Auburn University
Department of Economics
Working Paper Series

Energy Substitution, Production, and Trade in the US

Henry Thompson*

Auburn University*

AUWP 2010-06

This paper can be downloaded without charge from:

http://media.cla.auburn.edu/economics/workingpapers/

http://econpapers.repec.org/paper/abnwpaper/

Energy Substitution, Production, and Trade in the US

Henry Thompson

Auburn University

October 2010

Energy proves an essential input with robust comparative static effects in a factor proportions model of production for the US. Energy has a robust marginal product and significant substitution in a novel production function motivated by the definition of physical work. In this physical production function, energy and labor inputs interact separately with capital. The present data cover the years 1951 to 2008. One version of the model assumes an endogenous price of energy, and another endogenous energy imports at the world price. These comparative static models of production and trade have an array of policy implications.

Keywords: energy, substitution, production, trade

Thanks go to Henry Kinnucan for suggestions.

Contact: Economics, Comer Hall, Auburn University, AL 36849, 334-844-2910, thomph1@auburn.edu

Energy Substitution, Production, and Trade in the US

Energy as a primary factor of production may require little motivation. The present paper includes energy with capital and labor in a general equilibrium model with outputs of manufactures and services. The paper estimates a novel production function based on the definition of physical work separating interactions of energy and labor with capital. An error correction regression provides a reliable estimate for annual US data from 1951 to 2008.

Estimates of substitution elasticities with the physical production function differ from Cobb-Douglas. Energy has a weaker own price effect and labor a much stronger one. Capital proves a weak substitute for labor, and energy a strong substitute for capital. Excluding energy input understates the degree of own labor substitution.

The paper examines comparative static properties of factor proportions models with estimated substitution elasticities. One model has an endogenous price of energy and another energy imports at the exogenous world price.

The following section introduces the physical production function followed by sections on the data and estimates of substitution elasticities. Sections then develop the general equilibrium factor proportions model and present the two sets of simulations. The conclusion discusses implications for policies that include tariffs, subsidies, immigration, capital taxes, and energy subsidies.

1. A physical production function

Physical work equals force times distance suggesting the physical production function

$$Y = A(LK)^{\alpha_1}(EK)^{\alpha_2}. \tag{1}$$

Energy E and labor L interact separately as force with capital K to produce output Y as work. This functional form constrains exponents of the log linear production function to $Y = AL^{\alpha_1}E^{\alpha_2}K^{\alpha_1+\alpha_2}$. Euler's theorem with constant returns holds only if $\alpha_1 + \alpha_2 = ½$. Marginal products are

$$Y_K = (\alpha_1 + \alpha_2)Y/K \qquad Y_L = \alpha_1 Y/L \qquad Y_E = \alpha_2 Y/E. \tag{2}$$

First order conditions of cost minimization lead to the symmetric Hessian matrix,

$$\begin{pmatrix} 0 & Y_K & Y_L & Y_E \\ . & Y_{KK} & Y_{KL} & Y_{KE} \\ . & . & Y_{LL} & Y_{LE} \\ . & . & . & Y_{EE} \end{pmatrix} \begin{bmatrix} \partial\lambda \\ \partial K \\ \partial L \\ \partial E \end{bmatrix} = \begin{bmatrix} 0 \\ \partial r \\ \partial w \\ \partial e \end{bmatrix}. \tag{3}$$

as in Allen (1938) and Takayama (1993). Inverting (3) leads to cross price elasticities such as $\varepsilon_{Ke} = (\partial K/\partial e)(e/K)$ evaluated at estimated marginal products in the present application.

The cost minimization in (3) assumes exogenous input prices while input prices are endogenous in the general equilibrium. Similarly, factor prices exogenous to the firm are endogenous in the economy. The cost minimization in (3) is a given for the implicit national income maximization in the general equilibrium system (7) below.

The estimated physical production function (1) in natural log form is

$$\ln Y = \alpha_0 + \alpha_1(\ln L + \ln K) + \alpha_2(\ln E + \ln K) + \varepsilon \tag{4}$$

where ε is a white noise residual. Constant returns imply output elasticities equal factor shares $\partial \ln Y/\partial \ln L \equiv \varepsilon_L = \alpha_1$ for labor, $\varepsilon_E = \alpha_2$ for energy, and $\varepsilon_K = \alpha_1 + \alpha_2$ for capital.

