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Hyeongwoo Kim

Auburn University

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VECM Estimations of the PPP Reversion Rate Revisited: The Conventional Role of Relative Price Adjustment Restored

Hyeongwoo Kim*†

Auburn University

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Abstract

Cheung et al. (2004) use a vector error correction model that allows different speeds of convergence for nominal exchange rates and relative prices toward PPP. With the current float monthly data for five countries, they argue that the sluggish PPP reversion is primarily driven by nominal exchange rate adjustment rather than price adjustment, which is at odds with the conventional sticky-price models. Major findings of this paper are twofold. First, we show that it may be inappropriate to use short-horizon high frequency data in vector error correction models, even when both the nominal exchange rate and the relative price are not weakly exogenous. Second, using a long-horizon annual data set for 11 countries vis-à-vis the US, we find a significantly important role of relative prices in real exchange rate dynamics.

Keywords: Purchasing Power Parity, Convergence Rate, Half-Life, Impulse-Response, Variance Decomposition

JEL Classification: C32, F31

*Department of Economics, Auburn University, 337 Haley Center., Auburn, AL 36849. Tel: (334) 844-2928. Fax: (334) 844-4615. Email: gmmkim@gmail.com.

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1 Introduction

In the tradition of Dornbusch (1976), conventional rational expectations sticky price models implicitly assume the same convergence rates for nominal exchange rates and relative prices. In such models, real exchange rate deviations can be persistent as nominal prices adjust at a slow rate. However, as pointed out by Rogoff (1996), observed persistence of real exchange rate deviations is too high to be explained by such nominal rigidities.

In their recent study, Engel and Morley (2001) propose a state-space model that allows nominal exchange rates and relative prices to adjust at different speeds. A similar attempt was made by Cheung et al. (2004) who use a vector error correction model (VECM) of the nominal exchange rate and the relative price. Using the current float monthly data for five developed countries, they find that the PPP reversion rate is primarily driven by nominal exchange rate adjustment rather than relative price adjustment, which is at odds with the conventional view that addresses a dominant role of nominal prices (see, for example, Stockman 1987, Rogoff 1996, Obstfeld and Rogoff 2000).

The major contributions of this paper are twofold. First, we demonstrate that short-horizon high frequency data may not be a proper choice for vector error correction models, even when all the variables in the system are not weakly exogenous. It is shown that there may be no gains of using VECMs over the conventional univariate equation of real exchange rates when short-horizon low frequency data is used.

Second, we use long-horizon low frequency data for 11 developed countries with the US dollar as a base currency, and find a significantly important role of relative price adjustment in real exchange rate dynamics. We also find that relative prices often converge at a much slower rate than nominal exchange rates when a relative price shock occurs[1]. As additional evidence, we report variance decomposition analysis. We find that nominal exchange rates hardly explain variations of relative prices, while relative prices explain a great deal of variations of nominal exchange rates in the intermediate- to long-term.

The paper is organized as follows. In section 2, we outline our baseline vector error correction model and demonstrate the cases when it is indistinguishable from the conventional univariate

[1] Cheung et al. (2004) report convergence rates only when there is a nominal exchange rate shock. They find much faster convergence rate for relative prices than nominal exchange rates in that case. Our results confirm their results for this case.

equation of real exchange rates. We also provide pretest results that clearly show that the use of short-horizon high frequency data may not yield the gains of using vector error correction models. Section 3 reports the estimates for relative importance of relative prices and nominal exchange rates in PPP reversion. In section 4, we report the estimates for convergence rates and the corresponding half-life along with the nonparametric bootstrap confidence intervals. The variance decomposition estimates are also reported. Section 5 concludes.

2 The Econometric Model and Pretest Results

2.1 The Model

Let e_t be the log nominal exchange rate as the unit price of the foreign currency in terms of the domestic currency, and denote $\tilde{p}_t$ as the log relative price, $p_t - p_t^*$, where p_t and p_t^* are the log domestic price and the log foreign price, respectively. The log real exchange rate (s_t) is, then, defined as $e_t - \tilde{p}_t$.

When e_t and $\tilde{p}_t$ are individually $I(1)$, but cointegrated with the cointegrating vector $[1 \quad -1]$, the Granger Representation Theorem (Engle and Granger 1987) implies the following VECM of e_t and $\tilde{p}_t$.

$$\begin{bmatrix} \Delta e_t \\ \Delta \tilde{p}_t \end{bmatrix} = \mathbf{a} + \begin{bmatrix} \rho_1 \\ \rho_2 \end{bmatrix} s_{t-1} + \sum_{j=1}^{k} \begin{bmatrix} \beta_{11,j} & \beta_{12,j} \\ \beta_{21,j} & \beta_{22,j} \end{bmatrix} \begin{bmatrix} \Delta e_{t-j} \\ \Delta \tilde{p}_{t-j} \end{bmatrix} + \mathbf{C} \begin{bmatrix} u_t^e \\ u_t^{\tilde{p}} \end{bmatrix}, \qquad (1)$$

where $\mathbf{a}$ is a 2×1 vector of constants, s_{t-1} denotes the error correction term, and ρs are the convergence rates of e_t and $\tilde{p}_t$. $\mathbf{C}$ is a 2×2 contemporaneous matrix. More compactly,

$$\Delta \mathbf{y}_t = \mathbf{a} + \rho \beta' \mathbf{y}_{t-1} + \sum_{j=1}^{k} \mathbf{B}_j \Delta \mathbf{y}_{t-j} + \mathbf{C} \mathbf{u}_t, \qquad (2)$$

where $\beta' = [1 \quad -1]$ is the the known cointegrating vector.

It should be noted that the system (1) allows different convergence rates for e_t and $\tilde{p}_t$ toward PPP, while the conventional univariate equation approach of real exchange rates implicitly assumes the same convergence rate for e_t and $\tilde{p}_t$ ($\rho_1 = \rho_2 = \rho$). That is, the conventional single equation approach typically employs the following regression equation.

$$\Delta s_t = a + \rho s_{t-1} + \sum_{j=1}^{k} \beta_j \Delta s_{t-j} + u_t, \tag{3}$$

where u_t denotes a real exchange rate shock that is a composite shock of u_t^e and $u_t^{\tilde{p}}$ in (1).

However, it is well-known that the benefit from such generalization is limited when either e_t or $\tilde{p}_t$ is weakly exogenous ($\rho_1 = 0$ or $\rho_2 = 0$). We show that even when $\rho_1 \neq 0$ and $\rho_2 \neq 0$, there is no gain of using the VECM (in measuring speeds of reversion separately) over the univariate equation for the cases described below.

Remark: When $k = 0$, nominal exchange rate shocks and relative price shocks produce identical real exchange rate dynamics.

Assuming $k = 0$, let's rewrite (1) as follows.

$$\Delta e_t = a_1 + \rho_1 s_{t-1} + c_{11} u_t^e + c_{12} u_t^{\tilde{p}}$$
$$\Delta \tilde{p}_t = a_2 + \rho_2 s_{t-1} + c_{21} u_t^e + c_{22} u_t^{\tilde{p}},$$

where a_i is the i^{th} element of $\mathbf{a}$ and c_{ij} denotes the $(i,j)^{\text{th}}$ element of $\mathbf{C}$. Subtracting the second equation from the first one, we get the following.

$$\Delta e_t - \Delta \tilde{p}_t = (a_1 - a_2) + (\rho_1 - \rho_2) s_{t-1} + (c_{11} - c_{21}) u_t^e + (c_{12} - c_{22}) u_t^{\tilde{p}} \tag{4}$$

Let $\Delta s_t = \Delta e_t - \Delta \tilde{p}_t$, $a = a_1 - a_2$, $\rho = \rho_1 - \rho_2$, and $u_t = (c_{11} - c_{21}) u_t^e + (c_{12} - c_{22}) u_t^{\tilde{p}}$, then (4) reduces to (3)[2].

While it is empirically less important, it can be shown that each shock delivers identical dynamics of real exchange rates even when $k \geq 1$, if $\beta_{11,j} + \beta_{22,j} = \beta_{12,j} + \beta_{21,j}$, $\forall j = 1, \cdots, k$.

[2] This doesn't mean that the half-life estimates from (1) and (3) would be quantitatively identical, because the ρ estimate can be different from the $\rho_1 - \rho_2$ estimate in finite sample.

2.2 Pretest Results

In this section, we empirically test the cointegrating relation between e_t and $\tilde{p}_t$. When the cointegrating vector is known, the most straightforward way to verify the PPP hypothesis is to perform unit root tests on the real exchange rate s_t (Froot and Rogoff 1995).

