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**Abstract**

We show that non-linear dynastic altruism toward future generations yields non-monotonic relation between population growth and economic prosperity, which is polynomial in general. The exact shape of this non-monotonic relation depends on the concavity of parental altruistic utility. Hence, this work contributes to the recent line of modified R&D-based growth models, aimed to align theory with empirical evidence on non-linear relation between population growth and economic prosperity.

**JEL Classification**: : O-31, O-40
**Key-words**: Dynastic Altruism, Population Growth, Technological Progress

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# 1 Introduction

This work contributes to the recent line of modified R&D-based growth models, aimed to align theory with the ambiguous empirical findings regarding the relation between fertility, innovation, and economic growth[1]. We show that different specifications of parental altruism toward future generations, in particular linear vs. non-linear ones, yield corresponding alternative non-monotonic relations between population growth and economic prosperity.

Recent modifications to the R&D-based growth models aimed to remove the "weak scale-effect", which was presented in the second and third generation models - i.e. the counterfactual positive relation between population growth and economic growth[2]. This line of research incorporated human capital as productive input in the R&D sector, thereby, giving room to substitution between the quantity and quality of workers in overall labor supply. This substitution enables an increase in overall effective labor supply, and thereby growth rate, even for a constant or declining population of workers.

Several works in this literature have emphasized the role of dynastic altruism toward future generations, in determining the effect of population growth on economic prosperity. Dynastic altruism stimulates saving and investment in human capital. This positive effect of altruism on saving is increasing with population growth rate, and may overcome the negative diluting effect of population growth on human capital accumulation[3]; See for example Dalgaard and Kreiner (2001), Strulik (2005), Bucci (2008), and Bucci (2013)[4]. In these studies, parents' altruistic utility is linear in the number of offsprings (for a given per-child consumption level), and the effect of population growth on technological progress depends on the values of model parameters - i.e. it is monotonic given the parameters set.

We depart from the current literature by introducing nonlinear parental altruism in offspring number. We establish non-monotonic relation between population growth rate and economic growth rate, that is polynomial in general. In particular, we show that the non-monotonic relations between population growth rate and economic prosperity may vary from *U shape* to *hump shape*, depending on the concavity of parental altruistic utility. The hump shape relation is consistent with the empirical findings reported by Boikos et al.(2013) and Kelley and Schmidt (1995).

Our analysis of non-linear parental altruism follows the influential papers on fertility and economic growth by Barro and Becker (1989) and Becker et al.(1990): parents' "selfish" utility - from their own consumption, is higher than their altruistic utility - from the consumption of their offspring, and the degree of parental altruism for each child is decreasing with the number of kids[5]. This formulation implies also that for given per-child consumption level, altruistic utility is concave

[1] Recent summaries of the empirical literature can be found in Strulik et al. (2013) and Bucci (2015).

[2] See Jones (1999) for a compact summary of this literature.

[3] If human capital is not purely non-rival, population growth works to decrease per-capita human capital as the human capital of new born is lower than the average of existing workers.

[4] In Bucci (2008) and Bucci (2013) the total effect of population growth on economic prosperity depends also on the effect of technological progress on human capital accumulation, and on the returns to specialization, respectively.

[5] The microeconomic foundations for these works were laid in Becker and Barro (1988), and their broader implications to economic growth were summarized in Becker (1992).

in the number of children.

The remainder of the paper is organized as follows. Section 2 presents the detailed model. Section 3 analyzes the dynamic equilibrium and the effect of population growth on technological progress, and Section 4 concludes this study.

## 2 The Model

We extend Young's (1998) two-sector R&D model by incorporating population growth, human capital accumulation, and dynastic altruism. Time is discrete, and population grows at exogenous rate $n \geq 0$. Population size in each period is denoted $L_t = L_0(1+n)^t$, where $L_0$ is normalized to one. In each period, each worker is endowed with one unit of time.

### 2.1 Preferences

Consumer's lifetime utility is given by

$$U = \sum_{t=0}^{\infty} \rho^t (1+\theta n)^t \ln(c_t) \tag{1}$$

where $\rho, \theta \in (0,1)$ are the time preference and the degree of altruism, respectively. The current literature, focused on linear specification of the altruism factor, implying that $\theta$ is scalar; See for example Strulik (2005), Bucci (2008, 2013)[6]. Here, we let the degree of altruism per-child to depend on number of offspring, that is, $\theta \equiv \theta(n)$. Following Barro and Becker (1988, 1989), Becker et al.(1990) and Becker (1992), we assume $\theta(n) = \theta_0 n^{-\gamma}$, hence $\theta(n) n = \theta_0 n^{1-\gamma}$, where $\gamma, \theta_0 \in (0,1)$. The assumption $\theta_0 < 1$ implies that parent's "selfish" utility from her own consumption, has a higher weight than her altruistic utility from per-child consumption, in line with the latter references. To assure that (1) has finite values we assume $\rho(1+\theta n) < 1$. The aggregate instantaneous utility from consuming in (1), denoted $c$, is derived from utilizing $N$ differentiated varieties, denoted $c_i$, subject to a CES utility function

