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Finite Lifetimes, Population, and Growth

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Abstract

This work highlights principle differences in the predictions of R&D-based growth theory derived from the infinite horizon framework and the Overlapping Generations (OLG) framework of finitely living agents. In particular we show that the counterfactual positive effect of population growth on output growth presented in the second and third generation R&D-based growth models is eliminated in the corresponding OLG framework with finitely living agents. These differences arise because of the limiting effect of labor income on saving that presents only in the OLG framework. Our results indicate that the counterfactual relations between population and output growth rates presented in current R&D-based growth models are driven by their specific demographic structure.

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1 Introduction

This work shows that the prediction of R&D-based growth theory regarding the effect of population growth on output growth changes once analyzed in an Overlapping Generations (OLG) framework. In particular, we show that the counterfactual positive effect of population growth on output growth derived in the second and third generation growth models is eliminated once switching to the OLG demographic structure of finitely living agents.

The seminal contributions by Romer (1990), Grossman and Helpman (1991) and Aghion and Howitt (1992) established the foundations for modern - R&D based - growth theory. However, these canonical models were criticized for presenting a counterfactual scale-effect, which is a positive effect of population size on output growth (See Jones 1995). This shortcoming of the first-generation growth models provoked various corresponding modifications.

The following second and third generation models did not present scale-effect but rather proposed that population growth rate positively affects output growth. The removal of scale effect involved the introduction of decreasing returns in the innovation function (Jones 1995, Kortum 1997 and Segerstrom 1998), and two R&D sectors that perform both vertical quality improvements and horizontal variety expansion (Young 1998, Peretto 1998 and Howitt 1999).[1] Nonetheless, a positive stable correlation between population and output growth rates also could not be empirically validated (See Strulik et al. 2013 for a recent summary).

The next line of corresponding theoretical modifications shortly followed and is still being updated. A common element in this recent literature is the introduction of human capital as productive input in the R&D sector, which is endogenously accumulated. This modification enables substitution between the quantity and quality of workers, which makes increase in overall effective labor supply feasible even for constant or declining population of workers. The increase in effective labor supply is necessary for sustained increase in R&D efforts and, thereby, perpetual output growth, as pointed out by Strulik et al.(2013) in summarizing this literature:

"human capital growth can take over the role of population growth in R&D based growth models by predicting that productivity growth can be sustained with constant or declining population as long as human capital is accumulated rapidly enough"(p.414).

This line of research employed both the infinite horizon framework (Dalgaard and Kreiner 2001, Strulik 2005, Chu et al. 2013 and Bucci 2015), and more recently the overlapping generations framework as in Strulik et al.(2013) and Prettner (2014)[2]. In this paper we stress that the troubling results regarding the role of population in R&D driven Growth may not carry on to the Overlapping Generations (OLG), merely because of their different demographic structure. To this end we take a textbook R&D growth model with no human capital accumulation and place it in the OLG framework to derive comparable results.

[1] Jones (1999) provides a compact comparative summary of this literature.

[2] Strulik et al.(2013) where able to explain a non-monotonic relation between population and output growth rates by incorporating in the analysis parents' trade-off between the quantity and quality (i.e. human capital) of their children, which is central to unified growth theories - See Galor 2011 for review.

The principle cause for the different results derived in the OLG framework is the limiting effect of finite-lifetime on long-run saving. In the standard - Diamond (1965) type - OLG model the saving of young generations is bounded by their labor income. Jones and Manuelli (1990, 1992) showed that for this reason growth cannot sustain in the OLG framework under convex technologies that can support growth with infinitely living agent. We show that the production non-convexity applied in R&D-based models can sustain long-term balanced growth for finitely living agents where yielding unbounded (explosive) growth rates for infinitely living ones.

Population growth in the studied OLG model increases labor income and thus increases saving and investment. Nonetheless, at the same time it also increases demand for differentiated machines, hence it shifts investment away for innovation of new differentiated intermediate goods to a higher production level of old ones. We will show that in our reference model economy these two effects cancel out in the long run. Hence, the positive effect of population growth on output growth presented in the second and third generation growth model can be eliminated once placed in the OLG framework, having finite output growth rate that is neutral to population growth.

