Auburn University
Department of Economics
Working Paper Series

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AUWP 2011-02

Purchasing Power Parity and the Taylor Rule

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January 2011

Abstract

In the Kehoe and Midrigan (2007) model, the persistence parameter of the real exchange rate is closely related to the measure of price stickiness in the Calvo-pricing model. When we employ this view, Rogoff's (1996) 3 to 5 year consensus half-life implies that firms update their prices every 18 to 30 quarters on average. This is at odds with most estimates from U.S. aggregate data when single equation methods are applied to the New Keynesian Phillips Curve (NKPC), or when system methods are applied to Dynamic Stochastic General Equilibrium (DSGE) models that include the NKPC. It is well known, however, that there is a large degree of uncertainty around the consensus half-life of the real exchange rate. To obtain a more efficient estimator, this paper develops a system method that combines the Taylor rule and a standard exchange rate model to estimate half-lives. We use a median unbiased estimator for the system method with nonparametric bootstrap confidence intervals, and compare the results with those from the single equation method typically used in the literature. Applying the method to the real exchange rates of 18 developed countries against the U.S. dollar, we find that most of the half-life estimates from the single equation method fall in the range of 3 to 5 years with wide confidence intervals that extend to positive infinity. In contrast, the system method yields median-unbiased estimates that are typically shorter than one year with much sharper 95% confidence intervals, most of which range from 3 quarters to 5 years. These median unbiased estimates and the lower bound of the confidence intervals for the half-lives of real exchange rates are consistent with most estimates of price stickiness using aggregate U.S. data for the NKPC and DSGE models.

Keywords: Purchasing Power Parity, Calvo Pricing, Taylor Rule, Half-Life of PPP Deviations, Median Unbiased Estimator, Grid-t Confidence Interval

JEL Classification: C32, E52, F31

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1 Introduction

Reviewing the literature on Purchasing Power Parity (PPP), Rogoff (1996) found, using single equation methods, a remarkable consensus on 3 to 5 year half-life estimates of real exchange rate deviations from PPP. This is an important piece of Rogoff's "PPP puzzle" as the question of how one might reconcile highly volatile short-run movements of real exchange rates with an extremely slow convergence rate to PPP. This puzzle can be described in the context of the New Keynesian model with Calvo pricing. For example, Galí and Gertler (1999) use U.S. aggregate data, the unit labor cost and CPI, to estimate the New Keynesian Phillips curve (NKPC). Their preferred estimate implies that the average frequency of the price change is about 5 quarters. On the other hand, a single-good version of Kehoe and Midrigan's (2007) model can be used to find the implication of the 3 to 5 year half-life estimates from real exchange rate data for the same average pricing frequency (see Section 2 below). They imply 18 to 30 quarters. Thus, it is hard to reconcile Galí and Gertler's result with the extremely slow convergence rate found in Rogoff's remarkable consensus.

Using Rogoff's remarkable consensus as the starting point, many possible solutions to the PPP puzzle have been proposed in the literature.[1] One important example is Imbs, Mumtaz, Ravn, and Rey (2005), who point out that sectoral heterogeneity in convergence rates can cause upward bias in half-life estimates, and claim that this aggregation bias explains the PPP puzzle. While it is possible that the bias can solve the PPP puzzle under certain conditions, it is also possible that the bias is negligible under other conditions. For example, Chen and Engel (2005), Crucini and Shintani (2008), and Parsley and Wei (2007) have found negligible aggregation biases. Broda and Weinstein (2008) show that the aggregation bias of the form that Imbs, Mumtaz, Ravn, and Rey (2005) studied is small for their barcode data, even though the convergence coefficient rises as they move to aggregate indexes. These papers are about purely statistical findings.

Another delicate issue is how we should aggregate micro evidence of price stickiness for dynamic aggregate models, such as dynamic stochastic general equilibrium (DSGE) models, which Carvalho and Nechio (2008) have started to study. Thus, even though the aggregation bias is an important possibility, much more research seems necessary before we reach a consensus on whether or not the aggregation bias solves the PPP puzzle, and how we should aggregate for DSGE models.

[1] See Murray and Papell (2002) for a discussion of other solutions which take Rogoff's remarkable consensus as a starting point.

In this paper, we ask a different question: Should we really take Rogoff's remarkable consensus of 3-5 year half-life estimates as the starting point for the aggregate CPI data? The consensus may at first seem to support the reliability of these estimates, but Kilian and Zha (2002), Murray and Papell (2002), and Rossi (2005) have shown that the degree of uncertainty around these point estimates is huge. Murray and Papell (2002) conclude that singe equation methods provide virtually no information regarding the size of half-lives. Therefore, it is not clear if the true half-lives are as slow as Rogoff's remarkable consensus implies. If we apply a more efficient estimator to the real exchange rate data, we may find much faster convergence rates.

For the purpose of obtaining a more efficient estimator, we develop a system method that combines the Taylor rule and a standard exchange rate model in order to estimate the half-life of the real exchange rate. Several recent papers have provided empirical evidence in favor of exchange rate models with Taylor rules (see Mark 2005, Engel and West 2005, 2006, Clarida and Waldman 2007, Molodtsova and Papell 2007, and Molodtsova, Nikolsko-Rzhevskyy and Papell 2008). Therefore, a system method using an exchange rate model with the Taylor rule is a promising way to try to improve on single equation methods to estimate the half-lives.

Because standard asymptotic theory usually does not provide adequate approximations for the estimation of half-lives of real exchange rates, we use a nonparametric bootstrap method to construct confidence intervals. Median unbiased estimates based on the bootstrap are reported.

As we review in Section 5 below, the contrast between the single equation methods and our system method, in the context of PPP literature, corresponds with the contrast between single equation methods for the NKPC and system methods for DSGE models with the NKPC in the literature for closed economy models. Single equation methods such as Galí and Gertler's (1999) GMM yield small standard errors for the average price duration based on standard asymptotic theory. However, Kleibergen and Mavroeidis (2009), who take into account the weak identification problem of GMM, report that the upper bound of their 95% confidence interval for the price duration is infinity. The estimators of average price duration in system methods for DSGE models in Christiano, Eichenbaum, and Evans (2005) and Smets and Wouters (2007), among others, may be more efficient.

We apply the system method to estimate the half lives of real exchange rates of 18 developed countries against the U.S. dollar. Most of the estimates from the single equation method fall in the range of 3 to 5 years, with wide confidence intervals that extend to positive infinity. In contrast, the system method yields median unbiased estimates that are typically substantially shorter than 3 years with much sharper confidence intervals, most of which range from three quarters to 5 years.

In the recent papers of two-country exchange rate models with Taylor rules cited above, the authors assume that Taylor rules are adopted by the central banks of both countries. Because Taylor rules may not be used by some countries, we only assume that the Taylor rule is used by the home country, and remain agnostic about the monetary policy rule in the foreign country. None of these papers with Taylor rules estimates half-lives of real exchange rates.

Kim and Ogaki (2004), Kim (2005), and Kim, Ogaki, and Yang (2007) use system methods to estimate half-lives of real exchange rates. However, they use conventional monetary models without Taylor rules based on money demand functions. Another important difference of these works from the present paper is that their inferences are based on asymptotic theory, while ours is based on the grid bootstrap.

The rest of the paper is organized as follows. Section 2 describes our baseline model. We construct a system of stochastic difference equations for the exchange rate and inflation, explicitly incorporating a forward looking Taylor rule into the system. Section 3 explains our estimation methods. In Section 4, we report our empirical results. Section 5 reviews the current empirical NKPC literature in relation to our findings. Section 6 concludes.

2 The Model

2.1 Gradual Adjustment Equation

We start with a univariate stochastic process of real exchange rates. Let p_t be the log domestic price level, p_t^* be the log foreign price level, and e_t be the log nominal exchange rate as the price of one unit of the foreign currency in terms of the home currency. And we denote s_t as the log of the real exchange rate, $p_t^* + e_t - p_t$.

We assume that PPP holds in the long-run. Putting it differently, we assume that there exists a cointegrating vector $[1 \ -1 \ -1]'$ for a vector $[p_t \ p_t^* \ e_t]'$, where $p_t,\ p_t^*,$ and e_t are difference stationary processes. Under this assumption, the real exchange rate can be represented as the following stationary univariate autoregressive process of degree one.

$$s_{t+1} = d + \alpha s_t + \varepsilon_{t+1}, \tag{1}$$

where α is a positive persistence parameter that is less than one.

Admittedly, estimating half-lives of real exchange rates with an AR(1) specification may not be ideal, because the AR(1) model is mis-specified and will lead to an inconsistent estimator if the true data generating process is a higher order autoregressive process, AR(p). It is interesting to see, however, that Rossi (2005) reported similar half-life estimates from both models. Later in Section 4, we confirm that this is roughly the case for our exchange rate data. Thus, assuming AR(1) seems innocuous for the purpose of estimating the half life of most real exchange rates in our data. A different issue is whether or not the assumption of AR(1) is appropriate for other purposes such as examining the shape of the impulse responses (Steinsson, 2008). Even though this is an interesting question, we do not pursue this issue in the current paper.

Recently, Kehoe and Midrigan (2007) show that the persistence parameter α is closely related to a measure of price stickiness in Calvo (1983) pricing models. It can be shown that a single-good version of their model implies the stochastic process (1) for the real exchange rate where α equals the probability that firms do not adjust their prices in any given period. Along the line of Woodford (2007), Kim (2009) shows that (1) can be also derived from a similar model as Kehoe and Midrigan's (2007) with the Taylor Rule.

