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Bharat Diwakar and Gilad Sorek

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Bharat Diwakar and Gilad Sorek*†

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Abstract

We study two-sector R&D model with endogenous human capital accumulation. Allowing for fractional human capital spillover from parents to their offspring, which are subject to congestion in fertility rate, we establish non-monotonic relations between population growth and economic growth. These non-monotonic relations, which are polynomial in general, are determined by the base level of human capital spillover and the magnitude of the congestion effect: a *U shape* relation can arise under low congestion factor, whereas a *hump shape* may present for high congestion factor

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*Economics Department, Auburn University, Auburn Alabama. Emails: bzd0013@tigermail.auburn.edu, gms0014@auburn.edu

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1 Introduction

This work contributes to the literature on the role of population growth in modern, R&D-based, economic growth theory. By introducing human capital spillover within dynasties, from parents to their offspring, we establish non-monotonic relation between the rates of population growth and per-capita consumption growth.

The seminal contributions by Romer (1990), Grossman and Helpman (1991a,b) and Aghion and Howitt (1992) laid the foundations for modern, R&D-based, growth theory. However, these canonical models were criticized for presenting a counterfactual scale-effect, which is a positive effect of population size on output growth (See Jones 1995). This shortcoming of the first-generation growth models provoked various corresponding modifications.

The following second and third generation models did not present scale-effect but rather proposed that population growth rate positively affects economic growth. The removal of scale effect involved the introduction of decreasing returns in the innovation function (Jones 1995, Kortum 1997, and Segerstrom 1998), and two R&D sectors that perform both vertical quality improvements and horizontal variety expansion (Peretto 1998, Young 1998, and Howitt 1999)[1]. Nonetheless, this theoretical prediction also was not empirically validated[2].

The next line of corresponding theoretical modifications, to which the present work contributes, is still being updated. A common element in this recent literature is the introduction of human capital as productive input in the R&D sector. This specification induces substitution between the quantity and quality of workers, which enables an increase in overall effective labor supply and thereby enhancing economic growth, even with a constant or declining population of workers.

Within this line of research the assumed process of human capital formation is crucial to the relation between population growth and economic growth. More specifically, the potential diluting effect of population growth on the average human capital level was emphasized in the literature as hindering economic growth; See for example Dalgaard and Kreiner (2001), Strulik (2005), Bucci (2013), and Chu et al.(2013). In these papers the assumed diluting effect implies that young agents enter the labor force with zero human capital. However, other related studies abstracted this diluting effect entirely, following Lucas' (1988) exact formulation, meaning that newborns are inherited with the same level of human capital as their parents; See for example Tournemaine and Luangaram (2012) and Bucci (2015).

When discussing his own formulation of human capital accumulation, Lucas (1988) also emphasized the plausibility of fractional transmission of human capital within dynasties, from parent to their offspring: "..One needs to assume ... that the initial level each new member begins with is proportional to (not equal to!) the level already attained by older members of the family. This is simply one instance of a general fact that I will emphasize ... : that human capital accumulation is a social activity, involving groups of people in a way that has no counterpart in the accumulation of physical capital" (p.19).

[1] Jones (1999) provides a compact summary of this literature.

[2] Recent summaries of the empirical literature can be found in Strulik et al. (2013) and Bucci (2015).

The present work pursues and elaborates such intermediate approach regarding the diluting effect of newborns on human capital accumulation. First, we allow for fractional human capital spillover from parents to their offspring. That is, we consider the entire range between the two extreme cases presented in the aforementioned literature. This kind of spillover was widely considered in other strands of the literature on growth and human capital accumulation, without R&D-based innovations; See for example Becker et al.(1990), Galor and Tsiddon (1997), De-la Croix and Doepke (2004), and Fioroni (2010)[3].

Second, we consider congestion effects in the transmission of human capital within dynasties. That is the degree of human capital spillover from parents to their offspring is decreasing with the number of kids. The intuition that motivates this analysis is the following: we postulate that parental human capital spillover is transmitted through direct interaction between parents and their off springs in the household, where parenting time is not a pure public good.

Our framework is an extended version of Young's (1998) model of two-sector R&D, that incorporates population growth and human-capital accumulation. The analysis yields a rich set of possible relations between population growth and economic growth, including non-monotonic ones, depending on the assumed types of spillover. The welfare analysis shows that the rates of human capital accumulation and technological progress in the decentralized economy may deviate from the efficient ones, in various ways.

