both firms. I assume that $V > c_H$.

Firms are initially endowed with high-cost technology i.e., each firm incurs a unit production cost of c_H . Firms can invest in R&D and adoption of a cost-reducing technology. However, the outcome of the investment is uncertain, and the probability of success is positively related to the investment. In the first stage, firms simultaneously choose the probability of successful investment in cost-reducing technology $\mu_i \in [0, 1] \forall i = 1, 2$. In other words, a firm successfully adopts the low-cost technology with probability μ_i and remains high-cost type with probability $1 - \mu_i$. The cost of investment is given by $A\mu_i$ where $0 < A < (c_H - c_L)$ ¹. Depending on the nature of information sharing among firms there could be three alternative situations; (1) both investment and the actual outcome of investment are observed by the firms and consumers, (2) firms share information about investment but remain unaware of the final outcome of the investment made by the rival firm, and finally (3) firms do not share any information about each others' investments and the outcome of the investments. In the rest of the paper, I refer the first one as full information whereas the last two as incomplete information with observable and unobservable investment respectively. The realizations of production technology after investment are independent across firms, and there is

¹If $A > (c_H - c_L)$ then firms do not invest in the equilibrium under both full and incomplete information.

no spill over. In the next stage, firms choose prices simultaneously. Finally consumers observe the prices charged by the firms, decide whether to buy, and from which firm to buy.

3 The strategic investment

Under full information, firms are not only aware of rivals' investment decision but also get to know each others' investment outcome after the first stage. In the second stage price equilibrium if the firms are of same type then they aggressively compete and bring down the price to the respective marginal cost earning zero profit. However, the low cost type charges the marginal cost of the high type (c_H) and earns positive profit ($c_H - c_L$) when the rival is of high cost type whereas the high cost type charges its own marginal cost and earns zero profit. Thus, the *ex ante* expected profit of a firm i under full information is given by $\pi_i = \mu_i (1 - \mu_j) (c_H - c_L)$. The first stage expected profit maximization problem of a firm i under full information is

$$\begin{aligned} \max_{\mu_i} \pi_i - A\mu_i &= \max_{\mu_i} \mu_i [(1 - \mu_j) (c_H - c_L) - A] \\ \text{s.t. } 0 &\leq \mu_i \leq 1 \end{aligned}$$

Proposition 1 *Under full information Nash equilibriums, firms choose either $\mu_i^* = 1, \mu_j^* = 0$ or $\mu_i^* = \mu_j^* = \left(1 - \frac{A}{(c_H - c_L)}\right) \forall i, j = 1, 2$ where $i \neq j$.*

Proof. Under full information the reaction function i.e., $\mu_i^* = \arg \max_{\mu_i} \mu_i [(1 - \mu_j) (c_H - c_L) - A]$ and expected profit $\pi_i^* = \max_{\mu_1} \pi_i - A\mu_i$ are given by

$$\mu_i^* \begin{cases} = 1 & \text{if } 0 \leq \mu_j < \left(1 - \frac{A}{(c_H - c_L)}\right) \\ \in [0, 1] & \text{if } \mu_j = \left(1 - \frac{A}{(c_H - c_L)}\right) \\ = 0 & \text{if } 1 \geq \mu_j > \left(1 - \frac{A}{(c_H - c_L)}\right) \end{cases} \quad (1)$$

$$\pi_i^* = \begin{cases} (1 - \mu_j) (c_H - c_L) - A & \text{if } 0 \leq \mu_j < \left(1 - \frac{A}{(c_H - c_L)}\right) \\ 0 & \text{if } \mu_j = \left(1 - \frac{A}{(c_H - c_L)}\right) \\ 0 & \text{if } 1 \geq \mu_j > \left(1 - \frac{A}{(c_H - c_L)}\right) \end{cases} \quad (2)$$