By rearranging and taking conditional expectations, the equation (1) can be written by the following error correction model of real exchange rates with a *known* cointegrating relation described earlier.

$$\mathbb{E}_t \Delta p_{t+1} = b\left[\mu - (p_t - p_t^* - e_t)\right] + \mathbb{E}_t \Delta p_{t+1}^* + \mathbb{E}_t \Delta e_{t+1}, \tag{2}$$

where $\mu = \mathbb{E}(p_t - p_t^* - e_t)$, $b = 1 - \alpha$, $d = -(1-\alpha)\mu$, $\varepsilon_{t+1} = \varepsilon_{1,t+1} + \varepsilon_{2,t+1} - \varepsilon_{3,t+1} = (e_{t+1} - \mathbb{E}_t e_{t+1}) + (p_{t+1}^* - \mathbb{E}_t p_{t+1}^*) - (p_{t+1} - \mathbb{E}_t p_{t+1})$, and $\mathbb{E}_t \varepsilon_{t+1} = 0$. $E(\cdot)$ denotes the unconditional expectation operator while $\mathbb{E}_t(\cdot)$ is the conditional expectation operator on I_t, the economic agent's information set at time t.[2] Note that b is the convergence rate ($= 1 - \alpha$), which is a positive constant less than unity by construction.

2.2 The Taylor Rule Model

We assume that the uncovered interest parity (UIP) holds. That is,

$$\mathbb{E}_t \Delta e_{t+1} = i_t - i_t^*, \tag{3}$$

[2] A single-good version of Mussa's (1982) model implies this when we add a domestic price shock, $p_{t+1} - \mathbb{E}_t p_{t+1}$, that has a conditional expectation of zero given the information at time t.

where i_t and i_t^* are domestic and foreign interest rates, respectively.[3]

The central bank in the home country is assumed to continuously set its optimal target interest rate (i_t^T) by the following forward looking Taylor Rule.[4]

$$i_t^T = \bar{r} + \gamma_\pi \mathbb{E}_t \Delta p_{t+1} + \gamma_x x_t,$$

where $\bar{r}$ is a constant that includes a certain long-run equilibrium real interest rate along with a target inflation rate[5], and γ_π and γ_x are the *long-run* Taylor Rule coefficients on expected future inflation[6] ($\mathbb{E}_t \Delta p_{t+1}$) and current output deviations[7] (x_t), respectively. We also assume that the central bank attempts to smooth the interest rate by the following rule.

$$i_t = (1-\rho) i_t^T + \rho i_{t-1},$$

that is, the current actual interest rate is a weighted average of the target interest rate and the previous period's interest rate, where ρ is the smoothing parameter. Then, we can derive the forward looking version Taylor Rule equation with interest rate smoothing policy as follows.

$$i_t = (1-\rho)\bar{r} + (1-\rho)\gamma_\pi \mathbb{E}_t \Delta p_{t+1} + (1-\rho)\gamma_x x_t + \rho i_{t-1} \tag{4}$$

Combining (3) and (4), we obtain the following.

$$\begin{aligned} \mathbb{E}_t \Delta e_{t+1} &= (1-\rho)\bar{r} + (1-\rho)\gamma_\pi \mathbb{E}_t \Delta p_{t+1} + (1-\rho)\gamma_x x_t + \rho i_{t-1} - i_t^* \\ &= \iota + \gamma_\pi^s \mathbb{E}_t \Delta p_{t+1} + \gamma_x^s x_t + \rho i_{t-1} - i_t^*, \end{aligned} \tag{5}$$

[3]The UIP often fails to hold when one tests it by estimating a single regression equation, $\Delta e_{t+1} = \beta(i_t - i_t^*) + \varepsilon_{t+1}$. Therefore, it is not ideal to assume the UIP in our model, thus future research should remove this assumption. We believe, however, that our initial attempt should start with the UIP, because it is difficult to write an exchange rate model with the Taylor rule without the UIP for our purpose of getting more information from the model. Further, Taylor rule exchange rate models in the literature often assumes the UIP.

[4]We remain agnostic about the policy rule of the foreign central bank, because the Taylor rule may not be employed in some countries.

[5]See Clarida, Galí, and Gertler (1998, 2000) for details.

[6]It may be more reasonable to use real-time data instead of the final release data. However, doing so will introduce another complication as we need to specify the relation between the real-time price index and the consumer price index, which is frequently used in the PPP literature. Hence we leave the use of real-time data for future research.

[7]If we assume that the central bank responds to expected future output deviations rather than current deviations, we can simply modify the model by replacing x_t with $\mathbb{E}_t x_{t+1}$. However, this does not make any significant difference to our results.

where $\iota = (1-\rho)\bar{r}$ is a constant, $\gamma_\pi^s = (1-\rho)\gamma_\pi$ and $\gamma_x^s = (1-\rho)\gamma_x$ are *short-run* Taylor Rule coefficients.

Now, let's rewrite (2) as the following equation in level variables.

$$\mathbb{E}_t p_{t+1} = b\mu + \mathbb{E}_t e_{t+1} + (1-b)p_t - (1-b)e_t + \mathbb{E}_t p_{t+1}^* - (1-b)p_t^* \tag{2'}$$

Taking differences and rearranging it, (2') can be rewritten as follows.

$$\mathbb{E}_t \Delta p_{t+1} = \mathbb{E}_t \Delta e_{t+1} + \alpha \Delta p_t - \alpha \Delta e_t + \left[\mathbb{E}_t \Delta p_{t+1}^* - \alpha \Delta p_t^* + \eta_t\right], \tag{6}$$

where $\alpha = 1-b$ and $\eta_t = \eta_{1,t} + \eta_{2,t} - \eta_{3,t} = (e_t - \mathbb{E}_{t-1}e_t) + (p_t^* - \mathbb{E}_{t-1}p_t^*) - (p_t - \mathbb{E}_{t-1}p_t)$.

From (4), (5), and (6), we construct the following system of stochastic difference equations.

$$\begin{bmatrix} 1 & -1 & 0 \\ -\gamma_\pi^s & 1 & 0 \\ -\gamma_\pi^s & 0 & 1 \end{bmatrix} \begin{bmatrix} \mathbb{E}_t \Delta p_{t+1} \\ \mathbb{E}_t \Delta e_{t+1} \\ i_t \end{bmatrix} = \begin{bmatrix} \alpha & -\alpha & 0 \\ 0 & 0 & \rho \\ 0 & 0 & \rho \end{bmatrix} \begin{bmatrix} \Delta p_t \\ \Delta e_t \\ i_{t-1} \end{bmatrix} + \begin{bmatrix} \mathbb{E}_t \Delta p_{t+1}^* - \alpha \Delta p_t^* + \eta_t \\ \iota + \gamma_x^s x_t - i_t^* \\ \iota + \gamma_x^s x_t \end{bmatrix} \tag{7}$$

For notational simplicity, let's rewrite (7) in matrix form as follows.

$$\mathbf{A}\mathbb{E}_t \mathbf{y}_{t+1} = \mathbf{B}\mathbf{y}_t + \mathbf{x}_t, \tag{7'}$$

and thus,

$$\begin{aligned} \mathbb{E}_t \mathbf{y}_{t+1} &= \mathbf{A}^{-1}\mathbf{B}\mathbf{y}_t + \mathbf{A}^{-1}\mathbf{x}_t \\ &= \mathbf{D}\mathbf{y}_t + \mathbf{c}_t, \end{aligned} \tag{8}$$

where $\mathbf{D} = \mathbf{A}^{-1}\mathbf{B}$ and $\mathbf{c}_t = \mathbf{A}^{-1}\mathbf{x}_t$.[8] By eigenvalue decomposition, (8) can be rewritten as follows.

$$\mathbb{E}_t \mathbf{y}_{t+1} = \mathbf{V}\mathbf{\Lambda}\mathbf{V}^{-1}\mathbf{y}_t + \mathbf{c}_t, \tag{9}$$

where $\mathbf{D} = \mathbf{V}\mathbf{\Lambda}\mathbf{V}^{-1}$ and

$$\mathbf{V} = \begin{bmatrix} 1 & 1 & 1 \\ \frac{\alpha\gamma_\pi^s}{\alpha-\rho} & 1 & 1 \\ \frac{\alpha\gamma_\pi^s}{\alpha-\rho} & 1 & 0 \end{bmatrix}, \quad \mathbf{\Lambda} = \begin{bmatrix} \alpha & 0 & 0 \\ 0 & \frac{\rho}{1-\gamma_\pi^s} & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

[8] It is straightforward to show that $\mathbf{A}$ is nonsingular, and thus has a well-defined inverse.

Premultiplying (9) by $\mathbf{V}^{-1}$ and redefining variables,

$$\mathbb{E}_t \mathbf{z}_{t+1} = \mathbf{\Lambda} \mathbf{z}_t + \mathbf{h}_t, \tag{10}$$

where $\mathbf{z}_t = \mathbf{V}^{-1}\mathbf{y}_t$ and $\mathbf{h}_t = \mathbf{V}^{-1}\mathbf{c}_t$.

Note that, among non-zero eigenvalues in $\mathbf{\Lambda}$, α is between 0 and 1 by definition, while $\frac{\rho}{1-\gamma_\pi^s}$ $(= \frac{\rho}{1-(1-\rho)\gamma_\pi})$ is greater than unity as long as $1 < \gamma_\pi < \frac{1}{1-\rho}$. Therefore, if the long-run inflation coefficient γ_π is strictly greater than one, the system of stochastic difference equations (7) has a saddle path equilibrium, where rationally expected future fundamental variables enter in the exchange rate and inflation dynamics.[9] On the contrary, if γ_π is strictly less than unity, which might be true in the pre-Volker era in the US, the system would have a purely backward looking solution, where the solution would be determined by *past* fundamental variables and *any* martingale difference sequences.