Several theoretical papers have established ambiguous effect of population growth on technological progress, which depends on the strength of dynastic altruism toward future generations (Dalgaard and Kreiner 2001, Strulik 2005, Bucci 2013), the (potentially adverse) effect of specialization on the production complexity (Bucci 2015), the effect of technological progress on the stock of human capital - appreciation vs. depreciation (Bucci 2008). In these studies however, unlike in the present work, the effect of population growth on technological progress depends on the values of model parameters, and are monotonic given the parameters set.

Our work is closely related to the recent contribution by Boikos et al.(2013), which studies the effect of fertility on human capital accumulation, in a model with no R&D-based innovation and endogenous fertility. In their theoretical analysis the effect of population growth on human capital accumulation is allowed to be positive, negative, and non-monotonic. Hence, they allow for "negative dilution" effect of population growth on human capital accumulation, under which population growth enhances per-capita human-capital accumulation.

The theoretical part of their work shows that the overall effect of fertility on per-capita human capital accumulation and income growth depends crucially on the sign of the dilution effect, which is left unspecified (see discussion on p.50 and footnote there). Similar approach was taken also by Boucekkine and Fabbri (2013) and Marsiglio (2014), who assume unspecified and quadratic diluting effect, respectively, in models of physical capital accumulation and endogenous fertility[4].

[3] However, such spillover also present in recent R&D-driven growth models written in the Overlapping Generations framework; See for example Strulik et al.(2013) and Prettner (2014).

[4] Their focus however is on the implications of different types of dynastic altruism to optimal fertility rates.

All these studies were able to establish a *hump shape* relation between fertility growth and economic growth, consistent with the empirical findings reported by Boikos et al.(2013)[5], and Kelley and Schmidt (1995)[6]. In comparison with these studies, our results are derived in a full-fledged R&D-based growth model[7], based on a simple specification of the diluting effect, which has an intuitive economic interpretation. Namely, the diluting effect here is defined by fractional human-capital spillover from parent to their offspring, which is subject to congestion in the number of offspring. Hence, we assume that population growth always dilutes per-capita human capital accumulation, but not necessarily in a linear fashion. Yet, we establish non-monotonic polynomial relation between population growth and economic growth, which varies - from U shape to Hump shape, depending on the congestion factor in parental human capital spillover.

In another recent relevant paper by Prettner (2014), human capital is formed through public education system where higher fertility rate decreases schooling-intensity - i.e. per-student public spending. Prettner (2014) shows that for economies with developed public education system - in terms of spending level and teachers' productivity - there is a non-monotonic relation between fertility rate and economic growth: for initially low (high) rates increase in fertility has negative (positive) effect on economic growth. For economies with under-developed public education system the effect of fertility on population growth is definite-positive[8]. Prettner's (2014) results, derived in the overlapping generations framework, are similar to ours in the special case of no congestion in parental human-capital spillover, which we derive for infinitely living agents and decentralized human capital accumulation.

The paper is organized as follows. Section 2 presents the detailed model. Section 3 analyzes the dynamic equilibrium and the effect of population growth on technological progress. Section 4 presents welfare analysis for the model economy, and Section 5 concludes this study.

2 The Model

We extend Young's (1998) two-sector R&D model by adding population growth and human capital accumulation. Time is discrete, and population grows at exogenous rate $n \geq 0$. Population size in each period is denoted $L_t = L_0(1+n)^t$, where L_0 is normalized to one. In each period, each worker is endowed with one unit of time. To enhance exposition clarity, the analysis focuses first on exogenous human capital accumulation, and then human capital accumulation is endogenized through education choice.

[5] Growth in their model is driven by human capital accumulation, and the empirical analysis focuses, accordingly, on the relation between fertility rates and human capital accumulation, based on a panel data analysis for ninety-nine countries (both OECD and non-OECD), over the years 1960–2000.

[6] In an earlier study Boucekkine et al.(2002) derived such relation between fertility and economic growth in an overlapping generations model of vintage human capital.

[7] In concluding their study Boikos et al.(2013, p.57) propose this as a desirable extension to their analysis.

[8] In another work written in the OLG framework, Strulik et al.(2013) explain the non-monotonic relation between population and economic growth within an unified growth model that incorporates endogenous fertility along with transition from neoclassical technology to R&D-based growth.