Delgaard and Jensen (2009) showed that the effect of scale on output growth in OLG economy depends on the saving motive: Scale has positive effect on growth when the bequest motive is dominant but it may turn negative when the life-cycle motive dominates. This is because bequests transfer wealth over generations thereby relaxing the dependency of saving on labor income. Their work adds the bequest motive for saving, to an otherwise standard OLG model with capital externalities, and derives comparative statics with respect to the values of bequest motive parameter[3].

Our analysis departs from Delgaard and Jensen (2009) along two lines. First, we take a full fledge R&D-based model and analyze it in the OLG framework, to derive comparable results with the one derived in the infinite horizon framework. Our results indicate that in the economy with finitely living agent the effect of population growth on output growth depends on its relative change compared with innovation cost, which is abstracted in Delgaard and Jensen (2009) analysis of aggregate capital externalities. Second we consider the effect of population growth rate on output growth that is not addressed in Delgaard and Jensen (2009). In our analysis there are no altruistic or bequest motives in both frameworks - with finitely and infinitely living agents.

Our results confirm that the different demographic structures of the two power-horse models of macroeconomic analysis yield different dynamic properties and different relations between population and output growth rates.

The remainder of the paper is organized as follows. Section 2 presents the detailed model. Section 3 analyzes the dynamic equilibrium in comparison to the infinite horizon case. Section 4 extends the basic model to two R&D sectors, and Section 5 concludes this study.

[3] Strulik (2005) finds that altruistic motivation for saving determines the effect of population growth on output growth in an model of infinitely living agents and human capital accumulation.

2 The Model

We take the textbook model of variety expansion presented in Barro and Sala-I-Martin (2004) and Aghion Howitt (2009)[4], and modify it to the OLG framework. We will first solve the OLG model and then compare our results with those presented in Barro and Sala-I-Martin (2004) - "BS" hereafter.

2.1 Production and Innovation

Final output that can be used for consumption and investment is produced by perfectly competitive firms. Final good production takes labor and intermediate goods as inputs subject to the CRTS technology

$$Y_t = AL_t^{1-\alpha} \int_0^{N_t} K_{i,t}^{\alpha} \, \mathrm{d}i \quad , \alpha \in (0,1) \tag{1}$$

where Y_t is the final good, A is a productivity factor, L_t is the labor input, $K_{i,t}$ is the quantity of intermediate good i used in the final good production, and N_t is the number of different intermediate varieties, to which we refer as "machines"[5]. The price of the final good is normalized to one. The cost of producing each machine is one final output unit, and machines fully depreciate after one period of usage. Under symmetric equilibrium, all specialized machines are identical i.e. $K_{i,t} = K_t$ $\forall i$ and thus the output equation (1) becomes

$$Y_t = AN_t {K_t}^{\alpha} L_t^{1-\alpha} \tag{1a}$$

The labor market is perfectly competitive implying the wage, denoted w, equals the marginal product of labor

$$w_t = MPL_t = A(1-\alpha) N_t {K_t}^{\alpha} L_t^{-\alpha} \tag{2}$$

Hence labor income is

$$w_t L_t = A(1-\alpha) N_t {K_t}^{\alpha} L_t^{1-\alpha} = (1-\alpha)\, Y_t \tag{2a}$$

Final good producers set demand for each intermediate good as to maximize profit, denoted π

$$\max_{k_{i,t}} \pi_{i,t} = AL_{t,i}^{1-\alpha} \int_0^{N_t} K_{i,t}^{\alpha} \, \mathrm{d}i - \int_{i=1}^{N_t} p_{i,t} K_{i,t} \, \mathrm{d}i$$

[4] Chapters 6.1 and 3.4, respectively.

[5] The elasticity of substitution between different varieties is $\frac{1}{\alpha}$.

where $p_{i,t}$ is the price of intermediate good i. The first order condition for this profit maximization yields the following demand for each intermediate goods, denoted with upper script d

$$K_{i,t}^d = A^{\frac{1}{1-\alpha}} L_t \left(\frac{\alpha}{p_{i,t}}\right)^{\frac{1}{1-\alpha}} \tag{3}$$

The services of specialized machines are rented under patent protection. Given the demand function (3) the price of each machine is set to maximize the surplus function

$$\max_{p_{i,t}} \Pi_{i,t} = K_{i,t}^d (p_{i,t} - 1) = A^{\frac{1}{1-\alpha}} L_t \left(\frac{\alpha}{p_{i,t}}\right)^{\frac{1}{1-\alpha}} (p_{i,t} - 1)$$