2. Data and stationarity pretests

Annual data for 1951 through 2008 include real gross domestic product Y, real fixed capital assets K, and the labor force L all from the Department of Commerce (2010). Total Btu energy input E is from the Department of Energy (2010).

Figure 1 shows the mean weighted series. Output Y grows steadily with some irregularity and at a faster pace following the early 1980s. Capital K grows more regularly at a slightly increasing rate. The labor force L generally grows but occasionally declines. Energy input E grows at a relatively fast pace up to the energy crises in the middle of the 1970s when it declines before growing at a slower pace.

* Figure 1 *

The series are not stationary but are difference stationary as suggested by Figure 2 and shown in Table 1. Output Y is difference stationary by the Dickey-Fuller (1979) DF test with a constant and time trend. Capital K and labor L both have residual correlation in Durbin-Watson DW (1951) statistics but are difference stationary by augmented Dickey-Fuller ADF tests. Energy E is more volatile with ARCH(1) residual heteroskedasticity but is ADF difference stationary with six lags. The difference stationary variables suggest estimation with difference regressions or perhaps error correction models.

* Table 1 * Figure 2 *

3. Substitution in the physical production function

The empirical substitution literature relies on the constant elasticity of substitution of Arrow, Chenery, Minhas, and Solow (1961) and the translog production function of Christensen, Jorgensen, and Lau (1973). Thompson (2006) summarizes the literature on applied energy substitution.

Table 2 reports estimates of the physical production function. The regression on levels has residual correlation by the Durbin Watson DW test and heteroskedasticity by the ARCH(1) test. The series are not cointegrated according to the Engle-Granger (1987) EG test but the significant error correction coefficient suggests the variables are cointegrated.

* Table 2 *

Derived output elasticities including the error correction effect are 0.20 (0.12) for labor, 0.48 (0.12) for energy, and 0.68 (0.17) for capital with standard errors derived by error propagation. The sum of the coefficients 1.26 (0.24) marginally rejects constant returns. Relative to productivity, labor is overpaid and energy vastly underpaid. The error correction regression suggests no residual correlation or heteroskedasticity as verified by the lack of apparent structural breaks in the residual plot of Figure 3. This white noise residual suggests separate treatment of technology is unnecessary unlike estimates of capital-labor models.

* Figure 3 *

Marginal products and related second order terms evaluated at sample means lead to the Hessian matrix (3),

$$\begin{pmatrix} 0 & .5982 & .0448 & .4278 \\ . & -.0046 & .0003 & .0004 \\ . & . & -.0004 & .0002 \\ . & . & . & -.0030 \end{pmatrix}. \tag{5}$$

Invert (5) to derive symmetric partial derivatives of inputs with respect to input prices as elements of the inverse matrix

$$\begin{pmatrix} \partial K/\partial r & \partial L/\partial r & \partial E/\partial r \\ \partial K/\partial w & \partial L/\partial w & \partial E/\partial w \\ \partial K/\partial e & \partial L/\partial e & \partial E/\partial e \end{pmatrix} = \begin{pmatrix} -49.3 & 87.3 & 59.9 \\ . & -2381 & 127.5 \\ . & . & -97.0 \end{pmatrix}. \tag{6}$$

Cross price elasticities are evaluated at sample means and marginal products in (6).

Table 3 reports the derived substitution elasticities σ and compares them to Cobb-Douglas production σ_{CD}. As an example, the cross price elasticity of energy input with respect to the wage $\sigma_{Ew} = (\partial E/\partial w)w/E = (\partial E/\partial w)Y_L/\mu_E = (127.5 \text{ x } 0.0448)/72.7 = 0.08$ is twice the size of the Cobb-Douglas elasticity. Linear homogeneity is relaxed implying row sums of elasticities are not zero as with Cobb-Douglas.