$$c_t = \left( \sum_{i=1}^{N_t} c_{i,t}^{\frac{1}{\varepsilon}} \right)^{\varepsilon} \tag{1a}$$

with $\varepsilon = \frac{s}{s-1}$, and $s$ is the elasticity of substitution across all varieties. The consumption level of each variety is defined: $c_i = q_i x_i$, where $x_i$ and $q_i$ designate utilized quantity and quality, respectively.

[6] At the two extremes, with $\theta = 0$ or $\theta = 1$ preferences are of Millian or Benthamite type, respectively.

The assumed preferences imply the following instantaneous demand for each variety

$$x_{i,t}^{d} = q_{i,t}^{s-1} \left(\lambda p_{i,t}\right)^{-s} \left( \sum_{i=1}^{N_t} c_{i,t}^{\frac{1}{\varepsilon}} \right)^{\varepsilon} \tag{1b}$$

Where $\lambda$ is the Lagrange multiplier from the instantaneous utility maximization, i.e. the shadow value of the given periodic spending level. The logarithmic specification in (1) implies the standard Euler condition for optimal consumption smoothing, written in terms of aggregate spending, denoted $E$

$$\frac{E_{t+1}}{E_t} = \rho(1+\theta n)(1+r_{t+1}) \tag{2}$$

where $(1+r_{t+1})$ is the (gross) interest rate earned between periods $t$ and $t+1$.

## 2.2 Production and innovation

Effective labor supply is the sole input for production and innovation, and the wage rate is normalized to one. One unit of labor produces one unit of consumption good (regardless of its quality). Following Young (1998), innovation is certain and is subject to the cost function

$$f(q_{i,t+1}, \overline{q}_t) = \begin{cases} \exp\left(\phi \frac{q_{i,t+1}}{\overline{q}_t}\right) & q_{i,t+1} > q_{i,t} \\ \exp\left(\phi\right) & q_{i,t+1} \leq q_{i,t} \end{cases} \tag{3}$$

Innovation cost in sector $i$ is increasing with rate of quality improvement over the existing quality frontier - denoted $\overline{q}_t$, which is the highest quality to be already developed in the economy. As innovation is certain, vertical innovation (i.e. quality improvements) implies that the effective lifetime of each product is one period. Hence, each firm maximizes the following profit, denoted $\Pi$

$$\Pi_{i,t} = \frac{(p_{i,t+1}-1)x_{i,t+1}^{d} L_{t+1}}{1+r_{t+1}} - f(q_{i,t+1}, \overline{q}_t) \tag{4}$$

Maximizing (4) for $p_{i,t+1}$ yields the standard optimal monopolistic price $p^* = \varepsilon$, $\forall t, i$ . The first order condition for optimal quality choice is derived after plugging the optimal price and the demand function (1b) into (4)

$$\frac{1}{q_{i,t+1}^{*}} \frac{(\varepsilon-1)(s-1)\left(\lambda\varepsilon\right)^{-s} \left(\sum_{i=1}^{N_t} c_{i,t+1}^{\frac{1}{\varepsilon}}\right)^{\varepsilon} L_{t+1}}{1+r_{t+1}} = \frac{\phi}{\overline{q}_t} f(q_{i,t+1}^{*}, \overline{q}_t) \tag{5}$$

The asterisk superscript denotes optimally chosen values. Assuming free entry to the R&D sector implies that in equilibrium the profit in (4) equals zero. Combining this assumption with the optimality condition (5) we obtain the equilibrium rate of quality improvement $\forall_i : 1+g_q \equiv \frac{q_{t+1}^{*}}{\overline{q}_t} = \frac{s-1}{\phi}$. The cost parameter $\phi$ is assumed to be low enough to guarantee $g_q > 0$, and to make vertical competition between successive product generations redundant, i.e. $p^* < 1 + g_q$. As the rate of quality improvement is time invariant, so is the innovation cost. Hence, to enhance exposition, from hereafter we denote the actual innovation cost simply $f$. Notice that under symmetric equilibrium demand for each variety is $x_t^d = \frac{E_t}{\varepsilon N_t L_t} \forall_i$, and thus, the free entry condition implies that (4) can be written as

$$\frac{(1-\frac{1}{\varepsilon})\frac{E_{t+1}}{N_t+1}}{f} = 1 + r_{t+1} \tag{6}$$