We first implement two popular unit root tests for the 17 current float monthly CPI-based real exchange rates (s_t) with the US dollar as the base currency. The data set is obtained from the IFS CD-ROM, and the observations span from March of 1973 to December of 1998. Cheung et al. (2004) use the current float monthly real exchange rates for France, Germany, Italy, Japan, and the UK. Their DF-GLS tests (Elliott et al., 1996) reject the null of unit root for France, Germany, Italy, and the UK. Unfortunately, they did not explain how they chose the number of lags (k). However, as pointed out by Lopez et al. (2005), lag selection procedures are potentially very important in the unit root test literature. As recommended by Ng and Perron (2001), therefore, we implement ADF tests with the general-to-specific (GTS, Hall 1994) rule and DF-GLS tests with the modified Akaike Information Criteria (MAIC, Ng and Perron 2001). The results are reported in Table 1.

Insert Table 1

Our ADF tests were not able to reject the unit root null for any of the 17 real exchange rates. However, more powerful DF-GLS tests indeed reject the unit root hypothesis for France, Germany, Italy, Netherlands, and Norway. One may conclude, therefore, that we can implement estimations for the vector error correction model (1) for these countries, but this is not the case. It should be noted that the chosen ks are all zeros for the countries, and there is no benefit to using the VECM specification over the univariate equation approach as explained in the previous section. This clearly shows that it may not be appropriate to use short-horizon data for the VECM, since it would be indistinguishable from the univariate equation specification.

Figure 1 provides additional evidence against the use of monthly data in the VECM framework. Under the eyeball metric, one can see noticeably greater variation of relative prices in the long-horizon data set compared with those in the current float regime[3].

[3] Formally, one can implement a weak-exogeneity test for relative prices in each data set.

Insert Figure 1

Next, we implement the same unit root tests for the long-horizon annual data set for nominal exchange rates and CPIs. We use Taylor's (2002) over 100-year long data for the 15 developed countries[4]. The sample period is from 1880 to 1998 for all countries. The results are reported in Table 2.

Insert Table 2

Unlike the results with the short-horizon data set, our ADF tests reject the unit root null for 10 among 15 countries. With the DF-GLS tests, we are able to reject the null for one more country, Australia. More importantly, for ADF tests with the GTS, all chosen ks were greater than zero, which enables one to perform estimations with the VECM specification.

For a comparison with the empirical results by the VECM specification, we report the estimates and their 95% nonparametric bootstrap confidence intervals from the scalar error correction model (3) in Table 3. We also present the impulse-response function estimates a real exchange rate shock in Figure 2. We obtain slightly longer half-life estimates than the three- to five-year consensus half-life (Rogoff 1996). This is not unusual, though, because longer half-life estimates are often reported when we use an annual data set. We believe that such longer estimates are due to the time aggregation bias (Taylor 2001) but do not attempt to correct it, because we are interested in relative importance of the roles of relative price adjustment toward PPP[5].

Insert Table 3

Insert Figure 2

[4]We extended the original data set through 1998, and omitted Portugal, because its price level is the GDP deflator. Small number of missing data were filled by usual linear interpolation. Its electronically compiled data set can be downloaded at Michael Bordo's website (http://michael.bordo.googlepages.com/home3).

[5]Even if we correct for the bias, the relative importance of nominal exchange shocks and relative price shocks will remain the same.

3 The Roles of Relative Price and Nominal Exchange Rate Adjustments

In this section, we evaluate the relative contributions of relative price and nominal exchange rate adjustments toward PPP reversion. For this purpose, we consider the two types of structural shocks, a positive nominal exchange rate shock (u_t^e) and a negative relative price shock ($u_t^{\tilde{p}}$), that result in a positive shock to real exchange rate. Using conventional impulse-response analysis, we decompose the dynamic reversion path of the real exchange rate towards its long-run equilibrium into the relative price and nominal exchange rate adjustments.

We rewrite (2) as in the following level VAR($k+1$) form.

$$\mathbf{y}_t = \mathbf{a} + \sum_{j=1}^{k+1} \mathbf{\Gamma}_j \mathbf{y}_{t-j} + \mathbf{C}\mathbf{u}_t, \tag{5}$$

where

$$\begin{aligned}
\mathbf{\Gamma}_1 &= \mathbf{I}_2 + \rho\beta' + \mathbf{B}_1 \\
\mathbf{\Gamma}_j &= \mathbf{B}_{j+1} - \mathbf{B}_j,\ j = 2, \cdots, k \\
\mathbf{\Gamma}_{k+1} &= -\mathbf{B}_k
\end{aligned}$$

Then, under some regularity conditions, the following recursive relations hold (Pesaran and Shin 1996),

$$\begin{aligned}
\mathbf{D}_n &= \mathbf{\Gamma}_1 \mathbf{D}_{n-1} + \mathbf{\Gamma}_2 \mathbf{D}_{n-2} + \cdots + \mathbf{\Gamma}_{k+1} \mathbf{D}_{n-k-1}, \quad n = 1, 2, \cdots \qquad (6) \\
\mathbf{D}_0 &= \mathbf{I}_2 \text{ and } \mathbf{D}_n = 0 \text{ for } n < 0,
\end{aligned}$$

where

$$\mathbf{D}_n = \sum_{j=0}^{n} \mathbf{A}_j,\ \ \mathbf{\Delta y}_t = \sum_{i=0}^{\infty} \mathbf{A}_i \mathbf{C} \mathbf{u}_{t-i}$$

which measures the (cumulative) effects of $\mathbf{u}_t$ on the levels of $\mathbf{y}_{t+n}$.

Then, assuming that $\mathbf{C}$ is an upper-triangular matrix[6] obtained by the Cholesky decomposition

[6]That is, we assume that relative prices do not contemporaneously respond to a change in nominal exchange

of the variance-covariance matrix, the impulse-response functions are given as follows.

$$\begin{aligned} \phi_e^{\tilde{p}}(n) &= \mathbf{e}_1'\mathbf{D}_n\mathbf{C}\mathbf{e}_2,\ \phi_{\tilde{p}}^{\tilde{p}}(n) = \mathbf{e}_2'\mathbf{D}_n\mathbf{C}\mathbf{e}_2,\ \phi_s^{\tilde{p}}(n) = \beta'\mathbf{D}_n\mathbf{C}\mathbf{e}_2 \\ \phi_e^{e}(n) &= \mathbf{e}_1'\mathbf{D}_n\mathbf{C}\mathbf{e}_1,\ \phi_{\tilde{p}}^{e}(n) = \mathbf{e}_2'\mathbf{D}_n\mathbf{C}\mathbf{e}_1,\ \phi_s^{e}(n) = \beta'\mathbf{D}_n\mathbf{C}\mathbf{e}_1, \end{aligned} \tag{7}$$

where $\phi_i^j(n)$ denotes the response of variable i at time $t+n$ when a unit shock to variable j occurs at time t. $\mathbf{e}_1$ and $\mathbf{e}_2$ are 2×1 selection vectors.

Finally, we measure the relative contributions of relative price and nominal exchange rate adjustments at time $t+n$ when a shock occurs to relative prices at time t as follows[7].

$$d_e^{\tilde{p}}(n) = \frac{|\Delta\phi_e^{\tilde{p}}(n)|}{|\Delta\phi_e^{\tilde{p}}(n)| + |\Delta\phi_{\tilde{p}}^{\tilde{p}}(n)|},\ d_{\tilde{p}}^{\tilde{p}}(n) = 1 - d_e^{\tilde{p}}, \tag{8}$$

The relative contributions of the variables when there is a nominal exchange rate shock can be similarly obtained.

In Table 4, we report the relative contribution estimates as well as their standard errors[8] for $n = 1, 3, 5$, and 10. Unlike the results by Cheung et al. (2004) with short-horizon monthly data, our results with the long-horizon data imply a significantly important role of relative price adjustment when either shocks occur. Putting it differently, we do not see any dominant role of nominal exchange rate adjustment toward PPP.

Insert Table 4

In order to see what causes such different results, see Figures 3 and 4. Compared with the response function estimates by Cheung et al. (2004), the magnitude of relative price adjustments is not negligible at all. Therefore, no variable plays a dominant role in the PPP reversion to its long-run equilibrium value no matter which shock occurs. These results are largely consistent with

rates. Cheung et al. (2004) use the generalized impulse-response analysis proposed by Pesaran and Shin (1998), which requires that innovations are from a multivariate normal distribution. The results from both analysis were qualitatively the same.

[7] Cheung et al. (2004) use a slightly different method. However, our method correctly measures the relative contributions even when changes in response functions have different signs.