* Table 3 *

Cross price elasticities are weak with respect to wages but own labor substitution is elastic. In contrast, moderate cross price substitution between capital and energy has about the same order of magnitude as those own elasticities.

The physical production function has stronger own labor substitution but much weaker own energy substitution than Cobb-Douglas. The physical production function also reveals stronger substitution of energy for capital. Rising capital costs apparently encourages more energy efficient capital than revealed by Cobb-Douglas.

4. Factor Shares and Intensities

Table 4 presents the factor payment matrix with the sum of a row equal to factor income and the sum of a column equal to sector income. Assuming an input has the same price across sectors leads to industry shares λ_{ij} of factor distribution across sectors in Table 5. As an example 16% = \$787/\$4894 of labor is employed in manufactures. The service sector employs the majority of capital and labor as it produces 85% of national income. Manufactures appear energy intensive.

* Table 4 * Table 5 *

Factor shares θ_{ij} of inputs are from sector incomes. As an example, the labor share of income in services is 45% = \$787/\$1759. Labor receives about the same share of revenue in services and manufactures. Energy receives a much larger share of revenue in manufactures, and capital a somewhat smaller share. Comparing industry shares, manufactures appears energy intensive, services capital intensive, and labor a middle input.

Table 6 shows factor intensities. For instance, the ratio of industry shares between capital and labor is $\lambda_{Kj}/\lambda_{Lj} = (a_{Kj}x_j/K)/(a_{Lj}x_j/L) = (a_{Kj}/a_{Lj})(K/L)$. Similarly, factor share ratios indicate factor intensity up to the ratio of factor prices. The theory of factor intensity assumes unit inputs a_{ij}. Services are capital intensive relative to both labor and especially energy. Manufactures, generally considered capital intensive, is energy intensive relative to both capital and labor. Theoretical misconceptions arise from excluding energy input. Labor is the middle factor, intensive in manufactures relative to capital but intensive in services relative to energy.

* Table 6 *

5. Factor Proportions Model Simulations

The factor proportions model of Heckscher (1919) and Ohlin (1924) formalized by Samuelson (1953), Jones (1965), and Chipman (1966) and reviewed by Jones and Neary (1984) provides the theoretical foundation for the present simulations. Jones and Scheinkman (1977) and Chang (1979) develop comparative static properties of the model. Jones and Easton (1983) and Thompson (1983, 1985) analyze the factor proportions model with three inputs.

The model assumes full employment, cost minimization, and competitive pricing. The full employment condition is $Ax = v$ where A is the matrix of cost minimizing unit inputs, x the output vector, and v the input vector. Competitive pricing in each industry is $A^T w = p$ where w is the vector of factor prices and p the vector of product prices. Full employment is the first equation in the system (7) and competitive pricing the second.

The comparative static model in elasticity form where ′ represents percentage change is

$$\begin{pmatrix} \sigma & \lambda \\ \theta^T & 0 \end{pmatrix} \begin{pmatrix} w' \\ x' \end{pmatrix} = \begin{pmatrix} v' \\ p' \end{pmatrix} \qquad (7)$$

The substitution terms in Table 3 are the σ matrix, industry shares in Table 6 the λ matrix, and factor shares in Table 7 the θ matrix. Factor prices w and outputs x adjust endogenously to exogenous changes in factor endowments v and prices p.

Table 7 presents the inverse of the system matrix in (7) in an array of general equilibrium comparative static elasticities with the Cobb-Douglas model included for comparison. The upper left quadrant presents elasticities of factor prices with respect to endowment changes. Input prices are insensitive to changes in energy input E. Capital K and energy E are enemies, an increase in one lowering the return to the other. The strong negative effect of capital K on the energy price e may be evidence that newly installed capital tends to be more energy efficient. Capital and energy are moderate technical substitutes in production but general equilibrium complements, illustrating the importance of factor intensity relative to substitution as discussed in Thompson (1995). Elasticities in this quadrant are similar with the physical production function and Cobb-Douglas.

