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Abstract

We study the implications of patents in an overlapping generations model with horizontal innovation of differentiated physical capital. We show that within this demographic structure of finitely lived agents, weakening patent protection generates two contradicting effects on innovation and growth. Weakening patent protection lowers the (average) price of patented machines, thereby increasing machine utilization, output, aggregate saving, and investment. However, a higher demand for machines shifts investment away from the R&D activity aimed at inventing new machine varieties, toward the formation of physical capital. The growth maximizing level of patent protection is incomplete and we show that shortening patent length is more effective than loosening patent breadth in spurring growth. Shorter patent length has an additional positive effect on growth by decreasing investment in old patents. Finally, we show that the welfare implications of shortening patent breadth depend on consumer time preference and the degree of machine specialization.

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†This work integrates and extends the analyses presented by Diwakar and Sorek in earlier working papers titled "Finite Lifetimes, Patents' Length and Breadth, and Growth" and "Dynamics of Human Capital Accumulation, IPR Policy, and Growth" (circulated as Auburn University Department of Economics Working Papers: AU-WP #2016-11 and #2016-7, respectively). We have benefitted from comments by seminar participant at Auburn University, the 2016 Southern Economic Association conference in Washington DC, and the 2017 North American Econometric Society Summer Meeting in St. Louis MO.

1 Introduction

There is a relatively large literature on the role of patent policy in modern growth theory and the implications of patent strength to R&D-based growth and welfare. The current literature, however, is almost exclusively written about models with infinitely lived agents. This paper utilizes an overlapping generations model to highlight some unique implications of finite lifetimes to patent policy.

In an economy of finitely lived agents, the limited longevity sets a barrier to growth by inducing integenerational trade in productive assets. This point was emphasized by Jones and Manuelli (1992) in a model of physical capital accumulation, and by Chou and Shy (1993) in an endogenous growth model of variety expansion with no physical capital. Both studies employed the canonical Overlapping Generations (OLG) model pioneered by Samuelson (1958) and Diamond (1965), where saving and investment are constrained by labor income[1].

Jones and Manuelli (1992) showed that perpetual growth cannot prevail in the neoclassical OLG economy[2] due to the limited ability of the young to purchase capital held by the old. One of the remedies they consider to support sustained growth in such economy is direct income transfers from old to young. Chou and Shy (1993) emphasized that inter-generational trade in old patent slows down growth as investment in old patents crowds out innovative (R&D) investment in new varieties. They showed that due to this crowding-out effect, which is not present in infinitely-lived agent economy, shortening patent length enhances growth.

To the best of our knowledge, Sorek (2011) is the only other work to study the growth implications of patents in the OLG framework. However, this work focuses on the effect of patents' breadth and length on quality growth (i.e. vertical innovation), where differentiated consumption goods are only produced with labor (i.e. there is no physical capital as in Chou and Shy 1993). In Sorek's (2011) setup, the effect of patent policy on growth depends crucially on the elasticity of inter-temporal substitution, through the effect of the interest rate on life-cycle saving in the OLG model. This effect plays no role in the current analysis (though it is considered in the Appendix).

The present work studies an OLG economy that incorporates both variety expansion and physical capital accumulation, to highlight a unique mechanism through which loosening patents' strength spurs growth. Our analysis places the variety-expansion model proposed by Rivera-Batiz and Romer (1991)[3], into the canonical OLG demographic framework of Samuelson (1958) and Diomaond (1965). Previous works on Rivera-Batiz and Romer's (1991) model economy with infinitely lived agents concluded that growth is maximized with complete patent protection, that is, infinite patent length and complete patent breadth; See Kwan and Lai (2003), Cysne and Turchick

[1] More generally, in economies with finitely lived agents the accumulation of assets is limited by the agent's consumption horizon (longevity).

[2] In other words, the perpetual accumulation of physical capital per-capita.

[3] Barro and Sala-i-Martin (2004) and Aghion and Howitt (2008) adopted this framework as the textbook model of variety expansion; See chapters 6 and 8, respectively.

(2012), and Zeng et al. (2014).[4],[5]

In order to isolate the main effect under study from the aforementioned crowding-out effect[6], we first show that under infinite patent length growth is maximized with incomplete patent breadth. The mechanism at work behind this result involves the trade-off between the static and dynamic effects faced by the patents policy maker. Weakening patent breadth protection works to lower the price of patented machines (by weakening sellers' market power), which in turn increases demand for machines. With more machines being utilized, output and labor income are higher, thus increasing aggregate saving and investment. This is the positive static effect of loosening patent breadth protection on growth.[7] However, higher demand for machines shifts investment away from patents and innovation toward physical capital. This is the negative dynamic effect of weakening patent breadth protection on growth.

We show that the growth maximizing patent breadth depends negatively on the depreciation rate of capital due to the effect of the latter on machines' price. The lower the depreciation rate, the lower the price of physical capital and, therefore, the higher is the demand for physical capital. With initial lower machine prices, there is less potential for growth enhancement through further price decrease induced by loosening patent protection.

The effect of patent policy on growth we are highlighting here is not present in the counterpart model of infinitely living agents, where saving is not bounded by labor income. With infinitely lived agents, the growth rate is determined by the standard Euler condition[8], and thus the effect of patent protection strength on growth works solely through its positive impact on the returns to innovation and, thereby, the interest rate.

Next, we show that, for any positive depreciation rate on physical capital, shortening patent length is more effective in spurring growth than loosening patent breadth protection. Shortening patent length triggers the mechanism presented above while mitigating the crowding out effect as in Chou and Shy (1993).[9] Shortening patent length induces the same effect as loosening patent

[4] These studies differ only in their modelling approach of patent policy. The first two model patent policy through constant imitation rate, which can be also interpreted as stochastic patent duration, as will be explained in Section 4. The last study models patent policy along two (more natural) dimensions: deterministic patent duration and price regulation which is, technically, equivalent to our modeling approach of patent breadth. All these works assume the differentiated inputs are intermediate goods that are formed are formed in the same period they are being used, whereas we consider the differentiated inputs as investment goods (i.e. physical capital) that are formed one period ahead of utilization. Nonetheless, for the infinitely lived agents this assumption does not effect the implications of patent breadth for growth.

