Downside Risk at the Zero Lower Bound*

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Abstract

The zero lower bound endogenously generates downside risk for the economy. Higher uncertainty about the future amplifies this risk and induces significant precautionary saving by households. This decline in aggregate demand leads to sizable contractions in output and inflation at the zero lower bound. Optimal monetary policy implies further lowering real interest rates to help offset the precautionary behavior by households. However, the central bank must maintain a zero policy rate for an extended period of time and accept higher inflation risk in the future. Access to state-contingent government spending allows the monetary authority to more effectively stabilize the future price level.

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1 Introduction

Many policymakers and economists have cited increased uncertainty about the future as a key driver in generating the Great Recession and the subsequent slow recovery. For example, empirical work by Stock and Watson (2012) and Leduc and Liu (2014) argues that higher uncertainty accounts for a sizable fraction of the decline in real activity during the Great Recession. In addition, anecdotal evidence from Kocherlakota (2010) argues that higher uncertainty was a large drag on the economic recovery. However, this empirical and anecdotal evidence leaves key questions unanswered. Why does uncertainty matter for the recent macroeconomic outcomes? Bloom (2009) documents a variety of events that generate significant uncertainty about the future. Prior to the Great Recession, however, these events did not seem to spillover dramatically to the macroeconomy. In addition, if “uncertainty shocks” are important drivers of inflation and real activity, how should monetary and fiscal policy optimally respond when the economy faces higher uncertainty about the future?

In this paper, we argue that the zero lower bound on nominal interest rates plays a key role in transmitting the effects of uncertainty to the macroeconomy. The inability of the monetary authority to offset shocks endogenously generates downside risk for the economy. Increased uncertainty about the future amplifies this risk and endogenously lowers the conditional mean, increases the conditional volatility, and generates left-skewed distributions of outcomes at the zero lower bound. Thus, an uncertainty shock at the zero lower bound acts like a bad “news” shock about the future coupled with an endogenously-determined fall in the “worse-case scenario.” Using a simple general-equilibrium model, we show that a one-standard deviation uncertainty shock at the zero lower bound causes a 0.4% decline in the output gap. Away from the zero lower bound, the same uncertainty shock has only small effects as monetary policy is able to insulate the economy from the negative effects of higher uncertainty.

After showing why uncertainty shocks matter at the zero lower bound, we solve for the optimal responses of monetary and fiscal policy. Optimal monetary policy under commitment allows the central bank to attenuate much effects of the uncertainty shock at the zero lower bound. To minimize the downside risk when the economy hits the zero lower bound, the monetary authority commits to a higher price-level in the future. These expectations of higher inflation induce lower real interest rates, which help offset the precautionary saving
by households. However, the optimal policy requires maintaining a zero policy rate for an extended period of time. In addition, the central bank must accept higher inflation risk in the future to minimize the downside risk when the economy hits the zero lower bound. By committing to increase government spending if adverse shocks are realized, optimal fiscal policy is highly state-contingent and allows the monetary authority to more accurately stabilize the future price level.

To analyze the quantitative impact of uncertainty shocks, we calibrate and solve a representative-agent, dynamic stochastic general equilibrium model. We solve the model using a global solution method and model higher uncertainty about the future as an increase in the volatility of the aggregate shocks in the economy. Our calibrated model is consistent with unconditional standard deviations of the output gap, inflation, and the nominal interest rate. Our model can also match the stochastic volatility present in the data. Finally, the model can generate zero lower bound durations that are in line with the recent zero lower bound episode in the United States. Without uncertainty shocks and the zero lower bound, the model struggles to jointly match these features of the recent macroeconomic data.

We view our work as highly complementary to other recent work on the Great Recession and business-cycle models. For example, Christiano, Eichenbaum and Trabandt (2014) argues that the interaction between financial frictions and the zero lower bound is crucial for understanding the economics during the recent recession and recovery. However, they reach this conclusion using a model solution method which relies on certainty equivalence. By contrast, our work allows for uncertainty about future shocks to interact with the zero lower bound to generate downside risk. Our paper also extends the ideas of Ilut and Schneider (2014), which argues that exogenous shocks to the “worse-case” distribution of total factor productivity are a key driver of business cycles. The downside risk generated by the zero lower bound acts like a bad news shock combined with an endogenous decline in the worse-case outcome for the economy.
2 Intuition

This section formalizes the intuition from the Introduction using several key equations from a simple general-equilibrium model. For Section 2 only, we use Taylor series approximations of these equations to show why the zero lower bound is crucial in transmitting the negative effects of uncertainty. In addition, these approximations allow us to provide the intuition for the optimal monetary and fiscal policy at the zero lower bound. These approximations provide analytical tractability which is unavailable when examining the model equations in their original nonlinear form. In Section 4, we show that the intuition from these approximations is consistent with the computational results using the full nonlinear model.

2.1 Household Consumption Under Uncertainty

The household consumption Euler equation highlights why the zero lower bound is crucial in transmitting the effects of uncertainty shocks. Under constant relative risk aversion utility from consumption, the following equation links consumption $C_t$ by the representative household to the gross real interest rate $R_t$:

$$1 = E_t \left\{ \beta R_t \left( \frac{C_{t+1}}{C_t} \right)^{-\sigma} \right\},$$

where $\beta$ is the household discount factor and $\sigma$ is the risk aversion parameter in the household’s utility function. Using a third-order Taylor series approximation around the steady state, Appendix A shows Equation (1) can be written as follows:

$$c_t = E_t c_{t+1} - \frac{1}{\sigma} \left( r^r_t - r^r \right) - \frac{1}{2} \sigma \text{Var}_t c_{t+1} + \frac{1}{6} \sigma^2 \text{Skew}_t c_{t+1},$$

where lowercase variables denote the log of the respective variable, $r^r_t$ is the steady state net real interest rate, and $\text{Var}_t c_{t+1}$ and $\text{Skew}_t c_{t+1}$ denotes the conditional variance and skewness of future consumption. For any given real interest rate, households consume less if they expect a more volatile and negatively-skewed distribution of future consumption.