* Table 7 *

The lower left Rybczynski (1955) quadrant in Table 7 shows effects of changing factor endowments on outputs. Increases in energy E or capital K have output effects favoring their intensive sector. Energy has an elastic output effect on manufactures, and capital a similar elastic effect on services. Outputs fall for the other sector, especially for manufactures with increased capital. Labor effects on outputs are inelastic favoring manufactures. These endowment/output effects are similar for the physical production function and Cobb-Douglas.

Production frontier effects in the lower right quadrant of Table 7 are elastic, especially for manufactures. Thompson and Toledo (2007) show that production frontier elasticities diminish with disaggregation. The falling price of manufactures due to import competition is consistent with the decline of manufactures output. Production frontier effects for the price of services are strong for the physical production function relative to Cobb-Douglas.

Factor intensities account for the elastic Stolper-Samuelson (1941) effects in the upper right quadrant of Table 7 that differ considerably from Cobb-Douglas. The falling price of manufactures strongly lowers the price of energy and the capital return, and raises wages. The rising price of services strongly lowers the price of energy but raises wages and the capital return. Inelastic effects on the capital return make the effects on the real return to capital ambiguous. The strong effects on the price of energy are consistent with energy intensive manufactures. Cobb-Douglas production is misleading in that a falling price of manufactures lowers the capital return and raises wages.

6. The model with energy imports

With an exogenous price of energy in the global energy market, supply becomes endogenous. All energy input could be imported or there could be domestic energy input E_{dom} with energy imports E_{imp} supplying the difference with total input, $E_{imp} = E – E_{dom}$. At the perfectly inelastic E_{dom} the domestic price would be e_{dom} but the lower international energy price e implies imports. Domestic supply could be price sensitive as well but for present purposes focus on energy imports.

With the exogenous energy price, the comparative static system (7) is

$$\begin{pmatrix} \sigma_{KK} & \sigma_{KL} & 0 & \lambda_{KM} & \lambda_{KS} \\ \sigma_{LK} & \sigma_{LL} & 0 & \lambda_{LM} & \lambda_{LS} \\ \sigma_{EK} & \sigma_{EL} & -1 & \lambda_{EM} & \lambda_{ES} \\ \theta_{KM} & \theta_{LM} & 0 & 0 & 0 \\ \theta_{KS} & \theta_{LS} & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} r' \\ w' \\ E' \\ x_M' \\ x_S' \end{pmatrix} = \begin{pmatrix} K' - \sigma_{KE}e' \\ L' - \sigma_{LE}e' \\ -\sigma_{EE}e' \\ p_M' - \theta_{EM}e' \\ p_S' - \theta_{ES}e' \end{pmatrix}. \tag{8}$$

as in Thompson (1983). Partial derivative comparative static effects are found multiplying the inverse A^{-1} of this system matrix by the vector for each exogenous change. For K′, L′, p_M′, and p_S′ the inverse isolates these effects directly. For e′ the inverse is multiplied by the ceteris paribus exogenous vector $(-\sigma_{KE}, -\sigma_{LE}, -\sigma_{EE}, -\theta_{EM}, -\theta_{ES})^T$.

These comparative static results are in Table 8. Factor price equalization FPE holds in the upper left corner relating endowments and returns of the two domestic inputs. The same number of domestic inputs and exogenous traded product prices implies FPE. Similar freely trading economies will have the same wages and rent given endowments within the production cone.

* Table 8 *

The Stolper-Samuelson price effects in the upper right corner and Rybczynski endowment effects in the lower left corner are consistent with services that are labor intensive relative to domestic capital.

Energy input E in the middle row of Table 8 responds strongly to changes in exogenous variables with positive links to labor and the price of manufactures. Increased labor force L raises energy demand and manufactures output, increasing energy imports. The pattern of falling manufactures prices and rising services prices implies declining energy imports, or less energy imports than would have occurred.

An increase in the international price of energy lowers energy imports substantially, lowering manufactures but raising services outputs. The return to capital rises moderately while wages fall with a nearly elastic effect.

7. Conclusion

The present estimates show that energy input and substitution have played critical roles in the US economy since the middle of the 20th century. There is promise of a more prominent role over the coming decades. Models and estimates without energy input introduce misconception and misspecification. The present physical production function offers an alternative to log linear and translog production functions.

