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Measuring the Speed of Convergence of Stock Prices: A Nonparametric and Nonlinear Approach

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Abstract

This paper evaluates the speed of convergence across national stock markets employing a nonlinear, nonparametric stochastic model of relative stock prices. We use operational algorithms for estimating general measures of persistence of the relative stock price that are based on two statistical notions: the short memory in mean (SMM) and the short memory in distribution (SMD). Using G7 countries' stock indices, we obtain strong empirical evidence in favor of the contrarian strategy for France, Germany, Italy, and the UK relative to the US market, while our results imply quite limited usefulness of the strategy for Canada and Japan. Further, we obtain fairly fast convergence rates toward the equilibrium for the former group.

JEL Classification: C14; C22; F36; G11; G14

Keywords: Speed of Convergence; Contrarian Strategy; Short Memory in Mean; Short-Memory in Distribution; ϕ-mixing; Max Half-Life; Max Quarter-Life

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I Introduction

In the field of empirical financial economics, mean reversion properties of asset prices have been widely investigated to examine the usefulness of the contrarian investment strategy (DeBondt and Thaler, 1985) relative to the momentum investment strategy.

The contrarian strategy implicitly requires asset prices not to permanently deviate from its fundamental value path. When asset prices exhibit such properties, one may obtain excess returns by short-selling assets that have performed well and buying assets with relatively poor past performance. If asset prices are *not* mean-reverting, however, the momentum strategy may apply, that is, investors will need to buy better performing assets selling assets that perform poorly to obtain excess returns, because deviation of asset prices from its fundamental value path is permanent.

Empirical evidence on mean reversion in US stock prices is at best mixed. For instance, Fama and French (1988) and Poterba and Summers (1988) report some empirical evidence favoring the mean reversion hypothesis for the US stock returns. Many other researchers, however, question the validity/robustness of their findings.[1]

Mean reversion in the context of the international stock markets has also been actively investigated. For instance, Kasa (1992) reports that stock indices of 5 industrialized countries share a common world component (cointegration), while Richards (1995) finds no evidence of cointegration using stock index data for 16 OECD countries.

[1] See among others, Richardson (1993), McQueen (1992), Kim et al. (1991), Richardson and Stock (1989).

Balvers et al. (2000) employed a panel technique to deal with the mean-reverting hypothesis for 18 countries with well-developed stock markets. They find very strong evidence in favor of mean reversion.[2, 3] Further, they report fairly short half-life estimates for stock index deviations from the fundamental, about 3 years, which supports the usefulness of the contrarian strategy because stock price reversal may occur in the short- or intermediate-term investment horizon.[4]

Kim (2009), however, report a lot weaker evidence for the usefulness of the strategy when one controls for serial correlations and cross-section dependence. Further, he reported about 5 and 13 years half-life point estimates after correcting for bias when the Morgan Stanley Capital International (MSCI) World index and the US index are used for a reference index, respectively, which substantially weaken the practical usefulness of the contrarian strategy.

On the other hand, extremely slow convergence rates of asset price deviations from their fundamental values imply superior performance of the momentum strategy over the contrarian strategy. However, Taylor (2001) demonstrates that linear model based half-life estimates are *upward* biased when the true data generating process is nonlinear. It should be noted that nonlinear models have been successfully applied to a variety of financial data.[5] Note also that the risks associated with contrarian trading/arbitrage coupled with the transaction

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[2] Bhojraj and Swaminathan (2006) find some evidence in favor of the long-run contrarian strategy using stock index data for 38 countries that include both developed and less developed economies.

[3] It should be noted that Balvers et al. (2000) use panel tests that require cross-section independence among the sample countries, which may suffer from severe size distortion problems when the assumption fails to hold (see for example, Phillips and Sul, 2003). Kim (2009) finds very weak evidence for mean-reversion when panel tests that allow cross-section dependence are used.

[4] The term half-life refers to the time period sufficient for the deviations to half-way adjust to its long-run equilibrium value.

[5] See among others, Michael et al. (1997), Gallagher and Taylor (2001), Taylor et al. (2001), Peel and Taylor (2002), Chortareas et al.(2002), Sarno et al. (2004), McMillan (2008), and Kim and Moh (2010).

costs certainly suggests the potential for nonlinear structure, because arbitrage will occur only when deviations are large. See among others, Boswijk et al. (2007), Kim et al. (2009), Chen and Kim (2011), and Jawadi and Prat (2012) that employ nonlinear models to analyze stock price adjustment dynamics.[6]

To study the persistence properties of nonlinear stochastic models of relative stock prices across the G7 markets, we use more general time series concepts of the convergence toward the long-run equilibrium: short-memory-in-mean (SMM) and short-memory-in-distribution (SMD), which is closely related to the statistical notion of ϕ-mixing. SMM was proposed by Granger and Teräsvirta (1993) and Granger (1995) as an alternative to the *linear* notion of stationarity. Granger (1995) argued that SMM and SMD are better measures of persistence in more general models that nest linear models as a special case.

Our nonparametric approach has a number of advantages over other methods. First of all, our method does not require the knowledge on the parametric representation of transition functions nor any distributional assumptions. Hence, our method is less likely to generate specification problems. Second, because our method nests not only linear but also nonlinear stochastic processes, half-life estimations based on the present method are less likely to result in bias raised by Taylor (2001). Third, our approach provides more general notion of long-run equilibrium implied by SMD in addition to SMM.

Even though this approach is potentially very useful, it has been overlooked in the current empirical financial economics literature, because estimation and test methods using the concept of SMM/SMD are not well known to the profession yet.

[6] These nonlinear models require parametric specifications for the transition function across regimes. Typical choices include the threshold autoregressive model, the exponential smooth transition model, and the logistic smooth transition model.

The present paper employs the operational algorithms developed for the first time by El-Gamal and Ryu (2006) to investigate general measures of persistence using the notions of SMM and SMD.

Using the MSCI stock indices for the G7 countries from December 1969 to June 2011, we report empirical evidence in favor of the contrarian investment strategy for France, Germany, Italy, and the U.K. relative to the U.S., while relative stock prices of Canada and Japan seem consistent with the momentum strategy. For the first group countries, we find fairly fast convergence rates toward the long-run equilibrium, which strengthens the usefulness of the contrarian investment strategy. Put differently, our nonparametric analysis provides practically quite useful information no matter which investment strategies are employed.

The remainder of the paper is organized as follows. Section 2 presents the baseline model and explains key statistical notions. We also describe our operational algorithms. In Section 3, we describe the data and provide major empirical findings. Section 4 concludes.

II The Econometric Model

This section presents some useful definitions for our nonparametric model as an alternative to conventional linear models that are often employed in the current empirical financial economics literature. We also provide our nonparametric measures of persistence for a general Markovian univariate time series models.

1. Linear Model

We first consider a linear model for the relative stock price as a benchmark model.