The first order condition for this maximization problem yields the standard monopolistic pricing $p_{i,t} = \frac{1}{\alpha}$. Plugging this price in (3) and then substituting (3) into (1a) we obtain the following expression for total output

$$Y_t = A^{\frac{1}{1-\alpha}} \alpha^{\frac{2\alpha}{1-\alpha}} N_t L_t \tag{4}$$

Hence, output growth rate, denoted g_Y is a function of population growth rate n and the variety expansion rate, denoted g_N :

$$1 + g_{Y,t+1} \equiv \frac{Y_{t+1}}{Y_t} = \frac{A^{\frac{1}{1-\alpha}} \alpha^{\frac{2\alpha}{1-\alpha}} N_{t+1} L_{t+1}}{A^{\frac{1}{1-\alpha}} \alpha^{\frac{2\alpha}{1-\alpha}} N_t L_t} = (1+n)(1+g_{N,t+1}) \tag{4a}$$

Innovation technology follows conventional specification

$$\Delta N_t = (N_{t+1} - N_t) = \delta R_t N_t^\gamma \tag{5}$$

Where R is R&D investment in new varieties. Each new variety is immediately granted with internal effective patent protection. Equation (5) implies that the cost of new variety, denoted η_t is

$$\eta_t \equiv \frac{R_t}{(N_{t+1} - N_t)} = \frac{1}{\delta N_t^\gamma} \tag{5a}$$

Where $\gamma \in [0, 1]$. For $\gamma = 0$ this cost is time invariant, and for positive (negative) γ the per-variety innovation cost is declining (increasing) with economic development level, implying dynamic knowledge spillover ("fishing out" effect). First generation models assumed $\gamma = 1$ and second generation models assumed $0 < \gamma < 1$ (See Jones 1999).

2.2 Preferences

Lifetime utility from consumption over two periods follows the tractable logarithmic formulation[6]

$$u(c_1, c_2) = \ln c_1 + \rho \ln c_2 \tag{6}$$

where $\rho \in (0,1)$ is the subjective discount factor. Young agents allocate their labor income between consumption and saving, denoted s. The solution for standard optimal saving problem is $s_t = \frac{\rho}{1+\rho} w_t$, hence aggregate saving is $S_t = \frac{\rho}{1+\rho} w_t L_t$. Substituting the explicit expression for w_t from (2) we obtain

$$S_t = \frac{1}{1+\frac{1}{\rho}} (1-\alpha) A^{\frac{1}{1-\alpha}} N_t \alpha^{\frac{2\alpha}{1-\alpha}} L_t \tag{7}$$

3 Equilibrium and Growth Dynamics

The savings from labor income in (7) are allocated to three types of investment: buying patents over old technologies, inventing new varieties and physical capital (i.e. specialized machines). New and old varieties play equivalent role in production, having symmetric presentation (1). Hence the market value of old varieties equals the cost of inventing a new one - η_t.

The market for specialized machines clears as the supply of each variety equals the demand given in (3). Hence, aggregate investment in each period -denoted I_t - satisfies

$$I_t = N_{t+1} \left(\eta_t + A^{\frac{1}{1-\alpha}} L_{t+1} \alpha^{\frac{2}{1-\alpha}} \right) \tag{8}$$

Equalizing (7) to (8) we impose the equilibrium condition $I_t = S_t$, which yields the dynamic equation that governs variety expansion rate

$$1 + g_{N,t+1} = \frac{N_{t+1}}{N_t} = \frac{A^{\frac{1}{1-\alpha}} L_t (1-\alpha) \alpha^{\frac{2\alpha}{1-\alpha}}}{\left(\eta_t + A^{\frac{1}{1-\alpha}} \alpha^{\frac{2}{1-\alpha}} L_{t+1} \right) \left(1 + \frac{1}{\rho} \right)} \tag{9}$$

The growth rate (9) can be also written as

$$1 + g_{N,t+1} = \frac{A^{\frac{1}{1-\alpha}} (1-\alpha) \alpha^{\frac{2\alpha}{1-\alpha}}}{\left(\frac{\eta_t}{L_t} + A^{\frac{1}{1-\alpha}} (1+n) \alpha^{\frac{2}{1-\alpha}} \right) \left(1 + \frac{1}{\rho} \right)} \tag{9a}$$

We denote long-run growth rates $\widetilde{g} \equiv \lim_{t \to \infty} g$.