2.1 Preferences

Consumer's lifetime utility is given by

$$U = \sum_{t=0}^{\infty} \rho^t \ln (c_t) \tag{1}$$

where $\rho \in (0,1)$ is the subjective discount factor, and c is the per-capita instantaneous utility from consuming N differentiated products, i.e. "varieties", subject to a CES utility function

$$c_t = \left(\sum_{i=1}^{N_t} c_{i,t}^{\frac{1}{\varepsilon}} \right)^{\varepsilon} \tag{1a}$$

with $\varepsilon = \frac{s}{s-1}$, and s is the elasticity of substitution across all varieties. The consumption level of each variety c_i, is defined as $c_i = q_i x_i$, where x_i and q_i denote the consumed quantity and product quality, respectively. The assumed preferences imply the instantaneous demand for each variety

$$x_{i,t}^{d} = q_{i,t}^{s-1} (\lambda p_{i,t})^{-s} \left(\sum_{i=1}^{N_t} c_{i,t}^{\frac{1}{\varepsilon}} \right)^{\varepsilon} \tag{1b}$$

Where λ is the Lagrange multiplier from the instantaneous utility maximization (i.e. the shadow value of given periodic spending level). The logarithmic specification in (1) implies the standard Euler condition for optimal consumption smoothing, written in terms of aggregate spending, denoted E

$$\frac{E_{t+1}}{E_t} = \rho (1 + r_{t+1}) \tag{2}$$

where $(1 + r_{t+1})$ is the (gross) interest rate earned between periods t and $t+1$.

2.2 Production and innovation

We will start by analyzing a model with exogenous rate of human capital accumulation, subject to the following aggregate growth rate

$$1 + g_h \equiv \frac{h_t}{h_{t-1}} = \frac{(1 + \omega n)(1 + \widetilde{g}_h)}{1 + n} \tag{3}$$

where h_t is per-capita human capital, and the parameter $\omega \in (0,1)$ measures parental human-capital spillover. With constant population, i.e. $n = 0$, per-capita human capital grows at the rate $\widetilde{g}_h$. For positive population growth and $\omega = 1$ parental human capital spillover is complete, and thus population growth has no effect on the per-capita human capital level. For $\omega = 0$ population growth rate works as a full dilution factor over h_t. Our analysis focuses on the intermediate cases with fractional transmission of human capital from parents to their off springs. Furthermore, we consider nonlinear spillover, due to congestion in the number of offspring, that is, $\omega \equiv \omega(n)$ and $\omega'(n) < 0$. To enhance tractability, we focus on the following specification

$$\omega(n) = \omega_0 \exp(-\mu n) \tag{3a}$$

Where $\omega_0 \in (0,1)$, and $\mu \geq 0$ is the congestion factor. With $\mu = 0$, there is no congestion in human capital spillover. Notice that (3)-(3a) imply that population growth slows down the accumulation of per-capita human capital, that is $\frac{\partial g_h}{\partial n} < 0$ [9], hence we assume effective diluting. Aggregate human capital, denoted H, is defined as the product of population size and per-capita human capital

$$H_t = L_t h_t \tag{3b}$$

Effective labor supply is the sole input for production and innovation, and the wage rate is normalized to one. One unit of labor produces one unit of consumption good (regardless of its quality). We follow Young's (1998) specification of the innovation cost function

$$f(q_{i,t+1}, \overline{q}_t) = \begin{cases} \exp\left(\phi \frac{q_{i,t+1}}{\overline{q}_t}\right) & q_{i,t+1} > q_{i,t} \\ \exp(\phi) & q_{i,t+1} \leq q_{i,t} \end{cases} \tag{4}$$

Innovation cost in sector i is increasing with the rate of quality improvements over the quality frontier of the economy, i.e. the highest quality that was already developed - denoted $\overline{q}_t$. As innovation is assumed to be certain, vertical innovation (i.e. quality improvements) implies that the effective lifetime of each product is one period. Hence, each firm maximizes the profit

$$\Pi_{i,t} = \frac{(p_{i,t+1} - 1) x^d_{i,t+1} L_{t+1}}{1 + r_{t+1}} - f(q_{i,t+1}, \overline{q}_t) \tag{5}$$

Maximizing (5) for price $p_{i,t+1}$ yields the standard optimal monopolistic price $p^* = \varepsilon$, $\forall t, i$. The first order condition for optimal quality choice is derived after plugging the optimal price and the demand function (1b) into (5)

$$\frac{1}{q^*_{i,t+1}} \frac{(\varepsilon - 1)(s - 1)(\lambda\varepsilon)^{-s} \left(\sum_{i=1}^{N_t} c_{i,t+1}^{\frac{1}{\varepsilon}}\right)^{\varepsilon} L_{t+1}}{1 + r_{t+1}} = \frac{\phi}{\overline{q}_t} f(q^*_{i,t+1}, \overline{q}_t) \tag{5a}$$

The asterisk superscript denotes optimally chosen values for the variables in the decentralized economy. Assuming free entry to the R&D sector implies that in equilibrium the profit in (5) equals zero. Combining this assumption with the optimality condition (5a) we obtain the equilibrium rate of quality improvement

$$\forall_i : 1 + g_q \equiv \frac{q^*_{t+1}}{\overline{q}_t} = \frac{s - 1}{\phi} \tag{5b}$$

We assume the cost parameter ϕ is low enough to guarantee $g_q > 0$, and to make vertical competition between successive product generations redundant, i.e. $p^* < 1 + g_q \Rightarrow \varepsilon < \frac{s-1}{\phi}$.