The present models with three factors of production address a wide range of policies issues including import import protection, free trade agreements, capital taxes, immigration, and energy taxes or subsidies. Protection of manufactures raises the demand for energy, increasing either the price of energy or energy imports. Wages fall in the model with domestic energy but rise with international energy. For the capital return, results are opposite but with an ambiguous effect in the domestic energy model. Labor should favor protection of manufactures only with with international energy.

Free trade agreements generally lead to rising services prices for the US. The effect for labor is higher wages with domestic energy, but falling wages with international energy. The capital return rises with an ambiguous real effect in the model with domestic energy. Higher prices of services lower imports of international energy as the economy specializes away from energy intensive manufactures.

Capital taxes reduce the capital stock resulting in lower wages, reduced services output, and increased manufactures output. In the model with international energy, capital taxes have no net effects on wages as output adjustments relieve the labor market. The price of energy or energy imports rise with the reduced capital input that occurs with capital taxes.

Immigration lowers the wage and raises the capital return. In the model with domestic energy, the price of energy rises along with both outputs as immigrants disperse between sectors. In the model with international energy, services output falls while energy imports and manufactures output increase as immigrants go into manufacturing.

Tariffs on international energy lower energy imports with an elastic effect. Energy import spending inclusive of the tax falls. Production shifts strongly toward services. Wages fall but the capital return rises. Energy subsidies to raise domestic supply strongly favor manufactures, raise wages, and lower the return to capital. Effects on real incomes depend on the burden of the energy subsidies.

References

Allen, R.G.D., 1938, *Mathematical Analysis for Economists*, New York: St. Martin's Press.

Arrow Kenneth, H. B. Chenery, B. S. Minhas, and Robert Solow (1961) "Capital-Labor Substitution and Economic Efficiency," *Review of Economics and Statistics* 43, 225-50.

Chang, Winston (1979) "Some Theorems of Trade and General Equilibrium with Many Goods and Factors," *Econometrica* 47, 709-26.

Chipman, John (1966) "A Survey of the Theory of International Trade: Part 3, The Modern Theory," *Econometrica* 34, 18-76.

Christensen, Laurits, Dale Jorgensen, and Lawrence Lau (1973) "Transcendental Logarithmic Production Frontiers," *The Review of Economics and Statistics* 55, 28-45.

Department of Commerce (2010) *National Economic Accounts*, Bureau of Economic Analysis, www.bea.gov.

Department of Energy (2010) *Country Energy Profiles*, Energy Information Agency, www.eia.doe.gov.

Dickey, David and Wayne Fuller (1979) "Distribution of the Estimates for Autoregressive Time Series with a Unit Root," *Journal of the American Statistical Association* 74, 427-31.

Durbin, James, and Geoffrey Watson (1951) "Testing for Serial Correlation in Least Squares Regression, II" *Biometrika* 38, 159–179.

Engel, Robert and Clive Granger (1987) "Cointegration and Error-Correction: Representation, Estimation, and Testing," *Econometrica* 55, 251-76.

Heckscher, Eli (1919) "The Effect of Foreign Trade on the Distribution of Income," *Ekonomisk Tidskrift* 21, 497-512.

Jones, Ron (1965) "The Structure of Simple General Equilibrium Models," *Journal of Political Economy* 73, 57-72.

Jones, Ron and Stephen Easton (1983) "Factor Intensity and Factor Substitution in General Equilibrium," *Journal of International Economics* 15, 65-99.

Jones, Ron and Peter Neary (1984) "The Positive Theory of International Trade," in *Handbook of International Trade*, vol. 1, Ron Jones and Peter Kenen, eds, Amsterdam: North Holland.

Jones, Ron and José Scheinkman (1977) "The Relevance of the Two Sector Production Model in Trade Theory," *The Journal of Political Economy* 85, 909-35.

Ohlin, Bertil (1924) *The Theory of Trade*, translated in Harry Flam and June Flanders, *Heckscher-Ohlin Trade Theory*, MIT Press, 1991, 73-214.