Assuming γ_π is strictly greater than one, we can show that the solution to (7) satisfies the following relation (see Appendix A for the derivation).

$$\begin{aligned}\Delta e_{t+1} = \hat{\imath} + \frac{\alpha\gamma_\pi^s}{\alpha-\rho}\Delta p_{t+1} - \frac{\alpha\gamma_\pi^s}{\alpha-\rho}\Delta p_{t+1}^* + \frac{\alpha\gamma_\pi^s-(\alpha-\rho)}{\alpha-\rho} i_t^* \\ + \frac{\gamma_\pi^s(\alpha\gamma_\pi^s-(\alpha-\rho))}{(\alpha-\rho)\rho}\sum_{j=0}^{\infty}\left(\frac{1-\gamma_\pi^s}{\rho}\right)^j \mathbb{E}_t f_{t+j+1} + \omega_{t+1},\end{aligned} \tag{11}$$

where,

$$\hat{\imath} = \frac{\alpha\gamma_\pi^s-(\alpha-\rho)}{(\alpha-\rho)(\gamma_\pi^s-(1-\rho))}\iota,$$

$$f_t = -\left[i_t^* - \mathbb{E}_t\Delta p_{t+1}^*\right] + \frac{\gamma_x^s}{\gamma_\pi^s}x_t$$

$$\begin{aligned}\omega_{t+1} = \frac{\gamma_\pi^s(\alpha\gamma_\pi^s-(\alpha-\rho))}{(\alpha-\rho)\rho}\sum_{j=0}^{\infty}\left(\frac{1-\gamma_\pi^s}{\rho}\right)^j (\mathbb{E}_{t+1}f_{t+j+1} - \mathbb{E}_t f_{t+j+1}) \\ + \frac{\gamma_\pi^s}{\alpha-\rho}\eta_{t+1} - \frac{\alpha\gamma_\pi^s-(\alpha-\rho)}{\alpha-\rho}\upsilon_{t+1},\end{aligned}$$

and,

$$\mathbb{E}_t\omega_{t+1} = 0$$

[9]The condition $\gamma_\pi < \frac{1}{1-\rho}$ is easily met for all sample periods we consider in this paper.

Or, (11) can be rewritten with full parameter specification as follows.

$$\Delta e_{t+1} = \hat{\imath} + \frac{\alpha\gamma_\pi(1-\rho)}{\alpha-\rho}\Delta p_{t+1} - \frac{\alpha\gamma_\pi(1-\rho)}{\alpha-\rho}\Delta p^*_{t+1} + \frac{\alpha\gamma_\pi(1-\rho)-(\alpha-\rho)}{\alpha-\rho} i^*_t \qquad (11')$$
$$+ \frac{\gamma_\pi(1-\rho)(\alpha\gamma_\pi(1-\rho)-(\alpha-\rho))}{(\alpha-\rho)\rho}\sum_{j=0}^{\infty}\left(\frac{1-\gamma_\pi(1-\rho)}{\rho}\right)^j \mathbb{E}_t f_{t+j+1} + \omega_{t+1}$$

Here, f_t is a proxy variable that summarizes the fundamental variables such as foreign *ex ante* real interest rates and domestic output deviations.

Note that if γ_π is strictly less than unity, the restriction in (11) may not be valid, since the system would have a backward looking equilibrium rather than a saddle path equilibrium.[10] Put it differently, exchange rate dynamics critically depends on the size of γ_π. As mentioned in the introduction, however, we have some supporting empirical evidence for such a requirement for the existence of a saddle path equilibrium, at least for the post-Volker era. So we believe that our specification would remain valid for our purpose in this paper.

One related research has been recently put forward by Clarida and Waldman (2007), who investigate exchange rate dynamics when central banks employ Taylor rules in a small open economy framework proposed by Svensson (1999). In their paper, they derive the dynamics of real exchange rates by combining the Taylor Rule and the uncovered interest parity (or real interest parity), so that the real exchange rate is mainly determined by the *ex ante* real interest rate. In their model, the real interest rate follows an AR(1) process of which the autoregressive coefficient is a function of the Taylor rule coefficients. When the central bank responds to inflation more aggressively, the economy returns to its long-run equilibrium at a faster rate. Therefore, the half-life of PPP deviations is negatively affected by γ_π.

It should be noted that their model does not explicitly incorporate the commodity view of PPP in the sense that real exchange rate dynamics are mainly determined by the portfolio market equilibrium conditions. Unlike them, we combine Kehoe and Midrigan's (2007) model with the UIP as well as the Taylor Rule. Under this framework, no policy parameters can affect the half-life of the PPP deviations because real exchange rate persistence is mainly driven by firms' behavior. On the other hand, policy parameters do affect volatilities of inflation and the nominal exchange rate in our model. For example, the more aggressively the central bank responds to inflation, the less volatile inflation is, which leads to a less volatile nominal exchange

[10] If the system has a purely backward looking solution, the conventional structural Vector Autoregressive (SVAR) estimation method may apply.

rate.

One interesting feature arises when another policy parameter, ρ, varies. As the value for ρ increases, the volatility of Δp_{t+1} decreases. This is due to the uncovered interest parity condition. A higher value of ρ, higher interest rate inertia, implies that the central bank changes the nominal interest rate less. Therefore, Δe_{t+1} should change less due to the uncovered interest parity. When $\alpha = \rho$, it can be shown that after the initial cost-push shock, price does not change at all (see Appendix B). That is, Δp_{t+1} instantly jumps and stays at its long-run equilibrium value of zero. Hence, the convergence toward long-run PPP should be carried over by the exchange rate adjustments. When $\alpha < \rho$, price must decrease after the initial cost-push shock, since the nominal exchange rate movement is limited by the uncovered interest parity and domestic interest rate inertia.

3 Estimation Methods

We discuss two estimation strategies here: a conventional univariate equation approach and the GMM system method (Kim, Ogaki, and Yang, 2007).

3.1 Univariate Equation Approach

A univariate approach utilizes the equations (1) or (2). For instance, the persistence parameter α in (1) can be consistently estimated by the conventional least squares method under the maintained cointegrating relation assumption. Once we obtain the point estimate of α, the half-life of the real exchange rate can be calculated by $\frac{\ln(.5)}{\ln \alpha}$. Similarly, the regression equation for the convergence parameter b can be constructed from (2) as follows.

$$\Delta p_{t+1} = b\left[\mu - (p_t - p_t^* - e_t)\right] + \Delta p_{t+1}^* + \Delta e_{t+1} + \tilde{\varepsilon}_{t+1}, \tag{2"}$$

where $\tilde{\varepsilon}_{t+1} = -\varepsilon_{t+1} = -(e_{t+1} - \mathbb{E}_t e_{t+1}) - (p_{t+1}^* - \mathbb{E}_t p_{t+1}^*) + (p_{t+1} - \mathbb{E}_t p_{t+1})$ and $\mathbb{E}_t \tilde{\varepsilon}_{t+1} = 0$.

3.2 GMM System Method

Our second estimation strategy combines the equation (11) with (1). The estimation of the equation (11) is a challenging task, however, since it has an infinite sum of rationally expected discounted future fundamental variables. Following Hansen and Sargent (1980, 1982), we linearly project $\mathbb{E}_t(\cdot)$ onto Ω_t, the econometrician's information set at time t, which is a subset of I_t. Denoting $\hat{\mathbb{E}}_t(\cdot)$ as such a linear projection operator onto Ω_t, we can rewrite (11) as follows.

$$\begin{aligned}\Delta e_{t+1} = \hat{\imath} + \frac{\alpha\gamma_\pi^s}{\alpha-\rho}\Delta p_{t+1} - \frac{\alpha\gamma_\pi^s}{\alpha-\rho}\Delta p_{t+1}^* + \frac{\alpha\gamma_\pi^s-(\alpha-\rho)}{\alpha-\rho} i_t^* \\ + \frac{\gamma_\pi^s(\alpha\gamma_\pi^s-(\alpha-\rho))}{(\alpha-\rho)\rho}\sum_{j=0}^{\infty}\left(\frac{1-\gamma_\pi^s}{\rho}\right)^j \hat{\mathbb{E}}_t f_{t+j+1} + \xi_{t+1},\end{aligned} \tag{12}$$

where

$$\xi_{t+1} = \omega_{t+1} + \frac{\gamma_\pi^s(\alpha\gamma_\pi^s-(\alpha-\rho))}{(\alpha-\rho)\rho}\sum_{j=0}^{\infty}\left(\frac{1-\gamma_\pi^s}{\rho}\right)^j \left(\mathbb{E}_t f_{t+j+1} - \hat{\mathbb{E}}_t f_{t+j+1}\right),$$

and

$$\hat{\mathbb{E}}_t \xi_{t+1} = 0,$$

by the law of iterated projections.

For appropriate instrumental variables that are in Ω_t, we assume $\Omega_t = \{f_t, f_{t-1}, f_{t-2}, \cdots\}$. This assumption would be an innocent one under the stationarity assumption of the fundamental variable, f_t, and it can greatly lessen the burden in our GMM estimation by significantly reducing the number of coefficients to be estimated.

Assume, for now, that f_t be a zero mean covariance stationary, linearly indeterministic stochastic process so that it has the following Wold representation.

$$f_t = c(L)\nu_t, \tag{13}$$

where $\nu_t = f_t - \hat{\mathbb{E}}_{t-1} f_t$ and $c(L)$ is square summable. Assuming that $c(L) = 1 + c_1 L + c_2 L^2 + \cdots$ is invertible, (13) can be rewritten as the following autoregressive representation.

$$b(L) f_t = \nu_t, \tag{14}$$

where $b(L) = c^{-1}(L) = 1 - b_1 L - b_2 L^2 - \cdots$. Linearly projecting $\sum_{j=0}^{\infty}\left(\frac{1-\gamma_\pi^s}{\rho}\right)^j \mathbb{E}_t f_{t+j+1}$ onto Ω_t, Hansen and Sargent (1980) show that the following relation holds.

$$\sum_{j=0}^{\infty}\delta^j \hat{\mathbb{E}}_t f_{t+j+1} = \psi(L) f_t = \left[\frac{1-\left(\delta^{-1}b(\delta)\right)^{-1} b(L)L^{-1}}{1-(\delta^{-1}L)^{-1}}\right] f_t, \tag{15}$$

where $\delta = \frac{1-\gamma_\pi^s}{\rho}$.