Let p_t^i and f_t^i be the natural logarithm stock index and its fundamental value, respectively, for country i. If p_t^i is mean-reverting around f_t^i, that is, if they are cointegrated with the cointegrating vector $[1,-1]$, its stochastic process has the following error correction representation.

$$\Delta\left(p_{t+1}^i - f_{t+1}^i\right) = a^i - \lambda^i\left(p_t^i - f_t^i\right) + \varepsilon_{t+1}^i,$$

where $0 < \lambda^i < 1$ is the rate of convergence and ε_t^i is an idiosyncratic white noise shock. Even though f_t^i is not directly observable, we assume that it is known to have the following stochastic process.

$$f_t^i = c^i + p_t^w + v_t^i,$$

where c^i is an idiosyncratic fixed effect constant, p_t^w is the reference stock index, and v_t^i is a white noise process. These two equations jointly imply the following stationary autoregressive process for the relative stock price, $x_t^i = p_t^i - p_t^w$.

$$x_{t+1}^i = \mu^i + \rho^i x_t^i + \eta_{t+1}^i,$$

where $\mu^i = a^i + \lambda^i c^i$, $\rho^i = 1 - \lambda^i$, and $\eta_{t+1}^i = v_{t+1}^i - \left(1 - \lambda^i\right)v_t^i + \varepsilon_{t+1}^i$. Put it differently, this equation implies that the stock price deviation from the reference index is short-lived and eventually die out. Note that ρ^i is a measure of the persistence for x_t^i in this linear model representation.

Omitting the constant term and superscript i, consider the following representation for x_t which nests the previous linear representation as a special case.

$$x_{t+1} = m(x_t) + \varepsilon_{t+1}$$

Note that this equation implies $m(x_t)$ is the conditional expectation of x_{t+1} at time t given information set. In what follows, we extend this nonlinear representation into a framework that extends more than the first moment.

2. Non-linear and nonparametric model

This section introduces nonparametric measures of persistence for general non-linear model, which is based on the framework proposed by El-Gamal and Ryu (2006) for a first-order Markovian univariate time series $\{x_t\}$. Abandoning linearity in time series domain, we pursue nonlinearity in density domain instead. From the Chapman-Kolmogorov equations, we define transition probability kernel and the Markov operator, which can be approximated by a finite transition matrix. We also directly apply the consistent tests of ergodicity and mixing to our relative stock index data via Domowitz and El-Gamal (1993, 1996, 2001) .

We define the short memory in distribution (SMD) and the short memory in mean (SMM) as stated in El-Gamal and Ryu (2006).

Definition 1. The time series is said to have *Short Memory in Distribution* (SMD) if $F_s(x) \Rightarrow \overline{F}(x)$, as $s \uparrow \infty$ where $F_s(x) = \Pr(x_{t+s} \leq x \mid \mathrm{A}_t)$ is the cumulative distribution function of x_{t+s} conditional on the past information set $\mathrm{A}_t = \sigma(x_{t-j}; j \geq 0)$, and $\overline{F}$ be some fixed (unconditional) distribution function.

Definition 2. The time series is said to have the *Short Memory in Mean* (SMM) property if $\| E[x_{t+s} \mid \mathrm{A}_t] - E[x_{t+s}] \| < c_s$; $c_s \xrightarrow{s \uparrow \infty} 0$.

Note that SMM is equivalent to *mixing in mean* or *mixingales* as discussed in McLeish (1978) and Gallant and White (1988), while SMD shares a property of mixing.

We use the asymptotic independence notion of uniform or ϕ-mixing to study SMD and SMM. As shown by El-Gamal and Ryu (2006), we can calculate the SMD and SMM numerically. That is, we can get the finite grid analog $\phi_n(s)$ which converges to $\phi(s)$ as the grid size $n \uparrow \infty$. Similarly, we can also get the *grid* $MDM_n(s)$ which converges to the Maximum Distance in Mean, $MDM(s)$, the measure of SMM, as the grid size $n \uparrow \infty$. We provide detailed explanations on the numerical algorithms to compute our persistence measures in Appendix.[7]

The notion of half-life can now be replaced by the value of s at which $MDM_n(s) = 0.5 \times MDM_n(0)$, that is, the number of periods needed for the worst possible transitory shock from the unconditional mean to be cut in half. This notion may then be extended beyond half-life to consider *Max m-life* as the number of time periods before the worst possible shock would have shrunk to (1-m) of its original magnitude. Likewise, we define *Max quarter-life* by the number of time periods before the worst possible shock would have shrunk to 0.25, i.e., m = 0.75 of its initial one unit shock.[8]

For non-parametric estimation of $P_{T,n}$ using a kernel estimator, we begin with the estimated $\phi(s)$ and *Max m-life* using so-called Silverman's rule of thumb: $h_T = \sigma_T \ T^{-1/5}$, where σ_T is the standard deviation of our series. The estimated *Max*

[7] See El-Gamal and Ryu (2006) for more detailed description about the numerical calculation and convergence arguments of finite grids of SMD and SMM.
[8] This metric is an extension of the quarter-life that is introduced by Steinsson (2008), which is based on linear regression models. This additional measure of persistence can be used to see if the convergence takes place monotonically.

m-life with this bandwidth selection rule typically yielded quite less persistent dynamics which is in favor of the contrarian investment strategy. However, as El-Gamal and Ryu (2006) shows, such results may not be reliable because this selection rule tends to produce an over-smoothed estimate of the transition density, which results in downward bias in the estimates of $\phi(s)$ and *Max m-life*. Therefore, the rule of thumb tends to yield empirical support for the contrarian strategy.

Realizing this issue, we implement estimations for an array of the choice of the level of under-smoothing, *k*. That is, we modify the Silverman's rule of thumb as follows.

$$h_T = \left(\frac{\sigma_T}{k}\right) T^{-1/5}$$

And we report our estimation results for *k* ranging 1 to 10. We note our estimates for $\phi(s)$ (or *Max m-life*) often converge each other as *k* approaches to 10. We interpret such results as empirical findings that support the contrarian investment strategy.

III Empirical Results

1. Data and Summary Statistics

We use monthly frequency data from Morgan Stanley Capital International (MSCI) for stock market indices for the G7 countries: Canada, France, Germany, Italy, Japan, the UK, and the US. The observations span from December 1969 to June 2011. We use end-of-period observations rather than average data to avoid a time aggregation bias (Taylor, 2001). All stock prices are value-weighted index prices in the US dollar that include gross dividends. See Figure 1 for the log transformed stock indices.

Table 1 provides basic summary statistics of national stock index deviations from the US index as the reference index (e.g., Balvers et al., 2000; Kim, 2009). The mean values of the national index deviations relative to the US index range from -0.945 for Italy to 0.827 for Japan, and the standard deviations vary from 0.222 for the UK to 0.674 for Japan. Half of the deviation series have negative skewness, while for the rest the right tail is more pronounced than the left tail. All deviations have a leptokurtic (high peak) distribution. The Jarque-Bera statistic implies a non-normal distribution for all series at the 95% significance level.[9] Overall, these results support our nonparametric approach to study the relative stock index dynamics.