After defining a flexible-price version of Equation (2), Appendix A shows how to derive the following approximate higher-order version of a standard New-Keynesian IS Curve:

$$x_t = E_t x_{t+1} - \frac{1}{\sigma} \left( r^r_t - r^r \right) - \frac{1}{2} \sigma \text{Var}_t x_{t+1} - \frac{1}{6} \sigma^2 \text{Skew}_t x_{t+1},$$

where $x_t$ denotes the gap between equilibrium and flexible-price output and $r^r_t$ is the natural real interest rate that would prevail in the flexible-price economy. Iterating Equation (3) for-
ward and taking expectations at time $t$ implies the following solution for current output gap:

$$x_t = -\sum_{i=0}^{\infty} \left( E_t r^n_{t+i} - E_t r^r_{t+i} \right) - \frac{1}{2} \sigma \sum_{j=0}^{\infty} \text{Var}_t x_{t+1+j} - \frac{1}{6} \sigma^2 \sum_{k=0}^{\infty} \text{Skew}_t x_{t+1+k}$$

Equation (4) shows that the transmission of the uncertainty shocks to the macroeconomy depends crucially on monetary policy’s ability to stabilize the economy. In absence of the zero lower bound, the monetary authority can always fully offset the uncertainty shock by setting its nominal policy rate to close the gap between the real and natural real interest rates. In this scenario, the conditional variance and skewness of the output gap are zero since the monetary authority can stabilize the economy in all future states of the world. However, suppose the natural real rate becomes negative and the zero lower bound prevents the central bank from fully stabilizing the economy. Households internalize this reduced ability to offset future contractionary shocks throughout the zero lower bound episode. This asymmetric ability to stabilize the economy endogenously generates a more volatile and negatively-skewed distribution of future output gaps. Increased uncertainty about the future amplifies this downside risk and leads to a significantly negative output gap today through the precautionary saving by households.

The forward solution for the output gap also shows us the intuition for the optimal responses of the monetary and fiscal authorities facing higher uncertainty at the zero lower bound. Even when the monetary authority is constrained by the zero lower bound, they can still affect the output gap through the expected path of real interest rates. When the zero lower bound endogenously generates a riskier distribution of future consumption, the central bank needs to further lower the path of real interest rates to help offset the precautionary saving by households. By committing to a higher path for the price-level, the central bank generates expectations of higher inflation in the future. Since the nominal policy rate $r_t$ is zero, higher expected inflation $E_t \pi_{t+1}$ lowers the current real interest rate through the Fisher relation $r_t = E_t \pi_{t+1} + r^r_t$. Optimal fiscal policy can also help stabilize the economy by committing to increase government spending in a state-contingent matter. The fiscal authority can also commit to raising government spending if bad outcomes are realized in the future, households internalize that the stabilizing fiscal policy results in a less risky distribution of future output gaps.
2.2 From Intuition to Model Simulations

The intuition of this section argues that the zero lower bound plays a key role in transmitting the negative effects of uncertainty shocks. In the following section, we calibrate and solve a nonlinear model and show that the simulated zero lower bound scenarios are consistent with the intuition developed in this section. In addition, we solve for the optimal responses of monetary and fiscal policy under commitment.

3 Model

This section outlines the baseline dynamic stochastic general equilibrium model that we use my analysis. The baseline model shares many features with the models of Ireland (2003) and Ireland (2011). The model features optimizing households and firms and a central bank that systematically adjusts the nominal interest rate to offset adverse shocks in the economy. We allow for sticky prices using the quadratic-adjustment costs specification of Rotemberg (1982). The baseline model considers fluctuations in the discount factor of households, which have the interpretation as demand shocks.

3.1 Households

In the model, the representative household maximizes lifetime expected utility over streams of consumption $C_t$ and leisure $1 - N_t$. The household receives labor income $W_t$ for each unit of labor $N_t$ supplied in the representative intermediate goods-producing firm. The representative household also owns the intermediate goods firm and receives lump-sum dividends $D_t$. The household also has access to zero net supply nominal bonds $B_t$ and real bonds $B_t^R$. A nominal bond pays the gross one-period nominal interest rate $R_t$ while a real bond pays the gross one-period real interest rate $R_t^R$. The household divides its income from labor and its financial assets between consumption $C_t$ and the amount of the bonds $B_{t+1}$ and $B_{t+1}^R$ to carry into next period. The discount factor of the household $\beta$ is subject to shocks via the stochastic process $a_t$. An increase in $a_t$ induces households to consume more and work less for no technological reason. Thus, we interpret changes in the household discount factor as demand shocks for the economy.

The representative household maximizes lifetime utility by choosing $C_{t+s}, N_{t+s}, B_{t+s+1}$.
and $B^R_{t+s+1}$, for all $s = 0, 1, 2, \ldots$ by solving the following problem:

$$\max E_t \sum_{s=0}^{\infty} a_{t+s} \beta^s \left( \frac{(C^\eta_{t+s}(1-N_{t+s})^{1-\eta})^{1-\sigma}}{1-\sigma} \right)$$

subject to the intertemporal household budget constraint each period,

$$C_t + \frac{1}{R_t} \frac{B^R_{t+1}}{P_t} + \frac{1}{R^P_{t+1}} B^R_{t+1} \leq \frac{W_t}{P_t} N_t + \frac{B_t}{P_t} + \frac{D_t}{P_t} + B^R_t.$$

Using a Lagrangian approach, household optimization implies the following first-order conditions:

$$\eta a_t C^\eta_t (1-N_t)^{(1-\eta)(1-\sigma)} = \lambda_t$$  \hspace{1cm} (5)

$$ (1-\eta) a_t C^\eta_t (1-N_t)^{(1-\eta)(1-\sigma)-1} = \lambda_t \frac{W_t}{P_t}$$  \hspace{1cm} (6)

$$1 = E_t \left\{ \left( \beta \frac{\lambda_{t+1}}{\lambda_t} \right) \left( \frac{R_t P_t}{P_{t+1}} \right) \right\}$$  \hspace{1cm} (7)

$$1 = E_t \left\{ \left( \beta \frac{\lambda_{t+1}}{\lambda_t} \right) R^P_t \right\}$$  \hspace{1cm} (8)

where $\lambda_t$ denotes the Lagrange multiplier on the household budget constraint. Equations (5) - (6) represent the household intratemporal optimality conditions with respect to consumption and leisure, and Equations (7) - (8) represent the Euler equations for the one-period nominal and real bonds.

### 3.2 Intermediate Goods Producers

Each intermediate goods-producing firm $i$ rents labor $N_t(i)$ from the representative household in order to produce intermediate good $Y_t(i)$. Intermediate goods are produced in a monopolistically competitive market where producers face a quadratic cost of changing their nominal price $P_t(i)$ each period. Firm $i$ chooses $N_t(i)$, and $P_t(i)$ to maximize the discounted present-value of cash flows $D_t(i)/P_t(i)$ given aggregate demand $Y_t$ and price $P_t$ of the finished goods sector. The intermediate goods firms all have access to the same constant returns-to-scale Cobb-Douglas production function, subject to a fixed cost of production $\Phi$.