Rybczynski, Tadeusz (1955) “Factor Endowments and Relative Commodity Prices,” *Economica* 22, 336-41.

Samuelson, Paul (1953) “Prices of Factors and Products in General Equilibrium,” *Review of Economic Studies* 21, 1-20.

Stolper, Wolfgang and Paul Samuelson (1941) “Protection and Real Wages,” *Review of Economic Studies* 9, 58-73.

Takayama, Akira (1993) *Analytical Methods in Economics*, The University of Michigan Press.

Thompson, Henry (1983) “Trade and International Factor Mobility,” *Atlantic Economic Journal* 11, 45-8.

Thompson, Henry (1985) “Complementarity in a Simple General Equilibrium Production Model,” *Canadian Journal of Economics* 18, 616-21.

Thompson, Henry (1993) “The Magnification Effect with Three Factors,” *Keio Economic Studies* 30 57-64.

Thompson, Henry (1995) “Factor Intensity versus Factor Substitution in a Specified General Equilibrium Model,” *Journal of Economic Integration* 10, 283-97

Thompson, Henry (2006) “The Applied Theory of Energy Substitution in Production,” *Energy Economics* 28, 410-425.

Thompson, Henry (2010) “Wages in a Factor Proportions Time series Model of the US,” *Journal of International Trade and Economic Development* 19, 241-56.

Thompson, Henry and Hugo Toledo (2007) “General Equilibrium Production with Constant Elasticity of Substitution,”*Keio Economic Studies* 44, 27-36

Table 1. Stationarity Analysis

τ_{DF} -3.80	DF	ADF	ADF(6)
lnY	-2.83		
ϕ 6.73	6.01		
DW 1.65	1.68		
ARCH F	0.18		
lnK	-1.38	-1.72	
ϕ	1.01	4.40	
DW	1.18*	1.96	
ARCH F	0.91	0.48	
lnL	-2.16	-3.10	
ϕ	3.09	5.03	
DW	1.38*	1.98	
ARCH F	-1.61	-0.57	
lnE	-1.32	-1.01	-1.13
ϕ	6.05	3.35	2.20
DW	1.68	2.00	1.80
ARCH F	4.06*	2.64*	-0.20

Table 2. Physical Production Function

	Physical	ECM
constant	-1.28***	0.001
	(0.05)	(0.006)
K+L	0.49***	0.14
	(0.06)	(0.12)
K+E	0.15**	0.46***
	(0.06)	(0.12)
EC res		-0.13***
		(0.07)
R^2	.996	.425
DW	0.22*	1.59*
ARCH F	7.78*	-0.03
AIC	-139	-236
EG	-1.74	

Table 3. Substitution Elasticities

σ σ_{CD}	$\partial \ln K$	$\partial \ln L$	$\partial \ln E$
$\partial \ln r$	-0.52 -0.49	0.60 0.45	0.49 0.04
$\partial \ln w$	0.07 0.51	-1.23 -0.55	0.08 0.04
$\partial \ln e$	0.44 0.51	0.63 0.45	-0.57 -0.96

Table 4. US Factor Payment Matrix, 2006, $bil

	M	S	Factor
K	$730	$5379	$6109
L	$787	$4107	$4894
E	$242	$178	$420
Sector	$1759	$9664	$11423

Table 5. Industry Shares and Factor Shares

λ_{ij} θ_{ij}	M	S
K	0.12 0.42	0.88 0.56
L	0.16 0.45	0.84 0.42
E	0.58 0.14	0.42 0.02

Table 6. Factor Intensity Ratios

θ	M	S
K/L	0.8	1.1
K/E	0.2	2.1
L/E	0.3	2.0

Table 7. Comparative Static Factor Proportions Model

Physical CD	∂K	∂L	∂E		∂p_M	∂p_S
∂r	-0.36 -0.38	0.39 0.41	-0.04 0.05		0.79 -0.47	0.86 1.47
∂w	0.49 0.52	-0.54 -0.57	0.05 0.05		-1.46 0.28	1.56 0.72
∂e	-0.52 -0.54	0.57 0.60	-0.05 -0.05		9.62 7.80	-7.67 -6.80
∂x_M	-1.58 -1.65	0.72 0.79	1.86 1.86		14.9 15.3	-14.1 -15.3
∂x_S	1.10 0.91	0.18 0.39	-0.28 -0.30		-5.95 -2.83	5.64 2.83