[5] In another related work, Iwaisako and Futagami (2013) study the implications of patent policy for growth in a model of infinitely lived agents with physical capital. However, the role of physical capital is completely different than in the present analysis. They use homogenous (raw) physical capital, along with labor, as an input in the production of differentiated consumption goods - to which patent policy applies.

[6] The weakening of breadth protection over all patents evenly (as considered here), does not reduce the crowding-out effect induced by intergenerational trade in old patents.

[7] Since the old are the patent owners, this effect of weakening patent breadth protection is similar to income transfers from the old to the young considered by Jones and Manuelli (1992).Similarly, Uhlig and Yanagawa (1996) showed that reliance on capital-income taxation can also enhance growth.

[8] The familiar Euler condition is given by $\frac{\dot{c}}{c} = \frac{1}{\theta}\left(r - \rho\right)$, where c is per-capita consumption, θ is the inter-temporal elasticity of substitution, ρ is the time preference parameter and r is the interest rate. See for example equations (3),(14) and (15), in Zeng et al. (2014).

[9] This crowding-out reduction could be also achieved by weakening patent breadth protection gradually along

breadth protection by lowering the average price of machine varieties. The expiration of patent over a certain specialized machine results in competition among imitators of this specific variety, which brings its price down to marginal cost. Shorter patent length increases the fraction of competitive machine-industries, thus lowering average machines' price. Compared with Chou and Shy (1993) and Sorek (2011), who found that one-period patent length yields higher growth then infinite patents protection in OLG economy with no physical capital, we also find that one period patent length never maximizes growth in our model economy.

Our welfare analysis shows that enhancing growth by loosening patent breadth protection is preferred by all generations if time preference and the degree of substitution across machine varieties are sufficiently low. This result concurs with Chou and Shy's (1993) analysis of the welfare implications of patent length in their OLG model economy.[10]

Finally, in the last section of the analysis, we present an implication of our main finding for patent policy and economic development. We show that when labor productivity increases relative to innovation cost, due to human capital accumulation, the growth maximizing patent strength corresponds to labor productivity. Hence, as the economy develops, the growth maximizing patent strength is increasing as well. This result provides a normative case for the documented positive correlation between the strength of intellectual property rights (IPR) and economic development worldwide (See Eicher and Newiak 2013, and Chu et al. 2014).

Chu et al. (2014) presented the first analysis of stage-dependent optimal IPR, based on a trade-off between imitation from foreign direct investment (FDI) and reliance on domestic innovation. Our last result provides a complementary case for growth enhancing stage-dependent IPR policy for a closed economy (which is independent of the imitation motive). In an earlier analysis of the topic, Diwakar and Sorek (2016) provide evidence that major developing economies strongly restrict (physical) capital inflows.

The paper proceeds in a straightforward manner. Section 2 presents the model. Section 3 studies the implications of alternative patent policies to growth and welfare. Lastly, Section 4 concludes.

2 Model

Our model uses the variety expansion model with lab-equipment innovation technology and differentiated capital goods proposed by Rivera-Batiz and Romer (1991) together with Diamond's (1965) canonical OLG demographic structure. Each period two overlapping generations of measure L, the "young" and the "old, are economically active. Each agent is endowed with one unit of labor to be supplied inelastically when young. Old agents retire and consume their saving.

The benchmark model presented in this section assumes full patent protection - i.e. infinite patent duration and complete patent breadth protection, implying that in any period innovators can

patents' lifetime. Either way the market value of an old patent will decrease, freeing investment resources for R&D activity.

[10] See Propositions 3-4 on page 310 there.

charge the unconstrained monopolistic price for their patented machines. We study the implications of incomplete patent protection in Section 3.

2.1 Production and innovation

The final good Y is produced by perfectly competitive firms with labor and differentiated capital goods, to which we refer also as "specialized machines".

$$Y_t = AL_t^{1-\alpha} \int_0^{M_t} K_{i,t}^{\alpha}\, \mathrm{d}i \ , \tag{1}$$

where $\alpha \in (0,1)$, A is a productivity factor, L is the constant labor supply, $K_{i,t}$ is the utilization level of machine-variety i in period t, respectively, and M_t measures the number of available machine-varieties.[11] Machines are subject to the depreciation rate $\delta \in (0,1)$ per usage-period, and the price of the final good is normalized to one. Under symmetric equilibrium, utilization level for all machines is the same, i.e. $K_{i,t} = K_t \ \forall \ i$, and thus total output is

$$Y_t = AM_tK_t{}^{\alpha}L^{1-\alpha}. \tag{1a}$$

The representative (perfectly-competitive) firm in the final-good production sector employs specialized machines at the rental price p_i and labor at the market wage w, in order to maximize the profit function

$$\pi_{j,t} = AL_t^{1-\alpha} \int_0^{M_t} K_{i,t}^{\alpha}\, \mathrm{d}i - \int_{i=1}^{M_t} p_{i,t}K_{i,t}\, \mathrm{d}i - w_tL_t \ .$$

The labor market is perfectly competitive and the equilibrium wage and aggregate labor income are $w_t = Y_t = (1-\alpha)\, AM_tK_t{}^{\alpha}L^{1-\alpha}$ and $w_tL = A(1-\alpha)M_tK_t{}^{\alpha}L^{1-\alpha}$, respectively. The profit maximization with respect to each machine variety yields the familiar demand function: $K_{i,t}^{d} = A^{\frac{1}{1-\alpha}}L\left(\frac{\alpha}{p_{i,t}}\right)^{\frac{1}{1-\alpha}}$. Assuming symmetric equilibrium prices and plugging the latter expression back into (1a) we obtain

$$Y_t = A^{\frac{1}{1-\alpha}}M_tL\left(\frac{\alpha}{p_t}\right)^{\frac{\alpha}{1-\alpha}} . \tag{2}$$

Innovation technology follows lab-equipment specification, and the cost of a new blue print is η output units.