[8] Standard errors were obtained by 10,000 nonparametric residual-based bootstrap simulations at the point estimate for each country.

the Figure 1 in the sense that significantly greater variation of relative prices can be identified only when one uses long-horizon low frequency data.

Insert Figure 3

Insert Figure 4

4 Convergence Rate and Half-Life Estimates

We report our point estimates for (1) along with 95% nonparametric confidence intervals in Table 5. Corresponding half-life estimates with 95% confidence intervals are reported in Table 6. As we can see in Table 5, we obtain quite strong evidence of nominal exchange rate and relative price adjustments toward PPP. All ρ_1 and ρ_2 estimates exhibit correct signs with an exception of Italy when there is a relative price shock. Most confidence intervals were compact for either ρs or both. The implied ρ $(= \rho_1 - \rho_2)$ estimates were very similar as the point estimates in Table 3, which implies that the VECM framework works fairly well for estimating the persistency of real exchange rate deviations.

Cheung et al. (2004) report a very surprising empirical result with regard to the adjustment speeds of nominal exchange rates and relative prices when a nominal exchange rate shock occurs. They find that much faster convergence rates for relative prices than those for nominal exchange rates, which is at odds with the conventional sticky price models. That is, their results imply the sluggish reversion rate of real exchange rates are mainly due to slow adjustment of nominal exchange rate rather than price-stickiness. Engel and Morley (2001) provide similar evidence and stated that the real puzzle is why nominal exchange rate converges so slowly.

Unlike Cheung et al. (2004) who report half-life estimates only when there is a unit nominal exchange rate shock, we report the half-life estimates for the cases when each shock occurs[9].

When a nominal exchange rate shock occurs, we find fairly similar results as those by Cheung et al. (2004). In other words, we also find more sluggish adjustment rates for relative prices than

[9]For the half-life estimation method, refer to Cheung et al. (2004).

nominal exchange rates. However, the magnitude of changes in relative price toward the long-run equilibrium was much larger than that of Cheung et al. (2004) with the current float monthly frequency data.

When there is a (negative) relative price shock, we find more sluggish convergence rates for relative prices than nominal exchange rates for many countries. Interestingly, we find delayed responses of both nominal exchange rates and relative prices to a relative price shock for many countries. Such responses result in delayed overshooting for many real exchange rates (Eichenbaum and Evans 1995).

It should be noted that measuring half-lives by a conventional method may be misleading when relative prices continue to fall (see Figure 3 (h) for example), because the real exchange rate can continue to deviate even after the half-life point of the relative price. This explains why some of the half-life estimates of real exchange rates are longer than the both half-life estimates for nominal exchange rate and relative price.

It is also interesting to see relative price shocks lead to much longer deviations of real exchange rates than nominal exchange rate shocks. We view this result as consistent with the conventional sticky price models.

Insert Table 5

Insert Table 6

As further evidence in favor of the important role of relative price adjustment, we implemented the variance decomposition analysis, and report the results in Table 7. The standard errors were also obtained from 10,000 nonparametric residual-based bootstrap simulations.

One of the most notable findings is that nominal exchange rates play virtually no role in relative price variations, while relative prices serve as an attractor for nominal exchange rates for many countries especially in the intermediate to long-term. Putting it differently, a full picture of sluggish movement of nominal exchange rates may not be separately understood without considering potential effects from relative prices.

Insert Table 7

5 Conclusion

This paper investigates the relative contributions of nominal exchange rates and relative price adjustments toward PPP in a VECM framework. Using over hundred-year long data for 11 currencies against the US dollar, we find that relative price adjustments play an important role in real exchange rate dynamics.

Our results sharply contrast with those of Cheung et al. (2004) who reported a dominant role of nominal exchange rate adjustment with short horizon high frequency data for 5 real exchange rates. We demonstrate, however, their estimations might be flawed by a potentially inappropriate lag selection method.

One of the striking results of Cheung et al. (2004) was the finding of much slower convergence rates of nominal exchange rates than those of relative prices, which corroborate the findings of Engel and Morley (2001) who also use current float quarterly data for G7 countries. We find similar results, and our results confirm theirs even in long-horizon. However, when there is a relative price shock, we find opposite observations for many countries, which is consistent with the conventional rational expectations sticky price models.

Our variance decomposition analysis provides further evidence in support of relative prices as pivotal in real exchange rate dynamics. We find that relative prices explain a great deal of nominal exchange rate variations in the intermediate- to long-term, while nominal exchange rates hardly explain real exchange rate variations.

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Table 1. Unit Root Tests: Current Float Monthly Data

$$\Delta s_t = a + \rho s_{t-1} + \sum_{j=1}^{k} \beta_j \Delta s_{t-j} + \varepsilon_t$$

$$s_t = \beta' \mathbf{y}_t,\ \beta = [1\ \ -1]',\ \mathbf{y}_t = [e_t\ \ \tilde{p}_t]',\ \tilde{p}_t = p_t - p_t^*$$

	ADF Test		*DF-GLS Test*	
Country	k_{GTS}	ADF	k_{MAIC}	DF-GLS
Austria	6	-1.688	0	-1.183
Belgium	0	-1.610	0	-1.607
Canada	8	-0.375	0	0.883
Denmark	2	-1.926	0	-1.470
Finland	6	-2.005	0	-1.435
France	0	-1.886	0	-1.892*
Germany	0	-1.796	0	-1.799*
Greece	4	-1.531	4	-1.217
Italy	0	-1.833	0	-1.838*
Japan	0	-1.567	1	-0.535
Netherlands	0	-1.935	0	-1.745*
Norway	6	-1.859	0	-1.958*
Portugal	6	-1.244	0	-1.347
Spain	7	-2.036	0	-1.045
Sweden	6	-1.482	0	-1.257
Switzerland	0	-2.142	0	-0.977
UK	1	-2.330	0	-1.570

Note: i) Observations span from 1973M3 to 1998M12 for all countries. ii) For the Augmented Dickey-Fuller tests, the numbers of lags were chosen by the general-to-specific rule (Hall, 1994). iii) For the DF-GLS tests, the modified Akaike Information Criteria (Ng and Perror, 2001) was employed. iv) For the DF-GLS tests, s_t denotes the GLS detrended real exchange rates. v) *, **, and ***denote significance at the 10%, 5%, and 1% levels, respectively.

Table 2. Unit Root Tests: Long-Horizon Annual Data

$$\Delta s_t = a + \rho s_{t-1} + \sum_{j=1}^{k} \beta_j \Delta s_{t-j} + \varepsilon_t$$

$$s_t = \beta' \mathbf{y}_t,\ \beta = [1\ \ -1]',\ \mathbf{y}_t = [e_t\ \ \tilde{p}_t]',\ \tilde{p}_t = p_t - p_t^*$$

	ADF Test		*DF-GLS Test*	
Country	k_{GTS}	ADF	k_{MAIC}	DF-GLS
Australia	1	-2.548	0	-1.944*
Belgium	1	-4.168***	3	-2.423**
Canada	0	-1.154	0	-1.330
Denmark	6	-1.251	6	-1.258
Finland	1	-6.002***	8	-2.358**
France	2	-2.985**	6	-1.088
Germany	1	-2.944**	2	-2.011**
Italy	2	-4.286***	0	-3.366***
Japan	6	-0.056	2	0.307
Netherlands	1	-2.634*	2	-2.352**
Norway	1	-3.495***	5	-2.030**
Spain	1	-3.244**	3	-2.101**
Sweden	1	-3.724***	2	-2.358**
Switzerland	2	-1.491	2	-0.758
UK	4	-2.579*	4	-2.178**

Note: i) Observations span from 1880 to 1998 with the exceptions of Japan (1885-1998) and Switzerland (1892-1998). ii) For the Augmented Dickey-Fuller tests, the numbers of lags were chosen by the general-to-specific rule (Hall, 1994). iii) For the DF-GLS tests, the modified Akaike Information Criteria (Ng and Perror, 2001) was employed. iv) For the DF-GLS tests, s_t denotes the GLS detrended real exchange rates. v) *, **, and ***denote significance at the 10%, 5%, and 1% levels, respectively.