Each intermediate goods-producing firm maximizes discount cash flows using the household stochastic discount factor:

$$\max E_t \sum_{s=0}^{\infty} \left( \beta^s \frac{\lambda_{t+s}}{\lambda_t} \right) \left[ \frac{D_{t+s}(i)}{P_{t+s}} \right]$$
subject to the production function:

\[
[P_t(i)]^{-\theta} Y_t \leq N_t(i) - \Phi,
\]

where

\[
D_t(i) = \left[ \frac{P_t(i)}{P_t} \right]^{1-\theta} Y_t - \frac{W_t}{P_t} N_t(i) - \frac{\phi_P}{2} \left[ \frac{P_t(i)}{\Pi P_{t-1}(i)} - 1 \right]^2 Y_t
\]

The first-order conditions for the firm \( i \) are as follows:

\[
\frac{W_t}{P_t} N_t(i) = \Xi_t N_t(i) \tag{9}
\]

\[
\phi_P \left[ \frac{P_t(i)}{\Pi P_{t-1}(i)} - 1 \right] \left[ \frac{P_t}{\Pi P_{t-1}(i)} \right] = (1 - \theta) \left[ \frac{P_t(i)}{P_t} \right]^{-\theta} + \theta \Xi_t \left[ \frac{P_t(i)}{P_t} \right]^{-\theta-1}
\]

\[
+ \phi_P E_t \left\{ \left( \frac{\lambda_{t+1}}{\lambda_t} \right)^{Y_t+1} \left[ \frac{P_{t+1}(i)}{\Pi P_t(i)} - 1 \right] \left[ \frac{P_{t+1}(i)}{P_t(i)} \right] \right\}, \tag{10}
\]

where \( \Xi_t \) is the multiplier on the production function, which denotes the real marginal cost of producing an additional unit of intermediate good \( i \).

### 3.3 Final Goods Producers

The representative final goods producer uses \( Y_t(i) \) units of each intermediate good produced by the intermediate goods-producing firm \( i \in [0,1] \). The intermediate output is transformed into final output \( Y_t \) using the following constant returns to scale technology:

\[
\left[ \int_0^1 Y_t(i)^{\theta-1} di \right]^{\theta} \geq Y_t
\]

Each intermediate good \( Y_t(i) \) sells at nominal price \( P_t(i) \) and each final good sells at nominal price \( P_t \). The finished goods producer chooses \( Y_t \) and \( Y_t(i) \) for all \( i \in [0,1] \) to maximize the following expression of firm profits:

\[
P_t Y_t - \int_0^1 P_t(i) Y_t(i) di
\]

subject to the constant returns to scale production function. Finished goods-producer optimization results in the following first-order condition:

\[
Y_t(i) = \left[ \frac{P_t(i)}{P_t} \right]^{-\theta} Y_t
\]

The market for final goods is perfectly competitive, and thus the final goods-producing firm earns zero profits in equilibrium. Using the zero-profit condition, the first-order condition
for profit maximization, and the firm objective function, the aggregate price index $P_t$ can be written as follows:

$$P_t = \left[ \int_0^1 P_t(i)^{1-\theta} di \right]^{1/\theta}$$

### 3.4 Monetary Policy

We assume a cashless economy where the monetary authority sets the one-period net nominal interest rate $r_t = \log(R_t)$. Due to the zero lower bound on nominal interest rates, the central bank cannot lower its nominal policy rate below zero. In our baseline model, we assume that the monetary authority sets its policy rate according to the following simple policy rule:

$$r_t^d = r + \phi_{x_t} (\pi_t - \pi) + \phi_{x_t} x_t + \phi_{pl} (p_t - p^*),$$

$$r_t = \max(0, r_t^d), \quad (11)$$

where $r_t^d$ is the desired policy rate of the monetary authority, $r_t$ is the actual policy rate subject to the zero lower bound, $p_t$ is the log of the price level, $p^*$ is the constant price-level target of the central bank, and $x_t$ is the gap between current output and output in the equivalent flexible-price economy. With the exception of the price-level term, this rule acts as a standard Taylor (1993)-type policy rule. We include a response to price-level with a very small value for $\phi_{pl}$ in order to pin down the long-run price level in response to shocks. We discuss the rationale for this modeling choice in detail in Section 5.2. In practice, the small value for $\phi_{pl}$ implies that this additional term does not change the short-run response of the nominal interest rate to shocks.

### 3.5 Shock Processes

Shocks to the discount rate of households are the exogenous stochastic process in the baseline model. Large negative innovations to this process imply a large decline in aggregate demand, which forces the economy to encounter the zero lower bound. The stochastic process for these fluctuations is as follows:

$$a_t = (1 - \rho_a) a + \rho_a a_{t-1} + \sigma^a_{t-1} \varepsilon_t^a$$

$$\sigma^a_t = (1 - \rho_{\sigma^a}) \sigma^a + \rho_{\sigma^a} \sigma^a_{t-1} + \sigma^a \varepsilon_t^a$$

$\varepsilon_t^a$ is a first moment shock that captures innovations to the level of the household discount factors. We refer to $\varepsilon_t^a$ and as a second moment or “uncertainty” shock since it captures innovations to the volatility of the exogenous processes of the model. An increase
in the volatility of the shock process increases the uncertainty about its future time path. Both stochastic shocks are independent, standard normal random variables. We specify the stochastic process in levels, rather than in logs, to prevent the volatility $\sigma^a$ from impacting average value of $a_t$ through a Jensen’s inequality effect. In the model solution, $a_t$ always remains significantly greater than zero. To ensure that the volatility stays positive, we impose a lower bound $\underline{\sigma^a} = 0.0005$ on the volatility in both the model solution and simulations.

### 3.6 Equilibrium

In the symmetric equilibrium, all intermediate goods firms choose the same price $P_t(i) = P_t$ and employ the same amount of labor $N_t(i) = N_t$. Thus, all firms have the same cash flows and we define gross inflation as $\Pi_t = P_t/P_{t-1}$ and the markup over marginal cost as $\mu_t = 1/\Xi_t$. Therefore, we can model our intermediate-goods firms with a single representative intermediate goods-producing firm. To be consistent with national income accounting, we define a data-consistent measure of output $Y^d_t = C_t$. This assumption treats the quadratic adjustment costs as intermediate inputs. Fluctuations in household discount factors do not affect the equivalent flexible-price version of the baseline model. Therefore, we define the output gap as data-consistent output in deviation from its deterministic steady state $x_t = \ln(Y^d_t/Y^d)$. In addition, the gross natural real interest rate that would prevail in the equivalent flexible-price economy can be defined as $R^n_t = \beta^{-1}a_t(E_ta_{t+1})^{-1}$. Thus, shocks to the household discount factor act as fluctuations in the natural real rate for the economy.

### 3.7 Solution Method

To formally analyze the impact of the zero lower bound, we solve the model using the policy function iteration method of Coleman (1990) and Davig (2004). This global approximation method, as opposed to local perturbation methods such as linearization, allows us to model the occasionally binding zero lower bound constraint. This method discretizes the state variables on a grid and solves for the policy functions which satisfy all the model equations at each point in the state space. Appendix B contains the details of the policy function iteration algorithm.

### 3.8 Calibration

Table 1 lists the calibrated parameters of the model. We calibrate the model at quarterly frequency using standard parameters for one-sector models of fluctuations. Since the model
shares features with the estimated models of Ireland (2003) and Ireland (2011), we calibrate many of the parameters to match the estimates reported by those papers. To assist in numerically solving the model, we introduce a multiplicative constant into the production function to normalize output $Y$ to equal one at the deterministic steady state. We choose steady-state hours worked $N$ and the model-implied value for $\eta$ such that the model has a Frisch labor supply elasticity of two. Household risk aversion over the consumption-leisure basket $\sigma$ is 2. The value for $\sigma$ implies an intertemporal elasticity of substitution of 0.5, which is consistent with the empirical estimates of Basu and Kimball (2002). The fixed cost of production for the intermediate-goods firm $\Phi$ is calibrated to $(\mu - 1)Y$, which eliminates pure profits in the deterministic steady state of the model.