Combining (2) and (6) we obtain

$$E_t = \frac{fN_{t+1}}{(1-\frac{1}{\varepsilon})\rho(1+\theta n)} \tag{7}$$

and plugging (7) back into (6) yields the interest rate

$$1 + r_{t+1} = \frac{1 + g_{N,t+1}}{\rho(1+\theta n)} \tag{8}$$

where $1 + g_{N,t+1} \equiv \frac{N_{t+1}}{N_t}$.

### 2.3 Human capital formation

Human capital formation is subject to the conventional specification[7]

$$\begin{aligned} h_{t+1} &= \frac{(\xi e_t + 1 - \delta)\, h_t}{(1+n)} \\ &\Rightarrow \;\triangle h_{t+1} \equiv h_{t+1} - h_t = \left[\frac{(\xi e_t + 1 - \delta)}{(1+n)} - 1\right] h_t \end{aligned} \tag{9}$$

where $h$ is per-capita human capital, and $e \in (0, 1)$ is the time invested in human capital formation. Effective labor supply, denoted $H$, is defined as the product of population size and per-capita human capital: $H_t = L_t h_t$. Following (7) we define the growth rate of per-capita human capital $(1 + g_{h,t+1}) \equiv \frac{h_{t+1}}{h_t} = \frac{(\xi e_t+1-\delta)}{(1+n)}$, and the growth rate of effective labor supply

$$1 + g_{H,t+1} \equiv \frac{H_{t+1}}{H_t} = (1 + g_{h,t})\,(1+n) = (\xi e_t + 1 - \delta) \tag{10}$$

The return on investment in human capital should equal the return on R&D investment

$$1 + r_{t+1} = \frac{(\xi e_t + 1 - \delta)\, h_t}{e_t h_t} \tag{11}$$

Plugging the interest rate (8) in (11) yield time investment in education

$$\forall t : e^* = \frac{(1-\delta)}{\frac{(1+g_{N_{t+1}})}{\rho(1+\theta n)} - \xi} \tag{12}$$

[7] For $\delta, n = 0$ this formulation coincides with Lucas' (1988).

# 3 Population Growth and Economic Prosperity

Our analysis is confined to the stationary (steady state) equilibrium, implying that the growth rates of all variables are time invariant. Plugging (12) back into (10) yields

$$1 + g_H = \frac{(1-\delta)}{1 - \frac{\rho(1+\theta n)\xi}{1+g_N}} \tag{13}$$

The aggregate resources-uses constraint for the economy is defined by the allocation of labor supply over production, education, and R&D investment

$$(1-e^*)\,H_t = \frac{E_t}{\varepsilon N_t} + fN_{t+1} \tag{14}$$

Plugging (7) into (14) yields

$$\begin{aligned}(1-e^*)\,H_t &= \frac{fN_{t+1}}{(\varepsilon-1)(1+\theta n)\rho} + fN_{t+1} \\ \Rightarrow \quad N_{t+1} &= \frac{(1-e^*)\,H_t}{f\left[\frac{1}{(\varepsilon-1)(1+\theta n)\rho} + 1\right]}\end{aligned} \tag{15}$$

Hence, variety expansion rate equals the exogenous growth rate of effective labor supply, that is $(1+g_N) = (1+g_H)$, which, following (9)-(10), implies

$$1 + g_N = (1-\delta) + \xi\rho(1+\theta n) \tag{15a}$$

Observe that under symmetric equilibrium, equation (1a) can be written as

$$c_t = \left(\sum_{i=1}^{N_t} (q_t x_t)^{\frac{1}{\varepsilon}}\right)^{\varepsilon} = N_t^{\varepsilon} q_t x_t = N_t^{\varepsilon} q_t \frac{E_t}{L_t N_t \varepsilon}$$

Plugging (9) in the above expression yields per-capita consumption grows rate

$$1 + g_c \equiv \frac{c_t}{c_{t-1}} = \frac{L_{t-1} N_t^{\varepsilon-1} q_t N_{t+1}}{L_t N_{t-1}^{\varepsilon-1} q_{t-1} N_t} = \frac{(1+g_q)(1+g_N)^{\varepsilon}}{1+n} \tag{16}$$

Which can be also written as

$$1 + g_c = \frac{(1+g_q)\left[(1-\delta) + \xi\rho(1+\theta(n)\,n)\right]^{\varepsilon}}{1+n} \tag{16a}$$