[9] More generally, $\frac{\partial g_h}{\partial n}$ is negative as long as $\omega'(n) < 0$.

As the rate of quality improvement is time invariant, so is equilibrium innovation cost $f(q_{i,t+1}, \overline{q}_t) = e^{s-1}, \forall t, i$. Notice that under symmetric equilibrium demand for each variety is $x_t^d = \frac{E_t}{\varepsilon N_t} \forall_i$, and thus the free entry condition can be also written as

$$\frac{(\varepsilon - 1)\frac{E_{t+1}}{\varepsilon N_{t+1}}}{f} = (1 + r_{t+1}) \tag{6}$$

3 Equilibrium and Growth Dynamics

3.1 Exogenous human capital accumulation

Combining (2) and (6) we obtain

$$E_t = \frac{f N_{t+1}}{\left(1 - \frac{1}{\varepsilon}\right)\rho} \tag{7}$$

and plugging (7) back into (6) yields the interest rate for the assumed stationary equilibrium

$$\frac{1 + g_N}{\rho} = (1 + r_{t+1}) \tag{8}$$

where $1 + g_N \equiv \frac{N_{t+1}}{N_t}$. The aggregate resources-uses constraint for the economy is defined by the allocation of labor between production and R&D investment

$$H_t = \frac{E_t}{\varepsilon} + f N_{t+1} \tag{9}$$

Plugging (7) into (9) yields

$$\begin{aligned} H_t &= \frac{f N_{t+1}}{(\varepsilon - 1)\rho} + f N_{t+1} \\ \Rightarrow \quad N_{t+1} &= \frac{H_t}{f\left(\frac{1}{(\varepsilon-1)\rho} + 1\right)} \end{aligned} \tag{10}$$

Hence, variety expansion rate equals the exogenous growth rate of effective labor supply $(1 + g_N) = (1 + g_H)$, which, following (3)-(3a), implies:

$$(1 + g_N) = (1 + \widetilde{g}_h)\left[1 + \omega(n) n\right] \tag{10a}$$

Observe that under symmetric equilibrium, equation (1a) can be written as

$$c_t = \left(\sum_{i=1}^{N_t} (q_{i,t} x_{i,t})^{\frac{1}{\varepsilon}}\right)^{\varepsilon} = N_t^{\varepsilon} q_t x_t = N_t^{\varepsilon} q_t \frac{E_t}{L_t N_t \varepsilon}$$

After plugging (7) into C_t, the above expression implies that in the stationary equilibrium per-capita consumption grows at a constant rate

$$1+g_c \equiv \frac{c_t}{c_{t-1}} = \frac{L_{t-1}N_t^{\varepsilon-1}q_tN_{t+1}}{L_tN_{t-1}^{\varepsilon-1}q_{t-1}N_t} = \frac{(1+g_q)\,(1+g_N)^{\varepsilon}}{1+n} \tag{11}$$

Then we substitute (10a) and (3a) into (11) to rewrite

$$1+g_c = \frac{(1+g_q)\,(1+\widetilde{g}_h)^{\varepsilon}(1+\omega_0\exp(-\mu n)\,n)^{\varepsilon}}{1+n} \tag{11a}$$

Equation (11a) reveals the two opposing effect induced by population growth on per-capita consumption growth. The positive effect is due to the increase in aggregate human capital supply, which accelerates variety expansion - according to equation (10). This positive effect is generated through the spillover parameter and is then amplified by the preference parameter ε, which is decreasing with the elasticity of substitution across varieties - s. With lower s gains from faster variety expansion, driven by faster human capital accumulation are higher. The negative effect of population growth on per-capita consumption growth, which presents in the denominator of (11a), is the regular pure dilution effect.