Figure 1 and Table I around here

2. Linear Model Estimates

As a benchmark, we report persistence measure estimates from a linear augmented Dickey-Fuller regression in Table II. Since the least squares estimate for the persistence parameter is downward biased, we correct for the median bias by grid

[9] We use the critical values from Deb and Sefton (1996) to deal with a size distortion problem using an asymptotic chi-square distribution with two degrees of freedom.

bootstrap method (Hansen, 1999) with 10,000 nonparametric simulations on each of 51 fine grid points in the vicinity of the least squares point estimate.[10]

Our estimates are roughly consistent with the relative stock index graphs shown in Figure 2. For instance, the national stock index of Canada and Japan relative to the US index exhibit more persistent movements compared with other relative stock indices that show more frequent reversals or adjustments. Such eyeball metric is confirmed by much longer half-life estimates for Canada and Japan than those of France, Germany, Italy, and the UK.

It seems that these results roughly support the momentum investment strategy rather than the contrarian strategy, because the point estimate tends to imply very sluggish adjustment toward the long-run equilibrium under the linear stochastic model framework. The shortest half-life point estimate is about 16 months for the UK vis-à-vis the US. The longest one is the case of Canada relative to the US, where the half-life point estimate is positive infinity. Further, median bias-corrected 95% confidence intervals are quite wide and often extend to positive infinity for 4 out of 6 relative stock prices.

We note that, however, this seemingly very slow rate of adjustment does not necessarily implies strong support against the use of the contrarian investment strategy. As Taylor (2001) points out, if the true data generating process is nonlinear, statistical inference on the persistence of the stochastic process under the linear model framework may be incorrect because the persistence parameter estimate tends to be upward biased.

[10] Kim (2009) employs a similar framework, whereas Balvers et al. (2000) use Andrew's (1993) parametric bootstrap method.

Figure 2 around here

Table II around here

Our persistence measure estimates are not subject to this type of bias because our framework allows any type of Markovian transition functions, where the linear model is a special case. In what follows, we report quite short half-life estimates for 4 out of 6 relative stock prices, which are in favor of the contrarian investment strategy, while we find empirical support for the momentum strategy for the rest two relative stock prices.

3. Nonparametric Model Estimates

This section reports nonlinear statistical test results of mean reversion in terms of ergodicity and mixing proposed by Domowitz and El-Gamal (2001) for our relative stock index variables in Table III. We account for nonlinearity in the process describing the evolution of relative stock market return through nonparametric estimation of the transition densities that underlie our test statistics. This circumvents the problem of taking a stand with respect to the exact parametric specifications of the model, and eliminates the need for extra critical value computations.

The mean reversion problem is reformulated in terms of general ergodic failure, as opposed to the specific case of a unit root process. This is more appropriate in nonlinear settings, given the essential linearity of unit root analysis, discussed in Granger (1995). The test rejects the null hypothesis of ergodicity if the p-value of a single randomized test is smaller than a pre-specified value. We then determine the rejection of ergodicity null by the percentiles of the density of p-values which are less than or close to a pre-specified number, e.g., 5% or 10%.

Applying those tests, we reject the null hypothesis of ergodicity as shown in Table III for the time series of Italy and Japan since percentiles of the density of p-values are above 5% and 10%. We have very weak support for the null when the test applies to France, which may reflect the size distortion uncovered in the Monte Carlo experiments in Domowitz and El-Gamal (2001). For Canada, Germany and the UK, the percentages are close to 5% and 10%, respectively, so we conclude that the test fails to reject the null hypotheses of ergodicity. These test results overall show that the mean reversion took place for the case of Canada, Germany and UK implying relative usefulness of contrarian strategy, but not for the remaining countries. Although the result is not surprising given other research like Kim (2009), we note that this progression does not presume linearity of the underlying process at the outset, which may prove useful in other settings.

In contrast, for the mixing tests, the percentiles are close to 5% and 10%, respectively for all the countries, we conclude that the randomized tests fail to reject the null hypotheses of mixing. However, we know from Monte Carlo results of Domowitz and El-Gamal (2001) that for a sample size as small as $T = 499$ our estimate of P_n^t is over-smoothed, thus producing low power for those tests. Thus, we need to get a different approach to get a complete argument of mean reversion property in stock market returns.

Table III around here

Next, we report our Max half-life (MHL) estimates as well as Max quarter-life (MQL) for the smoothing parameter (*k*) ranging from 1 to 10 to see how robust our estimates are.

We first report our estimates for the SMM (mixingale) property in Table IV. We note that MHL and MQL estimates tend to increase as k increases. However, with exceptions of Canada and Japan, MHLs of each country's relative stock index converge each other as k approaches to 10. That is, as we can see in Figures 3 through 8, the MHL with $k = 10$ becomes an upper limit for these countries, while the MHL is not well-defined for Canada and Japan even when $k = 10$ (Figures 3 and 7).

Similarly, the MHL is not well-defined for those two countries when we investigate persistence based on the SMD property (Table V), while we obtain well-defined MHL for the rest. We again observe convergence for all normalized *MDM(s)* and $\phi(s)$ as k approaches to 10. Therefore, we do not think it necessary to try estimation with more values for k.

These findings suggest very strong support for the contrarian investment strategy for France, Germany, Italy, and the UK against the US, while the momentum strategy is supported for Canada and Japan vis-à-vis the US stock price. It should be noted that unlike the results from linear models that often suggest very sluggish adjustment rates, our nonparametric measures imply quite fast speed of adjustment when the contrarian strategy is supported. For instance, the MHL for the UK vs. the US ranges from 4 to 18 months (Table IV) and from 5 to 29 months for SMM and SMD, respectively. However, our linear model estimates are a lot longer, extending to a positive infinity (Table II).

We also investigate whether the adjustment dynamics exhibit non-monotonic patterns using a metric from Steinsson (2008) for linear models. Note that MHL should equal to MQL – MHL if the adjustment takes place monotonically. This idea can be formulated 2MHL minus MQL and we report our estimates in Tables IV and V. We obtain mostly negative values when convergence is made, which implies a slower adjustment in the second half than the speed of adjustment in the first half.[11]

Tables IV and V around here

Figures 3 through 8 around here

IV Concluding Remarks

We revisit the usefulness of the contrarian investment strategy relative to the momentum strategy in international stock markets. Previous studies that employ linear stochastic models often provide fairly weak empirical support for the contrarian strategy, finding very persistent dynamics of relative stock prices.[12]

This paper employs a nonlinear, nonparametric stochastic model of relative international stock prices that utilizes two statistical notions: the short memory in mean (SMM) and the short memory in distribution (SMD). This allow us to use very general measures of persistence that avoid potential upward bias that arises

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[11] Steinsson (2008) reports mostly positive estimates using the US real exchange rate data, which may be consistent with hump-shape dynamics.

[12] This may occur when the stochastic process exhibits a (near) unit-root process, when linear models are employed.

when linear models are used even though the true data generating process is nonlinear (Taylor, 2001).