[11] The elasticity of substitution between different varieties is $\frac{1}{\alpha}$.

2.2 Preferences

Lifetime utility of the representative agent born in period t is derived from consumption (denoted by c) over two periods, based on the logarithmic instantaneous-utility specification[12]

$$U_t = \ln c_{1,t} + \rho \ln c_{2,t} \ , \tag{3}$$

where $\rho \in (0,1)$ is the subjective discount factor. Young agents allocate their labor income between consumption and saving (denoted by s). The solution for the standard optimal saving problem is $s_t = \frac{w_t}{1+\rho^{-1}}$. Hence, aggregate saving is $S_t = \frac{w_t L}{1+\rho^{-1}}$, which after substituting the explicit expressions for w_t becomes

$$S_t = \frac{(1-\alpha) A^{\frac{1}{1-\alpha}} M_t L \left(\frac{\alpha}{p}\right)^{\frac{\alpha}{1-\alpha}}}{1+\rho^{-1}} \tag{4}$$

2.3 Equilibrium and growth

The patent owners of each machine variety borrow raw physical capital from savers/lenders at the rate $(\delta + r_t)$, where r_t is the net interest. They then transform each unit of raw capital into one specialized machine, at no cost, and the specialized machines are then rented to final output producers at the price p. Hence, given the demand for each machine as previously specified, the per-period surplus from each patented machine, denoted PS, is: $PS_{i,t} = [p_{i,t} - (\delta + r_t)] K^d_{i,t}$.

This surplus is maximized by the standard monopolistic price $p_{i,t} = \frac{\delta + r_t}{\alpha}$. Under infinite patent duration, all new and old varieties are priced equally and, therefore, share the same utilization level. As long as innovation takes place, the market value of old varieties equals the cost of inventing a new one, η. The gross rate of return on investment in patents is given by $1 + r_t = \frac{PS+\eta}{\eta}$. Notice that the numerator in the interest expression contains η because each and every period all patents held by old agents are sold to the young agents.

Plugging the explicit term for the surplus into the interest rate expression yields an implicit expression for the equilibrium interest:

$$\forall t : 1 + r = \frac{[p_{i,t} - (\delta + r_t)] K^d_{i,t} + \eta}{\eta} = \frac{(\delta + r)^{-\frac{\alpha}{1-\alpha}} \left(\frac{1}{\alpha} - 1\right) A^{\frac{1}{1-\alpha}} L \alpha^{\frac{2}{1-\alpha}} + \eta}{\eta}. \tag{5}$$

Equation (5) also defines the no-arbitrage condition that equalizes the net rate of return on investment in patents and investment in physical capital.

Lemma 1 *There exists a unique stationary interest rate, r^*, which solves (5).*

Proof. The left hand side of (2) is increasing linearly in r , from one (for $r = 0$) to infinity. The

[12] It is well known that under the assumed demographic structure, the logarithmic instantaneous utility implies that the saving (and investment) level is independent of the interest rate. In the Appendix, we consider the implications of the general CEIS preference form.

right hand side of (5) is decreasing in r from $\frac{(\delta)^{-\frac{\alpha}{1-\alpha}}\left(\frac{\lambda}{\alpha}-1\right)A^{\frac{1}{1-\alpha}}L\left(\frac{\alpha^2}{\lambda}\right)^{\frac{1}{1-\alpha}}+\eta}{\eta} > 1$ (for $r=0$) to zero (for $r\rightarrow\infty$). Hence, by the fixed point theorem, there exists a positive stationary interest rate r^* that solves (5). ■

As the right hand side of (5) is decreasing with the depreciation rate, so does the equilibrium interest rate, that is $\frac{\partial r^*}{\partial\delta} < 0$. For the case $\delta = 0$, equation (5) yields an explicit solution for the stationary equilibrium rate:

$$r = \left[\left(\frac{1}{\alpha}-1\right)\frac{L}{\eta}\right]^{1-\alpha} A\alpha^2, \text{ for } \delta = 0, \forall t.$$

Under the equilibrium interest rate, aggregate saving is allocated over investment in old and young patents and in physical capital, where the investment in physical capital is set to meet the demand for specialized machines.

$$I_t = M_{t+1}\left[\eta + A^{\frac{1}{1-\alpha}}L\left(\frac{\alpha^2}{r+\delta}\right)^{\frac{1}{1-\alpha}}\right] \tag{6}$$

Equation (5) implies that machines prices are stationary: $\forall t, i : p_{i,t} = \frac{\delta+r^*}{\alpha}$. Hence, Following (2), the output growth rate, denoted $g_{Y,t+1} \equiv \frac{Y_{t+1}}{Y_t}-1$, which coincides with per-capita output growth[13], is equal to the rate of machine-varieties expansion, i.e. $g_{Y,t+1} \equiv \frac{Y_{t+1}}{Y_t} - 1 = g_{M,t+1} \equiv \frac{M_{t+1}}{M_t} - 1$. Imposing the equilibrium condition $S = I$, we equalize (4) and (6), to derive the stationary rate of variety expansion which defines the output growth rate:

$$1 + g_y = \frac{\frac{1-\alpha}{1+\rho^{-1}}\left(\frac{\alpha^2}{r+\delta}\right)^{\frac{\alpha}{1-\alpha}}}{\widehat{\eta} + \left(\frac{\alpha^2}{r+\delta}\right)^{\frac{1}{1-\alpha}}} \tag{7}$$

where $\widehat{\eta} = \frac{\eta}{A^{\frac{1}{1-\alpha}}L}$. It can be shown that sufficiently low $\widehat{\eta}$ guarantees a positive growth rate, and we will generally assume that this the case.