Table 3. Scalar Error Correction Model Regressions

$$\Delta s_t = a + \rho s_{t-1} + \sum_{j=1}^{k} \beta_j \Delta s_{t-j} + \varepsilon_t$$

$$s_t = \beta' \mathbf{y}_t,\ \beta = [1\ \ -1]',\ \mathbf{y}_t = [e_t\ \ \tilde{p}_t]',\ \tilde{p}_t = p_t - p_t^*$$

Country	k	ρ	s.e.	CI	HL	CI
Australia	1	-0.105	0.041	[-0.251,-0.061]	6.780	[2.870,11.48]
Belgium	1	-0.221	0.052	[-0.358,-0.153]	3.483	[2.268,4.798]
Finland	1	-0.413	0.068	[-0.573,-0.311]	2.063	[1.539,2.609]
France	2	-0.137	0.045	[-0.305,-0.081]	4.604	[2.191,8.078]
Germany	1	-0.090	0.030	[-0.190,-0.055]	8.193	[4.247,13.02]
Italy	2	-0.247	0.057	[-0.406,-0.168]	3.621	[2.374,4.854]
Netherlands	1	-0.094	0.035	[-0.216,-0.056]	7.824	[3.543,12.87]
Norway	1	-0.129	0.036	[-0.244,-0.083]	5.895	[3.478,8.823]
Spain	1	-0.125	0.038	[-0.251,-0.078]	5.952	[3.196,9.296]
Sweden	1	-0.172	0.045	[-0.314,-0.112]	4.382	[2.564,6.494]
UK	4	-0.153	0.058	[-0.370,-0.087]	3.729	[2.098,5.922]

Note: i) Observations span from 1880 to 1998 for all countries. ii)The numbers of lags (k) were chosen by the general-to-specific rule (Hall, 1994). iii) The 95% confidence intervals were obtained by getting 2.5% and 97.5% percentiles from 10,000 residual-based bootstrap simulations (Efron and Tibshirani, 1993). iv) Half-Life estimates were obtained from the impulse-response functions. v) The 95% confidence intervals for the half-life estimates were also obtained by 2.5% and 97.5% percentiles from 10,000 residual-based bootstrap simulations.

Table 4. Relative Contributions of Nominal Exchange Rate and Relative Price Adjustments to PPP Reversion

		$j=1$			$j=3$			$j=5$			$j=10$		
Country	shock	d_e	$d_{\tilde{p}}$	s.e.	d_e	$d_{\tilde{p}}$	s.e.	d_e	$d_{\tilde{p}}$	s.e.	d_e	$d_{\tilde{p}}$	s.e.
Australia	e	0.303	0.697	0.183	0.397	0.603	0.205	0.400	0.600	0.212	0.400	0.600	0.214
	$\tilde{p}$	0.215	0.785	0.257	0.406	0.594	0.229	0.401	0.599	0.214	0.400	0.600	0.214
Belgium	e	0.497	0.503	0.164	0.586	0.414	0.135	0.586	0.414	0.161	0.571	0.429	0.191
	$\tilde{p}$	0.571	0.429	0.233	0.609	0.391	0.165	0.591	0.409	0.155	0.579	0.421	0.192
Finland	e	0.811	0.189	0.118	0.866	0.134	0.167	0.549	0.451	0.211	0.860	0.140	0.191
	$\tilde{p}$	0.869	0.131	0.088	0.876	0.124	0.100	0.487	0.513	0.221	0.777	0.223	0.170
France	e	0.472	0.528	0.263	0.352	0.648	0.281	0.224	0.776	0.285	0.307	0.693	0.282
	$\tilde{p}$	0.639	0.361	0.125	0.527	0.473	0.193	0.478	0.522	0.237	0.059	0.941	0.276
Germany	e	0.510	0.490	0.208	0.388	0.612	0.191	0.461	0.539	0.186	0.483	0.517	0.204
	$\tilde{p}$	0.131	0.869	0.256	0.443	0.557	0.228	0.463	0.537	0.192	0.482	0.518	0.204
Italy	e	0.477	0.523	0.150	0.685	0.315	0.097	0.796	0.204	0.160	0.564	0.436	0.191
	$\tilde{p}$	0.692	0.308	0.131	0.946	0.054	0.250	0.163	0.837	0.221	0.819	0.181	0.213
Netherlands	e	0.045	0.955	0.192	0.657	0.343	0.175	0.696	0.304	0.178	0.694	0.306	0.181
	$\tilde{p}$	0.983	0.017	0.247	0.600	0.400	0.195	0.681	0.319	0.178	0.694	0.306	0.180
Norway	e	0.687	0.313	0.208	0.747	0.253	0.140	0.729	0.271	0.149	0.686	0.314	0.207
	$\tilde{p}$	0.325	0.675	0.192	0.968	0.032	0.154	0.789	0.211	0.143	0.715	0.285	0.185
Spain	e	0.057	0.943	0.221	0.562	0.438	0.203	0.592	0.408	0.218	0.596	0.404	0.235
	$\tilde{p}$	0.725	0.275	0.163	0.809	0.191	0.183	0.658	0.342	0.224	0.598	0.402	0.233
Sweden	e	0.707	0.293	0.158	0.801	0.199	0.133	0.810	0.190	0.156	0.811	0.189	0.183
	$\tilde{p}$	0.480	0.520	0.203	0.958	0.042	0.107	0.850	0.150	0.133	0.815	0.185	0.183
UK	e	0.963	0.037	0.177	0.900	0.100	0.122	0.467	0.533	0.200	0.184	0.816	0.258
	$\tilde{p}$	0.885	0.115	0.189	0.916	0.084	0.174	0.294	0.706	0.234	0.112	0.888	0.262

Note: i) Observations span from 1880 to 1998 for all countries. ii) The standard errors were obtained from 10,000 residual-based bootrap simulations.

Table 5. Vector Error Correction Model Regressions: Error Correction Terms Estimates

$$\Delta \mathbf{y}_t = \mathbf{a} + \rho\beta'\mathbf{y}_{t-1} + \sum_{j=1}^{k} \mathbf{B}_j \Delta \mathbf{y}_{t-j} + \mathbf{C}\mathbf{u}_t$$

$\mathbf{y}_t = [e_t \ \ \tilde{p}_t]'$, $\tilde{p}_t = p_t - p_t^*$, $\rho = [\rho_1 \ \ \rho_2]'$, $\mathbf{u}_t = [u_t^e \ \ u_t^{\tilde{p}}]'$, $\mathbf{C}$ is an upper-triangular matrix.

		Nominal Exchange Rate Equation			*Relative Price Equation*			Implied ρ
Country	k	ρ_1	s.e.	CI	ρ_2	s.e.	CI	$\rho_1 - \rho_2$
Australia	1	-0.033	0.039	[-0.153, 0.025]	0.048	0.019	[0.021,0.107]	-0.081
Belgium	1	-0.114	0.065	[-0.267,-0.007]	0.075	0.016	[0.042,0.116]	-0.189
Finland	1	-0.396	0.055	[-0.538,-0.297]	0.051	0.058	[-0.056,0.176]	-0.447
France	2	-0.090	0.056	[-0.276, 0.000]	0.060	0.041	[-0.034,0.171]	-0.150
Germany	1	-0.035	0.031	[-0.127, 0.012]	0.049	0.015	[0.025,0.092]	-0.083
Italy	2	-0.318	0.073	[-0.515,-0.200]	-0.095	0.063	[-0.244,0.053]	-0.223
Netherlands	1	-0.045	0.035	[-0.159, 0.002]	0.036	0.016	[0.011,0.082]	-0.080
Norway	1	-0.108	0.036	[-0.208,-0.044]	0.026	0.019	[-0.006,0.077]	-0.134
Spain	1	-0.083	0.039	[-0.205,-0.027]	0.048	0.019	[0.011,0.100]	-0.131
Sweden	1	-0.145	0.043	[-0.274,-0.080]	0.028	0.020	[-0.009,0.082]	-0.173
UK	4	-0.049	0.061	[-0.249, 0.055]	0.070	0.025	[0.028,0.146]	-0.119

Note: i) Observations span from 1880 to 1998 for all countries. ii) The numbers of lags (k) were chosen by the general-to-specific rule (Hall, 1994). iii) The 95% confidence intervals were obtained by getting 2.5% and 97.5% percentiles from 10,000 residual-based bootrap simulations (Efron and Tibshirani, 1993).

Table 6. Vector Error Correction Model Regressions: Half-Life Estimates

$$\Delta \mathbf{y}_t = \mathbf{a} + \rho\beta' \mathbf{y}_{t-1} + \sum_{j=1}^{k} \mathbf{B}_j \Delta \mathbf{y}_{t-j} + \mathbf{C}\mathbf{u}_t$$

$\mathbf{y}_t = [e_t \;\; \tilde{p}_t]'$, $\tilde{p}_t = p_t - p_t^*$, $\rho = [\rho_1 \;\; \rho_2]'$, $\mathbf{u}_t = [u_t^e \;\; u_t^{\tilde{p}}]'$, $\mathbf{C}$ is an upper-triangular matrix.