The crucial parameters in our calibration are the parameters that control the stochastic process for the demand shocks. These parameters control the amount and time-variation in uncertainty about the future faced by the economy. We calibrate autoregressive coefficients in both the level and uncertainty shock to be 0.85. For the first-moment shock, this value is similar to the maximum likelihood estimates of Ireland (2003) and Ireland (2011). The unconditional volatility $\sigma^a = 0.02$ and a one standard deviation uncertainty shock raises the volatility of the shock process by one percentage point $\sigma^{a^2} = 0.01$. In Section 4.5, we show that our calibrated model is consistent with both the unconditional and stochastic volatility using data on standard macroeconomic aggregates. In addition, we show that our model can generate lower bound episodes of similar duration to most recent episode in the United States.

### 3.9 Transmission of Precautionary Saving to Macroeconomy

Before examining the computational results, this section shows how precautionary saving by households lowers output and inflation in the macroeconomy. As we discuss in the previous sections, a more volatile and negatively-skewed expected distribution of consumption induces precautionary saving by the representative household. Since consumption and leisure are both normal goods, lower consumption also induces “precautionary labor supply,” or a desire for the household to supply more labor for a given level of the real wage. Figure 1 illustrates this effect graphically in real wage and hours worked space. Through the forward-looking marginal utility of wealth denoted by $\lambda_t$, an increase in uncertainty shifts the household labor supply curve outward through a wealth effect. If prices adjust slowly to changing marginal costs, however, firm markups over marginal cost rise when the household increases its desired
labor supply. For a given level of the real wage, an increase in markups decreases the demand for labor from firms. As Basu and Bundick (2012) discusses, the increase in markups depends crucially on the behavior of the monetary authority. At the zero lower bound, the central bank is unable to offset the increase in markups using its nominal interest rate. Thus, the precautionary saving by households leads to higher markups and lower output and inflation when the economy is stuck at the zero lower bound.

4 Uncertainty Shocks and the Zero Lower Bound

4.1 Traditional Impulse Responses

Figure 2 plots the traditional impulse responses to a one standard deviation uncertainty shock for our model at the stochastic steady state of the model variables. Holding the level of the discount factor shock constant, a 50 percent increase in the volatility of the demand shock causes about a 5 basis point decline in the output gap and inflation. In our following analysis of the zero lower bound, we focus on the relative amount that the zero lower bound amplifies the effects of an uncertainty shock compared to this impulse response at the stochastic steady state.

To compute a traditional impulse response of an uncertainty shock at the zero lower bound, we generate two time paths for the economy. In the first time path, we simulate a large negative first moment demand shock, which causes the zero lower bound to bind for about six quarters. In the second time path, we simulate the same large negative first moment demand shock, but also simulate a one-standard-deviation uncertainty shock. We compute the percent difference between the two time paths as the traditional impulse response to the uncertainty shock at the zero lower bound. Figure 2 also plots the effects of an uncertainty shock under this zero lower bound scenario. When the monetary authority is constrained by the zero lower bound, a one standard deviation uncertainty shock produces a 0.4 percent negative output gap and a 0.25 percent decline in inflation. Thus, the zero lower bound amplifies the uncertainty shock by an order of magnitude.

4.2 Generalized Impulse Responses

Figure 2 shows that the zero lower bound greatly amplifies the negative effects of the uncertainty shock. However, these traditional impulse responses mask the underlying reasons
why the uncertainty shock causes larger contractions at the zero lower bound. Therefore, we now compute the generalized impulse responses as advocated by Koop, Pesaran and Potter (1996). These alternative impulse responses show the distributions of possible outcomes in the future that households expect when making their decisions. Figure 3 plots the mean and 80% prediction intervals both at and away from the zero lower bound.

Away from the zero lower bound, the distribution of possible outcomes for output and inflation remains symmetric and experiences only a modest increase in volatility. Recall Equation (2) from Section 2:

\[ x_t = E_t x_{t+1} - \frac{1}{\sigma} (r_t^e - r_t^n) - \frac{1}{2} \sigma \text{Var}_t x_{t+1} - \frac{1}{6} \sigma^2 \text{Skew}_t x_{t+1} \]

Under its Taylor (1993)-type policy rule, the central bank helps stabilize the economy by responding to movements in inflation and the output gap. Households understand that the central bank will offset adverse shocks using its nominal policy rate. Thus, the monetary authority greatly curtails the spillover of the uncertainty shock to the macroeconomy and the expected volatility of the output gap only modestly increases. Since the central bank is also largely unaffected by the zero lower bound, their ability to offset shocks is nearly symmetric and thus the conditional mean and skewness remain largely unchanged.

At the zero lower bound, however, the asymmetric ability of the central bank to offset shocks endogenously generates downside risk for the economy. Increased uncertainty about the future amplifies this risk and endogenously lowers the conditional mean, increases the conditional volatility, and generates left-skewed distributions of outcomes at the zero lower bound. Households internalize these possible future outcomes, which induces significant precautionary saving by households. This decline in aggregate demand leads to sizable contractions in output and inflation at the zero lower bound. In addition, the duration of the zero lower bound episode is highly uncertain and may persist even four years after the initial shock to the economy. Although the uncertainty shock is relatively short-lived with a half-life of about 4 quarters, the amplification and propagation provided by the zero lower bound causes the downside risk to persist throughout the zero lower bound episode.

4.3 Optimal Monetary Policy Under Commitment

Under a simple monetary policy rule, higher uncertainty at the zero lower bound can have large negative effects on the economy. In this section, we show how an optimal policy maker
under commitment responds when the economy faces significant uncertainty about the future. Appendix C.1 outlines the optimal policy problem and its associated solution. The top panel of Figure 4 plots the previous results under the simple policy rule and the bottom panel shows generalized impulse responses under optimal monetary policy under commitment.

Under optimal policy, the central bank can remove much of the downside risk generated by the zero lower bound. When faced with higher uncertainty at the zero lower bound, the central bank commits to a higher path for the price-level in the future. Since the nominal policy rate is zero, higher expected inflation lowers the current real interest rate through the Fisher relation. Using the intuition from Equation (2), the zero lower bound endogenously generates higher volatility. Since the zero lower bound remains a real constraint on policy, the central bank cannot fully stabilize the economy in the future. However, the central bank can lower the path of real interest rates in an effort to offset the precautionary saving by households. Figure 4 shows that the central bank can attenuate much of the risk posed by the zero lower bound using expectations about future real interest rates. Under the optimal policy, the distribution of the output gap is much less volatile and lack much of the skewness of the results under the simple policy rule.