Three Nash equilibriums and respective expected profits under full information are

$$\mu_i^* = 1, \mu_j^* = 0 \text{ and } \pi_i^* = (c_H - c_L) - A, \pi_j^* = 0 \forall i, j = 1, 2 \text{ where } i \neq j \quad (3)$$

$$\mu_i^* = \mu_j^* = \left(1 - \frac{A}{(c_H - c_L)}\right) \text{ and } \pi_i^* = \pi_j^* = 0 \quad (4)$$

■

Figure 1 illustrates the reaction functions and full information equilibriums of the two stage game. In the interior equilibrium (E_3 in Figure 1) where both firms invest strictly positive amount, each firm earns zero expected profit; however, it is not a stable equilibrium². Whereas in the other two possible Nash equilibriums (E_1 and E_2 in Figure 1) only one firm invests a strictly positive amount (i.e., $A\mu_i = A$); the investing firm earns strictly positive expected profit, but non-investing firm earns zero profit.

Next I consider the case where firms do not exchange information about actual outcome of the investment in the cost-reducing technology; however, the first stage investment decisions are observed by all. Formally, this leads to a two stage Bayesian game. First, I discuss the second stage subgame where firms choose prices simultaneously with the private knowledge of their own production technology³. Without any loss of generality, I assume that $\mu_i \geq \mu_j \forall i, j = 1, 2$ where $i \neq j$ i.e., firm i is more likely to successfully adopt the low-cost technology than firm j .

Lemma 1 (Price equilibrium under incomplete information) *The high-cost type of firm i charges a price equal to its own unit production cost c_H and low-cost type randomizes over $[(1 - \mu_j)c_H + \mu_j c_L, c_H]$ with probability distributions*

$$F_i(p) = \frac{1}{\mu_i} - \frac{(1 - \mu_j)(c_H - c_L)}{\mu_i(p - c_L)} \text{ and } F_j(p) = \frac{1}{\mu_j} - \frac{(1 - \mu_j)(c_H - c_L)}{\mu_j(p - c_L)}$$

$\forall i, j = 1, 2$ where $i \neq j$. The ex ante expected profits are

$$\pi_i = \mu_i(1 - \mu_j)(c_H - c_L) \text{ and } \pi_j = \mu_j(1 - \mu_j)(c_H - c_L). \quad (5)$$

The above lemma implies that there does not exist any Bayesian price equilibrium in pure strategies. The low-cost type has competitive advantage over the high-cost type since $V - c_L > V - c_H$; thus, if the investing firm becomes low-cost type it enjoys market power and steals all business in the state when the rival (investing firm) remains high-cost type, but also has an incentive

²For $\epsilon > 0$, if firm j chooses $\left(1 - \frac{A}{(c_H - c_L)}\right) - \epsilon$ then firm i deviates away from $\left(1 - \frac{A}{(c_H - c_L)}\right)$ and does not have any incentive to revert back as firm i earns higher profit at any $\mu_i^* < \left(1 - \frac{A}{(c_H - c_L)}\right)$. Similar argument can be made for firm j for a given deviation by firm i .

³Spulber (1995) and Routledge (2010) consider Bertrand price competition under asymmetric information about rival's cost when firms face downward sloping market demand.

to undercut the rival in case it is of low-cost type too. In the unique price equilibrium, the low-cost type randomizes price over an interval $[\underline{p}, c_H]$ to balance these incentives. The equilibrium profit of a low-cost type is $\pi_L = (1 - \mu_j)(c_H - c_L)$ for any price $p \in [\underline{p}, c_H]$. This yields the lower bound of the mixed strategy price support $\underline{p} = (1 - \mu_j)c_H + \mu_j c_L$. If $\mu_i > \mu_j$, low cost type of firm i has mass point on the upper bound; in other words, firm i has higher probability of charging the upper bound (c_H) of the price distribution than firm j does. Note that the low-type of firm j earns the same profit (π_L), but there is no mass point on the upper bound (c_H) for firm j . Also, if $\mu_i = \mu_j$ then there is no mass point on the upper bound of low-cost type of either firm.