3 Patents

We now explore the implications of patent policy to growth and welfare. The growth implications of patent breadth protection under infinite patent length are studied first. We then demonstrate the greater effectiveness of finite patent length in spurring economic growth. Lastly, we examine welfare and the issue of stage-dependent patent policy.

3.1 Patent breadth and growth

We model patent breadth protection with the parameter λ, which limits the ability of patent holders to charge the unconstrained monopolistic price: $p(\lambda) = \lambda p^* = \frac{\lambda(\delta+r_t)}{\alpha}$ where $\lambda \in (\alpha, 1)$, and thus

[13] As both total population and the labor force are constant.

$p(\lambda) \in \left(1, \frac{1}{\alpha}\right)$. One can think of $p(\lambda)$ as the maximal price a patent holder can set and still deter competition by imitators. Weaker breadth protection lowers the cost of imitation, thereby imposing a lower deterrence price on patent holders.[14] When $\lambda = 1$, patent breadth protection is complete and patent holders can charge the unconstrained monopolistic price $p = \frac{\delta + r_t}{\alpha}$. With zero protection $\lambda = \alpha$, patent holders lose their market power completely and sell at marginal cost. Note that as patent breadth protection is weakened, machines' price is reduced and demand for each machine-variety is increasing. Under this patent breadth policy, the equilibrium interest rate in equation (5) modifies to

$$\forall t : 1 + r = \frac{(\delta + r)^{-\frac{\alpha}{1-\alpha}} \left(\frac{\lambda}{\alpha} - 1\right) A^{\frac{1}{1-\alpha}} L \left(\frac{\alpha^2}{\lambda}\right)^{\frac{1}{1-\alpha}} + \eta}{\eta}. \tag{8}$$

$$\text{For } \delta = 0, \forall t : r = \left[\left(\frac{\lambda}{\alpha} - 1\right)\frac{L}{\eta}\right]^{1-\alpha} A\left(\frac{\alpha^2}{\lambda}\right). \tag{8a}$$

Lemma 2 *The stationary equilibrium interest rate r^* is increasing with patent breadth protection, i.e.* $\frac{\partial r^*}{\partial \lambda} > 0$

Proof. Differentiating the right hand side for λ yields a positive derivative for any $\lambda < 1$. Hence, the value of r^*, which solves (8), is increasing with patent breadth protection λ. ■

Lemma 1 implies that loosening patent breadth protection decreases machines' price, $p(\lambda) = \frac{\lambda(\delta + r^*)}{\alpha}$, through capping the monopolistic markup and by decreasing the marginal cost (of capital) on which this mark up builds. Thus, loosening patent breadth protection increases the demand for each machine variety. This increase in demand for machines has positive effect on aggregate saving (4), for a given variety span:

$$S_t = \frac{(1-\alpha) A^{\frac{1}{1-\alpha}} M_t L \left(\frac{\alpha}{p(\lambda)}\right)^{\frac{\alpha}{1-\alpha}}}{1 + \rho^{-1}}.$$

This is the positive static effect of loosening patent breadth protection on aggregate saving (for a given variety span M_t) and, thereby, innovation and growth. However, for a given level of saving, the increased demand for machines works to shift investment toward physical capital and away from patents. This is the dynamic negative effect of loosening patent breadth protection on innovation and growth. From equation (6) we have:

$$I_t = M_{t+1}\left[\eta + A^{\frac{1}{1-\alpha}} L \left(\frac{\alpha}{p(\lambda)}\right)^{\frac{1}{1-\alpha}}\right].$$

Plugging $p(\lambda) = \frac{\lambda(\delta + r^*)}{\alpha}$ in the above saving and investment expressions and imposing the aggregate constraint $S = I$, we obtain

[14] Similar modeling approach for patent breadth protection was used (among others) by Goh and Olivier (2002), Iwaisako and Futagami (2013), and Chu et al. (2016).

$$1+g_y=\frac{1-\alpha}{1+\rho^{-1}}\frac{\psi^{\frac{\alpha}{1-\alpha}}}{\widehat{\eta}+\psi^{\frac{1}{1-\alpha}}}. \quad (9)$$

Where $\widehat{\eta}=\frac{\eta}{A^{\frac{1}{1-\alpha}}L}$ (as before), and $\psi\equiv\frac{\alpha^2}{\lambda(\delta+r^*)}$. Finally, we denote the growth maximizing policy λ^{**}.

Proposition 1 *For any positive depreciation rate, the growth-maximizing patent breadth is positive but incomplete, and is decreasing with depreciation rate and with . That is* $\forall\delta>0:\alpha<\lambda^{**}<1$ *and* $\frac{\partial\lambda^{**}}{\partial\delta}<0$.

Proof. Differentiating (9) for ψ reveals that the growth rate is increasing with ψ, if $\frac{\alpha}{1-\alpha}\widehat{\eta}>\psi^{\frac{1}{1-\alpha}}$, that is $\frac{\alpha}{1-\alpha}\widehat{\eta}>\left[\frac{\alpha^2}{\lambda(\delta+r^*)}\right]^{\frac{1}{1-\alpha}}$. Hence, under this condition the growth rate is decreasing in λ. Plugging the interest rate in (8a), i.e. for $\delta=0$, in the latter condition yields equality for $\lambda=1$, implying that this condition holds any $\delta>0$ (evaluated at $\lambda=1$). Hence, $\lambda^{**}<1$. For $\lambda=\alpha$ equation (8) yields $r^*=0$, $\forall\delta$. Then, setting $r^*=0$ in (9) yields negative growth rate, $g_y<0$, $\forall\delta$. Hence, $\alpha<\lambda^{**}<1$. Because the interest rate is increasing with δ, the degree of patent breadth protection that maximizes growth, to satisfy $\left[\frac{\alpha^2}{\lambda^{**}(\delta+r^{**})}\right]^{\frac{1}{1-\alpha}}=\frac{\alpha}{1-\alpha}\widehat{\eta}$, is decreasing with the depreciation rate, i.e. $\frac{\partial\lambda^{**}}{\partial\delta}<0$. ■