Differentiating (16a) for $n$ shows that the sign of $\frac{\partial g_c}{\partial n}$ depends on the sign of $\frac{\varepsilon(1-\gamma)\theta_0 n^{-\gamma}(1+n)}{\frac{(1-\delta)}{\xi\rho}+1+\theta_0 n^{1-\gamma}} - 1$. The latter expression is positive (negative) if the following (reverse) inequality holds

$$\varepsilon(1-\gamma)\,n^{-\gamma} - \left[1 - \varepsilon(1-\gamma)\right] n^{1-\gamma} > \frac{1}{\theta_0}\left(\frac{1-\delta}{\xi\rho} + 1\right) \tag{17}$$

**Proposition 1** *With $\varepsilon(1-\gamma) \leq 1$ the relation between $g_c$ and $n$ is* hump-shape.

**Proof.** For $\varepsilon(1-\gamma) \in (0,1)$ the left side of (17) is decreasing with $n$: starting from plus infinity for $n \to 0$, and turns to negative values for $n > \frac{\varepsilon(1-\gamma)}{1-\varepsilon(1-\gamma)}$. Hence, for $\varepsilon(1-\gamma) \in (0,1)$, the sign of $\frac{\partial g_c}{\partial n}$ is positive (negative) for low (high) fertility rates implying that $g_c(n)$ follows a hump shape. ■

Following (17), with $\varepsilon(1-\gamma) \leq 1$ (i.e. $\gamma \geq \frac{1}{s}$), per-capita consumption growth rate is maximized for $n = \frac{\varepsilon(1-\gamma)}{1-\varepsilon(1-\gamma)}$. Hence, as $\gamma$ increases (decreases) the range of $n$ for which $\frac{\partial g_c}{\partial n} > 0$ is shrinking (widening). Pushing the value of $\gamma$ to its lower limits yields a qualitatively different non-monotonic shape of $g_c(n)$, presented in Proposition 2.

**Proposition 2** *For $\gamma = 0$ and $\varepsilon < \frac{1}{\theta_0}\left(\frac{1-\delta}{\xi\rho} + 1\right)$ the relation between $g_c$ and $n$ follows* U shape.

**Proof.** Following the proof of Proposition 1, for $\gamma = 0$ the sign of $\frac{\partial g_c}{\partial n}$ depends on the sign of $\frac{\varepsilon\theta_0(1+n)}{\frac{(1-\delta)}{\xi\rho}+1+\theta_0 n} - 1$, which is increasing with $n$. For $\varepsilon < \frac{1}{\theta_0}\left(\frac{1-\delta}{\xi\rho} + 1\right)$, the sign of $\frac{\varepsilon\theta_0(1+n)}{\frac{(1-\delta)}{\xi\rho}+1+\theta_0 n} - 1$ is negative (positive) for low (high) values of $n$, implying a non-monotonic, *U shape* relation between $g_c$ and $n$. ■

For $\gamma = 0$ and $\varepsilon > \frac{1}{\theta_0}\left(\frac{(1-\delta)}{\xi\rho} + 1\right)$, we get $\frac{\partial g_c}{\partial n} > 0\ \forall n$, that is a positive monotonic effect of population growth rate on per-capita consumption growth. At the intermediate values range of $\gamma$, the function $g_c(n)$ presents a more complicated shape. For $\varepsilon(1-\gamma) > 1$ (i.e. $\gamma < \frac{1}{s}$), the left-hand side of (17) has a minimum at $n = \frac{\gamma\varepsilon}{\varepsilon(1-\gamma)-1}$, which is increasing with $\gamma$. For sufficiently low value on the right-hand side of condition (17), it will hold for any $n$, implying that $\frac{\partial g_c}{\partial n} > 0$ , $\forall n$. High values on the right-hand side of condition (17), guarantee that it holds for sufficiently low and high population growth rates, implying that as $n$ increases, $g_c(n)$ reaches a maximum first, then it starts decreasing and reaches a minimum.

# 4 Conclusions

This study contributes to the literature on the role of population in R&D-driven growth, by adding to the few recent studies that established non-monotonic relations between population growth and economic prosperity. We have demonstrated that different types of non-monotonic relations between population growth and economic prosperity may arise, based on alternative specifications of dynastic altruism. In particular, the shape of these non-monotonic relations depends on the concavity of parental utility from the number of their children.

We have shown that under the linear parental altruism, the effect of population growth on economic growth may follow a *U shape*, whereas under non-linear specification of altruism it may follow a *hump shape*, consistently with the aforementioned empirical findings.

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