At any price $p \in [(1 - \mu_j)c_H + \mu_j c_L, c_H]$ the low cost type firm can sell to the entire market if either the rival is of high-cost type or it is not undercut by the low-cost rival and thus, earns expected profit of (π_L); this implies $(1 - \mu_j)(p - c_L) + (1 - F_j(p))\mu_j(p - c_L) = (1 - \mu_j)(c_H - c_L)$ and $(1 - \mu_i)(p - c_L) + (1 - F_i(p))\mu_i(p - c_L) = (1 - \mu_j)(c_H - c_L)$. Thus, I get

$$F_i(p) = \frac{1}{\mu_i} - \frac{(1 - \mu_j)(c_H - c_L)}{\mu_i(p - c_L)} \text{ and } F_j(p) = \frac{1}{\mu_j} - \frac{(1 - \mu_j)(c_H - c_L)}{\mu_j(p - c_L)}.$$

Note that $F_i(p) = F_j(p)$ if $\mu_i = \mu_j$. The high-cost type charges its own marginal cost and earns zero profit in the equilibrium i.e., $\pi_H = 0$. I calculate the expected profits of firm i and firm j as $\pi_i = \mu_i \pi_L + (1 - \mu_i) \pi_H = \mu_i (1 - \mu_j)(c_H - c_L)$ and $\pi_j = \mu_j \pi_L + (1 - \mu_j) \pi_H = \mu_j (1 - \mu_j)(c_H - c_L)$ respectively. To be more precise, in this case,

$$\begin{aligned} \pi_i &= \mu_i (1 - \mu_i)(c_H - c_L) \text{ if } \mu_i \leq \mu_j \\ &= \mu_i (1 - \mu_j)(c_H - c_L) \text{ if } \mu_i \geq \mu_j. \end{aligned}$$

In the first stage, firm i chooses μ_i to maximize expected profit from investment given that rival firm j has chosen μ_j . In other words, firm i solves the following constrained expected profit maximization problem:

$$\max_{\mu_i} \pi_i - A\mu_i = \begin{cases} \max_{\mu_i} \mu_i [(1 - \mu_i)(c_H - c_L) - A] \text{ s.t. } 0 \leq \mu_i < \mu_j \\ \max_{\mu_i} \mu_i [(1 - \mu_j)(c_H - c_L) - A] \text{ s.t. } \mu_j < \mu_i \leq 1 \end{cases} \quad (6)$$

The following proposition describes the Bayesian Nash investment equilibrium under incomplete information with observable investment.

Proposition 2 *Under incomplete information with observable investment, firms choose $\mu_i^* = 1$*

and $\mu_j^* = \frac{1}{2} \left(1 - \frac{A}{(c_H - c_L)} \right) \forall i, j = 1, 2$ where $i \neq j$ in the Bayesian Nash investment equilibrium.

Proof. Suppose $\arg \max_{\mu_i} \pi_i - A\mu_i = \mu_i^*$ and $\pi_i^* = \max_{\mu_i} \pi_i - A\mu_i$. For the first part of (6)

$$\mu_i^* = \begin{cases} \mu_j & \text{if } \mu_j \leq \frac{1}{2} \left(1 - \frac{A}{(c_H - c_L)} \right) \\ \frac{1}{2} \left(1 - \frac{A}{(c_H - c_L)} \right) & \text{if } \mu_j > \frac{1}{2} \left(1 - \frac{A}{(c_H - c_L)} \right) \end{cases} \quad (7)$$

$$\pi_i^* = \begin{cases} \mu_j [(1 - \mu_j)(c_H - c_L) - A] & \text{if } \mu_j \leq \frac{1}{2} \left(1 - \frac{A}{(c_H - c_L)} \right) \\ \frac{[(c_H - c_L) - A]^2}{4(c_H - c_L)} & \text{if } \mu_j > \frac{1}{2} \left(1 - \frac{A}{(c_H - c_L)} \right) \end{cases} \quad (8)$$