Using monthly frequency stock prices from G7 countries, we obtain favorable empirical evidence supporting the contrarian strategy for France, Germany, Italy, and the UK. For these countries, we find fairly small MHL estimates for both SMM and SMD that are robust to the choice of bandwidth. For the rest of G7 countries, Canada and Japan, relative to the US, we report empirical results that favor the momentum strategy as we find weak evidence of SMM and SMD.

Appendix: Numerical algorithms for computing *Max m-life* and $\phi(s)$ functions

We now turn to the actual numerical algorithms used in this paper for computing our measures of SMD and SMM for known models and estimated transition densities. We begin with the assumption of having a known transition matrix $P_n(.,.)$ on an $n \times n$ grid.

Algorithm A: $MDM_n(s)$

1. Fix the grid $\boldsymbol{x} = (x_1, ..., x_n)$.
2. Compute the invariant measure f_n^* by iterating on $P_n^s f$ for any initial f, and $s = 1, 2, ...$, until convergence (in the sup norm) is made.
3. Compute the unconditional expectation $\mu = E_{f_n^*}[x_{t+s}] = \boldsymbol{x}' f_n^*$.
4. Define $MDM_n(0) = \max(\mu, 1-\mu)$.
5. For each s, and each point on the grid $\{x_i\}_{i=1}^n$, compute the conditional expectation $\mu_i(s) = E_{P_n^s \delta\{x_i\}}[x_{t+s} \mid x_t = x_i] = \boldsymbol{x}' P_n^s \delta_{\{x_i\}}$. Then, compute $MDM_n(s) = \max_i(|\mu_i(s) - \mu|)$.
6. Normalize $MDM_n(s)$ by defining $1 - m_n(s) = MDM_n(s) / MDM_n(0)$.
7. Plot *Max m-life* as against $(1 - m_n(s))$.

Algorithm B: $\phi_n(s)$

1. Perform steps 1-2 of Algorithm A.
2. For each s, and each point on the grid $\{x_i\}_{i=1}^n$, compute $\phi_{n,i}(s) = \| f_n^* - P_n^s \delta_{\{x_i\}} \|$.
3. Set $\phi_n(s) = \max_i(\phi_{n,i}(s))$.
4. Plot $\phi_n(s)$ against s.

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Figure 1. MSCI National Stock Prices vs. US Stock Price (dashed line)