	Nominal Exchange Rate Shock						*Relative Price Shock*					
Country	HL_s	CI	HL_e	CI	$HL_{\tilde{p}}$	CI	HL_s	CI	HL_e	CI	$HL_{\tilde{p}}$	CI
Australia	5.431	[2.260,9.510]	7.559	[0.371,17.19]	4.312	[1.303,8.333]	11.22	[4.246,19.64]	9.614	[0.153,23.38]	12.54	[0.067,24.89]
Belgium	3.512	[2.282,4.786]	3.972	[2.221,6.458]	2.884	[1.631,4.207]	5.449	[1.119,8.454]	4.337	[0.255,9.182]	8.675	[0.033,10.33]
Finland	1.759	[1.175,2.455]	2.108	[1.415,2.812]	0.616	[0.097,5.253]	2.411	[1.556,3.374]	2.079	[1.350,2.787]	0.819	[0.188,2.852]
France	4.848	[2.231,8.500]	9.549	[0.079,14.36]	3.660	[0.308,12.69]	1.290	[0.000,7.313]	1.404	[0.534,2.574]	1.492	[0.283,5.308]
Germany	7.932	[4.019,12.47]	13.95	[0.041,22.84]	5.343	[2.413,8.619]	9.412	[3.170,15.69]	7.704	[0.237,17.88]	11.68	[0.038,21.34]
Italy	4.237	[2.429,5.690]	3.515	[2.059,4.924]	2.713	[0.571,5.933]	2.979	[1.362,5.482]	1.397	[0.233,7.555]	0.556	[0.106,5.174]
Netherlands	7.302	[3.104,12.25]	10.49	[3.347,22.61]	3.215	[0.687,7.533]	11.13	[3.853,19.75]	10.75	[0.342,25.55]	12.08	[0.048,23.55]
Norway	4.615	[2.785,6.739]	4.813	[2.878,7.217]	4.059	[0.729,9.562]	9.988	[5.919,16.13]	7.307	[4.423,10.98]	0.471	[0.038,11.56]
Spain	6.116	[3.233,9.495]	8.199	[0.142,16.69]	4.132	[1.121,7.318]	4.200	[0.959,9.168]	1.966	[0.860,6.931]	0.372	[0.050,17.79]
Sweden	3.878	[2.240,5.827]	4.246	[2.320,6.648]	2.715	[0.411,8.664]	6.546	[2.853,10.19]	4.415	[1.810,7.264]	0.553	[0.064,10.39]
UK	3.525	[1.794,5.969]	2.180	[0.573,15.74]	6.741	[2.076,11.23]	8.554	[2.868,15.00]	0.759	[0.093,10.14]	12.28	[0.041,21.49]

Note: i) Observations span from 1880 to 1998 for all countries. ii) The numbers of lags (k) were chosen by the general-to-specific rule (Hall, 1994). iii) Half-Life estimates were obtained from the impulse-response functions. iv) The 95% confidence intervals for the half-life estimates were also obtained by 2.5% and 97.5% percentiles from 10,000 residual-based bootrap simulations (Efron and Tibshirani, 1993).

Table 7. Variance Decomposition of j-Step Ahread Forecast Error

		$j=1$			$j=3$			$j=5$			$j=10$		
Country	x	u_t^e	$u_t^{\tilde{p}}$	s.e.	u_t^e	$u_t^{\tilde{p}}$	s.e.	u_t^e	$u_t^{\tilde{p}}$	s.e.	u_t^e	$u_t^{\tilde{p}}$	s.e.
Australia	e	0.998	0.002	0.029	0.998	0.002	0.065	0.997	0.003	0.092	0.971	0.029	0.155
	$\tilde{p}$	0.005	0.995	0.004	0.020	0.980	0.016	0.041	0.959	0.032	0.106	0.894	0.082
Belgium	e	0.998	0.002	0.070	0.979	0.021	0.154	0.896	0.104	0.194	0.656	0.344	0.219
	$\tilde{p}$	0.002	0.998	0.002	0.018	0.982	0.012	0.049	0.951	0.031	0.126	0.874	0.082
Finland	e	0.898	0.102	0.069	0.483	0.517	0.109	0.249	0.751	0.081	0.116	0.884	0.063
	$\tilde{p}$	0.008	0.992	0.013	0.022	0.978	0.036	0.022	0.978	0.043	0.019	0.981	0.046
France	e	0.794	0.206	0.090	0.553	0.447	0.167	0.452	0.548	0.195	0.376	0.624	0.235
	$\tilde{p}$	0.007	0.993	0.012	0.036	0.964	0.048	0.063	0.937	0.084	0.120	0.880	0.159
Germany	e	0.999	0.001	0.021	0.999	0.001	0.059	0.997	0.003	0.086	0.962	0.038	0.145
	$\tilde{p}$	0.001	0.999	0.002	0.009	0.991	0.011	0.032	0.968	0.029	0.129	0.871	0.088
Italy	e	0.508	0.492	0.209	0.158	0.842	0.129	0.078	0.922	0.060	0.083	0.917	0.060
	$\tilde{p}$	0.000	1.000	0.008	0.015	0.985	0.024	0.038	0.962	0.041	0.067	0.933	0.062
Netherlands	e	0.988	0.012	0.035	0.977	0.023	0.072	0.982	0.018	0.084	0.963	0.037	0.143
	$\tilde{p}$	0.005	0.995	0.004	0.023	0.977	0.017	0.043	0.957	0.032	0.092	0.908	0.074
Norway	e	0.983	0.017	0.025	0.988	0.012	0.071	0.917	0.083	0.136	0.470	0.530	0.205
	$\tilde{p}$	0.000	1.000	0.001	0.002	0.998	0.007	0.005	0.995	0.017	0.017	0.983	0.046
Spain	e	0.944	0.056	0.055	0.840	0.160	0.132	0.746	0.254	0.171	0.583	0.417	0.203
	$\tilde{p}$	0.001	0.999	0.003	0.012	0.988	0.014	0.031	0.969	0.031	0.083	0.917	0.077
Sweden	e	0.993	0.007	0.033	0.915	0.085	0.118	0.726	0.274	0.172	0.358	0.642	0.162
	$\tilde{p}$	0.001	0.999	0.002	0.005	0.995	0.008	0.009	0.991	0.016	0.018	0.982	0.033
UK	e	0.931	0.069	0.088	0.951	0.049	0.114	0.783	0.217	0.176	0.648	0.352	0.227
	$\tilde{p}$	0.001	0.999	0.002	0.002	0.998	0.007	0.006	0.994	0.014	0.047	0.953	0.058

Note: i) Observations span from 1880 to 1998 for all countries. ii) The variance of j-step ahead forecast error is $Var(x_{t+j} - E_t(x_{t+j}))$. iii) The standard errors were obtained from 10,000 residual-based bootrap simulations.

Figure 1: Relative Prices and Nominal Exchange Rates in Annual and Monthly Frequencies