For actual estimation, we assume that f_t can be represented by a finite order AR(r) process, that is, $b(L) = 1 - \sum_{j=1}^{r} b_j L^j$, where $r < \infty$.[11] Then, it can be shown that the coefficients of $\psi(L)$ can be computed recursively (see Sargent 1987) as follows.

$$\psi_0 = (1 - \delta b_1 - \cdots - \delta^r b_r)^{-1}$$

$$\psi_r = 0$$

$$\psi_{j-1} = \delta\psi_j + \delta\psi_0 b_j,$$

where $j = 1, 2, \cdots, r$. Then, we obtain the following two orthogonality conditions.

$$\begin{aligned}\Delta e_{t+1} = \hat{\imath} + \frac{\alpha\gamma_\pi^s}{\alpha-\rho}\Delta p_{t+1} - \frac{\alpha\gamma_\pi^s}{\alpha-\rho}\Delta p_{t+1}^* + \frac{\alpha\gamma_\pi^s - (\alpha-\rho)}{\alpha-\rho} i_t^* \\ + \frac{\gamma_\pi^s(\alpha\gamma_\pi^s - (\alpha-\rho))}{(\alpha-\rho)\rho}\left(\psi_0 f_t + \psi_1 f_{t-1} + \cdots + \psi_{r-1} f_{t-r+1}\right) + \xi_{t+1},\end{aligned} \tag{16}$$

$$f_{t+1} = k + b_1 f_t + b_2 f_{t-1} + \cdots + b_r f_{t-r+1} + \nu_{t+1}, \tag{17}$$

where k is a constant scalar and $\hat{\mathbb{E}}_t \nu_{t+1} = 0$.[12][13]

Finally, the system method (GMM) estimation utilizes all aforementioned orthogonality conditions, (2"), (16), and (17). That is, a GMM estimation can be implemented by the following $2(p+2)$ orthogonality conditions.

$$\mathbb{E}\mathbf{x}_{1,t}(s_{t+1} - d - \alpha s_t) = 0 \tag{18}$$

$$\hat{\mathbb{E}}\mathbf{x}_{2,t-\tau}\left(\begin{array}{c}\Delta e_{t+1} - \hat{\imath} - \frac{\alpha\gamma_\pi^s}{\alpha-\rho}\Delta p_{t+1} + \frac{\alpha\gamma_\pi^s}{\alpha-\rho}\Delta p_{t+1}^* - \frac{\alpha\gamma_\pi^s-(\alpha-\rho)}{\alpha-\rho} i_t^* \\ -\frac{\gamma_\pi^s(\alpha\gamma_\pi^s-(\alpha-\rho))}{(\alpha-\rho)\rho}\left(\psi_0 f_t + \psi_1 f_{t-1} + \cdots + \psi_{r-1} f_{t-r+1}\right)\end{array}\right) = 0 \tag{19}$$

$$\hat{\mathbb{E}}\mathbf{x}_{2,t-\tau}(f_{t+1} - k - b_1 f_t - b_2 f_{t-1} - \cdots - b_r f_{t-r+1}) = 0, \tag{20}$$

[11] We can use conventional Akaike Information criteria or Bayesian Information criteria in order to choose the degree of such autoregressive processes.

[12] Recall that Hansen and Sargent (1980) assume a zero-mean covariance stationary process. If the variable of interest has a non-zero unconditional mean, we can either demean it prior to the estimation or include a constant but leave its coefficient unconstrained. West (1989) showed that the further efficiency gain can be obtained by imposing additional restrictions on the deterministic term. However, the imposition of such an additional restriction is quite burdensome, so we simply add a constant here.

[13] In actual estimations, we normalized (16) by multiplying $(\alpha-\rho)$ to each side in order to reduce nonlinearity.

where $\mathbf{x}_{1,t} = (1\ s_t)'$, $\mathbf{x}_{2,t} = (1\ f_t)'$, and $\tau = 0, 1, \cdots, p$.[14][15]

3.3 Median Unbiased Estimator and Grid-t Confidence Intervals

We correct for the bias in our α estimates by a GMM version of the grid-t method proposed by Hansen (1999) for the least squares estimator. It is straightforward to generate pseudo samples for the orthogonality condition (20) by the conventional residual-based bootstrapping. However, there are some complications in obtaining samples directly from (18) and (19), since p_t^* is treated as a forcing variable in our model. We deal with this problem as follows.

In order to generate pseudo samples for the orthogonality conditions (18) and (19), we denote $\tilde{p}_t$ as the relative price index $p_t - p_t^*$. Then, (2") and (16) can be rewritten as follows.

$$\Delta\tilde{p}_{t+1} = b\mu - b(\tilde{p}_t - e_t) + \Delta e_{t+1} + \tilde{\varepsilon}_{t+1}$$

$$\begin{aligned}\Delta e_{t+1} = \hat{\imath} &+ \frac{\alpha\gamma_\pi^s}{\alpha-\rho}\Delta\tilde{p}_{t+1} + \frac{\alpha\gamma_\pi^s - (\alpha-\rho)}{\alpha-\rho} i_t^* \\ &+ \frac{\gamma_\pi^s(\alpha\gamma_\pi^s - (\alpha-\rho))}{(\alpha-\rho)\rho}\left(\psi_0 f_t + \cdots + \psi_{r-1} f_{t-r+1}\right) + \xi_{t+1}\end{aligned}$$

Or, in matrix form,

$$\begin{aligned}\begin{bmatrix}\Delta\tilde{p}_{t+1}\\ \Delta e_{t+1}\end{bmatrix} &= \mathbf{C} + \mathbf{S}^{-1}\begin{bmatrix}-(1-\alpha)\\ 0\end{bmatrix}[\tilde{p}_t - e_t] \\ &+ \mathbf{S}^{-1}\begin{bmatrix}0\\ \frac{\alpha\gamma_\pi^s-(\alpha-\rho)}{\alpha-\rho} i_t^* + \frac{\gamma_\pi^s(\alpha\gamma_\pi^s-(\alpha-\rho))}{(\alpha-\rho)\rho}\cdot \\ \left(\psi_0 f_t + \cdots + \psi_{r-1} f_{t-r+1}\right)\end{bmatrix} + \mathbf{S}^{-1}\begin{bmatrix}\tilde{\varepsilon}_{t+1}\\ \xi_{t+1}\end{bmatrix},\end{aligned} \tag{21}$$

where $\mathbf{C}$ is a vector of constants and $\mathbf{S}$ is $\left[1\ \ -1 \vdots -\frac{\alpha\gamma_\pi^s}{\alpha-\rho}\ \ 1\right]$.

Then, treating each grid point $\alpha \in [\alpha_{\min}, \alpha_{\max}]$ as a true value, we can generate pseudo samples of $\Delta\tilde{p}_{t+1}$ and Δe_{t+1} by the conventional bootstrapping.[16] The level variables $\tilde{p}_t$ and e_t are obtained by numerical integration. It should be noted that all other parameters are treated

[14] p does not necessarily coincide with r.
[15] In actual estimations, we use the aforementioned normalization again.
[16] The historical data were used for the initial values and the foreign interest rate i_t^*.

as nuisance parameters (η).[17] Following Hansen (1999), we define the grid-t statistic at each grid point $\alpha \in [\alpha_{\min}, \alpha_{\max}]$ as follows.

$$t_n(\alpha) = \frac{\hat{\alpha}_{\mathrm{GMM}} - \alpha}{se(\hat{\alpha}_{\mathrm{GMM}})}, \tag{22}$$

where $se(\hat{\alpha}_{\mathrm{GMM}})$ denotes the robust GMM standard error at the GMM estimate $\hat{\alpha}_{\mathrm{GMM}}$. Implementing GMM estimations for B bootstrap iterations at each of N grid point of α, we obtain the (β quantile) grid-t bootstrap quantile functions, $q^*_{n,\beta}(\alpha) = q^*_{n,\beta}(\alpha, \eta(\alpha))$. Note that each function is evaluated at each grid point α rather than at the point estimate.[18]

Finally, we define the 95% grid-t confidence interval as follows.

$$\{\alpha \in R : q^*_{n,2.5\%}(\alpha) \leq t_n(\alpha) \leq q^*_{n,97.5\%}(\alpha)\}, \tag{23}$$

and the median unbiased estimator is,

$$\alpha_{MUE} = \alpha \in R, \text{ s.t. } t_n(\alpha) = q^*_{n,50\%}(\alpha) \tag{24}$$

4 Empirical Results

This section reports estimates of the persistence parameter α (or convergence rate parameter b) and their implied half-lives from the aforementioned two estimation strategies.

We use CPIs to construct real exchange rates with the US$ as a base currency. We consider 19 industrialized countries that provide 18 real exchange rates.[19] For interest rates, we use quarterly money market interest rates that are short-term interbank call rates rather than conventional short-term treasury bill rates, since we incorporate the Taylor Rule in the model where a central bank sets its target short-term market rate. For output deviations, we consider two different measures of output gaps, quadratically detrended real GDP gap (see Clarida, Galí,

[17] See Hansen (1999) for detailed explanations.

[18] If they are evaluated at the point estimate, the quantile functions correspond to the Efron and Tibshirani's (1993) bootstrap-t quantile functions.

[19] Among 23 industrialized countries classified by IMF, we dropped Greece, Iceland, and Ireland due to lack of reasonable number of observations. Luxembourg was not included because it has a currency union with Belgium.

and Gertler 1998) and unemployment rate gaps (see Boivin 2006).[20][21] The data frequency is quarterly and from the IFS CD-ROM. The sample period is from 1979:III to 1998:IV for Eurozone countries, and from 1979:III to 2003:IV for the rest of the countries.

The reason that our sample period starts from 1979.III is based on empirical evidence on the US Taylor Rule. As discussed in Section II, the inflation and exchange rate dynamics may greatly depend on the size of the central bank's reaction coefficient to expected inflation. We showed that the rationally expected *future* fundamental variables appear in the exchange rate and inflation dynamics only when the long-run inflation coefficient γ_π is strictly greater than unity. Clarida, Galí, and Gertler (1998, 2000) provide important empirical evidence for the existence of a structural break in the US Taylor Rule. Put it differently, they show that γ_π was strictly less than one during the pre-Volker era, while it became strictly greater than unity in the post-Volker era.