Proposition 1 *If population is growing (not decreasing) and innovation cost is not decreasing (increasing) population growth rate has no effect on long run output growth rate. That is* $\forall\ n \cdot \gamma \geq 0$ *:* $\frac{\partial \widetilde{g}_Y}{\partial n} = 0$*. If population and innovation cost grow at the same rate population growth has positive effect on long term growth. That is if* $n, \gamma \neq 0$ *and* $\forall t : \frac{\eta_t}{L_t} = a > 0$ *then* $\frac{\partial \widetilde{g}_Y}{\partial n} > 0$.

[6] In the Appendix we validate our main results for the general CRRA instantaneous utility function $u(c_t) = \frac{c_t^{1-\theta}}{1-\theta}$.

Proof. $\forall\ n \cdot \gamma \geq 0$ the term $\frac{\eta_t}{L_t}$ in (9a) is decreasing over time hence $1+\ \widetilde{g}_N = \frac{1-\alpha}{\alpha^2(1+n)\left(1+\frac{1}{\rho}\right)}$, which by (4a) implies that output growth rate is neutral to population growth: $1+\ \widetilde{g}_Y = (1+n)\left(1+\ \widetilde{g}_N\right) = \frac{1-\alpha}{\alpha^2\left(1+\frac{1}{\rho}\right)}$. Only for the limit case where population and innovation cost share the same growth rate we get $1+\ \widetilde{g}_Y = \frac{A^{\frac{1}{1-\alpha}}(1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}}}{\left(\frac{\eta_t}{(1+n)L_t}+A^{\frac{1}{1-\alpha}}\alpha^{\frac{2}{1-\alpha}}\right)\left(1+\frac{1}{\rho}\right)}$, which is increasing with n. ■

If innovation cost is increasing with variety expansion, i.e. $\gamma < 0$, population should grow at least as fast to sustain long term growth[7]. We turn now to compare our results from the OLG economy of finitely living agents with the one obtained in a corresponding model with infinitely living agent. BS (p.296) solve a model economy that uses the same production and innovation technologies for $\gamma = 0$, with consumers who maximize utility over infinite lifetime horizon: $U = \int_0^\infty e^{-\rho t}\frac{c^{1-\theta}-1}{1-\theta}dt$ [8]. The output growth rate for this economy is given by

$$g_Y = \frac{1}{\theta}\left[A^{\frac{1}{1-\alpha}}\left(\frac{L}{\eta}\right)\left(\frac{1-\alpha}{\alpha}\right)\alpha^{\frac{2}{1-\alpha}}-\rho\right] \tag{BS.1}$$

Equation (BS.1) implies that in the economy of infinitely-living agents output growth rate can be positive and bounded only if innovation cost grows proportionally to population size. If population grows faster than innovation cost the output growth rate is unbounded. By contrast, Proposition 1 implies that in the OLG economy growth rate is positive and bounded as long as population does not grow slower than innovation cost. In both models output growth is not sustainable if innovation cost grows faster than population (see BS Section 6.1.8 p. 303). When discussing the scale effect and population growth BS (p.302 there) assume that innovation cost increases proportionally to population size to derive the modified finite output growth rate

$$g_Y = \frac{1}{\theta}\left[n+\frac{\alpha(1-\alpha)}{\eta}-\rho\right] \tag{BS.2}$$

The output growth rate in (BS.2) is increasing with population growth rate, as in other second generation growth models. Proposition 1 implies that in the OLG economy this is only a limit case result and that whenever population grows faster than innovation cost population growth rate has no effect on the finite long run output growth rate.

4 Two-sector R&D

Here we allow innovation to take place both on the horizontal dimension and the vertical dimension, in the spirit of the third-generation growth models, as Young (1998) and Howitt (1999).

[7] Note that scale effect presents in our model only if population and innovation cost are stationary, that is $\frac{\partial \widetilde{g}_Y}{\partial L} > 0$ iff $n = \gamma = 0$.

[8] Where θ is the inter-temporal elasticity of substitution which was so far assumed to equal one in our analysis, by equation (6). Our solution for the general CRRA instantaneous utility function (with $\theta \neq 1$) is in the Appendix.