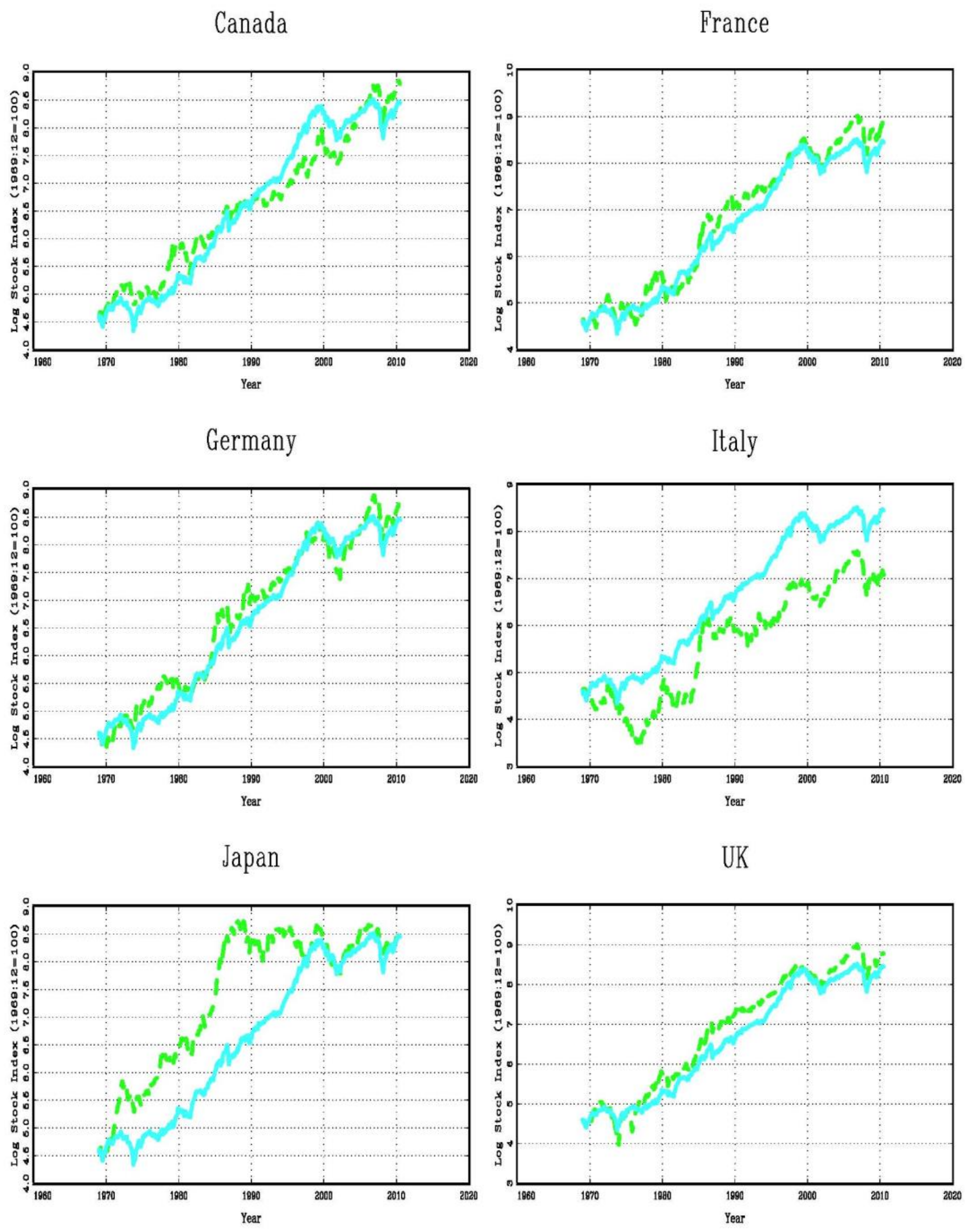


Figure 2. Log National Stock Prices relative to US Stock Price

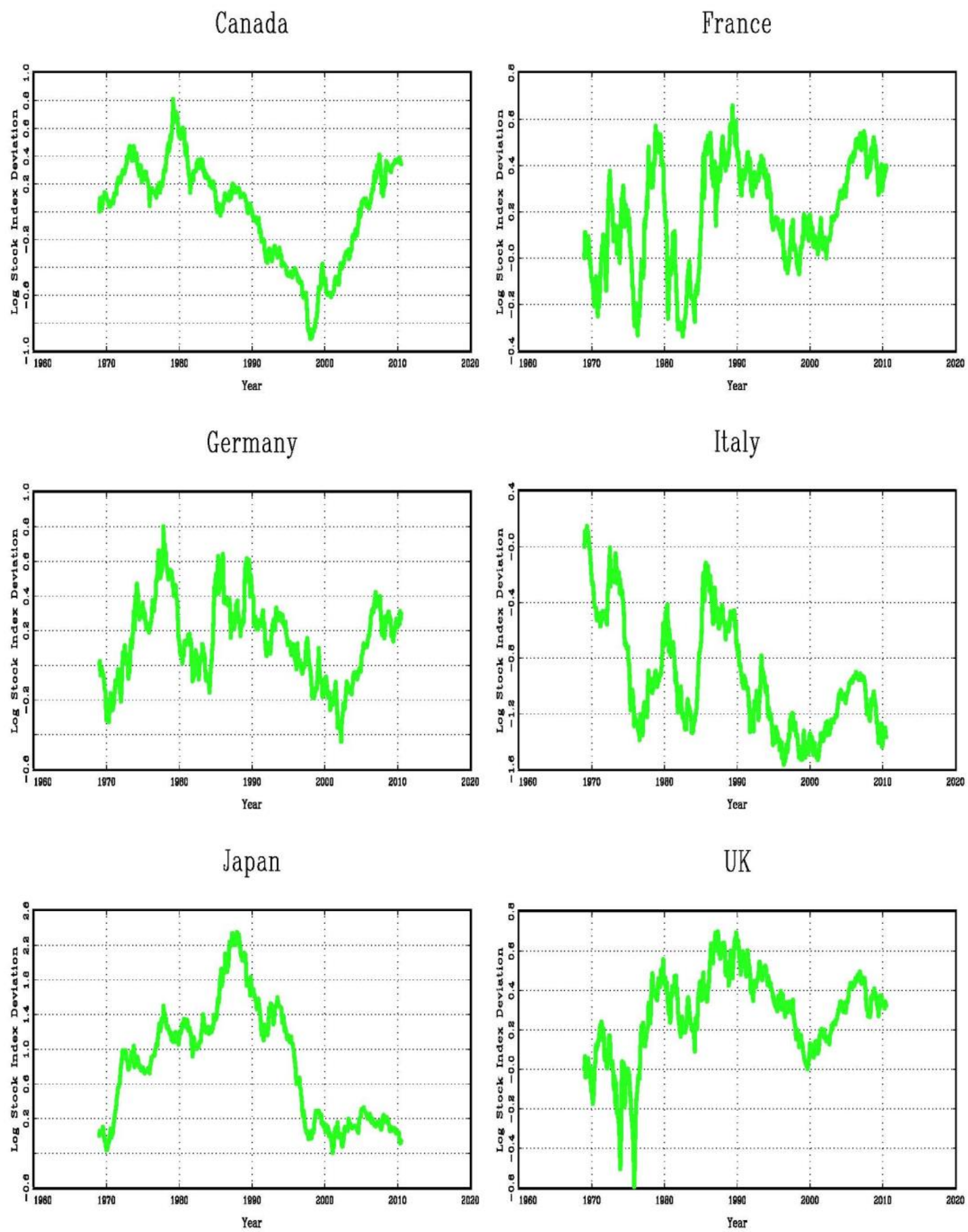


Figure 3. SMM and SMD Properties: Canada vs. US

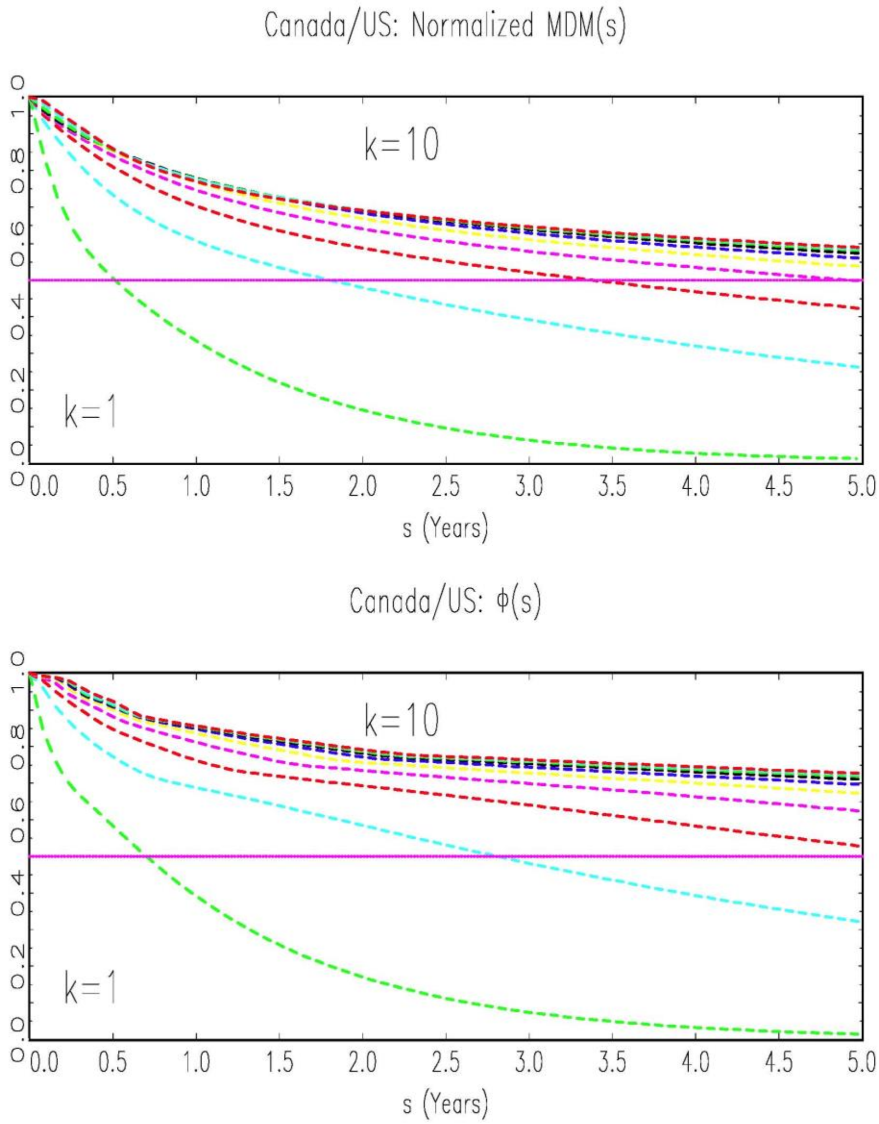
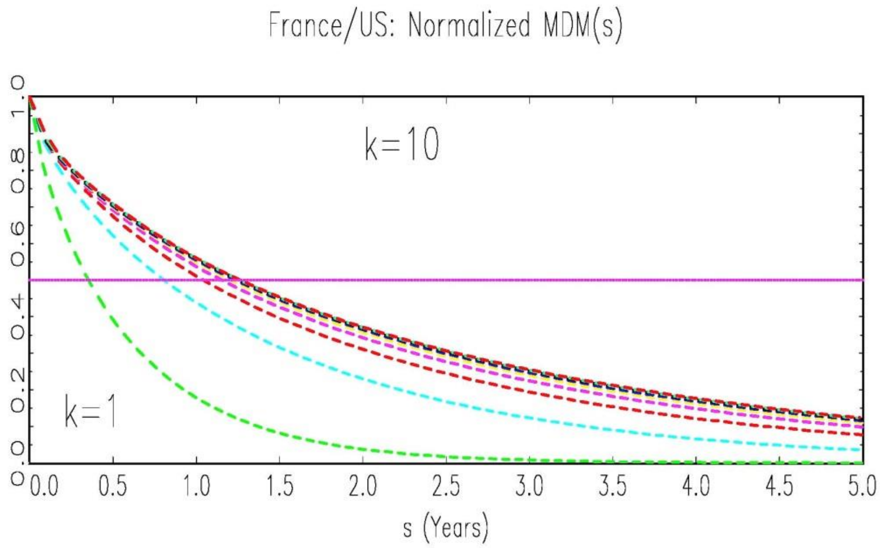