Differentiating (11a) for n shows that $\frac{\partial g_c}{\partial n}$ is positive (negative) if the following (reverse) inequality holds

$$\varepsilon\,(1+n)\,(1-\mu n) - n > \frac{\exp(\mu n)}{\omega_0} \tag{11b}$$

Proposition 1 *With exogenous human capital accumulation, for sufficiently high ω_0 and μ, the function $g_c(n)$ is hump shape. That is for sufficiently strong base spillover and congestion effect, economic growth first accelerates with population growth rate and then slows down.*

Proof. For $\mu > 0$, the right hand side of (11b) is increasing with n. For $\varepsilon\,(1-\mu) < 1$, the left hand side of (11b) is monotonically decreasing with n, and hence, for sufficiently high n it is guaranteed that (11b) does not hold, that is $\frac{\partial g_c}{\partial n} < 0$. If $\omega_0 > \frac{1}{\varepsilon}$, condition (11b) holds for $n = 0$. Hence under these conditions, $\frac{\partial g_c}{\partial n}$ is positive (negative) under sufficiently low (high) population growth rates ■

For $\varepsilon\,(1-\mu) < 1$ and $\omega_0 < \frac{1}{\varepsilon}$, the function $g_c(n)$ is monotonically decreasing. Under lower values of μ, for which $\varepsilon\,(1-\mu) > 1$, the left-hand side of (11b) is increasing with n up to $n = \frac{\varepsilon(1-\mu)-1}{2\varepsilon\mu}$, and then starts decreasing (for high n values). Then, $g_c(n)$ still follows a hump shape for $\omega_0 > \frac{1}{\varepsilon}$. However, for $\omega_0 < \frac{1}{\varepsilon}$, condition (11b) holds only for intermediate values of n, implying that $g_c(n)$ is first decreasing with n - to a local minimum, and then it is increasing to a local maximum from where it is monotonically decreasing. Hence, within this parameters set the shape of $g_c(n)$ follows a co-sine shape, which combines U shape with Hump shape. As the value of μ decreases the range of the U shape is expanding.

Proposition 2 *With exogenous human capital accumulation, for sufficiently low ω_0 and μ the function, $g_c(n)$ follows U shape. That is for sufficiently weak base spillover and congestion effect, economic growth first slows down with population growth rate and then accelerates.*

Proof. For the limit case $\mu = 0$ (11b) is modified to $\varepsilon + n(\varepsilon - 1) > \frac{1}{\omega_0}$. The right-hand side of this condition is increasing with n. For $\omega_0 > \frac{1}{\varepsilon}$ the latter inequality does (not) hold for sufficiently high (low) n, implying that $\frac{\partial g_c}{\partial n}$ is negative (positive) for low (high) values of n ■

For $\mu = 0$ and $\omega_0 > \frac{1}{\varepsilon}$ we have $\forall n > 0 : \frac{\partial(1+g_c)}{\partial n} > 0$, that is $g_c(n)$ is monotonically increasing.

3.2 Endogenous human capital accumulation

We turn now to incorporate endogenous human capital accumulation in the model, subject to the conventional specification

$$\begin{aligned} h_{t+1} &= \frac{(1+\omega n)(\xi e_t + 1 - \delta) h_t}{(1+n)} \qquad (12) \\ &\Rightarrow \triangle h_{t+1} \equiv h_{t+1} - h_t = \left[\frac{(1+\omega n)(\xi e_t + 1 - \delta)}{(1+n)} - 1\right] h_t \end{aligned}$$

where $e \in (0,1)$ is the time invested in human capital formation, δ is a depreciation rate, and ξ captures the productivity of the human capital formation technology[10]. Equation (12) implies that $1 + g_h \equiv \frac{h_{t+1}}{h_t} = \frac{(1+\omega n)(\xi e_t + 1 - \delta)}{(1+n)}$, and following (3b) we obtain

$$1 + g_H \equiv \frac{H_{t+1}}{H_t} = (1+g_h)(1+n) = (1+\omega n)(\xi e_t + 1 - \delta) \qquad (13)$$

The return on investment in human capital should equal the return on R&D investment defined in (5)

$$1 + r_{t+1} = \frac{(\xi e_t + 1 - \delta) h_t}{e_t h_t} \qquad (14)$$

Plugging the interest rate (8) in (14) and rearranging yields

$$\forall t : e^* = \frac{1-\delta}{\frac{1+g_N}{\rho} - \xi} \qquad (15)$$

and plugging (15) back into (13) yields

$$1 + g_H = \frac{(1+\omega n)(1-\delta)}{1 - \frac{\xi \rho}{1+g_N}} \qquad (16)$$

Modifying the resources-uses constraint (10) for the time invested in human capital formation yields

$$(1 - e^*) H_t = \frac{f N_{t+1}}{(\varepsilon - 1)\rho} + f N_{t+1} \qquad (17)$$