Consider the second part of (6).

$$\mu_i^* \begin{cases} = 1 & \text{if } 0 \leq \mu_j < \left(1 - \frac{A}{(c_H - c_L)} \right) \\ \in \left[1 - \frac{A}{(c_H - c_L)}, 1 \right] & \text{if } \mu_j = \left(1 - \frac{A}{(c_H - c_L)} \right) \\ = \mu_j & \text{if } \left(1 - \frac{A}{(c_H - c_L)} \right) < \mu_j \leq 1 \end{cases} \quad (9)$$

$$\pi_i^* = \begin{cases} (1 - \mu_j)(c_H - c_L) - A > 0 & \text{if } 0 \leq \mu_j < \left(1 - \frac{A}{(c_H - c_L)} \right) \\ 0 & \text{if } \mu_j = \left(1 - \frac{A}{(c_H - c_L)} \right) \\ \mu_j [(1 - \mu_j)(c_H - c_L) - A] < 0 & \text{if } \left(1 - \frac{A}{(c_H - c_L)} \right) < \mu_j \leq 1. \end{cases} \quad (10)$$

To find the best response function of firm i for any given μ_j , I compare the derived expected profits of firm i in (8) and (10). Note that $(1 - \mu_j)(c_H - c_L) - A > \mu_j [(1 - \mu_j)(c_H - c_L) - A]$ which means that if $0 \leq \mu_j \leq \frac{1}{2} \left(1 - \frac{A}{(c_H - c_L)} \right)$ the best response of firm i is $\mu_i^* = 1$. Also $(1 - \mu_j)(c_H - c_L) - A \geq \frac{[(c_H - c_L) - A]^2}{4(c_H - c_L)}$ for $\mu_j \leq \left(1 - \frac{((c_H - c_L) + A)^2}{4(c_H - c_L)^2} \right)$ which implies that $\mu_i^* = 1$ if $\frac{1}{2} \left(1 - \frac{A}{(c_H - c_L)} \right) \leq \mu_j \leq \left(1 - \frac{((c_H - c_L) + A)^2}{4(c_H - c_L)^2} \right)$. However, $(1 - \mu_j)(c_H - c_L) - A \leq \frac{[(c_H - c_L) - A]^2}{4(c_H - c_L)}$ for $\mu_j \geq \left(1 - \frac{((c_H - c_L) + A)^2}{4(c_H - c_L)^2} \right)$; thus, best response of firm i is $\mu_i^* = \frac{1}{2} \left(1 - \frac{A}{(c_H - c_L)} \right)$ if $\left(1 - \frac{((c_H - c_L) + A)^2}{4(c_H - c_L)^2} \right) \leq \mu_j \leq 1$. To summarize, the reaction function of firm i under incomplete information is given by

$$\mu_i^* = \begin{cases} 1 & \text{if } 0 \leq \mu_j \leq \left(1 - \frac{((c_H - c_L) + A)^2}{4(c_H - c_L)^2} \right) \\ \frac{1}{2} \left(1 - \frac{A}{(c_H - c_L)} \right) & \text{if } \left(1 - \frac{((c_H - c_L) + A)^2}{4(c_H - c_L)^2} \right) \leq \mu_j \leq 1 \end{cases} \quad (11)$$

$\forall i, j = 1, 2$ where $i \neq j$. Two asymmetric Bayesian Nash equilibriums of the investment game under incomplete information are

$$\mu_i^* = 1, \mu_j^* = \frac{1}{2} \left(1 - \frac{A}{(c_H - c_L)} \right) \quad (12)$$

which yield the following *ex ante* expected profits for firm i and firm j

$$\pi_i^* = \frac{(c_H - c_L) - A}{2}, \pi_j^* = \frac{[(c_H - c_L) - A]^2}{4(c_H - c_L)}. \quad (13)$$