We implement similar GMM estimations for (4) as in Clarida, Galí, and Gertler (2000)[22][23] with longer sample period and report the results in Table 1 (see the note on Table 1 for detailed explanation). We use two output gap measures for three different sub-samples. Most coefficients were highly significant and specification tests by J-test were not rejected.[24] More importantly, our requirement for the existence of a saddle path equilibrium met for the post-Volker era rather than the pre-Volker era. Therefore, we may conclude that this provides some empirical justification for the choice of our sample period.

Insert Table 1 Here

We report our GMM version median unbiased estimates and the 95% grid-t confidence intervals in Table 2. We implemented estimations using both gap measures, but report the full estimates with unemployment gaps in order to save space.[25] We chose $N = 30$ and $B =$

[20] We also tried same analysis with the cyclical components of real GDP series from the HP-filter with 1600 of smoothing parameter. The results were quantitatively similar.

[21] The unemployment gap is defined as a 5 year backward moving average subtracted by the current unemployment rate. This specification makes its sign consistent with that of the conventional output gap.

[22] They used GDP deflator inflation along with the CBO output gaps (and HP detrended gaps).

[23] Unlike them, we assume that the Fed targets current output gap rather than future deviations. However, this doesn't make any significant changes to our results. And we include one lag of interest rate rather than two lags for simplicity.

[24] J-test statistics are available upon request.

[25] The results with quadratically detrended real GDP gaps were quantitatively similar.

500 totaling 15,000 GMM simulations for each exchange rate. We chose $p = r = 8$ by the conventional Bayesian Information Criteria., and standard errors were adjusted using the QS kernel estimator with automatic bandwidth selection in order to deal with unknown serial correlation problems. For comparison, we report the corresponding estimates by the least squares in Table 3.

We note that the system method provides much shorter half-life estimates compared with ones from the single equation method (see Tables 2 and 3). The median value of the half-life estimate was 3.42 years from the univariate estimations after adjusting for the median bias using the grid-t bootstrap. However, the median value of the GMM median unbiased estimates was still below 1 year, 0.94 year, when we correct for the bias.[26] Our estimates are roughly consistent with the average half-life estimates from the micro-data evidence by Crucini and Shintani (2008)[27] and the differences of the point estimates for different countries are very similar to those of Murray and Papell (2002) for most countries.[28] J-test accepts our model specification for all countries with an exception of the UK.

We also notice that our median-unbiased point estimate $\hat{\alpha}_{\text{GMM,MUE}}$ is consistent with the price-stickiness parameter estimates by Galí and Gertler (1999) who use the New Keynesian Phillips Curve specification with Calvo pricing. Recall that a single-good version model by Kehoe and Midrigan (2007) or Kim (2009) implies that α coincides with the Calvo probability parameter.

Regarding efficiency, we obtained substantial efficiency gains from the system method over the single equation method. Murray and Papell (2002) report a version of the grid-α confidence intervals (Hansen, 1999)[29] of which upper limits of their half-life estimates are infinity for every exchange rates they consider. Based on such results, they conclude that single equation methods may provide virtually no useful information due to wide confidence intervals.

Our grid-t confidence intervals from the single equation method were consistent with such a

[26] Without bias correction, the median value of the half-life estimate was 2.59 years from the univariate estimations and 0.90 year from the system method. All estimates and the conventional 95% bootstrap confidence intervals are available from authors upon request.

[27] For the OECD countries, their baseline half-life estimates for traded good prices were 1.5 years, while 1.58 and 2.00 years for all and non-traded good prices.

[28] The exceptions to this similarity are Japan and the UK, as our point estimates for the countries are much smaller than others. Using the same sample period of Murray and Papell (2002), however, we obtained the α estimates of 0.89 and 0.82 for Japan and the UK, respectively. Therefore, these exceptions seem to have arisen from the difference in the sample periods.

[29] Their confidence intervals are constructed following Andrews (1993) and Andrews and Chen (1994), which are identical to the Hansen's (1999) grid-α confidence intervals if we assume that the errors are drawn from the empirical distribution rather than the i.i.d. normal distribution.

view (see Table 3). The upper limits are infinity for most real exchange rates. However, when we implement estimations by the system method, our 95% GMM version grid-t confidence intervals were very compact. Our results can be also considered as great improvement over Kim, Ogaki, and Yang (2007) who acquired limited success in efficiency gains.

Insert Table 2 Here

Insert Table 3 Here

Lastly, we compare univariate half-life estimates from an AR(1) specification with those from a more general AR(p) specification. Following Rossi (2005), we choose the number of lags by the modified Akaike Information criteria (MAIC, Ng and Perron, 2001) with a maximum 12 lags. We also estimate the lag length by the modified Bayesian Information criteria (MBIC, Ng and Perron, 2001), which yields $p = 1$ for most real exchange rates. The MAIC chooses $p = 1$ for 6 out of 18 real exchange rates. For the remaining 12 real exchange rates, we implement the impulse-response analysis to estimate the half-lives of PPP deviations. As can be seen in Table 4, allowing higher order AR(p) processes results in very different half-life estimates from those of the AR(1) specification for some countries such as Italy, Portugal, and Spain. This implies that one has to be careful in interpreting the results based on AR(1) models for these exchange rates. For many other real exchange rates, however, the half-life estimates do not change much implying that the AR(1) process is not a bad approximation.

Insert Table 4 Here

5 Comparisons with Estimates based on the New Keynesian Phillips Curve

As discussed in Section 2, α in the real exchange rate autoregression in Equation (1) is the Calvo (1983) probability that a firm must keep its price unchanged in a given period in a single-good version of Kehoe and Midrigan's (2007) model. We denote this probability by θ. Even though $\alpha = \theta$ in our interpretation, the AR coefficient can be different from θ in other models. In this section, we review various methods of estimating θ for the NKPC and compare the results from U.S. quarterly data with our estimates of the probability. For comparisons, note that the average time over which a price is fixed is $(1-\theta)\sum_{k=0}^{\infty} k\theta^{k-1} = 1/(1-\theta)$.

A classic method to estimate θ is a single equation method that applies GMM to the NKPC as in Galí and Gertler (1999) and Eichenbaum and Fisher (2007). Galí and Gertler's preferred estimates of θ are about 0.8, implying an average duration of about 5 quarters. Eichenbaum and Fisher's estimates of θ are also about 0.8 for the baseline model, but are lower, around 0.6 with the implied average duration of about 2.5 quarters, when the model is modified. A recurring problem with this method is the weak identification problem as surveyed by Kleibergen and Mavroeidis (2009). The 95% confidence interval using their recommended method gives a lower bound of two quarters and an upper bound of infinity for the average price duration.

Another single equation method is the minimum distance method applied to the NKPC as in Sbordone (2002, 2005). The minimum distance estimator is also subject to the weak identification problem according to Magnusson and Mavroeidis (2009). Their 95% confidence intervals give a lower bound average duration of about 3.3 quarters to an upper bound of infinity. The minimum distance method gives sharper results than GMM.

Thus, the single equation methods for the NKPC yield results that are similar to the single equation methods for the real exchange rate half-lives, and both those confidence intervals are very wide.

System methods to estimate θ in the literature use DSGE models with the NKPC. Christiano, Eichenbaum, and Evans (2005) use a minimum distance estimator for the DSGE model, and obtain a point estimate of θ of 0.6 for the benchmark model. Their estimate implies the average duration of 2.5 quarters. At this point, it is not clear whether or not the tight confidence intervals they report based on asymptotic theory is subject to the weak identification problem.

Another popular system method is the Bayesian analysis of DSGE models with the NKPC. The posterior mode of θ in Smets and Wouters (2007) is 0.65, implying the average duration of about 2.9 quarters. Del Negro and Schorfheide (2008) show that posterior mean estimates of θ depend on priors and range from 0.56 to 0.84.

It is interesting to compare these estimates from aggregate data with evidence from Micro data. Nakamura and Steinsson (2008) use a substantially more detailed data set than Bils and

Klenow (2004), and find that the median duration of prices excluding sales was between 8 and 11 months in 1998-2005. However, given that the frequency of price changes differs dramatically across goods in these and other micro studies, aggregating these results for aggregate structural models is a challenge.

6 Conclusion

After recognizing that the degree of uncertainty for estimating the half lives of real exchange rates from single equation methods is huge, we proposed a system method that combines the Taylor rule and a standard exchange rate model, then estimated the half-lives of the real exchange rates of 18 developed countries against the U.S.

We used two types of nonparametric bootstrap methods to construct confidence intervals: the standard bootstrap and Hansen's (1999) grid bootstrap. The standard bootstrap evaluates bootstrap quantiles at the point estimate of the AR(1) coefficient, which implicitly assumes that the bootstrap quantile functions are constant functions. This assumption does not hold for the AR model, and Hansen's grid bootstrap method, which avoids this assumption, has better coverage properties. In our applications, we often obtain very different confidence intervals for these two methods.[30] Therefore, the violation of the assumption is deemed quantitatively important.

When we use the grid bootstrap method, most of the (approximately) median unbiased estimates from the single equation method fall in the range of 3 to 5 years with wide confidence intervals that extend to positive infinity. In contrast, the system method yields median unbiased estimates that are typically substantially less than one year with much sharper confidence intervals, most of which range from 3 quarters to 5 years.

These results indicate that monetary variables from the exchange rate model based on the Taylor rule provide useful information about the half-lives of the real exchange rates. The estimators from the system method are much sharper in the sense that confidence intervals are much narrower than those from a single equation method. Approximately median unbiased estimates of the half-lives are typically about one year, which is much more reasonable than consensus 3 to 5 years from single equation methods. It is also interesting to see that our half-life estimates imply about 4 to 6 quarters of average price duration in the context of the Calvo pricing model. Our 95% confidence intervals of half-lives of the real exchange rates are

[30]Results from standard bootstrap are available upon request.

consistent with most of the estimates of average price durations for aggregate U.S. data for the NKPC and DSGE models.