Under optimal policy, however, the monetary authority must maintain a zero policy rate for an extended period of time. To generate higher inflation in the future at the zero lower bound, the central bank promises to maintain a zero policy rate for a period of time after the initial shock to the economy diminishes. Figure 4, however, shows that the central bank may by forced to maintain a zero policy rate for a very long period of time to help attenuate the effects of higher uncertainty. Under the optimal policy, the economy may not escape the zero lower bound even several years after the initial shock.

In addition, the central bank accepts higher inflation risk in the future to help offset the downside risk generated by the zero lower bound. Figure 4 shows that the distribution of possible outcomes for inflation now displays a slight positive skewness under the optimal policy. By committing to a higher price level in response to adverse fluctuations, the monetary authority is able to minimize much of the downside risk to short-run output and inflation. However, the economy may be continually buffeted by shocks and the central bank may have to keep promising additional inflation in the future. The top panel of Figure 5
shows the implications for the medium-run price level. Even under optimal policy, higher uncertainty at the zero lower bound translates into significant uncertainty about the future price level.

The impulse responses for inflation and the price level provide the key insight into the trade-offs faced by the central bank. When an uncertainty shock hits at the zero lower bound, the central bank understands that the economy faces a great deal of risk because the nominal interest rate is constrained. However, the central bank also understands that the higher uncertainty is not permanent. Therefore, they optimally choose higher and more volatile inflation after the uncertainty shock subsides in order to limit the downside risk when the shock volatility is very high. Thus, the monetary policymaker faces a trade-off between the distribution of medium-run inflation and the distribution of short-run output and inflation when the economy hits the zero lower bound. However, optimal monetary policy does not fully eliminate the risk posed by the zero lower bound. If the volatility of shocks hitting the economy is high, the economy may still experience large fluctuations and fail to escape the zero lower bound for an extended period.

4.4 Optimal Fiscal and Monetary Policy

Even under optimal monetary policy, higher uncertainty at the zero lower bound can cause significant fluctuations for the economy. We now show that fiscal policy may play a significant role in stabilizing the economy in this scenario. We assume that the policymaker has access to government spending financed via lump-sum taxation. Appendix C.2 outlines the optimal policy problem when monetary and fiscal policy are jointly optimal. Figure 5 shows the generalized impulse responses under optimal monetary policy and jointly optimal monetary and fiscal policy.

Access to state-contingent government spending helps stabilize the real economy during period of heightened uncertainty and allows for more effective stabilization of the future price level. At the onset of the uncertainty shock, optimal government spending only slightly increases on average. However, the distribution of possible outcomes for government spending is volatile and slightly positively-skewed after the uncertainty shock. Thus, when the nominal interest rate is constrained, the optimal policymaker uses government spending in a state-contingent manner to help offset shocks that hit the economy. In addition, after the uncertainty shock subsides, the optimal policy slightly lowers government spending on
average but continues to use it to offset shocks. Access to this additional policy instrument allows the policymaker to assume less inflation risk and provides more effective stabilization of the future price level.

4.5 Empirical and Model-Implied Moments

Our simple macroeconomic model shows that higher uncertainty can have significant negative effects at the zero lower bound. The key parameters in our analysis are the stochastic processes for the demand shocks hitting the economy. Thus, we want to ensure that our calibration is reasonable and consistent with macroeconomic data. Therefore, we now compare some of the moments of the simulated model with their counterparts in the data. This exercise allows us to examine whether the distribution of outcomes the representative household uses in evaluating their decisions is in line with the macroeconomic data. To evaluate the simple model, we use data on the output gap, inflation, and the nominal federal funds rate. We measure potential output using the Congressional Budget Office estimate and compute the output gap as the percent deviation between actual and potential output. We use the annualized quarterly percent change in the GDP deflator as our measure of inflation.

We evaluate our simple model using three key features of the recent macroeconomic data. First, we assess the model’s ability to match the unconditional volatility in the data as measured by the sample standard deviation. Second, we evaluate the amount of stochastic volatility in both the data and in the model. To assess the model fit, we want to use a model-free and non-parametric measure of stochastic volatility. To estimate the stochastic volatility, we generate a time-series of the 5-year rolling estimates of the standard deviation for each data series. Then, we compute the standard deviation of the time-series of these estimates. This simple measure provides an estimate of amount of stochastic volatility. If the actual data were homoskedastic, the estimate of the 5-year rolling standard deviations should show little volatility and the resulting statistic would be zero. Finally, we examine the model’s ability to generate zero lower bound scenarios that are consistent with the most recent macroeconomic data. Table 2 shows the results for the empirical and model-implied moments as well as the small sample 90% bootstrapped confidence intervals.

The calibration of our baseline model is consistent with both the unconditional and stochastic volatility in the recent data. We are able to closely match the unconditional volatility of inflation and the standard deviations of the output gap and nominal interest
rate in the data lie well within the confidence intervals of the model. Our baseline model well-approximates the stochastic volatility in the output gap and the nominal interest rate. The stochastic volatility for inflation in the model falls slightly below the actual data, but the data also falls well inside the confidence interval. Finally, in our 30 year sample of data, the model generates the time at the zero lower bound that is similar to the recent U.S. zero lower bound episode.

Without both uncertainty shocks and the amplification provided by the zero lower bound, the calibrated model struggles to match the macroeconomic data. The third column of Table 2 sets the shock volatility constant at its steady state value and the fourth column solves the model without imposing the zero lower bound. Without stochastic shock volatility, both the unconditional and stochastic volatility in the macro aggregates fall dramatically. In almost all of the moments, the empirical moment now falls outside of the confidence interval of the model-implied moment. In addition, the model without uncertainty shocks struggles to generate significant periods of time at the zero lower bound. Similarly, the model struggles to match the data on the output gap and inflation if the zero lower bound is not imposed. If the monetary authority is fully unconstrained, the unconditional and stochastic volatility greatly decline and the moments in the data again fall outside the model-implied confidence intervals. Taken together, Table 2 shows that the model struggles to jointly match these features of the recent macroeconomic data without both uncertainty shocks and the zero lower bound.

5 Discussion and Connections with Literature

5.1 Connections with Existing Literature

These results help generalize the results of Eggertsson and Woodford (2003) and Levin et al. (2010) to an environment where the monetary authority aims to stabilize the economy under uncertainty. Both of these papers examine optimal monetary policy using a standard linearized New-Keynesian model. Using the model of Eggertsson and Woodford (2003), Woodford (2012) emphasizes that credible forward guidance must not let the price-level shift down in response to adverse fluctuations in the economy. When the economy faces significant uncertainty about the future, we additionally show that the central bank must also be concerned with the expected distribution of the price-level. In particular, optimal monetary
policy under uncertainty does not allow downside risk into the expected distribution of the price-level. Levin et al. (2010) shows that the expected value of the output gap and inflation after the natural rate becomes positive acts a terminal condition during the zero lower bound episode. They show that this “forward guidance vector” pins down the path of output and inflation during the initial contraction in a perfect foresight setting. The results of this paper suggest that the entire distribution of output and inflation after the initial contraction pins down the distribution of output and inflation during the zero lower bound period. In an environment of increased uncertainty, optimal monetary policy accepts a more volatile and positively-skewed inflation risk in the medium-run to minimize the short-run downside risk to output and inflation.