Proposition 2 *The maximal growth rate that can be achieved with incomplete patent breadth protection is unique:* $1+g_y^{**}=\frac{\alpha^\alpha(1-\alpha)^{2-\alpha}}{(1+\rho^{-1})\widehat{\eta}^{1-\alpha}}$

Proof. By setting $\psi=\psi^{**}\equiv\left(\frac{\alpha}{1-\alpha}\widehat{\eta}\right)^{1-\alpha}$ in the growth equation (9). ■

3.2 Patent length and growth

We turn to study the implications of patent length for growth, under complete patent breadth protection. We study stochastic patent length, assuming that each period a fraction $1-\pi$ of the existing patents expire, where $\pi\in(0,1)$.[15][16] However, all new patents are certain to grant patent for one period (which will expire with probability $1-\pi$ in the second period). This means that the actual lifetime of a patent, denoted T, is the value of $E(T)=1+\frac{\pi}{1-\pi}$ for all new and old patented technologies. Under this specification, the stationary fraction of patented industries, μ, is

$$\mu=\frac{\pi+g}{1+g}\Rightarrow 1-\mu=\frac{1-\pi}{1+g}. \quad (10)$$

Applying (12) to (1) we write the modified output equation:

[15] This formulation has two practical advantages. First, it is a continuous policy instrument although time in this model is discrete. Second, it greatly enhances tractability.

[16] This modelling approach follows Helpman (), Kwan and Lai (2003), and Rubens and Turchick (2012). Their original interpretation was that a fraction π of the patented technologies are being imitated due to a lack of patent-protection enforcement.

$$Y_t = A^{\frac{1}{1-\alpha}} M_t L \left[\frac{\pi + g}{1+g} \left(\frac{\alpha^2}{\delta + r_t} \right)^{\frac{\alpha}{1-\alpha}} + \frac{1-\pi}{1+g} \left(\frac{\alpha}{\delta + r_t} \right)^{\frac{\alpha}{1-\alpha}} \right]. \tag{11}$$

Aggregate saving is still a constant fraction of total output: $S_t = \frac{(1-\alpha)}{1+\rho^{-1}} Y_t$, and the modified investment equation is

$$I_t = M_{t+1} \left[\frac{\pi + g}{1+g} \eta + \frac{\pi + g}{1+g} A^{\frac{1}{1-\alpha}} L \left(\frac{\alpha^2}{\delta + r_{t+1}} \right)^{\frac{1}{1-\alpha}} + \frac{1-\pi}{1+g} A^{\frac{1}{1-\alpha}} L \left(\frac{\alpha}{\delta + r_{t+1}} \right)^{\frac{1}{1-\alpha}} \right]. \tag{12}$$

Imposing $I_t = S_t$ yields the following implicit equation for the stationary-growth rate:

$$1 + g = \frac{\frac{(1-\alpha)}{1+\rho^{-1}} \left[\left(\frac{\alpha^2}{\delta + r_t} \right)^{\frac{\alpha}{1-\alpha}} + \frac{1-\pi}{\pi+g} \left(\frac{\alpha}{\delta + r_t} \right)^{\frac{\alpha}{1-\alpha}} \right]}{\widehat{\eta} + \left(\frac{\alpha^2}{\delta + r_t} \right)^{\frac{1}{1-\alpha}} + \frac{1-\pi}{\pi+g} \left(\frac{\alpha}{\delta + r_t} \right)^{\frac{1}{1-\alpha}}} = \frac{\frac{(1-\alpha)}{1+\rho^{-1}} \psi_t^{\frac{\alpha}{1-\alpha}} \left(1 + \frac{1-\pi}{\pi+g} \alpha^{\frac{-\alpha}{1-\alpha}} \right)}{\widehat{\eta} + \psi_{t+1}^{\frac{1}{1-\alpha}} \left(1 + \frac{1-\pi}{\pi+g} \alpha^{\frac{-1}{1-\alpha}} \right)}. \tag{13}$$

Where $\psi_t \equiv \frac{\alpha^2}{\lambda(\delta + r_t)}$, as before. This equation has only one positive root, and for $\pi = 1$ it coincides with (7). The interest rate under the current patent policy is given by

$$\forall t : 1 + r = \frac{(\delta + r)^{-\frac{\alpha}{1-\alpha}} \left(\frac{\lambda}{\alpha} - 1 \right) \left(\frac{\alpha^2}{\lambda} \right)^{\frac{1}{1-\alpha}} + \pi \widehat{\eta}}{\widehat{\eta}}. \tag{14}$$

The stationary equilibrium interest rate that satisfies (14), r^*, is increasing with the stochastic patent length π, and for $\pi = 1$ it coincides with (5).

Remark 1 *Setting* $\pi = (1 - \delta)$ *in (14) yields* $\delta + r^* = \left(\frac{1-\alpha}{\alpha \widehat{\eta}} \right)^{1-\alpha} \alpha^2$. *Thus, by Proposition 2 we have:* $\psi (\pi = 1 - \delta, \lambda = 1) = \psi^{**} (\pi = 1, \lambda^{**}) \equiv \left(\frac{\alpha}{1-\alpha} \widehat{\eta} \right)^{1-\alpha}$.