■

Figure 2 depicts the reaction functions of firms (denoted by (11)). Observe that, there are two asymmetric⁴ Bayesian Nash equilibriums (represented by E_4 and E_5 in Figure 2). In particular, one firm chooses μ_i such that it becomes low-type with probability one; it also implies that the firm invests maximum possible amount (A). Whereas the other firm invests less, remains high-cost type with a strictly positive probability, and earns less profit. Both firms make strictly positive investment to generate uncertainty about the cost structure and thus, in turn, earn strictly positive expected profit. Note that firms do not behave in the similar fashion in the asymmetric equilibrium under full information. It is because under full information full disclosure of final outcome of R&D investment reveals the type of the firms and thus, there is no uncertainty left that can generate positive expected profit. Further, increase in cost differential ($c_H - c_L$) increases market power and profitability of the low-cost type which in turn, creates higher strategic incentive to invest.

Finally, I study the equilibrium investment behavior when the probability of successful investment in cost reducing technology is also a private knowledge i.e., firms do not disclose their investment behavior to the rival. In particular, firms choose probability of success simultaneously in the first stage and do not disclose the decision to each other. A firm comes to know its own type but is unaware of both the probability of success and the actual outcome of the investment made by the rival. In the next stage, firms choose prices simultaneously. I solve the game by backward induction. Note that in this multi-stage imperfect information game, the nature of pricing equilibrium outcomes of the second stage is similar to that of the incomplete information one discussed in Lemma 1.

Proposition 3 *Under incomplete information with unobservable investment, firms choose $\mu_i^* = \mu_j^* = \left(1 - \frac{A}{(c_H - c_L)}\right) \forall i, j = 1, 2$ where $i \neq j$ in the Bayesian Nash investment equilibrium.*

Proof. Suppose $\mu_i = \mu_j = \bar{\mu}$ is a Nash equilibrium. Given $\mu_j = \bar{\mu}$, if firm i deviates to $\mu_i \neq \bar{\mu}$ then the rival firm j does not observe this deviation and believes that firm i has chosen $\bar{\mu}$. Thus,

⁴It is easy to prove why symmetric equilibrium does not exist. Assume that $\mu_i = \mu_j = \tilde{\mu}$. Given $\mu_j = \tilde{\mu}$, firm i has a strictly positive incentive to deviate to $\mu_i > \tilde{\mu}$ since firm i earns higher expected profit if it decides to invest more than its rival.

the low-cost type of firm i randomizes over a price interval $p \in [(1 - \bar{\mu})c_H + \bar{\mu}c_L, c_H]$ and earns $(1 - \bar{\mu})(c_H - c_L)$. If it deviates to μ_i , the *ex ante* expected profit of firm i i.e., $\pi_i = \mu_i\pi_L + (1 - \mu_i)\pi_H$, is given by

$$\pi_i = \mu_i(1 - \bar{\mu})(c_H - c_L).$$

The expected profit from deviation is maximized at

$$\mu_i \begin{cases} = 1 & \text{if } 0 \leq \bar{\mu} < \left(1 - \frac{A}{(c_H - c_L)}\right) \\ \in [0, 1] & \text{if } \bar{\mu} = \left(1 - \frac{A}{(c_H - c_L)}\right) \\ = 0 & \text{if } 1 \geq \bar{\mu} > \left(1 - \frac{A}{(c_H - c_L)}\right) \end{cases}$$

Similarly, if $\mu_i = \bar{\mu}$ I can find the profit from deviation for firm j and the value of μ_j that maximizes the expected profit from deviation. Thus, neither firm has no incentive to deviate if $\mu_i^* = \mu_j^* = \bar{\mu} = \left(1 - \frac{A}{(c_H - c_L)}\right)$. Next, I check whether $\mu_i = 1$ and $\mu_j = 0$ is a Nash equilibrium. In this case, $p_i = p_j = c_H$, $\pi_i = [(c_H - c_L) - A]$ and $\pi_j = 0$. Given $\mu_i = 1$, if firm j deviates i.e., $\mu_j > 0$ then it earns strictly positive profit; further, this expected profit from deviation is maximized at $\mu_j = 1$. Therefore, I can conclude that $\mu_i = 1$ and $\mu_j = 0$ is not a Nash equilibrium. ■