Our paper is a first step toward moving to a system method with the exchange rate model based on the Taylor rule. We followed most of the papers in the literature with this type of the model by using the uncovered interest parity to connect the Taylor rule to the exchange rate. Because the uncovered interest parity for short-term interest rates is rejected by the data, one future direction is to modify the model by removing the uncovered interest parity. This is a challenging task because no consensus has emerged as to how the deviation from the uncovered interest parity should be modeled. Even though the AR(1) specification seems to be a good approximation for most real exchange rates, it is possible that more general AR(p) models yield quite different half-lives for some exchange rates. This is another challenging task in our system approach because it is not easy to obtain informative saddle-path solutions for a higher order system of difference equations.

A Derivation of (11)

Since $\mathbf{\Lambda}$ in (10) is diagonal, assuming $0 < \alpha < 1$ and $1 < \gamma_\pi < \frac{1}{1-\rho}$, we can solve the system as follows.

$$z_{1,t} = \sum_{j=0}^{\infty} \alpha^j h_{1,t-j-1} + \sum_{j=0}^{\infty} \alpha^j u_{t-j} \tag{a1}$$

$$z_{2,t} = -\sum_{j=0}^{\infty} \left(\frac{1-\gamma_\pi^s}{\rho} \right)^{j+1} \mathbb{E}_t h_{2,t+j} \tag{a2}$$

$$z_{3,t} = h_{3,t-1} + \upsilon_t, \tag{a3}$$

where u_t and υ_t are *any* martingale difference sequences.

Since $\mathbf{y}_t = \mathbf{V}\mathbf{z}_t$,

$$\begin{bmatrix} \Delta p_t \\ \Delta e_t \\ i_{t-1} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ \frac{\alpha\gamma_\pi^s}{\alpha-\rho} & 1 & 1 \\ \frac{\alpha\gamma_\pi^s}{\alpha-\rho} & 1 & 0 \end{bmatrix} \begin{bmatrix} z_{1,t} \\ z_{2,t} \\ z_{3,t} \end{bmatrix} \tag{a4}$$

From first and second rows of (a4), we get the following.

$$\Delta e_t = \frac{\alpha\gamma_\pi^s}{\alpha-\rho}\Delta p_t - \frac{\alpha\gamma_\pi^s - (\alpha-\rho)}{\alpha-\rho} z_{2,t} - \frac{\alpha\gamma_\pi^s - (\alpha-\rho)}{\alpha-\rho} z_{3,t} \tag{a5}$$

Now, we find the analytic solutions for $\mathbf{z}_t$. Since $\mathbf{h}_t = \mathbf{V}^{-1}\mathbf{c}_t$,

$$\mathbf{h}_t = \frac{1}{1-\gamma_\pi^s} \begin{bmatrix} -\frac{\alpha-\rho}{\alpha\gamma_\pi^s-(\alpha-\rho)} & \frac{\alpha-\rho}{\alpha\gamma_\pi^s-(\alpha-\rho)} & 0 \\ \frac{\alpha\gamma_\pi^s}{\alpha\gamma_\pi^s-(\alpha-\rho)} & -\frac{\alpha\gamma_\pi^s}{\alpha\gamma_\pi^s-(\alpha-\rho)} & 1 \\ 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} \mathbb{E}_t\Delta p_{t+1}^* - \alpha\Delta p_t^* + \eta_t + \iota + \gamma_x^s x_t - i_t^* \\ \gamma_\pi^s(\mathbb{E}_t\Delta p_{t+1}^* - \alpha\Delta p_t^* + \eta_t) + \iota + \gamma_x^s x_t - i_t^* \\ \gamma_\pi^s(\mathbb{E}_t\Delta p_{t+1}^* - \alpha\Delta p_t^* + \eta_t) + \iota + \gamma_x^s x_t - \gamma_\pi^s i_t^* \end{bmatrix},$$

and thus,

$$h_{1,t} = -\frac{\alpha-\rho}{\alpha\gamma_\pi^s - (\alpha-\rho)} \left(\mathbb{E}_t\Delta p_{t+1}^* - \alpha\Delta p_t^* + \eta_t \right) \tag{a6}$$

$$h_{2,t} = \frac{1}{1-\gamma_\pi^s} \left[\frac{\rho\gamma_\pi^s}{\alpha\gamma_\pi^s - (\alpha-\rho)} (\mathbb{E}_t\Delta p_{t+1}^* - \alpha\Delta p_t^* + \eta_t) + \iota + \gamma_x^s x_t - \gamma_\pi^s i_t^* \right] \tag{a7}$$

$$h_{3,t} = -i_t^* \tag{a8}$$

Plugging (a6) into (a1),

$$z_{1,t} = -\frac{\alpha-\rho}{\alpha\gamma_\pi^s-(\alpha-\rho)}\sum_{j=0}^{\infty}\alpha^j\left(\Delta p^*_{t-j}-\alpha\Delta p^*_{t-j-1}+\eta_{t-j-1}\right)+\sum_{j=0}^{\infty}\alpha^j u_{t-j} \tag{a9}$$
$$= -\frac{\alpha-\rho}{\alpha\gamma_\pi^s-(\alpha-\rho)}\Delta p^*_t+\sum_{j=0}^{\infty}\alpha^j u_{t-j}-\frac{\alpha-\rho}{\alpha\gamma_\pi^s-(\alpha-\rho)}\sum_{j=0}^{\infty}\alpha^j\eta_{t-j-1}$$

Plugging (a7) into (a2)[31],

$$\begin{aligned}
z_{2,t} &= -\frac{\gamma_\pi^s}{\alpha\gamma_\pi^s-(\alpha-\rho)}\sum_{j=0}^{\infty}\left(\frac{1-\gamma_\pi^s}{\rho}\right)^j\left(\mathbb{E}_t\Delta p^*_{t+j+1}-\alpha\mathbb{E}_t\Delta p^*_{t+j}+\mathbb{E}_t\eta_{t+j}\right)\\
&\quad-\frac{1}{\rho}\sum_{j=0}^{\infty}\left(\frac{1-\gamma_\pi^s}{\rho}\right)^j\left(\iota+\gamma_x^s\mathbb{E}_t x_{t+j}-\gamma_\pi^s\mathbb{E}_t i^*_{t+j}\right)\\
&= \frac{\alpha\gamma_\pi^s}{\alpha\gamma_\pi^s-(\alpha-\rho)}\Delta p^*_t-\frac{\gamma_\pi^s}{\alpha\gamma_\pi^s-(\alpha-\rho)}\eta_t-\frac{\iota}{\gamma_\pi^s-(1-\rho)}\\
&\quad-\frac{\gamma_\pi^s}{\rho}\sum_{j=0}^{\infty}\left(\frac{1-\gamma_\pi^s}{\rho}\right)^j\mathbb{E}_t\Delta p^*_{t+j+1}-\frac{\gamma_\pi^s}{\rho}\sum_{j=0}^{\infty}\left(\frac{1-\gamma_\pi^s}{\rho}\right)^j\left(\frac{\gamma_x^s}{\gamma_\pi^s}\mathbb{E}_t x_{t+j}-\mathbb{E}_t i^*_{t+j}\right)
\end{aligned}$$

Then, denoting f_t as $-\left(i^*_t-\mathbb{E}_t\Delta p^*_{t+1}\right)+\frac{\gamma_x^s}{\gamma_\pi^s}x_t=-\left(i^*_t-\mathbb{E}_t\Delta p^*_{t+1}\right)+\frac{\gamma_x}{\gamma_\pi}x_t$,

$$z_{2,t}=\frac{\alpha\gamma_\pi^s}{\alpha\gamma_\pi^s-(\alpha-\rho)}\Delta p^*_t-\frac{\gamma_\pi^s}{\alpha\gamma_\pi^s-(\alpha-\rho)}\eta_t-\frac{\iota}{\gamma_\pi^s-(1-\rho)}-\frac{\gamma_\pi^s}{\rho}\sum_{j=0}^{\infty}\left(\frac{1-\gamma_\pi^s}{\rho}\right)^j E_t f_{t+j} \tag{a10}$$

Finally, plugging (a8) into (a3),

$$z_{3,t}=-i^*_{t-1}+\upsilon_t \tag{a11}$$

Now, plugging (a10) and (a11) into (a5),

$$\begin{aligned}
\Delta e_t &= \frac{\alpha\gamma_\pi^s}{\alpha-\rho}\Delta p_t-\frac{\alpha\gamma_\pi^s}{\alpha-\rho}\Delta p^*_t+\frac{\gamma_\pi^s}{\alpha-\rho}\eta_t+\frac{\alpha\gamma_\pi^s-(\alpha-\rho)}{(\alpha-\rho)(\gamma_\pi^s-(1-\rho))}\iota\\
&\quad+\frac{\gamma_\pi^s(\alpha\gamma_\pi^s-(\alpha-\rho))}{(\alpha-\rho)\rho}\sum_{j=0}^{\infty}\left(\frac{1-\gamma_\pi^s}{\rho}\right)^j\mathbb{E}_t f_{t+j}+\frac{\alpha\gamma_\pi^s-(\alpha-\rho)}{\alpha-\rho}i^*_{t-1}-\frac{\alpha\gamma_\pi^s-(\alpha-\rho)}{\alpha-\rho}\upsilon_t
\end{aligned} \tag{a12}$$

[31] We use the fact $\mathbb{E}_t\eta_{t+j}=0,\ j=1,2,\cdots$.