Figure 4. SMM and SMD Properties: France vs. US



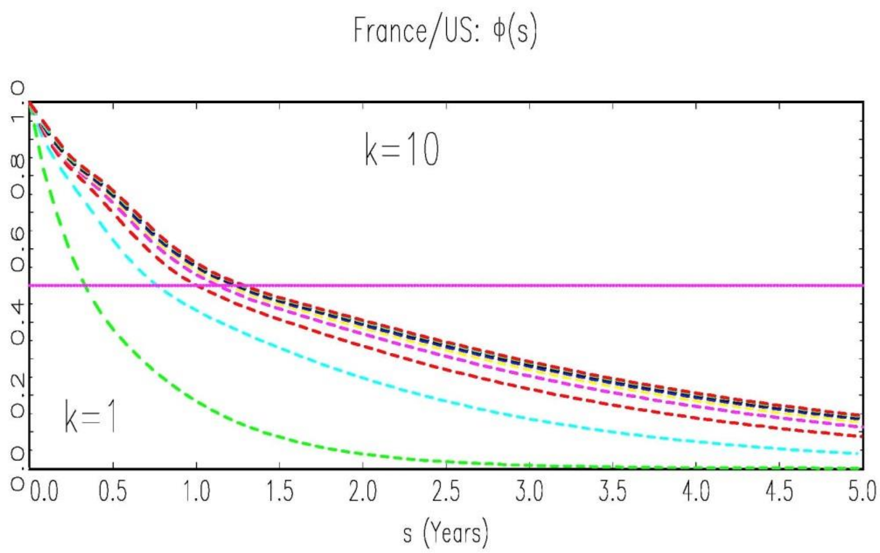
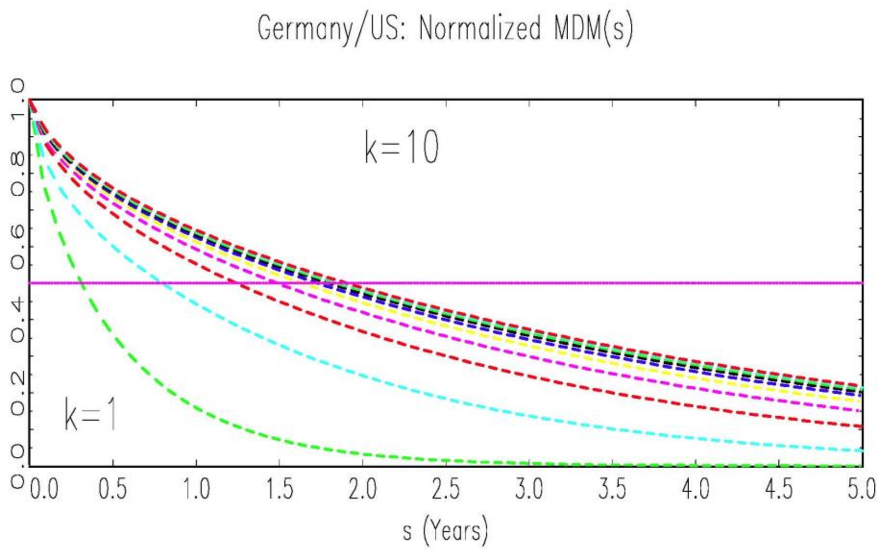