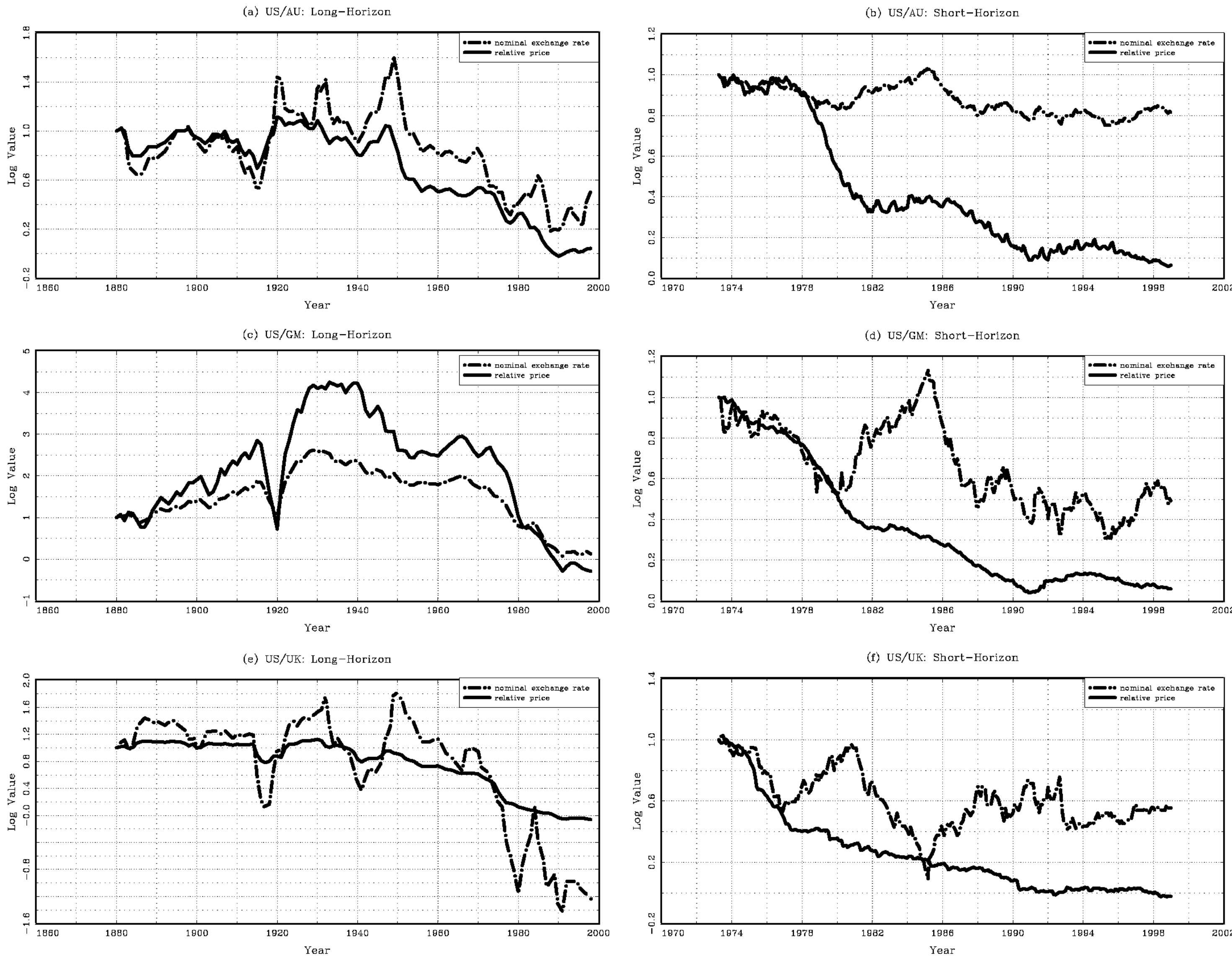
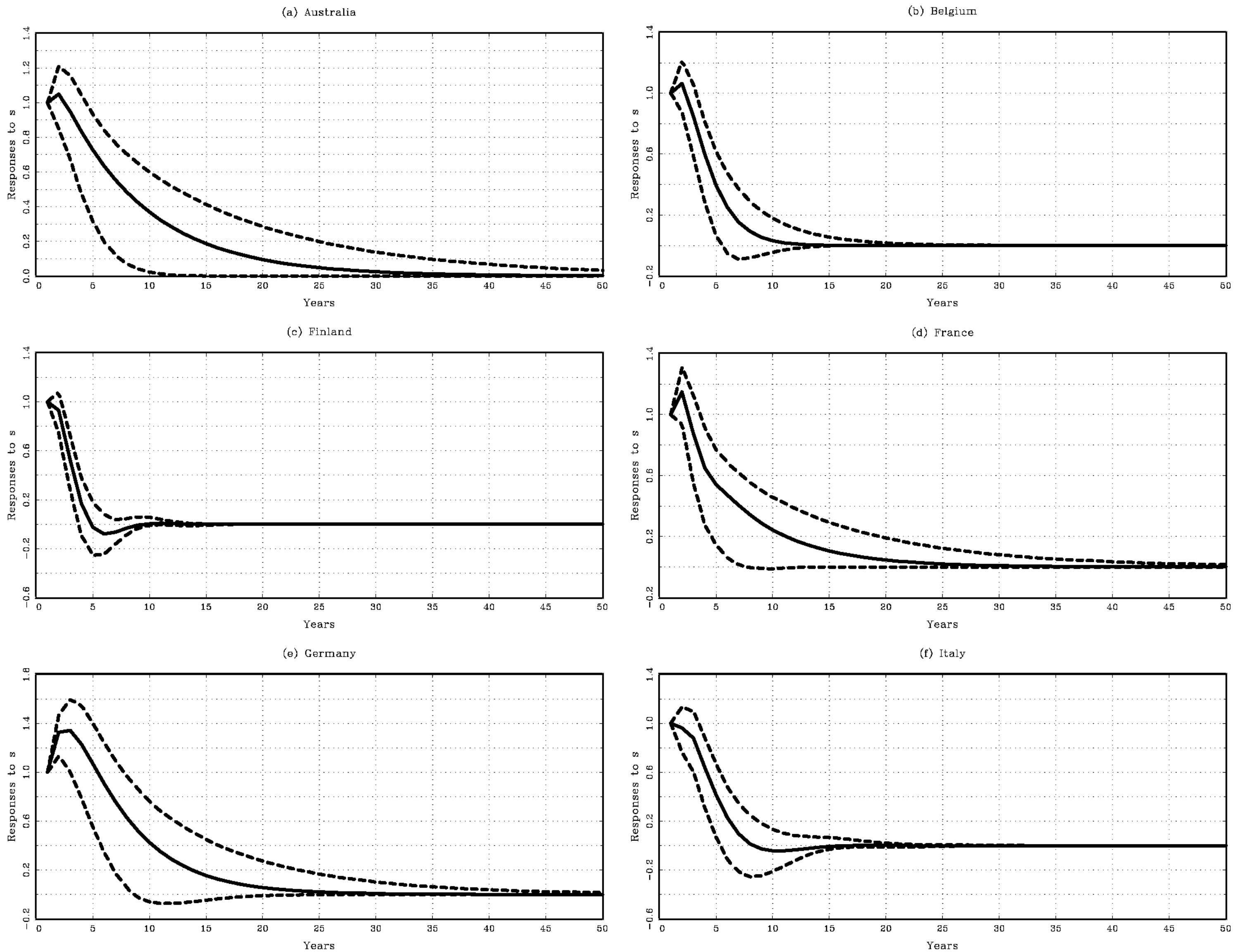


Figure 2: Responses to a Unit Real Exchange Rate Shock



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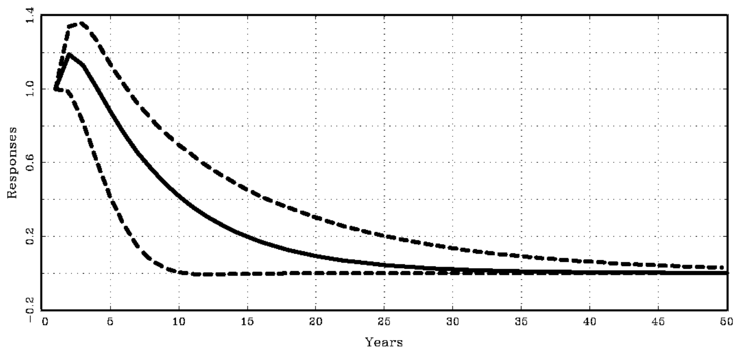

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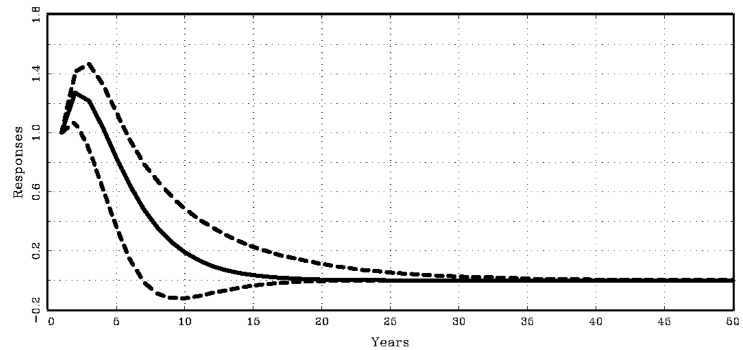

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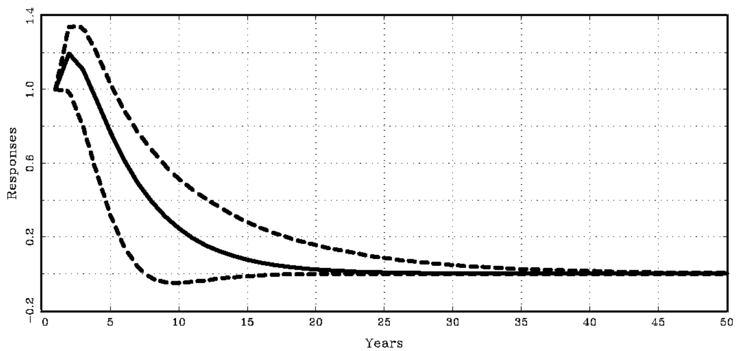