Updating (a12) once and applying law of iterated expectations,

$$\Delta e_{t+1} = \hat{\imath} + \frac{\alpha\gamma_\pi^s}{\alpha-\rho}\Delta p_{t+1} - \frac{\alpha\gamma_\pi^s}{\alpha-\rho}\Delta p_{t+1}^* + \frac{\alpha\gamma_\pi^s - (\alpha-\rho)}{\alpha-\rho} i_t^* \\ + \frac{\gamma_\pi^s(\alpha\gamma_\pi^s - (\alpha-\rho))}{(\alpha-\rho)\rho}\sum_{j=0}^{\infty}\left(\frac{1-\gamma_\pi^s}{\rho}\right)^j \mathbb{E}_t f_{t+j+1} + \omega_{t+1}, \tag{a13}$$

where

$$\hat{\imath} = \frac{\alpha\gamma_\pi^s - (\alpha-\rho)}{(\alpha-\rho)(\gamma_\pi^s - (1-\rho))}\iota,$$

$$\omega_{t+1} = \frac{\gamma_\pi^s(\alpha\gamma_\pi^s - (\alpha-\rho))}{(\alpha-\rho)\rho}\sum_{j=0}^{\infty}\left(\frac{1-\gamma_\pi^s}{\rho}\right)^j (\mathbb{E}_{t+1} f_{t+j+1} - \mathbb{E}_t f_{t+j+1}) \\ + \frac{\gamma_\pi^s}{\alpha-\rho}\eta_{t+1} - \frac{\alpha\gamma_\pi^s - (\alpha-\rho)}{\alpha-\rho}\upsilon_{t+1},$$

and,

$$\mathbb{E}_t \omega_{t+1} = 0$$

B The Solution When $\alpha = \rho$

When α equals ρ, we have the following system of difference equations.

$$\begin{bmatrix} 1 & -1 & 0 \\ -\gamma_\pi^s & 1 & 0 \\ -\gamma_\pi^s & 0 & 1 \end{bmatrix} \begin{bmatrix} \mathbb{E}_t \Delta p_{t+1} \\ \mathbb{E}_t \Delta e_{t+1} \\ i_t \end{bmatrix} = \begin{bmatrix} \rho & -\rho & 0 \\ 0 & 0 & \rho \\ 0 & 0 & \rho \end{bmatrix} \begin{bmatrix} \Delta p_t \\ \Delta e_t \\ i_{t-1} \end{bmatrix} + \begin{bmatrix} \mathbb{E}_t \Delta p_{t+1}^* - \rho \Delta p_t^* + \eta_t \\ \iota + \gamma_x^s x_t - i_t^* \\ \iota + \gamma_x^s x_t \end{bmatrix}, \tag{b1}$$

which can be represented by the following.

$$\mathbb{E}_t \mathbf{y}_{t+1} = \mathbf{V} \mathbf{\Lambda} \mathbf{V}^{-1} \mathbf{y}_t + \mathbf{c}_t, \tag{b2}$$

where

$$\mathbf{V} = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}, \ \mathbf{\Lambda} = \begin{bmatrix} \rho & 0 & 0 \\ 0 & \frac{\rho}{1-\gamma_\pi^s} & 0 \\ 0 & 0 & 0 \end{bmatrix}, \ \mathbf{V}^{-1} = \begin{bmatrix} -1 & 1 & 0 \\ 1 & -1 & 1 \\ 0 & 1 & -1 \end{bmatrix}$$

The system yields the same eigenvalues, $\alpha = \rho$ and $\frac{\rho}{1-(1-\rho)\gamma_\pi}$. Therefore, when γ_π is greater than one, we have the saddle-path equilibrium as before. By pre-multiplying both sides of (b2) by $\mathbf{V}^{-1}$, we get,

$$\mathbb{E}_t \mathbf{z}_{t+1} = \mathbf{\Lambda} \mathbf{z}_t + \mathbf{h}_t, \tag{b3}$$

where $\mathbf{V}^{-1}\mathbf{y}_t = \mathbf{z}_t$ and $\mathbf{V}^{-1}\mathbf{c}_t = \mathbf{h}_t$.

We solve the system as follows.

$$z_{1,t} = \sum_{j=0}^{\infty} \rho^j h_{1,t-j-1} + \sum_{j=0}^{\infty} \rho^j u_{t-j} \tag{b4}$$

$$z_{2,t} = -\sum_{j=0}^{\infty} \left(\frac{1-\gamma_\pi^s}{\rho} \right)^{j+1} \mathbb{E}_t h_{2,t+j} \tag{b5}$$

$$z_{3,t} = h_{3,t-1} + \upsilon_t, \tag{b6}$$

where u_t and υ_t are *any* martingale difference sequences.

Since $\mathbf{y}_t = \mathbf{V}\mathbf{z}_t$,

$$\begin{bmatrix} \Delta p_t \\ \Delta e_t \\ i_{t-1} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} z_{1,t} \\ z_{2,t} \\ z_{3,t} \end{bmatrix} \tag{b7}$$

Now, we find the analytical solutions for $\mathbf{z}_t$. Since $\mathbf{h}_t = \mathbf{V}^{-1}\mathbf{c}_t$,

$$\mathbf{h}_t = \begin{bmatrix} -1 & 1 & 0 \\ 1 & -1 & 1 \\ 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} (\mathbb{E}_t \Delta p^*_{t+1} - \rho \Delta p^*_t + \eta_t) + \iota + \gamma^s_x x_t - i^*_t \\ \gamma^s_\pi (\mathbb{E}_t \Delta p^*_{t+1} - \rho \Delta p^*_t + \eta_t) + \iota + \gamma^s_x x_t - i^*_t \\ \gamma^s_\pi (\mathbb{E}_t \Delta p^*_{t+1} - \rho \Delta p^*_t + \eta_t) + \iota + \gamma^s_x x_t - \gamma^s_\pi i^*_t \end{bmatrix},$$

thus,

$$h_{1,t} = -(1-\gamma^s_\pi)\left(\mathbb{E}_t \Delta p^*_{t+1} - \rho \Delta p^*_t + \eta_t\right) \tag{b8}$$

$$h_{2,t} = \mathbb{E}_t \Delta p^*_{t+1} - \rho \Delta p^*_t + \eta_t + \iota + \gamma^s_x x_t - \gamma^s_\pi i^*_t \tag{b9}$$

$$h_{3,t} = -(1-\gamma^s_\pi) i^*_t \tag{b10}$$

From (b4) and (b8),

$$\begin{aligned} z_{1,t} &= -(1-\gamma^s_\pi)\sum_{j=0}^{\infty} \rho^j \left(\Delta p^*_{t-j} - \rho \Delta p^*_{t-j-1} + \eta_{t-j-1}\right) + \sum_{j=0}^{\infty} \rho^j u_{t-j} \qquad \text{(b11)} \\ &= -(1-\gamma^s_\pi)\Delta p^*_t + \sum_{j=0}^{\infty} \rho^j u_{t-j} - (1-\gamma^s_\pi)\sum_{j=0}^{\infty} \rho^j \eta_{t-j-1} \end{aligned}$$

From (b5) and (b9),

$$\begin{aligned} z_{2,t} = &- \sum_{j=0}^{\infty} \left(\frac{1-\gamma^s_\pi}{\rho}\right)^{j+1} \left(\mathbb{E}_t \Delta p^*_{t+j+1} - \rho \mathbb{E}_t \Delta p^*_{t+j} + \mathbb{E}_t \eta_{t+j} + \iota + \gamma^s_x \mathbb{E}_t x_{t+j} - \gamma^s_\pi \mathbb{E}_t i^*_{t+j}\right) \quad \text{(b12)} \\ &= (1-\gamma^s_\pi)\Delta p^*_t - \left(\frac{1-\gamma^s_\pi}{\rho}\right)\eta_t - \frac{(1-\gamma^s_\pi)\iota}{\rho - (1-\gamma^s_\pi)} \\ &- \gamma^s_\pi \sum_{j=0}^{\infty} \left(\frac{1-\gamma^s_\pi}{\rho}\right)^{j+1} \left(\mathbb{E}_t \Delta p^*_{t+j+1} + \frac{\gamma^s_x}{\gamma^s_\pi}\mathbb{E}_t x_{t+j} - \mathbb{E}_t i^*_{t+j}\right) \end{aligned}$$

Denoting f_t as $-\left(i^*_t - \mathbb{E}_t \Delta p^*_{t+1}\right) + \frac{\gamma^s_x}{\gamma^s_\pi} x_t = -\left(i^*_t - \mathbb{E}_t \Delta p^*_{t+1}\right) + \frac{\gamma_x}{\gamma_\pi} x_t$,

$$z_{2,t} = (1-\gamma^s_\pi)\Delta p^*_t - \gamma^s_\pi \sum_{j=0}^{\infty} \left(\frac{1-\gamma^s_\pi}{\rho}\right)^{j+1} \mathbb{E}_t f_{t+j} - \left(\frac{1-\gamma^s_\pi}{\rho}\right)\eta_t - \frac{(1-\gamma^s_\pi)}{\rho-(1-\gamma^s_\pi)}\iota \tag{b13}$$

From (b6) and (b10),

$$z_{3,t} = -(1-\gamma^s_\pi) i^*_{t-1} + \upsilon_t \tag{b14}$$

From (b7), (b13), and (b14),

$$\Delta p_t = (1-\gamma_\pi^s)\Delta p_t^* - \gamma_\pi^s \sum_{j=0}^{\infty} \left(\frac{1-\gamma_\pi^s}{\rho}\right)^{j+1} \mathbb{E}_t f_{t+j} \\ - \left(\frac{1-\gamma_\pi^s}{\rho}\right)\eta_t + \frac{(1-\gamma_\pi^s)}{(1-\gamma_\pi^s)-\rho}\iota - (1-\gamma_\pi^s) i_{t-1}^* + \upsilon_t \tag{b15}$$

Updating (b15) once and applying the law of iterated expectations,

$$\Delta p_{t+1} = \hat{\iota} + (1-\gamma_\pi^s)\Delta p_{t+1}^* - (1-\gamma_\pi^s) i_t^* - \gamma_\pi^s \sum_{j=0}^{\infty} \left(\frac{1-\gamma_\pi^s}{\rho}\right)^{j+1} \mathbb{E}_t f_{t+j} + \omega_{t+1}, \tag{b16}$$

where

$$\hat{\iota} = \frac{(1-\gamma_\pi^s)}{(1-\gamma_\pi^s)-\rho}\iota,$$

$$\omega_{t+1} = -\gamma_\pi^s \sum_{j=0}^{\infty} \left(\frac{1-\gamma_\pi^s}{\rho}\right)^{j} (\mathbb{E}_{t+1} f_{t+j+1} - \mathbb{E}_t f_{t+j+1}) - \left(\frac{1-\gamma_\pi^s}{\rho}\right)\eta_{t+1} + \upsilon_{t+1},$$

and

$$\mathbb{E}_t \omega_{t+1} = 0$$

Note that there is no inertia for the domestic inflation in this solution, since there is no backward looking component. Put it differently, when there is a shock, Δp_{t+1} instantly *jumps* to its long-run equilibrium.