The modified production function is given by

$$Y_t = AL_t^{1-\alpha} \int_0^{N_t} (q_{i,t}K_{i,t})^{\alpha}\,\mathrm{d}i, \quad 0<\alpha<1 \tag{10}$$

Where q_i is a productivity factor for variety i . Under symmetric equilibrium (10) becomes

$$Y_t = AL_t^{1-\alpha}(q_tK_t)^{\alpha}N_t \tag{10a}$$

and the modified demand for each intermediate variety is

$$K_{i,t}^d = \left(\frac{A\alpha q_{i,t}^{\alpha}}{p_{i,t}}\right)^{\frac{1}{1-\alpha}} L_t \tag{11}$$

The innovation cost function is borrowed from Young (1998)

$$f(q_{i,t+1},\overline{q}_t) = \begin{cases} e^{\phi\frac{q_{i,t+1}}{\overline{q}_t}} & q_{i,t+1} > q_{i,t} \\ e^{\phi} & q_{i,t+1} \leq q_{i,t} \end{cases} \tag{12}$$

The average product quality of current period, denoted $\overline{q}_t$, sets the benchmark quality level for all product - existing and new varieties in the next period. That is both vertical and horizontal knowledge spillover are assumed. In the presence of certain vertical innovation the effective lifetime of each product generation is one period. Hence each innovating firm maximizes the current value of one period surplus stream against the innovation cost

$$\Pi_{i,t} = \frac{(p_{i,t+1}-1)\left(\frac{A\alpha}{p_{i,t+1}}\right)^{\frac{1}{1-\alpha}} L_{t+1} q_{i,t+1}^{\frac{\alpha}{1-\alpha}}}{1+r_{t+1}} - f(q_{i,t+1},\overline{q}_t) \tag{13}$$

Maximizing (13) for $p_{i,t+1}$ yields the standard optimal monopolistic price $p^* = \frac{1}{\alpha}$, $\forall t, i$. The first order condition for optimal quality choice $q_{i,t+1}^*$ is

$$\frac{\alpha}{1-\alpha}\frac{\left(\frac{1}{\alpha}-1\right)\left(A\alpha^2\right)^{\frac{1}{1-\alpha}} L_{t+1}\left(q_{i,t+1}^*\right)^{\frac{\alpha}{1-\alpha}-1}}{1+r_{t+1}} = f'(q_{i,t+1}^*,\overline{q}_t) \tag{13a}$$

Assuming free entry to the R&D sector implies that in equilibrium the profit in (13) equals zero. Combining this latter assumption with the optimality condition (13a) and plugging in the explicit innovation cost (12), we obtain the optimal quality choice

$$\forall_i : 1+g_q \equiv \frac{q_{t+1}^*}{\overline{q}_t} = \frac{\alpha}{\phi(1-\alpha)} \tag{14}$$

We assume the cost parameter ϕ is low enough to guarantee $g_q > 0$ and to make vertical competition redundant, i.e. $p^* < 1 + g_q$. As the rate of quality improvement is time invariant, so is equilibrium innovation cost $f(q_{i,t+1}, \overline{q}_t) = e^{\frac{\alpha}{1-\alpha}}, \forall t, i$.

4.1 Finite lifetimes

Applying (11) and (14) to (10a) we write output growth rate as

$$1 + g_{Y,t+1} = \frac{Y_{t+1}}{Y_t} = (1+n)\left(\frac{\alpha}{\phi(1-\alpha)}\right)^{\frac{\alpha}{1-\alpha}} \frac{N_{t+1}}{N_t} \tag{15}$$

Aggregate savings and investment are given by

$$\begin{aligned} S_t &= \tfrac{\rho}{1+\rho}(1-\alpha)A^{\frac{1}{1-\alpha}} L_t N_t \alpha^{\frac{2\alpha}{1-\alpha}} q_t^{\frac{\alpha}{1-\alpha}} \\ I_t &= N_{t+1}\left(e^{\frac{\alpha}{1-\alpha}} + A^{\frac{1}{1-\alpha}} L_{t+1} \alpha^{\frac{2}{1-\alpha}} q_{t+1}^{\frac{\alpha}{1-\alpha}}\right) \end{aligned}$$