Figure 5. SMM and SMD Properties: Germany vs. US



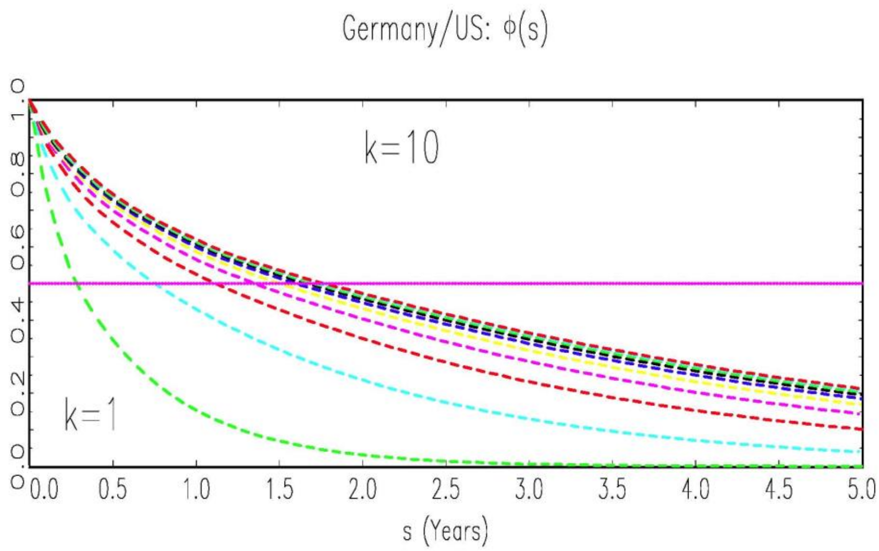


Figure 6. SMM and SMD Properties: Italy vs. US

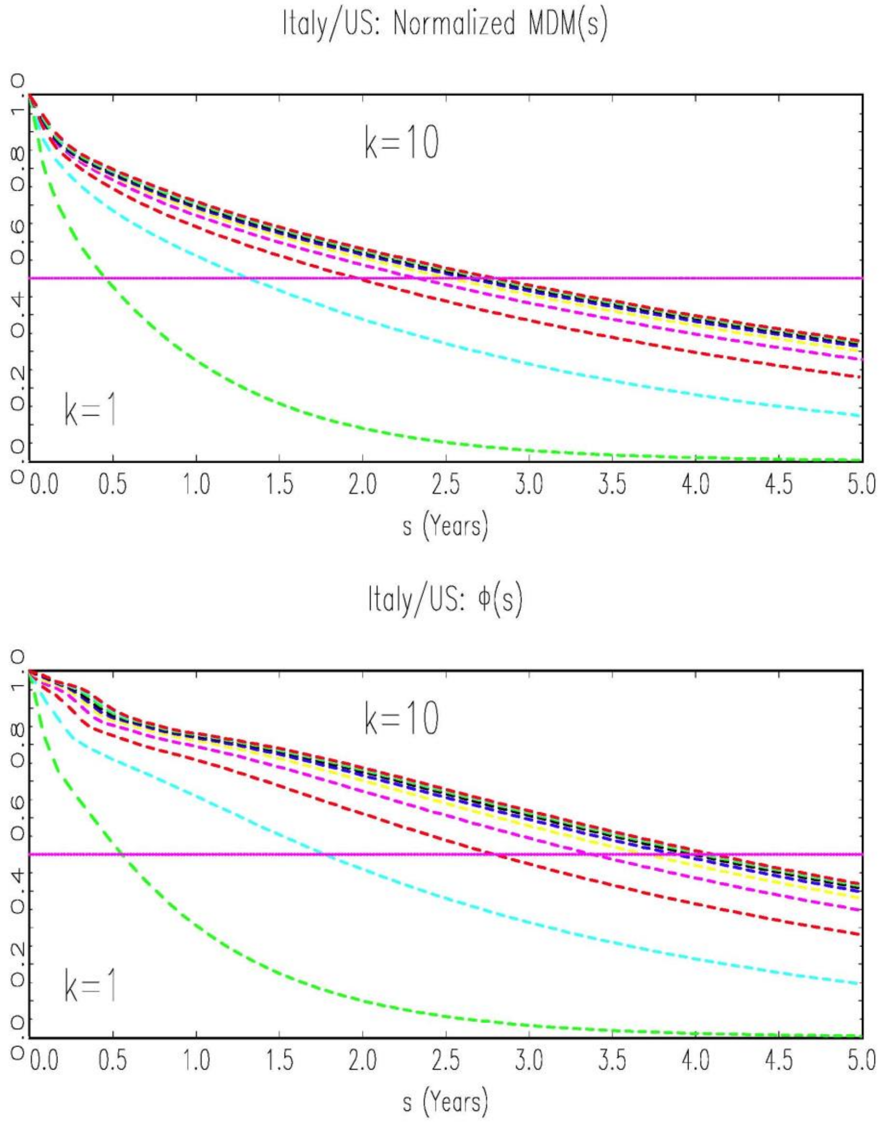
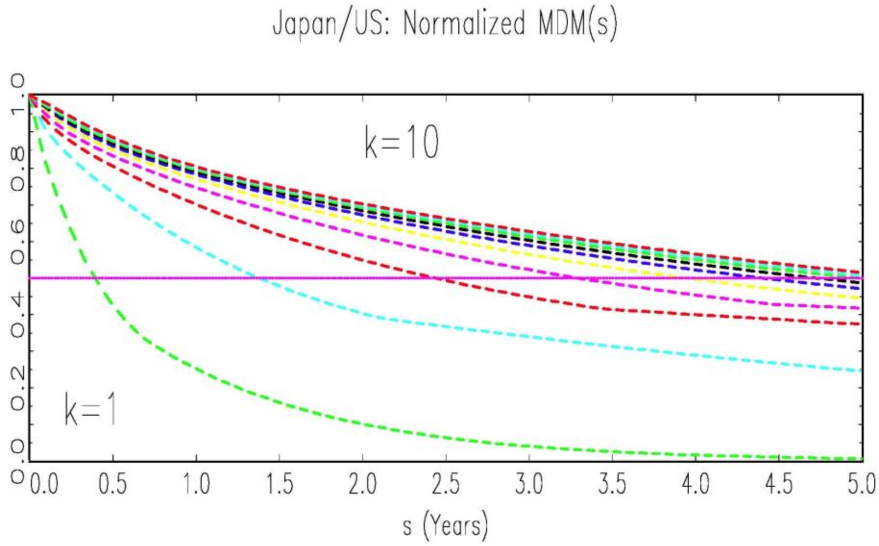