Applying the implicit function theorem to (13) we obtain the following expression for $\frac{\partial g}{\partial \pi}$:

$$\frac{-\frac{(1-\alpha)\psi^{\frac{\alpha}{1-\alpha}}}{1+\rho^{-1}} \left[\frac{\alpha \left| \frac{\partial \psi}{\partial \pi} \right| \left(1 + \frac{1-\pi}{\pi+g} \alpha^{\frac{-\alpha}{1-\alpha}} \right)}{(1-\alpha)\psi} + \frac{(1+g)\alpha^{\frac{-\alpha}{1-\alpha}}}{(\pi+g)^2} \right] B + \psi^{\frac{1}{1-\alpha}} \left[\frac{\left| \frac{\partial \psi}{\partial \pi} \right| \left(1 + \frac{1-\pi}{\pi+g} \alpha^{\frac{-1}{1-\alpha}} \right)}{(1-\alpha)\psi} + \frac{(1+g)\alpha^{\frac{-1}{1-\alpha}}}{(\pi+g)^2} \right] A}{\frac{(1-\alpha)\psi^{\frac{\alpha}{1-\alpha}} (1-\pi)\alpha^{\frac{-\alpha}{1-\alpha}}}{(1+\rho^{-1})(\pi+g)^2} B - \frac{\psi^{\frac{1}{1-\alpha}} (1-\pi)\alpha^{\frac{-1}{1-\alpha}}}{(\pi+g)^2} A + \left[\widehat{\eta} + \psi^{\frac{1}{1-\alpha}} \left(1 + \frac{1-\pi}{\pi+g} \alpha^{\frac{-1}{1-\alpha}} \right) \right]^2} \tag{15}$$

Where A and B are the numerator and denominator in the right hand side of (13), respectively. Based on the above remark and equation (15), we obtain the following proposition.

Proposition 3 *For any positive depreciation rate, finite patent length can yield a higher growth than incomplete patent breadth protection.*

Proof. Substituting the $\psi = \psi^{**} = \left(\frac{\widehat{\eta}\alpha}{1-\alpha}\right)^{1-\alpha}$ into (13) yields the growth rate obtained in Proposition 2. However, substituting $\psi = \left(\frac{\widehat{\eta}\alpha}{1-\alpha}\right)^{1-\alpha}$ into (15) shows that both the numerator and denominator are positive for any positive depreciation rate, that $\forall\delta > 0 : \frac{\partial g}{\partial \pi}|_{\pi=1-\delta} > 0$. For zero depreciation rate the numerator is zero and the denominator is still positive That is, for $\delta = 0 : \frac{\partial g}{\partial \pi}|_{\pi=1-\delta} = 0$. Hence, for any positive depreciation rate, growth under finite patent length can be enhanced beyond the maximal rate defined in Proposition 2, by increasing expected patent length beyond 1+ $\frac{1-\delta}{\delta}$. That is $\pi^{**} > 1-\delta$. ■

3.3 Patents and Welfare

This section will briefly explore the welfare implications of loosening patent protection. To maintain tractability we focus on patent breadth protection. We follow Chou and Shy (1993) in comparing the lifetime utility of all living generations under alternative stationary patent policies. Substituting the explicit expressions for c_1 and c_2, implied by the saving equations in Subsection 2.2, into the lifetime utility function (3) yields the indirect lifetime utility of the representative consumer who was born in period t:

$$U_t = \ln\left[\frac{(1-\alpha)M_t A^{\frac{1}{1-\alpha}}\left(\frac{\alpha}{p}\right)^{\frac{\alpha}{1-\alpha}}}{1+\rho}\right] + \rho\ln\left[\frac{\rho(1-\alpha)M_t A^{\frac{1}{1-\alpha}}\left(\frac{\alpha}{p}\right)^{\frac{\alpha}{1-\alpha}}}{1+\rho}(1+r^*)\right]. \tag{16}$$

Equation (16) implies that $U_t = U_{t-1} + (1+\rho)\ln(1+g)$, and thus

$$U_t = U_0 + t(1+\rho)\ln(1+g). \tag{16a}$$

where U_0 is given by evaluating (16) for M_0. By Propositions 1, for any positive depreciation rate, the stationary growth rate g is maximized with incomplete breadth protection. Hence, by (16), a sufficient condition for incomplete breadth protection to increase the lifetime utility of all generations who are born in the present and future periods, is having $\frac{\partial U_0}{\partial\lambda}{}_{\lambda=1} < 0$. The sign of the derivatives $\frac{\partial U_0}{\partial\lambda}$ depends on the sign of the following expression:

$$\left[\frac{\rho}{1+r} - \frac{\alpha}{1-\alpha}\frac{(1+\rho)}{(\delta+r^*)}\right]\frac{\partial r}{\partial\lambda} - \frac{\alpha}{1-\alpha}\frac{(1+\rho)}{\lambda}.$$

However, Lemma 1 implies $\frac{\partial r^*}{\partial\lambda} > 0$. Hence, having a negative term in the brackets of (16a) is sufficient condition to assure $\frac{\partial U_0}{\partial\lambda} < 0$. This sufficient condition holds if $\frac{\rho}{1+\rho} < \frac{\alpha}{1-\alpha}$.[17] This result is summarized in our last proposition.

Proposition 4 *Higher growth under incomplete patent breadth protection benefits all generations if time preference and the degree of substitution across varieties are sufficiently low, such that* $\frac{\rho}{1+\rho} < \frac{\alpha}{1-\alpha} \Leftrightarrow \frac{1}{\alpha} < \frac{1}{\rho} + 2$.

[17] Recall that the elasticity of substitution across variety is $\frac{1}{\alpha}$.

Proposition 4, which relies on comparison between two alternative stationary policies concurs with the results obtained in the welfare analysis of Chou and Shy (1993). However, the direct transitional impact of loosening patent breadth policy at a certain period will not yield Pareto improvement even if the above proposition holds. At period zero, the amount of available machines is already pre-determined, and thus, decreasing their price can not increase their utilization level.

Hence, the positive effect on aggregate saving will not prevail, and only the negative effect on second-period consumption (due to lower interest rate) will be at work. Therefore, in this case transfers from the next young generation (to be born in period one) to the current young generation will be required to maintain Pareto improvement. However, the complete analysis of this issue falls beyond the scope of the current study.