Observe that the investment equilibrium described in the above proposition is identical to the symmetric investment equilibrium under full information. It is precisely because the gain from deviation are the same in both cases. However, unlike full information firms earn strictly positive profit in this case. Moreover, there is no asymmetric investment equilibrium. The reason is as follows. Since the rival cannot observe a firm's actual investment, the firm with higher investment (under asymmetric case) can take this advantage and reduce its investment by making the rival believe that it is still the aggressive investor.

4 Discussion

One of the objectives of this paper is to compare the incomplete information outcomes with that of the full information. Under incomplete information both firms always invest strictly positive amount in cost reducing technology and earn strictly positive expected profit. It is primarily because incomplete information about each others actual investment outcome reduces aggressive price competition among firms; as a result firms enjoy more market power compared to full information. Further, one can conclude that both firms together make more investment in cost reducing

technology when the actual outcome of the investment is pure private knowledge if $(c_H - c_L) \leq 3A$.

Social surplus is maximized when a firm charges its own marginal cost. Thus, the expected total surplus is equal to

$$(\mu_i \mu_j + \mu_i (1 - \mu_j) + \mu_j (1 - \mu_i)) (V - c_L) + (1 - \mu_i) (1 - \mu_j) (V - c_H) - A (\mu_i + \mu_j)$$

which is maximized at

$$\mu_i^S = \mu_j^S = \left(1 - \frac{A}{(c_H - c_L)}\right)$$

$\forall i, j = 1, 2$ where $i \neq j$. Thus, the symmetric full information and incomplete information (with unobservable investment) equilibriums are socially optimal. However, incomplete information equilibrium will be less desirable for the consumers as consumer surplus is definitely lower under incomplete information.

References

- [1] Jansen, J., 2010. Strategic Information Disclosure and Competition for an Imperfectly Protected Innovation. *The Journal of Industrial Economics* 58, 349-372.
- [2] Routledge, R., 2010. Bertrand competition with cost uncertainty. *Economics Letters* 107, 356-359.
- [3] Spulber, D., 1995. Bertrand competition when rivals' costs are unknown. *Journal of Industrial Economics* 43, 1-11.
- [4] Thomas, C., 1997. Disincentives for cost-reducing investment. *Economic Letters* 57, 359-363.