Plugging the interest rate (8) in (17) and rearranging yields

[10] With constant population and no depreciation equation (13) falls back to Lucas' (1988) original formulation: $\triangle h_t = \xi(e_{t-1}) h_{t-1}$.

$$N_{t+1} = \frac{(1-e^*)\,H_t}{f\left[\frac{1}{(\varepsilon-1)\rho}+1\right]} \tag{17a}$$

Hence, the aggregate human-capital stock and the varieties span share the same growth rate, as in Section 3. Imposing $(1+g_H)=(1+g_N)$ in (16) and simplifying we obtain

$$1+g_N = (1-\delta)(1+\omega n)+\xi\rho \tag{18}$$

Hence, following (11)-(11a), per-capita consumption growth rate remains $1+g_c = \frac{(1+g_q)(1+g_h)^{\varepsilon}(1+\omega n)^{\varepsilon}}{1+n}$, which can be written explicitly as[11]

$$1+g_c = \frac{(1+g_q)\left[(1-\delta)(1+\omega_0\exp(-\mu n)\cdot n)+\xi\rho\right]^{\varepsilon}}{1+n} \tag{19}$$

Equation (19) shows that the effect of population growth on per-capita consumption growth under endogenous human capital accumulation is very similar to the one presented in equation (11a), for exogenous rate of human capital accumulation. Nonetheless, here, the effect of population growth rate on per-capita consumption growth depends also on the technological parameters of human capital formation, and the time preference parameter.

Following (19), $\frac{\partial g_c}{\partial n}$ is positive (negative) if the following (reverse) inequality holds

$$\varepsilon > \frac{1+\omega_0\exp(-\mu n)\cdot n+\frac{\xi\rho}{(1-\delta)}}{\omega_0\exp(-\mu n)\cdot(1-\mu n)(1+n)} \tag{19a}$$

Proposition 3 *With endogenous human capital accumulation, for $\mu>0$ and $\varepsilon > \frac{1}{\omega_0}\left(1+\frac{\xi\rho}{(1-\delta)}\right)$, the relation between population growth and per-capita consumption growth follows a hump shape.*

Proof. Condition (19a) does not hold for $n>\frac{1}{\mu}$, as the denominator turns negative, but it does hold for sufficiently low n if $\varepsilon > \frac{1}{\omega_0}\left(1+\frac{\xi\rho}{(1-\delta)}\right)$. Hence, under these conditions $g_c(n)$ is non-monotonic and follows a hump shape ∎

Proposition 4 *With endogenous human capital accumulation, for sufficiently low congestion effect and $\varepsilon < \frac{1}{\omega}\left(1+\frac{\xi\rho}{(1-\delta)}\right)$, the relation between g_c and n follows non-monotonic U shape.*

Proof. For the limit case $\mu=0$, condition (19a) becomes $\varepsilon > \frac{1+\omega_0 n+\frac{\xi\rho}{(1-\delta)}}{\omega_0(1+n)} \Rightarrow \varepsilon+n(\varepsilon-1) > \frac{1+\frac{\xi\rho}{(1-\delta)}}{\omega}$. The latter condition holds for sufficiently high n , but it does not hold for sufficiently low (yet non-negative) n if $\omega_0 < \frac{1+\frac{\xi\rho}{(1-\delta)}}{\varepsilon}$. Hence, under these conditions $\frac{\partial g_c}{\partial n}$ is negative (positive) for sufficiently low (high) values of n, implying that $g_c(n)$ is U shaped ∎

Having $\varepsilon > \frac{1+\frac{\xi\rho}{(1-\delta)}}{\omega}$ implies that $\forall n>0 : \frac{\partial g_c}{\partial n} > 0$, that is positive monotonic relation between population growth and economic growth. The relation between g_c and n established in Proposition

[11] Following (3a),(13) ,(16) and (18).

4 is similar to the one presented in Prettner (2014) for an OLG economy with public education system. In Prettner's work, as in the present study, high productivity in human capital formation (interpreted there as teachers' productivity and schooling efficiency) is needed to obtain such non-monotonic relation. In addition, his result also requires a high level of public spending on schooling, which is set exogenously, whereas Proposition 4 above is derived for decentralized investment in education, chosen by the households.