Figure 7. SMM and SMD Properties: Japan vs. US



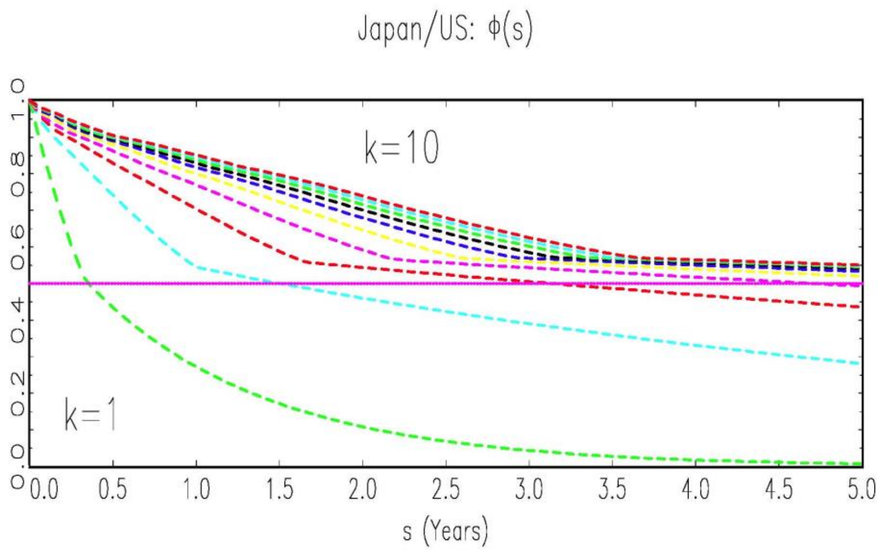


Figure 8. SMM and SMD Properties: UK vs. US

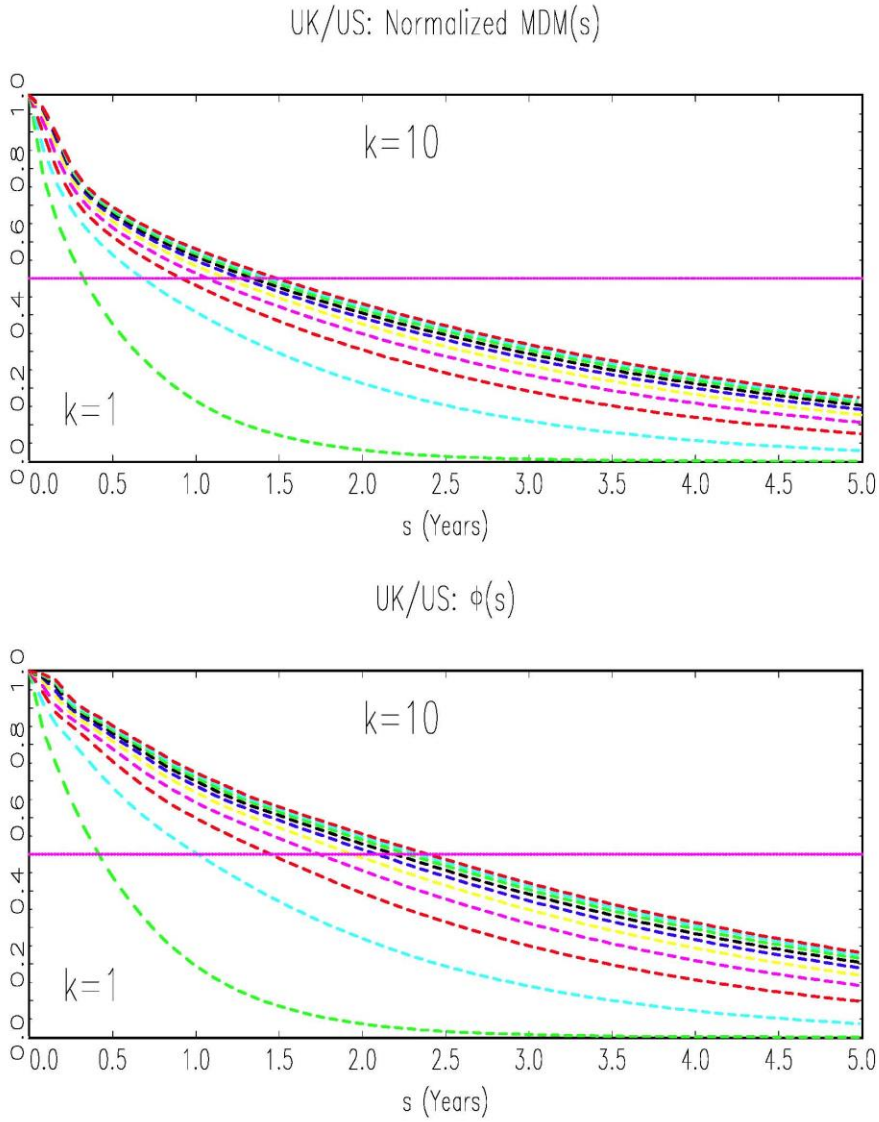


Table I. Summary Statistics of Relative Stock Prices

	Mean	StdDev	Min	Max	Skewness	Kurtosis	JB
Canada	0.009	0.362	-0.915	0.811	-0.163	4.203	32.23
France	0.190	0.232	-0.339	0.659	-0.235	4.859	76.32
Germany	0.147	0.235	-0.441	0.803	-0.171	4.118	28.39
Italy	-0.945	0.408	-1.567	0.151	0.114	4.273	34.70
Japan	0.827	0.674	-0.201	2.356	0.012	4.411	41.30
UK	0.281	0.222	-0.595	0.699	0.155	6.510	257.6

Note: i) Relative stock prices are defined as the log national stock index minus the log US stock index. ii) JB denotes the Jarque-Bera statistics. We obtained the statistics for the residual of each series after filtering out with an AR(1) specification.

Table II. Half-Life Estimation from a Linear Model

	ρ	Conf. Interval	Half-Life (month)	Conf. Interval
Canada	1.000	[0.986,1.008]	∞	[50.62, ∞)
France	0.979	[0.956, 1.008]	32.72	[15.55, ∞)
Germany	0.980	[0.957, 1.008]	34.91	[15.65, ∞)
Italy	0.981	[0.963, 1.004]	35.78	[18.47, ∞)
Japan	0.999	[0.987, 1.007]	480.2	[55.08, ∞)
UK	0.978	[0.956, 1.006]	31.50	[15.58, ∞)

Note: i) ρ denotes the persistence parameter from a linear augmented Dickey-Fuller regression equation, that is, $x_t = c + \rho x_{t-1} + \sum_{j=1}^{k} \beta_j \Delta x_{t-j} + e_t$. ii) The lag parameter (k) is chosen by the general-to-specific rule with 12 maximum number of lags. iii) The point estimate and the 95% confidence interval was constructed by Hansen's (1999) grid bootstrap method to correct for median bias. For this, 10,000 bootstrap simulations on each of 51 grid points were implemented.

Table III. Ergodicity and Mixing Test Results

(a) Ergodicity Test		
Country	% p-values < 0.05	% p-values < 0.10
Canada	6	11
France	13	21
Germany	6	10
Italy	22	29
Japan	23	33
UK	4	9
(b) Mixing Test		
Country	% p-values < 0.05	% p-values < 0.10
Canada	2	6
France	3	7
Germany	2	5
Italy	3	6
Japan	5	11
UK	3	8

Note: i) These are randomized tests proposed by Domowitz and El-Gamal (2001). ii) The numbers in the table are the percentage of rejections at the 5% and the 10% significance level from 1,000 independent randomized runs.

Table IV. Max Half-Life and Max Quarter-Life: Short Memory in Mean

(a) Max Half-Life (month)			
Country	$k = 1$	$k = 10$	Convergence
Canada	7	>60	No
France	5	16	Yes
Germany	4	23	Yes
Italy	6	34	Yes
Japan	5	>60	No
UK	4	18	Yes

(b) Max Quarter-Life (month)			
Country	$k = 1$	$k = 10$	Convergence
Canada	16	>60	No
France	10	37	Yes
Germany	9	55	Yes/No
Italy	14	>60	No
Japan	13	>60	No
UK	9	46	No

(c) 2MHL – MQL (month)			
Country	$k = 1$	$k = 10$	Convergence
Canada	-2	n.a.	No
France	0	-5	Yes
Germany	-1	-9	Yes/No
Italy	-2	n.a.	No
Japan	-3	n.a.	No
UK	-1	-10	No

Note: i) We estimate Max Half-Life (MHL) and Max Quarter-Life (MQL) for the smoothing parameter k raning 1 through 10. ii) We denote "Yes" in the last column when the m-life estimates converge as k approaches to 10, that is, when greater values for k produces no substantial difference in MHL and MQL estimates of the normalized Maximal Distance Measure (MDM). iii) $2MHL - MQL$ is adopted from Steinsson (2008). Zero values for $2MHL - MQL$ imply monotonic adjustment process towards the long-run equilibrium. Negative values occur when $MHL < MQL - MHL$.

Table V. Max Half-Life and Max Quarter-Life: Short Memory in Distribution

(a) Max Half-Life (month)

Country	$k = 1$	$k = 10$	Convergence
Canada	9	>60	No
France	4	16	Yes
Germany	4	22	Yes
Italy	7	50	Yes
Japan	5	>60	No
UK	5	29	Yes

(b) Max Quarter-Life (month)

Country	$k = 1$	$k = 10$	Convergence
Canada	19	>60	No
France	10	42	Yes
Germany	9	53	Yes/No
Italy	15	>60	Yes
Japan	14	>60	No
UK	11	57	Yes/No

(c) 2MHL - MQL (month)

Country	$k = 1$	$k = 10$	Convergence
Canada	-1	n.a.	No
France	-2	-10	Yes
Germany	-1	-9	Yes/No
Italy	-1	n.a.	Yes
Japan	-4	n.a.	No
UK	-1	1	Yes/No

Note: i) We estimate Max Half-Life (MHL) and Max Quarter-Life (MQL) for the smoothing parameter k raning 1 through 10. ii) We denote "Yes" in the last column when the m-life estimates converge as k approaches to 10, that is, when greater values for k produces no substantial difference in MHL and MQL estimates of the normalized Maximal Distance Measure (MDM). iii) $2MHL - MQL$ is adopted from Steinsson (2008). Zero values for $2MHL - MQL$ imply monotonic adjustment process towards the long-run equilibrium. Negative values occur when $MHL < MQL - MHL$.