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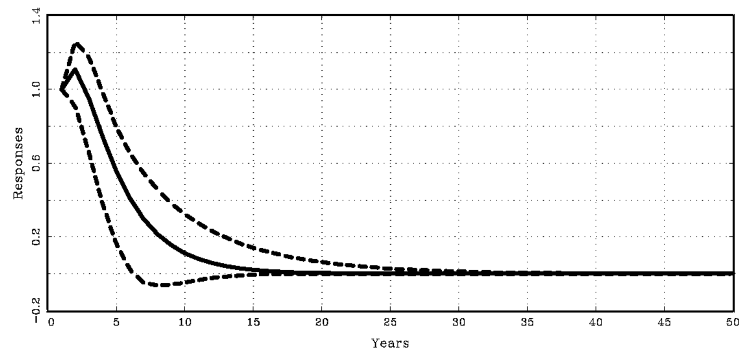

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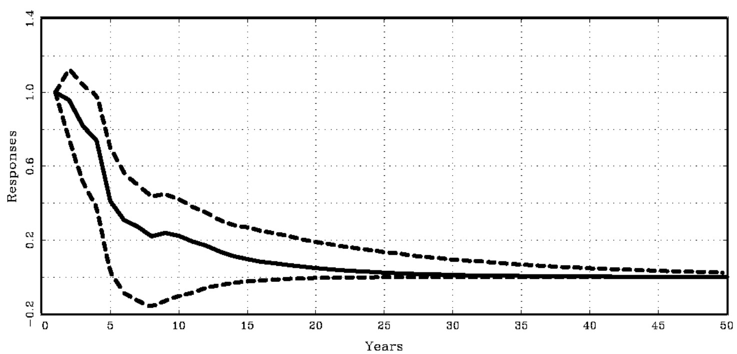
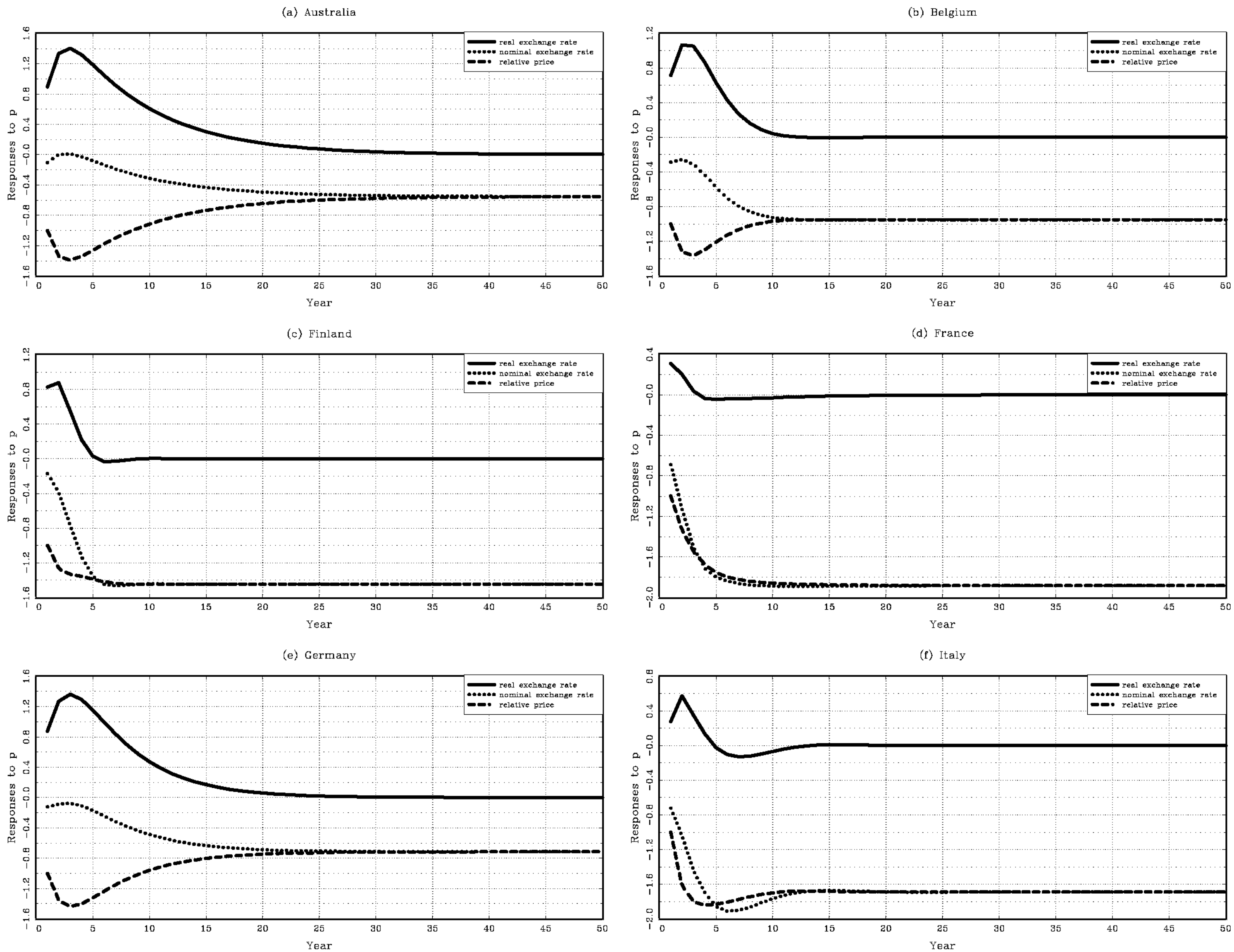


Figure 3: Responses to a Unit Relative Price Shock



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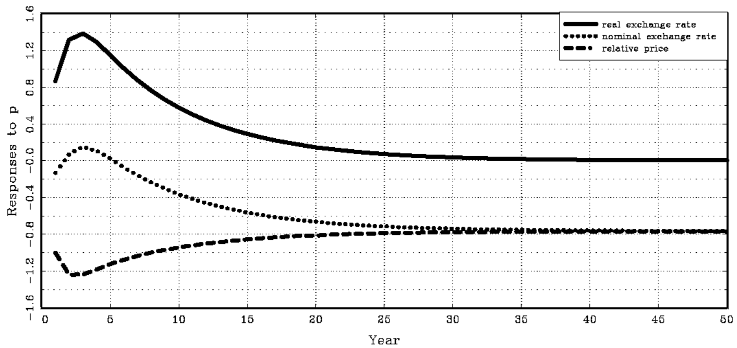

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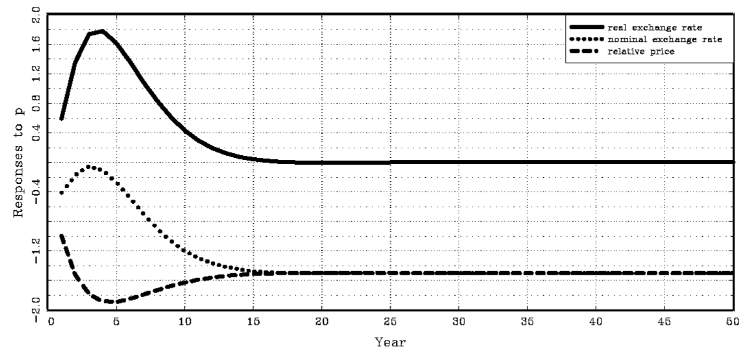

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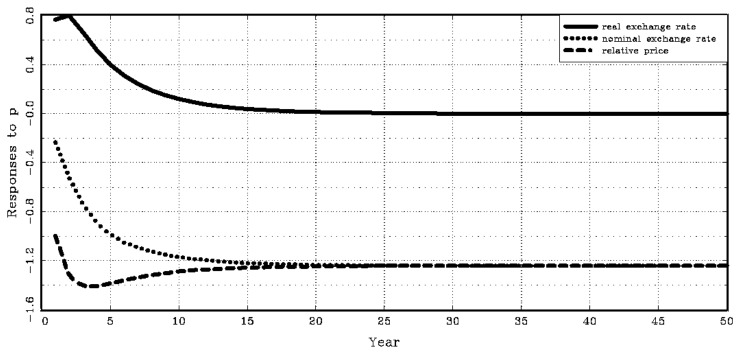

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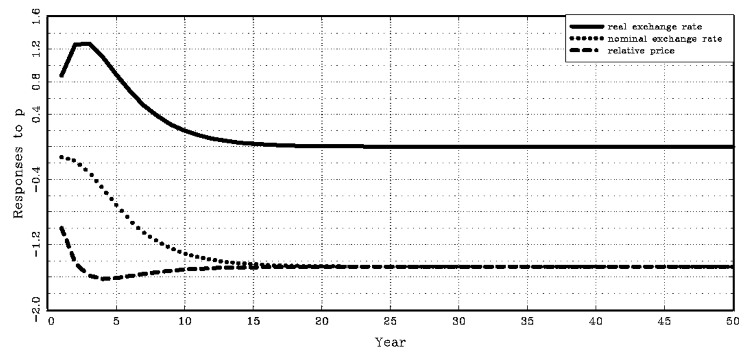