4 Welfare Analysis

In Young's (1998) original model, growth is driven solely by vertical (quality improving) innovation, that is slower than the social optimum (see p.59 there). We turn now to evaluate the welfare performance of our extended version of Young's model. The social planner maximizes (1) along the balanced growth path, implying the following objective function

$$U = \frac{1}{1-\rho}\left(\ln c_0 + \frac{\rho \ln (1+g_c)}{1-\rho}\right) \tag{20}$$

This maximization problem is still subject to the resources-uses constraint (17) and the implied explicit expression for $(1+g_c)$ in (19). Imposing these restrictions on (20) we obtain the constrained objective function[12]

$$U = \frac{1}{1-\rho}\left(\begin{array}{c} \ln\left[\frac{N_0^{\varepsilon} q_0[(1-e)h_0 - fN_0(1+\omega n)(\xi e+1-\delta)]}{\varepsilon N_0}\right] + \\ + \frac{\rho \ln\left[\frac{\frac{1}{\phi}\ln f((1+\omega n)(\xi e+1-\delta))^{\varepsilon}}{1+n}\right]}{1-\rho} \end{array} \right) \tag{20a}$$

After normalizing all initial values to unity, we derive the first order conditions with respect to investment in education and quality improvements

$$\frac{\partial U}{\partial e} : \frac{1+\xi f\,(1+\omega n)}{((1-e^{**}) - f\,(1+\omega n)\,(\xi e^{**}+1-\delta))} = \frac{\rho}{1-\rho}\left[\frac{\xi\varepsilon}{\xi e^{**}+1-\delta}\right] \tag{21}$$

$$\frac{\partial U}{\partial f} : \frac{(1+\omega n)\,(\xi e+1-\delta)}{((1-e) - f^{**}\,(1+\omega n)\,(\xi e+1-\delta))} = \frac{\rho}{1-\rho}\frac{1}{f^{**}\ln f^{**}} \tag{22}$$

The superscript with double asterisk denotes the solution values for the maximization of (20a). Combining conditions (21)-(22) yields the efficient investment in quality

$$\xi\,(1+\omega n) = \frac{1}{f^{**}\left[\varepsilon\,(\ln f^{**}) - 1\right]} \tag{23}$$

The efficient investment in quality improvement decreases with the productivity of human capital formation ξ and the degree of human capital spillover ω, and increases with the elasticity across varieties, s. The first optimization condition can be written as

[12] Here, ω can be any of the specification of human capital spillovers considered in Section 3. Following the innovation function (4), the quality growth rate is given by $\frac{\ln f}{\phi}$.

$$\left(\frac{1}{\rho}-1\right)\ln f^{**}+1=\frac{\xi\left(1-e^{**}\right)}{\xi e^{**}+1-\delta}\Rightarrow e^{**}=\frac{\varepsilon\ln f-1-\frac{1-\delta}{\xi}\left[\left(\frac{1}{\rho}-1\right)(\ln f)+1\right]}{\left(\frac{1}{\rho}-1\right)\ln f+\varepsilon\ln f} \tag{24}$$

The efficient investment in education implies the following rate of human capital accumulation

$$1+g_H=\frac{(1+\omega n)(\varepsilon\ln f-1)(\xi+1-\delta)}{\left(\frac{1}{\rho}-1\right)(\ln f)+\varepsilon\ln f}=\frac{1+\frac{1-\delta}{\xi}}{f\ln f\left[\left(\frac{1}{\rho}-1\right)+\varepsilon\right]} \tag{24a}$$

Proposition 5 *The growth rates of human capital accumulation and the product's quality improvements may deviate from the efficient one in various ways. Overall efficiency is achieved* iff $\frac{1+\frac{1-\delta}{\xi}}{f^{**}\ln f^{**}\left[\left(\frac{1}{\rho}-1\right)+\varepsilon\right]}=(1-\delta)(1+\omega n)+\xi\rho$

Proof. Comparing (23) with (5a) shows that the market will provide efficient rate of quality improvements only if, $\xi(1+\omega n)=\frac{1}{\exp^{s-1}(s-1)}$. Hence, generally, the rate of quality improvements in the decentralized economy can be higher or lower than the efficient one. Comparing (24a) with (16) implies that the rate of human capital accumulation in the market is efficient only if

$$\frac{1+\frac{1-\delta}{\xi}}{f^{**}\ln f^{**}\left[\frac{1}{\rho}-1+\varepsilon\right]}=(1-\delta)(1+\omega n)+\xi\rho$$

Clearly, this condition may hold only for a very specific set of parameter ■

5 Conclusions

In this work we have established a polynomial relation between population growth and economic growth, building on the notion of human-capital spillover from parents to their off springs. We have shown that the shape of the non-monotonic relation between population growth and economic growth can be altered and even inverted in the presence of congestion in human-capital spillover. Our findings contribute to the recent literature that is aimed to modify R&D-based model to remove the counterfactual definite positive effect of population growth on technological progress, and economic growth ("weak scale effect").