Table 8. Comparative Static Elasticities with an Exogenous Price of Energy

	∂K	∂L		∂e^*		∂p_M	∂p_S
∂r	0	0		0.69		-5.87	6.17
∂w	0	0		-0.95		7.68	-5.73
∂E	-10.1	11.1		-19.6		188	-150
∂x_M	-20.4	21.4		-29.6		365	-294
∂x_S	3.90	-2.90		4.52		-58.2	47.3

Figure 1. Trends in Output and Inputs

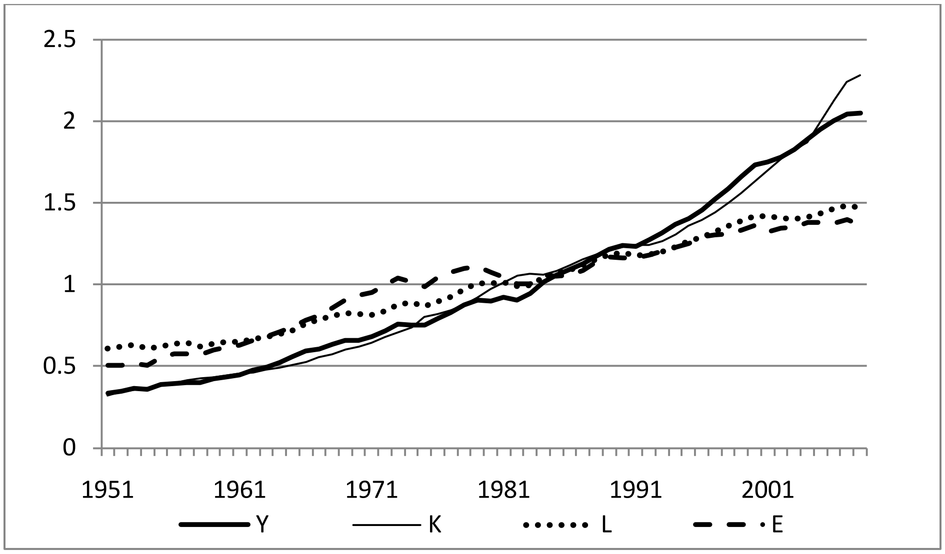

Figure 2. Differences in Inputs and Output

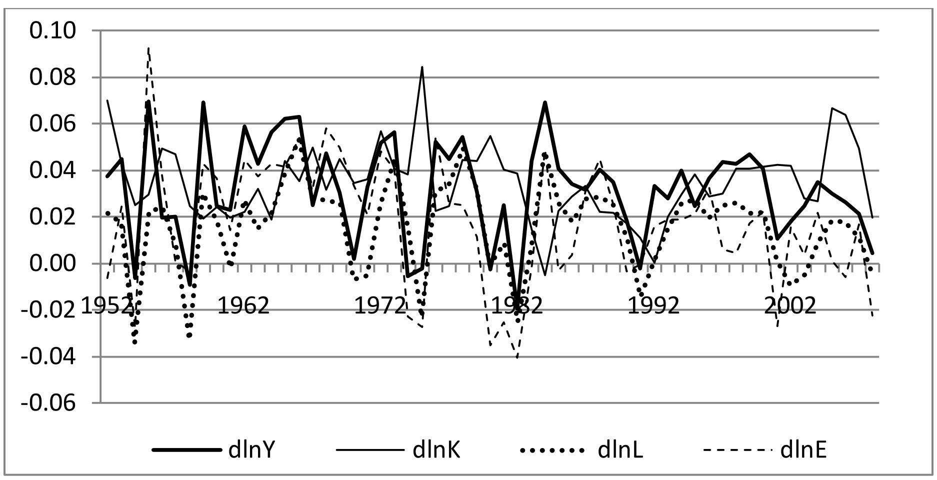

Figure 3. Residual of the ECM