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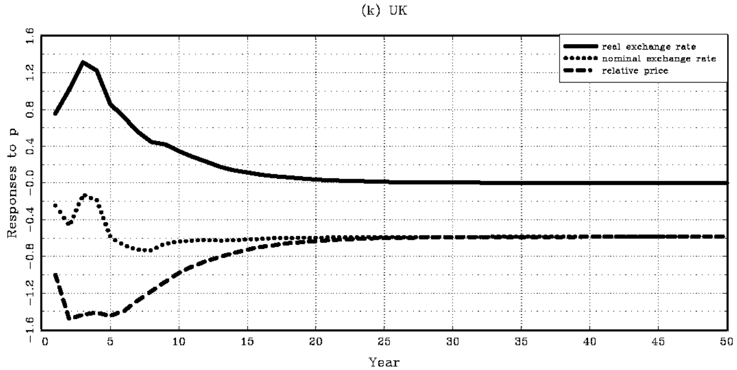
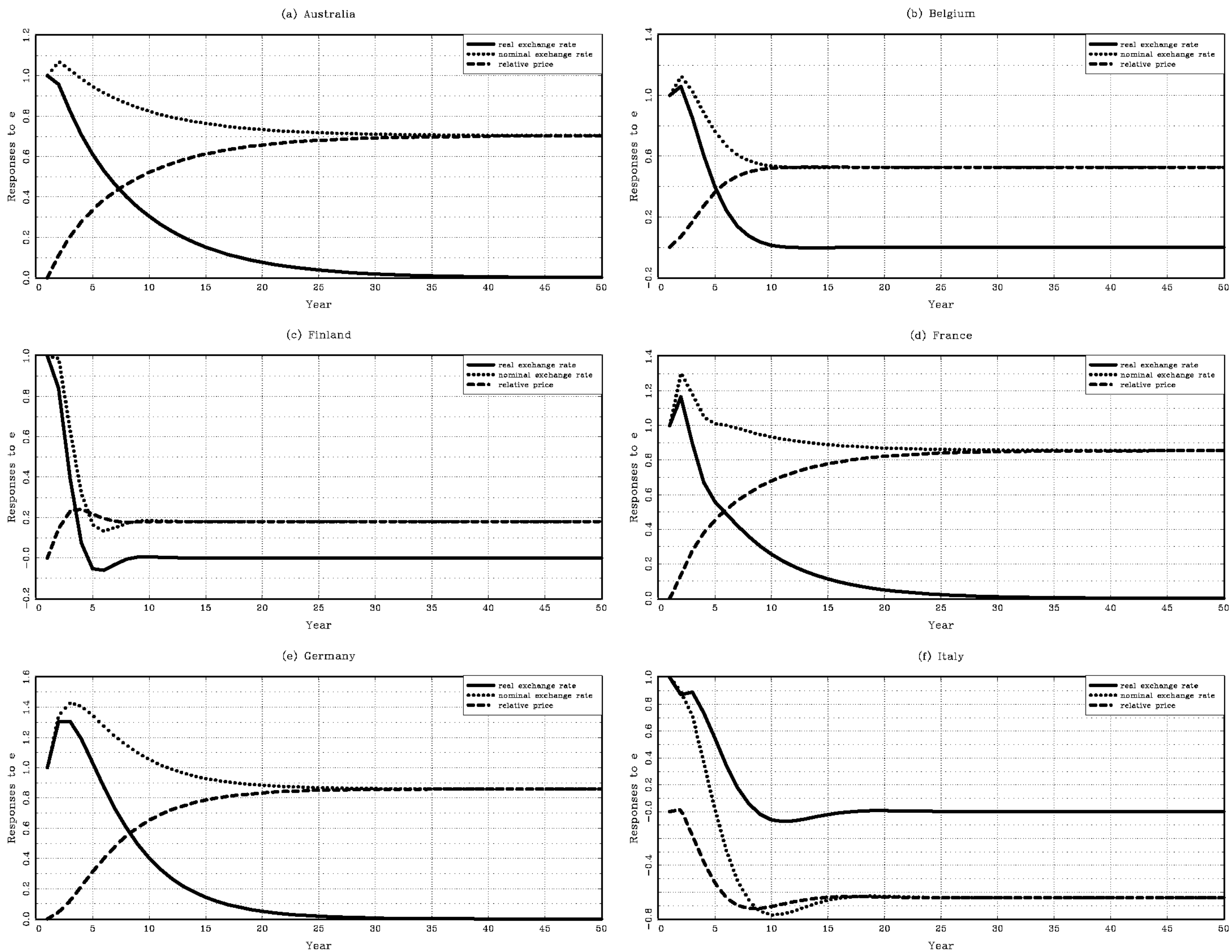


Figure 4: Responses to a Unit Nominal Exchange Rate Shock



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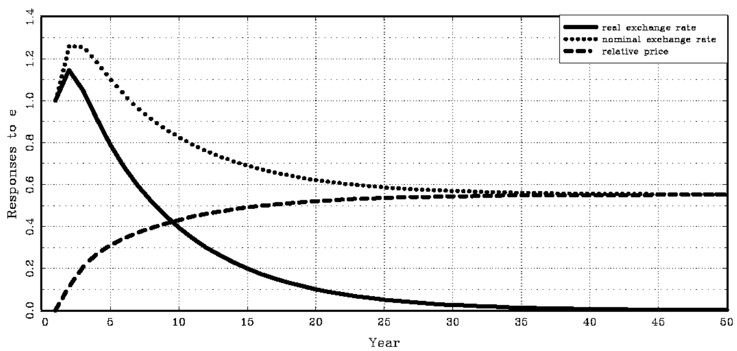

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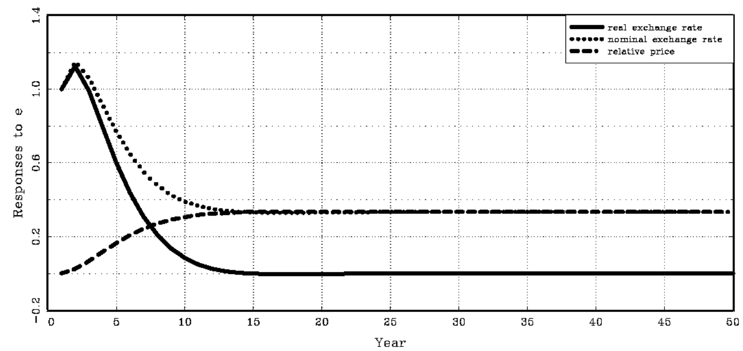

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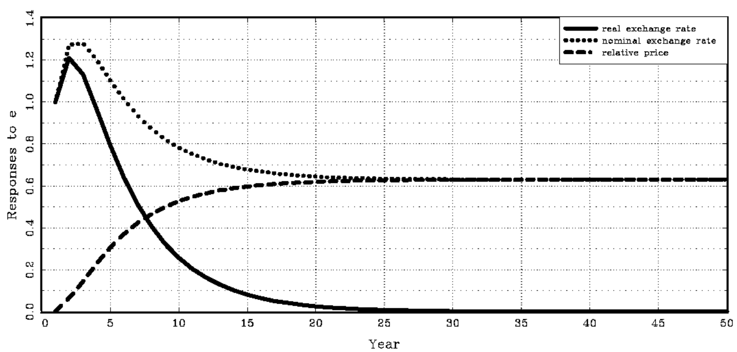

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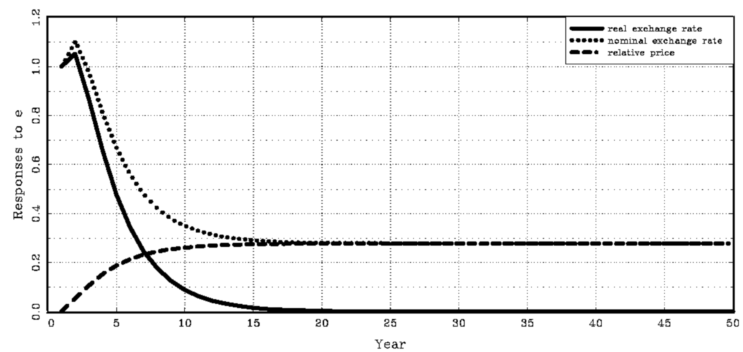

(k) UK