In particular, this work adds to the few recent studies that established non-monotonic relation between population growth and economic growth. We have shown that under sufficient congestion impact, the effect of population growth on economic growth may follow a hump shape, that is consistent with the empirical finding of Boikos et al.(2013) and Kelley and Schmidt (1995). Finally, we have shown that the rates of human capital accumulation and products' quality improvements in the decentralized economy may deviate in various ways from the welfare maximizers.

Subsequent research is called to explore the implications of endogenous fertility rates to the results derived in this work, including the potential for equilibria multiplicity that was pointed out but not fully explored by Boikos et al.(2013, p.49).

References

[1] Aghion P., Howitt P., 1992. A model of growth through creative destruction. Econometrica 60, 323–51

[2] Becker,G.S., Murphy, K.M., Tamura, R., 1990. Human capital, fertility, and economic growth. Journal of Political Economy 98, S12-S37

[3] Boikos S., Bucci A., Stengos T., 2013. Non-monotonicity of fertility in human capital accumulation and economic growth. Journal of Macroeconomics 38, 44–59

[4] Boucekkine, R., De la Croix, D., Licandro, O., 2002. Human vintage capital, demographic trends and endogenous growth. Journal of Economic Theory 104, 340–375

[5] Boucekkine, R., Fabbri, G., 2013. Assessing Parfit's repugnant conclusion within a canonical endogenous growth set-up. Journal of Population Economics. 26, 751–767

[6] Bucci, A., 2008. Population growth in a model of economic growth with human capital accumulation and horizontal R&D. Journal of Macroeconomics 30, 1124–1147

[7] Bucci A., 2013. Returns to specialization, competition, population, and growth. Journal of Economic Dynamics and Control 37, 2023–2040

[8] Bucci A., 2015. Product proliferation, population, and economic growth. Journal of Human Capital. 9. 170-197

[9] Chu A. C., Cozzi, G., & Liao, C.-H., 2013. Endogenous fertility and human capital in a Schumpeterian growth model. Journal of Population Economics, 26, 181–202

[10] Dalgaard C.J, Kreiner C.T, 2001. Is declining productivity inevitable?. Journal of Economic Growth 6, 187–203

[11] De La Croix, D., Doepke, M., 2004. Public versus private education when differential fertility matters. Journal of Development Economics 73, 607-629

[12] Fioroni T., 2010. Child mortality and fertility: public vs private. Journal of Population Economics 23, 73-97

[13] Galor O., Tsiddon D., 1997. The distribution of human capital and economic growth. Journal of Economic Growth 2, 93-124

[14] Grossman G.M., Helpman E.,1991a. Innovation and growth in the global economy. MIT press. Cambridge. MA.

[15] Grossman G.M., Helpman, E., 1991b. Quality ladders in the theory of growth. Review of Economic Studies 58, 43-61

[16] Howitt P., 1999. Steady endogenous growth with population and R&D inputs growing. Journal of Political Economy 107, 715-730

[17] Jones C.I., 1995. R&D-based models of economic growth. Journal of Political Economy 103, 759-784

[18] Jones C.I., 1999. Growth: with or without scale effects. American Economic Review. 89, 139-144

[19] Kelley A, Schmidt R.M., 1995. Aggregate population and economic growth correlations: the role of the components of demographic changes. Demography 32, 543–555

[20] Kortum S.S., 1997. Research, Patenting, and Technological Change. Econometrica 65, 1389–419

[21] Lucas R.E. 1988. On the mechanics of economic development. Journal of Monetary Economics 22, 3-42

[22] Marsiglio S., 2014. Reassessing Edgeworth's conjecture when population dynamics is stochastic. Journal of Macroeconomics 42, 130-140

[23] Peretto P.F., 1998. Technological change and population growth. Journal of Economic Growth 3, 283–311

[24] Prettner K., 2014. The non-monotonous impact of population growth on economic prosperity. Economics Letters 124, 93–95

[25] Romer P., 1990. Endogenous technological progress. Journal of Political Economy 98, S71-S102.

[26] Segerstrom, P., 1998. Endogenous growth without scale effects. American Economic Review. 88 (5). 1290-1310

[27] Strulik H., 2005. The role of human capital and population growth in R&D-based models of economic growth. Review of International Economics 13, 129–145

[28] Strulik H., Prettner K., Prskawetz A., 2013. The past and future of knowledge-based growth. Journal of Economics Growth 18, 411–437

[29] Tournemaine, F., Luangaram P., 2012. R&D, human capital, fertility, and growth. Journal of Population Economics 25, 923-953

[30] Young A., 1998. Growth without scale effects. Journal of Political Economy 106, 41-63