3.4 Stage-dependent patent policy

Proposition 2 implies that the growth maximizing patent policy depends on the value of $\widehat{\eta} \equiv \frac{\eta}{A^{\frac{1}{1-\alpha}}L}$. This term can be interpreted as innovation-cost per effective labor supply, denoted $H \equiv A^{\frac{1}{1-\alpha}}L$.[18] However, the value of these parameters may be associated with the economy's development stage. Labor productivity is typically increasing along the course of economic development through the accumulation of human capital. Similarly, the literature on R&D driven growth has considered alternative endogenous dynamics of the innovation cost due to inter-temporal knowledge spillover. This works to decrease per-variety invention cost and the stepping on toes ("fishing out") effect (which works to increase invention cost as R&D efforts increase), and distance from the frontier.[19]

Proposition 2 suggests that the growth-maximizing patent protection is decreasing with $\widehat{\eta}$. In this subsection, we attempt to formalize this result. Adding the time subscript to the relevant parameters, we re-write the output and growth equations

$$Y_t = M_t H_t \left[\frac{\alpha}{p_t\left(\lambda_t\right)}\right]^{\frac{\alpha}{1-\alpha}} \tag{17}$$

$$1 + g_{M,t+1} = \frac{(1-\alpha)}{1+\rho^{-1}} \frac{H_t \left[\frac{\alpha}{p_t(\lambda_t)}\right]^{\frac{\alpha}{1-\alpha}}}{\eta + H_{t+1}\left[\frac{\alpha}{p_{t+1}(\lambda_{t+1})}\right]^{\frac{1}{1-\alpha}}} \tag{17a}$$

where $p_t\left(\lambda_t\right) = \frac{\lambda_t(\delta+r_t)}{\alpha}$, as before, and the interest rate follows the modified no-arbitrage condition

$$1 + r^*_{t+1} = \frac{(\delta + r^*_{t+1})^{\frac{-\alpha}{1-\alpha}} \left(\frac{\lambda_{t+1}}{\alpha} - 1\right) \lambda_{t+1}^{\frac{-1}{1-\alpha}} \alpha^{\frac{2}{1-\alpha}} + \widehat{\eta}_{t+1}}{\widehat{\eta}_{t+1}} \tag{17b}$$

Equation (17) implies the following growth rate of per-capita output

[18] If $A^{\frac{1}{1-\alpha}}$ is interpreted as labor augmented productivity factor we can write (1) as : $Y = MK^{\alpha}\left(A^{\frac{1}{1-\alpha}}L\right)^{1-\alpha}$.

[19] See Jones (1995) and Segerstrom (1998), for example. Jones (1999) provides a compact summary of the topic.

$$1+g_{y,t+1}=\left(1+g_{M,t+1}\right)\left(1+g_{H,t+1}\right)\left(1+g_{p(\lambda),t+1}\right)^{\frac{-\alpha}{1-\alpha}} \tag{18}$$

Combining equations (18) with (17a) yields

$$1+g_{y,t+1}=\frac{(1-\alpha)}{1+\rho^{-1}}\frac{\psi_{t+1}^{\frac{\alpha}{1-\alpha}}}{\widehat{\eta}_{t+1}+\psi_{t+1}^{\frac{1}{1-\alpha}}} \tag{18a}$$

Notice that the growth equation (18a) depends only on the patent policy expected to prevail in period $t+1$.

Proposition 5 *The growth maximizing policy,* $\psi_t^{**}\equiv\left(\frac{\alpha}{1-\alpha}\widehat{\eta}_t\right)^{1-\alpha}$*, implies that patent breadth is decreasing with* $\widehat{\eta}_{t+1}$*.*

Proof. Proposition 1 implies that this growth rate is maximized with $\psi_{t+1}^{**}\equiv\left(\frac{\alpha}{1-\alpha}\widehat{\eta}_{t+1}\right)^{1-\alpha}$. Rewriting the interest rate expression (17b) we obtain: $\left(\frac{r_{t+1}^*}{\delta+r_{t+1}^*}\frac{\alpha}{\lambda_{t+1}-\alpha}\widehat{\eta}_{t+1}\right)^{1-\alpha}=\underbrace{\frac{\alpha^2}{\lambda_{t+1}\left(\delta+r_{t+1}^*\right)}}_{=\psi_{t+1}}$.

Hence, the growth maximizing condition is satisfied with $\frac{r_{t+1}^*}{\delta+r_{t+1}^*}=\frac{\lambda_{t+1}-\alpha}{1-\alpha}$. Clearly, for $\delta=0$ growth is maximized by the stationary policy of complete breadth protection. However, for any $\delta>0$, as $\widehat{\eta}_{t+1}$ is decreasing (increasing), the left hand side of the latter condition is also increasing. Then, in order to restore the equality patent breadth protection should also increase (decrease). Then, by strengthening patent breadth protection, the right hand side is increasing while the left hand side is increasing at a lower rate. ■

4 Conclusion

This work proposes a contribution to the literature on patent policy and economic growth by exploring the implications of patent policy in an OLG framework with physical capital. We have highlighted a novel mechanism through which weakening patent protection can enhance growth, which is unique to the OLG demographic structure of finitely lived agents. This mechanism involves a trade-off between the effect of patent strength on aggregate saving and investment and the allocation of total investment between patent ownership and physical capital. This positive effect can be induced by either shortening patent length or loosening patent breadth protection. However, shortening patent length also mitigates the crowding out effect of trade in old patents on R&D investment. Hence, shortening patent length can be more effective at generating growth than loosening patent breadth protection. These effects are not present in similar models with infinitely lived agents. Consequently, growth in these models is maximized with eternal patent life and complete patent breadth protection.