5.2 Contractionary Bias in the Nominal Interest-Rate

In addition to the precautionary working mechanism, increases in uncertainty at the zero lower bound can produce an additional source of fluctuations. This additional amplification mechanism, which Basu and Bundick (2012) defines as the contractionary bias in the nominal interest rate distribution, can dramatically affect the economy when uncertainty increases at the zero lower bound. The contractionary bias emerges when the zero lower bound prevents the monetary authority from attaining its inflation goal on average. For this discussion, assume monetary policy sets its desired policy rate using the following simple rule:

\[ r^d_t = r + \phi \pi_t (\pi_t - \pi) \]  
\[ r_t = \max (0, r^d_t) \]  

For a given monetary policy rule, the volatility of the exogenous shocks determines the volatility of inflation. Through the monetary policy rule in Equation (15), the volatility of inflation dictates the volatility of the desired nominal policy rate. However, since the zero lower bound left-truncates the actual policy rate distribution, more volatile desired policy rates lead to higher average actual policy rates. Figure 6 illustrates this effect by plotting hypothetical distributions of the nominal interest rate under low and high levels of exogenous shock volatility. The plot shows that the average actual policy rate is an increasing function of the volatility of the exogenous shocks when monetary policy follows a simple Taylor (1993)-type rule. Reifschneider and Williams (2000) first discuss this phenomenon and Mendes (2011) analytically proves this result using a simple New-Keynesian model.
Changes in the contractionary bias caused by higher uncertainty have dramatic general-equilibrium effects on the economy. Figure 7 plots the average Fisher relation \( r = \pi + r^r \) and the average policy rule under both high and low levels of volatility. The upper-right intersection of the monetary policy rule and the Fisher relation dictates the normal general-equilibrium average levels of inflation and the nominal interest rate. An increase in volatility shifts the policy rule inward and increases the average nominal interest rate for a given level of inflation. Higher volatility thus raises average real interest rates, since it implies a higher level of the nominal interest rate for a given level of inflation. All else equal, higher real interest rates discourage output and put downward pressure on the average level of inflation in the economy. Appendix B of Basu and Bundick (2012) provides numerical evidence that small changes in the contractionary bias caused by higher expected volatility can have dramatic effects on the model economy. For example, Basu and Bundick (2012) shows that a small 0.25 percentage point increase in uncertainty about future demand produces a 0.35 percent decrease in aggregate output when monetary policy follows a simple Taylor (1993)-type rule at the zero lower bound.

Throughout this paper, we follow Reifschneider and Williams (2000) and focus on specifications for monetary policy that remove this alternative mechanism. Adding a small weight on the price-level in our simple policy rule automatically removes the contractionary bias by offsetting any deflation with equivalent inflation in the future. This modeling strategy allows us to isolate the effects of the precautionary behavior by households and show how it makes the economy less responsive to the expected path of interest rates. Without these corrections, an increase in the contractionary bias caused by higher uncertainty implies that the monetary authority misses its unconditional inflation target simply because the zero lower bound binds in a few more states of the world.

This discussion of the contractionary bias helps clarify the economic mechanisms at work in some recent papers in the literature. Recent work by Nakata (2012) and Johannsen (2013) show that higher demand and fiscal uncertainty at the zero lower bound greatly depresses the economy. Both papers use nonlinear New-Keynesian models and assume that monetary policy follows a Taylor (1993)-type rule subject to the zero lower bound. However, neither of these papers make any adjustments for the contractionary bias. Therefore, their results contain the effects of both the contractionary bias and precautionary working mechanisms. While quantitatively large, we argue that the effects of the contractionary bias channel
emerge as a technical consequence of examining changes in uncertainty under a particular simple monetary policy rule. In addition, Bundick (2014) argues that the the uncorrected Taylor (1993)-type rule probably does not represent the actual conduct of Federal Reserve policy at the zero lower bound. Therefore, while these papers also examine the effects of uncertainty at the zero lower bound, they primarily rely on a very different economic mechanism to generate their results.

6 Conclusions

The aim of this paper is to show how uncertainty about the future can affect the ability of the monetary authority to stabilize the economy. In the absence of the zero lower bound, monetary policy could simply alleviate the contractionary effects of uncertainty by lowering its nominal policy rate. When the monetary authority encounters the zero lower bound, the central bank must rely on expectations about the future path of policy. This paper shows that the monetary authority must commit to a more expansive policy when the future is more uncertain. This study emphasizes that policymakers must consider the entire distribution of possible outcomes when evaluating trade-offs at the zero lower bound.
Technical Appendix

A Derivation of Equations From Intuition Section

This section provides a detailed derivation of the equations from Section 2. Using the consumption Euler equation in Equation (1), complete the following steps to derive Equation (2):

1. Multiply and divide the right side of the Euler equation by the steady state values of the real interest rate $R^R$ and consumption $C$ raised to the power $-\sigma$. Apply the natural logarithm and exponential functions inside the conditional expectations. Denote $\tilde{X}_t = \log(X_t/X)$ to write the variables in log-deviations from steady state.

$$1 = E_t \left\{ \beta R_t^R \left( \frac{C_{t+1}}{C_t} \right)^{-\sigma} \right\} = E_t \left( \frac{R_t^R}{R^R} \right) \left( \frac{C_t}{C} \right)^{\sigma} \left( \frac{C_{t+1}}{C} \right)^{-\sigma}$$

$$1 = E_t \left\{ \exp \left( \log \left( \frac{R_t^R}{R^R} \right) - \sigma \log \left( \frac{C_{t+1}}{C} \right) + \sigma \log \left( \frac{C_t}{C} \right) \right) \right\}$$

$$1 = E_t \left\{ \exp \left( \hat{R}_t^R + \sigma \hat{C}_t - \sigma \hat{C}_{t+1} \right) \right\}$$

2. Reorganize, divide by the time $t$ variables, and take the logarithm of both sides.

$$1 = E_t \left\{ \exp \left( \hat{R}_t^R + \sigma \hat{C}_t \right) \exp \left( -\sigma \hat{C}_{t+1} \right) \right\}$$

$$\left( \exp \left( \hat{R}_t^R + \sigma \hat{C}_t \right) \right)^{-1} = E_t \left\{ \exp \left( -\sigma \hat{C}_{t+1} \right) \right\}$$

$$-\hat{R}_t^R - \sigma \hat{C}_t = \log \left( E_t \left\{ \exp \left( -\sigma \hat{C}_{t+1} \right) \right\} \right)$$

3. Replace $\exp \left( -\sigma \hat{C}_{t+1} \right)$ with its Taylor series expansion around $\hat{C}_{t+1} = 0$ and take conditional expectations at time $t$.