On the contrary, Δe_{t+1} does have inertia. From (b7),

$$\Delta e_t = z_{1,t} + \Delta p_t \tag{b17}$$

Plug (b11) into (b17) and update it once to get,

$$\Delta e_{t+1} = \Delta p_{t+1} - (1-\gamma_\pi^s)\Delta p_{t+1}^* + \sum_{j=0}^{\infty} \rho^j u_{t-j+1} - (1-\gamma_\pi^s) \sum_{j=0}^{\infty} \rho^j \eta_{t-j}, \tag{b18}$$

where Δp_{t+1} contains rational expectation of future fundamentals as defined in (b16). Note that Δe_{t+1} exhibits inertia due to the presence of the martingale difference sequences.

In a nutshell, in the special case of $\rho = \alpha$, domestic inflation instantly jumps to its long-run equilibrium and all the convergence will be carried over by the exchange rate adjustments

Acknowledgement

Special thanks go to Lutz Kilian, Christian Murray, David Papell, Paul Evans, Eric Fisher, and Henry Thompson for helpful suggestions. We also thank seminar participants at Bank of Japan, Keio University, University of Houston, University of Michigan, Texas Tech University, Auburn University, University of Southern Mississippi, the 2009 ASSA Meeting, the 72nd Midwest Economics Association Annual Meeting, the Midwest Macro Meeting 2008, and the 2009 NBER Summer Institute (EFSF).

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Table 1. GMM Estimation of the US Taylor Rule Estimation

Deviation	Sample Period	γ_π $(s.e.)$	γ_x $(s.e.)$	ρ $(s.e.)$
Real GDP	1959:Q1-2003:Q4	1.466 (0.190)	0.161 (0.054)	0.820 (0.029)
	1959:Q1-1979:Q2	0.605 (0.099)	0.577 (0.183)	0.708 (0.056)
	1979:Q3-2003:Q4	2.517 (0.306)	0.089 (0.218)	0.806 (0.034)
Unemployment	1959:Q1-2003:Q4	1.507 (0.217)	0.330 (0.079)	0.847 (0.028)
	1959:Q1-1979:Q2	0.880 (0.096)	0.217 (0.072)	0.710 (0.057)
	1979:Q3-2003:Q4	2.435 (0.250)	0.162 (0.078)	0.796 (0.034)

Notes: i) Inflations are quarterly changes in log CPI level $(\ln p_t - \ln p_{t-1})$. ii) Quadratically detrended gaps are used for real GDP output deviations. iii) Unemployment gaps are 5 year backward moving average unemployment rates minus current unemployment rates. iv) The set of instruments includes four lags of federal funds rate, inflation, output deviation, long-short interest rate spread, commodity price inflation, and M2 growth rate.

Table 2. GMM Median Unbiased Estimates and 95% Grid-t Confidence Intervals

Country	$\hat{\alpha}_{\text{GMM}}$	$\text{CI}_{\text{grid-}t}$	HL	HL $\text{CI}_{\text{grid-}t}$	J (pv)
Australia	0.884	[0.837,0.943]	1.404	[0.977,2.953]	5.532 (0.700)
Austria	0.804	[0.786,0.826]	0.793	[0.721,0.904]	8.173 (0.417)
Belgium	0.816	[0.794,0.844]	0.852	[0.751,1.019]	7.942 (0.439)
Canada	1.000	[0.967,1.000]	∞	[5.109, ∞)	4.230 (0.836)
Denmark	0.937	[0.874,1.000]	2.675	[1.290, ∞)	6.272 (0.617)
Finland	0.948	[0.897,1.000]	3.235	[1.587, ∞)	7.460 (0.488)
France	0.799	[0.777,0.822]	0.772	[0.688,0.885]	8.517 (0.385)
Germany	0.786	[0.767,0.809]	0.721	[0.652,0.819]	9.582 (0.296)
Italy	0.832	[0.806,0.864]	0.945	[0.805,1.181]	4.228 (0.836)
Japan	0.754	[0.729,0.782]	0.613	[0.549,0.706]	9.800 (0.279)
Netherlands	0.838	[0.798,0.883]	0.984	[0.766,1.388]	6.638 (0.576)
New Zealand	0.805	[0.786,0.828]	0.799	[0.718,0.918]	6.874 (0.550)
Norway	0.873	[0.785,0.971]	1.271	[0.716,5.983]	8.225 (0.412)
Portugal	0.792	[0.779,0.806]	0.741	[0.694,0.803]	6.132 (0.633)
Spain	0.896	[0.856,0.943]	1.581	[1.114,2.954]	6.738 (0.565)
Sweden	1.000	[0.945,1.000]	∞	[3.088, ∞)	7.107 (0.525)
Switzerland	0.831	[0.795,0.870]	0.937	[0.755,1.240]	9.136 (0.331)
UK	0.778	[0.756,0.806]	0.690	[0.620,0.801]	17.49 (0.025)
Median	0.832	[0.795,0.867]	0.941	[0.753,1.211]	-

Notes: i) The US$ is the base currency. ii) Unemployment gaps are used for output deviations. iii) Sample periods are 1979.II-1998.IV (78 observations) for Eurozone countries and are 1979.II-2003.IV (98 observations) for non-Eurozone countries. iv) $\text{CI}_{\text{grid-}t}$ denotes the 95% confidence intervals that were obtained by 500 residual-based bootstrap replications on 30 grid points (Hansen, 1999). v) J denotes the J-statistic and pv is its associated p-values.

Table 3. Univariate Median Unbiased Estimates and Grid-t Confidence Intervals

Country	$\hat{\alpha}_{LS}$	$CI_{grid\text{-}t}$	HL	HL $CI_{grid\text{-}t}$
Australia	0.972	[0.891,1.000]	6.173	[1.494, ∞)
Austria	0.945	[0.866,1.000]	3.087	[1.205, ∞)
Belgium	0.924	[0.847,1.000]	2.203	[1.045, ∞)
Canada	1.000	[0.946,1.000]	∞	[3.122, ∞)
Denmark	0.942	[0.866,1.000]	2.886	[1.200, ∞)
Finland	0.959	[0.883,1.000]	4.107	[1.390, ∞)
France	0.931	[0.847,1.000]	2.432	[1.044, ∞)
Germany	0.950	[0.852,1.000]	3.349	[1.078, ∞)
Italy	0.943	[0.859,1.000]	2.932	[1.138, ∞)
Japan	0.952	[0.886,1.000]	3.511	[1.428, ∞)
Netherlands	0.936	[0.839,1.000]	2.619	[0.990, ∞)
New Zealand	0.959	[0.923,0.997]	4.089	[2.174,61.29]
Norway	0.934	[0.851,1.000]	2.529	[1.073, ∞)
Portugal	0.975	[0.913,1.000]	6.765	[1.904, ∞)
Spain	0.959	[0.898,1.000]	4.129	[1.604, ∞)
Sweden	0.959	[0.891,1.000]	4.089	[1.497, ∞)
Switzerland	0.951	[0.862,1.000]	3.481	[1.168, ∞)
UK	0.932	[0.845,1.000]	2.442	[1.028, ∞)
Median	0.951	[0.866,1.000]	3.415	[1.203, ∞)

Notes: i) The US$ is the base currency. ii) Sample periods are 1979.II-1998.IV (78 observations) for Eurozone countries and are 1979.II-2003.IV (98 observations) for non-Eurozone countries. iii) $CI_{grid\text{-}t}$ denotes the 95% confidence intervals that were obtained by 500 residual-based bootstrap replications on 30 grid points (Hansen, 1999).

Table 4. Univariate Median Unbiased Half-Life Estimates: AR(1) vs. AR(p)

Country	p_{MAIC}	p_{MBIC}	$\text{HL}_{\text{AR(1)}}$	HL_{IRF}
Australia	1	1	6.173	6.173
Austria	1	1	3.087	3.087
Belgium	4	1	2.203	2.884
Canada	6	1	∞	∞
Denmark	4	1	2.886	3.883
Finland	6	2	4.107	3.631
France	1	1	2.432	2.432
Germany	6	1	3.349	3.386
Italy	3	1	2.932	∞
Japan	1	1	3.511	3.511
Netherlands	6	1	2.619	2.882
New Zealand	9	1	4.089	3.895
Norway	1	1	2.529	2.529
Portugal	6	1	6.765	∞
Spain	2	1	4.129	12.13
Sweden	4	4	4.089	3.387
Switzerland	1	1	3.481	3.481
UK	3	1	2.442	3.129
Median	3.5	1	3.415	3.496

Notes: i) p_{MAIC} and p_{MBIC} denote the lag length chosen by the modified Akaike Information criteria and the modified Bayesian Information criteria (Ng and Perron, 2001) with maximum 12 lags, respectively. ii) $\text{HL}_{\text{AR(1)}}$ refers the half-life point estimates with an AR(1) specification and was replicated from Table 3 for a comparison purpose. iii) HL_{IRF} deontes the half-life point estimates obtained from the impulse-response function with the lag length chosen by p_{MAIC}. HL_{IRF} with p_{MBIC} is not reported because the estimates are virtually the same as $\text{HL}_{\text{AR(1)}}$. iv) We correct the median bias of each autoregressive coefficient for higher order AR(p) conditioning on all other coefficients.