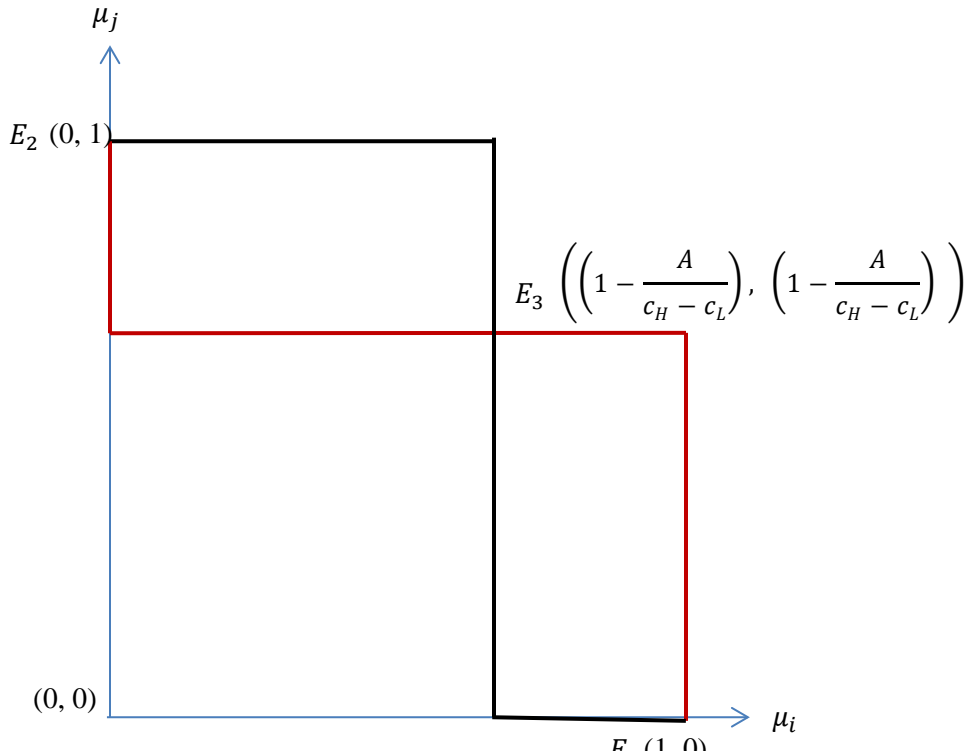


Figure 1

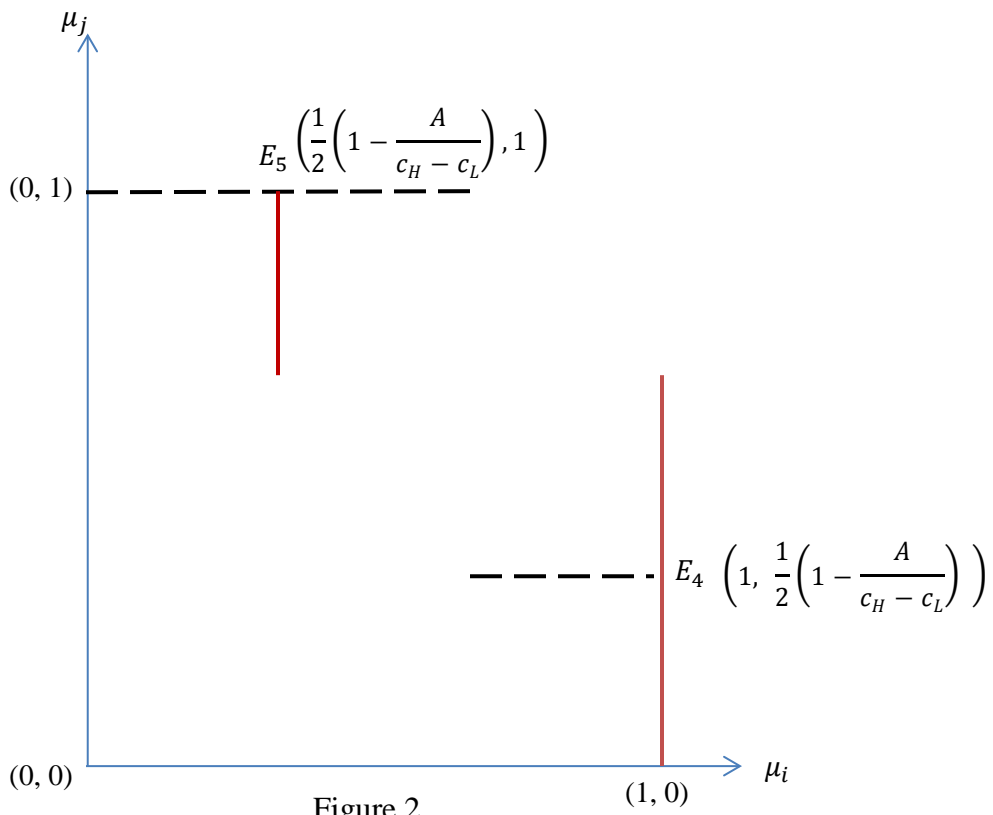


Figure 2