$$-\hat{R}_t^R - \sigma \hat{C}_t = \log \left( E_t \left\{ 1 - \sigma \hat{C}_{t+1} + \frac{1}{2} \sigma^2 \hat{C}_{t+1}^2 - \sigma^3 \hat{C}_{t+1}^3 + \ldots \right\} \right)$$

$$-\hat{R}_t^R - \sigma \hat{C}_t = \log \left( 1 - \sigma E_t \hat{C}_{t+1} + \frac{1}{2} \sigma^2 E_t \hat{C}_{t+1}^2 - \sigma^3 E_t \hat{C}_{t+1}^3 + \ldots \right)$$
4. Define 

\[ Z = \sigma_{t}E_{t}\hat{C}_{t+1} - \frac{1}{2}\sigma^{2}E_{t}\hat{C}_{t+1}^{2} + \sigma^{3}E_{t}\hat{C}_{t+1}^{3} - O\left(C_{t+1}^{4}\right) \]

and use the Taylor series expansion of \( \log(1 - Z) = -Z - (1/2)Z^{2} - (1/3)Z^{3} - O\left(Z^{4}\right) \) to expand the previous equation. To compute a third-order approximation, drop all terms that are fourth-order or above. Reorganize the remaining terms to form the conditional variance and conditional skewness:

\[ \text{Var}_{t}\hat{C}_{t+1} = E_{t}\hat{C}_{t+1}^{2} - \left(E_{t}\hat{C}_{t+1}\right)^{2} \]

\[ \text{Skew}_{t}\hat{C}_{t+1} = E_{t}\hat{C}_{t+1}^{3} - 3E_{t}\hat{C}_{t+1}^{2}E_{t}\hat{C}_{t+1} + \left(E_{t}\hat{C}_{t+1}\right)^{3} \].

5. Denote variables in logs using lowercase letters and normalize steady state consumption \( C \) to equal one to derive Equation (2):

\[ c_{t} = E_{t}c_{t+1} - \frac{1}{\sigma}\left(r_{t}^{e} - r^{e}\right) - \frac{1}{2}\sigma\text{Var}_{t}c_{t+1} + \frac{1}{6}\sigma^{2}\text{Skew}_{t}c_{t+1} \]

6. Define a flexible-price version of the previous equation. Subtract the flexible-price version from the actual approximated Euler equation in Step 5. Note in a model with only discount factor shocks, flexible-price consumption is constant since prices always fully adjust. Define the data-consistent output gap \( x_{t} \) as the deviation of consumption from steady state:

\[ x_{t} = E_{t}x_{t+1} - \frac{1}{\sigma}\left(r_{t}^{e} - r_{t}^{n}\right) - \frac{1}{2}\sigma\text{Var}_{t}x_{t+1} - \frac{1}{6}\sigma^{2}\text{Skew}_{t}x_{t+1} \]

**B Numerical Solution Method**

To analyze the impact of uncertainty at the zero lower bound, we solve the model using the policy function iteration method of Coleman (1990). This global approximation method allows us to model the occasionally-binding zero lower bound constraint. This section provides the details of the algorithm when monetary policy follows the simple policy rule in Equation (11) and (12). The algorithm is implemented using the following steps:

1. Discretize the state variables of the model: \( \{a_{t} \times \sigma_{t}^{a} \times P_{t-1}\} \)

2. Conjecture initial guesses for the policy functions of the model \( N_{t} = N(a_{t}, \sigma_{t}^{a}, P_{t-1}) \), \( \Pi_{t} = \Pi(a_{t}, \sigma_{t}^{a}, P_{t-1}) \), \( R_{t} = R(a_{t}, \sigma_{t}^{a}, P_{t-1}) \), and \( R_{t}^{R} = R^{R}(a_{t}, \sigma_{t}^{a}, P_{t-1}) \).
3. For each point in the discretized state space, substitute the current policy functions into the equilibrium conditions of the model. Use interpolation and numerical integration over the exogenous state variable \( a_t \) to compute expectations for each Euler equation. This operation generates a nonlinear system of equations. The solution to this system of equations provides an updated value for the policy functions at that point in the state space. The solution method enforces the zero lower bound for each point in the state space and in expectation.

4. Repeat Step (3) for each point in the state space until the policy functions converge and cease to be updated.

We implement the policy function iteration method in FORTRAN using the nonlinear equation solver DNEQNF from the IMSL numerical library. When monetary policy follows an alternative specification, the state variables and Euler equations are adjusted appropriately.

C Optimal Policy

C.1 Optimal Monetary Policy Under Commitment

The optimal monetary policy maker under commitment aims to maximize the representative household’s utility subject to the constraints of the economy. Some of the constraints include expectations of future variables. Following Khan, King and Wolman (2003), we introduce lagged Lagrange multipliers to make the solutions time-invariant. The augmented
Lagrangian for the optimal policy problem under commitment can be written as follows:

\[
L = \min_{\{\omega_{t+s}\}_{s=0}^{\infty}} \max_{\{d_{t+s}\}_{s=0}^{\infty}} \mathbb{E}_t \left\{ \sum_{s=0}^{\infty} \beta^s \left( \frac{a_{t+s}}{a_t} \right) \left( \frac{C_{t+s}^\eta (1 - N_{t+s})^{1-\eta} 1-\sigma}{1-\sigma} \right) \right. \\
+ \omega_{1t+s} \left( Y_{t+s} - C_{t+s} - \frac{\phi_P}{2} \left( \frac{\Pi_{t+s}}{\Pi} - 1 \right)^2 Y_{t+s} \right) \\
+ \omega_{2t+s} \left( N_{t+s} - \Phi - Y_{t+s} \right) \\
+ \omega_{3t+s} \left( W_{t+s}^R - \frac{1 - \eta}{\eta} C_{t+s} (1 - N_{t+s})^{-1} \right) \\
+ \omega_{4t+s} \left( (\theta - 1) - \theta W_{t+s}^R + \phi_P \left( \frac{\Pi_{t+s}}{\Pi} - 1 \right) \left( \frac{\Pi_{t+s}}{\Pi} \right) \right) \left( a_{t+s} C^\eta (1-\sigma)^{-1} (1 - N_{t+s})^{(1-\eta)(1-\sigma)} Y_{t+s} \right) \\
- \omega_{4t+s-1} \left( \phi_P \left( \frac{\Pi_{t+s}}{\Pi} - 1 \right) \left( \frac{\Pi_{t+s}}{\Pi} \right) \right) \left( a_{t+s} C^\eta (1-\sigma)^{-1} (1 - N_{t+s})^{(1-\eta)(1-\sigma)} Y_{t+s} \right) \\
+ \omega_{5t+s} \left( a_{t+s} C^\eta (1-\sigma)^{-1} (1 - N_{t+s})^{(1-\eta)(1-\sigma)} R_{t+s}^{-1} \right) \\
- \omega_{5t+s-1} \left( a_{t+s} C^\eta (1-\sigma)^{-1} (1 - N_{t+s})^{(1-\eta)(1-\sigma)} \Pi_{t+s}^{-1} \right) \\
+ \omega_{6t+s} \left( R_{t+s} - 1 \right) \right\},
\]