Finally, we have also presented an important implication of the main mechanism under study to patent policy and economic development. A stage-dependent patent policy for which patent strength is increasing over the course of economic development may be growth maximizing. This result provides a normative case for the often observed positive correlation between patent strength and economic development around the world.

References

[1] Aghion P., Howitt P., 2009. The economics of growth. MIT Press. Cambridge. MA

[2] Barro R.J., Sala-i-Martin, X., 2004. Economic growth, Second Edition. MIT Press. Cambridge. MA

[3] Chu A.C., Cozzi G., Galli S., 2014. Stage-dependent intellectual property rights. Journal of Development Economics 106, 239–249

[4] Chu A., Furukawa Y., Ji L., 2016. Patents, R&D subsidies, and endogenous market structure in a Schumpeterian economy. Southern Economic Journal 82, 809–825

[5] Chou C., Shy O., 1993. The crowding-out effects of long duration of patents. RAND Journal of Economics 24, 304–312

[6] Cysne R., Turchick D., 2012. Intellectual property rights protection and endogenous economic growth revisited. Journal of Econonomic Dynamics & Control 36, 851–861

[7] Diwakar B., Sorek G., 2016. Dynamics of Human Capital Accumulation, IPR Policy, and Growth, Auburn University Economics Working papers Series, AUWP 2016-11, available at: http://cla.auburn.edu/econwp/Archives/2016/2016-11.pdf

[8] Diamond P., 1965. National debt in a neoclassical growth model. American Economic Review 55, 1126–1150

[9] Eicher T.S., Newiak M., 2013. Intellectual property rights as development determinants. Canadian Journal of Economics 46, 4–22

[10] Engelhardt, G.V., Kumar, A., 2009. The elasticity of inter temporal substitution: new evidence from 401(k) participation. Economics Letters 103, 15–17.

[11] Goh A.T., Olivier J., 2002. Optimal patent protection in a two-sector economy. International Economic Review 43, 1191–1214

[12] Iwaisako T., Futagami K., 2013. Patent protection, capital accumulation, and economic growth. Economic Theory 52, 631–68

[13] Hall R., 1988. Intertemporal substitution in consumption. Journal of Political Economy 96, 339–357

[14] Jones L. E., Manuelli R.E., 1992. Finite Lifetimes and Growth. Journal of Economic Theory 58, 171-197

[15] Kwan Y.K., Lai E.L.-C., 2003.Intellectual property rights protection and endogenous economic growth. Journal of Economic Dynamics & Control 27, 853–873

[16] Ogaki M., Reinhart C.M., 1998. Measuring intertemporal substitution: the role of durable goods. Journal of Political Economy 106, 1078–1098

[17] Rivera-Batiz L., Romer P.M., 1991. Economic Integration and Endogenous Growth. Quarterly Journal of Economics 106, 531-56

[18] Samuelson P.A., 1958. An exact consumption-loan model of interest with or without the social contrivance of money. Journal of Political Economy 66, 467–82

[19] Sorek G., 2011. Patents and quality growth in OLG economy. Journal of Macroeconomics 33, 690-699

[20] Uhlig H., Yanagawa N, 1996. Increasing the capital income tax may lead to faster growth. European Economic Review 40, 1521–1540

[21] Zeng J., Zhang J., Fung, M. K.-Y, 2014. Patent length and price regulation in an R&D growth model with monopolistic competition. Macroeconomic Dynamics 18, 1-22

Appendix: CEIS utility

We turn here to consider the implication of the general CEIS instantaneous utility to our previous result, considering the following lifetime utility form:

$$U = \frac{c_t^{1-\theta}}{1-\theta} + \rho \frac{c_{t+1}^{1-\theta}}{1-\theta} \tag{A.1}$$

where $\frac{1}{\theta}$ is the elasticity of inter-temporal substitution, and for $\theta = 1$ (26) falls back to the logarithmic form (3). The modified solution for the standard optimal saving problem is $s_t = \frac{w_t}{1+\rho^{-\frac{1}{\theta}}(1+r)^{\frac{\theta-1}{\theta}}}$. Hence, aggregate saving now is $S_t = \frac{w_t L}{1+\rho^{-\frac{1}{\theta}}(1+r)^{\frac{\theta-1}{\theta}}}$ .Substituting the explicit expressions for w_t into S_t and equalizing to aggregate investment given in (18) yields the growth equation

$$1 + g_y = \frac{1-\alpha}{1+\frac{1}{\rho}(1+r)^{1-\frac{1}{\theta}}} \frac{\left(\frac{\alpha^2}{\lambda(\delta+r)}\right)^{\frac{\alpha}{1-\alpha}}}{\widehat{\eta} + \left(\frac{\alpha^2}{\lambda(\delta+r)}\right)^{\frac{1}{1-\alpha}}} \tag{A.2}$$

As it is well known, in the standard OLG framework the effect of interest rate on saving depends on the inter-temporal elasticity of substitution: it is positive (negative) if $\theta < 1$ ($\theta > 1$). Hence, because the interest rate is increasing with patent protection, the positive impact of decreasing patent breadth on growth is diminishing with the inter-temporal elasticity of substitution. More specifically, for $\theta < 1$ all our results remain (and will hold for a larger set of parameters) as a decrease in the interest rate by itself stimulates saving and investment (this is an additional effect was not induced under the logarithmic utility form). However, as θ increases beyond one, the decrease in the interest rate due to loosening patent breadth protection will work to hinder growth, countering the positive effects that were defined in Proposition 1. For sufficiently high value of θ this direct interest effect may dominate the over all impact of loosening patent protection on innovation and growth. Nevertheless, the empirical literature commonly suggests that θ is lower than one, and thus supporting the relevance of our main findings[20]. The welfare analysis for $\theta \neq 1$ turns out being intractable.

[20] See for example Hall (1988), Beaudry and Wincoop (1996), Ogaki and Reinhart (1998), Engelhardt and Kumar (2009).