where \(d_t = \{Y_t, C_t, N_t, W_t^R, \Pi_t, R_t\}\) is the set of decision variables and \(\omega_t = \{\omega_{1t}, \omega_{2t}, \omega_{3t}, \omega_{4t}, \omega_{5t}, \omega_{6t}\}\) is the vector of Lagrange multipliers. The final constraint imposes the zero lower bound constraint since the gross nominal policy rate \(R_t\) must be greater than or equal to one. After solving for the first-order conditions, the optimal policy problem is solved using the algorithm outlined in Appendix B. To determine the equilibrium real interest rate \(R_t^R\), we also include the Euler equation for a zero net supply real bond as well. The algorithm solves for the policy functions for \(N_t = N(a_t, \sigma_t^a, \omega_{4t-1}, \omega_{5t-1})\), \(\Pi_t = \Pi(a_t, \sigma_t^a, \omega_{4t-1}, \omega_{5t-1}), R_t = R(a_t, \sigma_t^a, \omega_{4t-1}, \omega_{5t-1}), R_t^R = R^R(a_t, \sigma_t^a, \omega_{4t-1}, \omega_{5t-1}), \omega_{4t} = \omega_4(a_t, \sigma_t^a, \omega_{4t-1}, \omega_{5t-1}), \text{ and } \omega_{5t} = \omega_5(a_t, \sigma_t^a, \omega_{4t-1}, \omega_{5t-1})\) on a discretized state space for \(\{a_t \times \omega_{4t-1} \times \omega_{5t-1}\}\).
C.2 Optimal Fiscal and Monetary Policy Under Commitment

To solve for jointly optimal fiscal and monetary policy under commitment, we make two changes to the previous Lagrangian from Section C.1. We include utility from government spending in the period utility function of the representative household:

\[
\frac{(C_t^{1-\eta} (1 - N_t^{1-\eta})^{1-\sigma})}{1 - \sigma} + \psi G_t^{1-\sigma},
\]

where we choose \( \psi \) to pin down steady state \( G/Y \) to be 20 percent. In addition, the national income accounting identity in the first constraint now includes government spending:

\[
Y_t = C_t + G_t + \phi P \left( \frac{\Pi_t}{\Pi_t - 1} \right)^2 Y_t,
\]

where the set of decision variables is \( d_t = \{ Y_t, C_t, N_t, W_t, \Pi_t, R_t, G_t \} \).
References


Table 1: Calibration of Model Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Calibrated Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>Household Discount Factor</td>
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<tr>
<td>$\phi_P$</td>
<td>Adjustment Cost to Changing Prices</td>
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<tr>
<td>$\Pi$</td>
<td>Steady State Inflation Rate</td>
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</tr>
<tr>
<td>$\sigma$</td>
<td>Parameter Affecting Household Risk Aversion</td>
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<tr>
<td>$\eta$</td>
<td>Consumption Share in Period Utility Function</td>
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<tr>
<td>$\theta$</td>
<td>Elasticity of Substitution Intermediate Goods</td>
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<tr>
<td>$\rho_a$</td>
<td>Preference Shock Persistence</td>
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<td>$\rho_{s\sigma}$</td>
<td>Uncertainty Shock Persistence</td>
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<tr>
<td>$\sigma$</td>
<td>Steady State Shock Volatility</td>
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<tr>
<td>$\sigma_{s\sigma}$</td>
<td>Uncertainty Shock Volatility</td>
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Figure 1: Transmission of Precautionary Labor Supply to Macroeconomy
Table 2: Empirical and Model-Implied Moments

<table>
<thead>
<tr>
<th>Moment</th>
<th>Data 1984 - 2013</th>
<th>Baseline Model</th>
<th>No Stochastic Shock Volatility</th>
<th>No Zero Lower Bound</th>
</tr>
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<tbody>
<tr>
<td>Unconditional Volatility</td>
<td></td>
<td></td>
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<td></td>
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<tr>
<td>$x$</td>
<td>2.52</td>
<td>1.67</td>
<td>0.92</td>
<td>1.24</td>
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<tr>
<td></td>
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<td>(0.80, 1.81)</td>
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<tr>
<td>$\pi$</td>
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<td>0.98</td>
<td>0.60</td>
<td>0.80</td>
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<tr>
<td></td>
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<td>(0.54, 1.14)</td>
<td></td>
</tr>
<tr>
<td>$r$</td>
<td>2.91</td>
<td>2.40</td>
<td>1.86</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(1.68, 3.25)</td>
<td>(1.50, 2.26)</td>
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<tr>
<td>Stochastic Volatility</td>
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<tr>
<td>$x$</td>
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<td>0.71</td>
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<td></td>
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<td>(0.12, 0.37)</td>
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<td>$\pi$</td>
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<td>0.38</td>
<td>0.14</td>
<td>0.27</td>
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<tr>
<td>$r$</td>
<td>0.74</td>
<td>0.72</td>
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<tr>
<td></td>
<td>(0.40, 1.15)</td>
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<tr>
<td>Quarters at Zero Lower Bound</td>
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<tr>
<td></td>
<td>(3, 35)</td>
<td>(1, 20)</td>
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</table>

Note: Unconditional volatility is measured with the sample standard deviation. Stochastic volatility is measured by the standard deviation of the time-series estimate for the 5-year rolling standard deviation. The economy is considered at the zero lower bound if the policy rate falls below 25 basis points. The 90% small sample bootstrapped confidence intervals are given in parenthesis.
Figure 2: Traditional Impulse Responses to Demand Uncertainty Shock

Note: The output gap, price level, and shock volatility responses are plotted as percent deviations. The real interest rate and inflation are plotted in annualized percent deviations. The nominal interest rate is plotted in annualized percent.
Figure 3: Generalized Impulse Responses to Demand Uncertainty Shock

Away From Zero Lower Bound

Output Gap

Inflation

Nominal Interest Rate

At Zero Lower Bound

Output Gap

Inflation

Nominal Interest Rate

Note: The output gap response is plotted in percent deviations. The inflation response is plotted in annualized percent deviations and the nominal interest rate is plotted in annualized percent.
Figure 4: Generalized Impulse Responses Under Optimal Monetary Policy

Note: The output gap response is plotted in percent deviations. The inflation response is plotted in annualized percent deviations and the nominal interest rate is plotted in annualized percent.
Figure 5: Generalized Impulse Responses Under Optimal Fiscal and Monetary Policy

Note: The output gap and government spending responses are plotted in percent deviations. The inflation response is plotted in annualized percent deviations and the nominal interest rate is plotted in annualized percent.
Figure 6: Nominal Interest Rate Distribution with Zero Lower Bound Constraint

Figure 7: Taylor (1993) Rules & Fisher Relation with Zero Lower Bound