Imposing the resources uses constraint $S_t = I_t$ we obtain the rate of variety expansion g_N

$$1 + g_{N,t+1} \equiv \frac{N_{t+1}}{N_t} = \frac{\frac{\rho}{1+\rho} A^{\frac{1}{1-\alpha}}(1-\alpha) L_t \alpha^{\frac{2\alpha}{1-\alpha}} q_t^{\frac{\alpha}{1-\alpha}}}{e^{\frac{\alpha}{1-\alpha}} + L_{t+1} A^{\frac{1}{1-\alpha}} \left(\alpha^2 q_{t+1}^{\alpha}\right)^{\frac{1}{1-\alpha}}} \tag{16}$$

Hence, by (15), output growth rate is

$$1 + g_{Y,t+1} = (1+n)\left(\frac{\alpha}{\phi(1-\alpha)}\right)^{\frac{\alpha}{1-\alpha}} \frac{\frac{\rho}{1+\rho}(1-\alpha) L_t A^{\frac{1}{1-\alpha}} \alpha^{\frac{2\alpha}{1-\alpha}} q_t^{\frac{\alpha}{1-\alpha}}}{e^{\frac{\alpha}{1-\alpha}} + L_{t+1} A^{\frac{1}{1-\alpha}} \alpha^{\frac{2}{1-\alpha}} q_{t+1}^{\frac{\alpha}{1-\alpha}}} \tag{17}$$

Proposition 2 $\forall n \cdot (1+g_q)^{\frac{\alpha}{1-\alpha}} > 1 : \frac{\partial \widetilde{g}_Y}{\partial n} = 0$, *that is long run output growth is independent of population growth rate.* $\forall n > \frac{1}{1+g_q} : \widetilde{g}_Y = 0$, *that is a negative population growth cannot sustain long run output growth if it is too low compared with the rate of quality improvements.*

Proof. Rewrite (17) as

$$1 + g_{Y,t+1} = (1+n)\left(\frac{\alpha}{\phi(1-\alpha)}\right)^{\frac{\alpha}{1-\alpha}} \frac{\frac{\rho}{1+\rho} A^{\frac{1}{1-\alpha}}(1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}}}{\left(\frac{e}{q_t}\right)^{\frac{\alpha}{1-\alpha}} \frac{1}{L_t} + (1+n) A^{\frac{1}{1-\alpha}} \alpha^{\frac{2}{1-\alpha}} \left(\frac{\alpha}{\phi(1-\alpha)}\right)^{\frac{\alpha}{1-\alpha}}}$$

Hence $\forall n \cdot (1+g_q)^{\frac{\alpha}{1-\alpha}} > 1 : 1 + \widetilde{g}_Y = \frac{\rho}{1+\rho} \frac{(1-\alpha)}{\alpha^2}$. ■

4.2 Infinite lifetime

Suppose now that the same technologies are used by infinitely living agents who maximize their lifetime utility from consumption

$$U = \sum_{t=0}^{\infty} \rho^t Ln\left(c_t\right) \tag{18}$$

Where c_t is per-capita consumption[9]. The Euler condition for optimal consumption smoothing is

$$\frac{c_{t+1}}{c_t} = \rho\left(1 + r_{t+1}\right) \tag{19}$$

Denoting aggregate consumption C_t we rewrite the Euler condition

$$\frac{\frac{C_{t+1}}{L_{t+1}}}{\frac{C_t}{L_t}} = \rho\left(1 + r_{t+1}\right) \Rightarrow \frac{C_{t+1}}{C_t} = \rho\left(1 + n\right)\left(1 + r_{t+1}\right) \tag{19a}$$

The free entry condition imposed on (13) implies that the interest rate is given by

$$1 + r_{t+1} = \frac{\left(\frac{1}{\alpha} - 1\right) L_{t+1} A^{\frac{1}{1-\alpha}} \alpha^{\frac{2}{1-\alpha}} q_{t+1}^{\frac{\alpha}{1-\alpha}}}{e^{\frac{\alpha}{1-\alpha}} + L_{t+1} A^{\frac{1}{1-\alpha}} \alpha^{\frac{2}{1-\alpha}} q_{t+1}^{\frac{\alpha}{1-\alpha}}} \tag{20}$$

For $n > 0$ the long run interest rate converges to the markup $\frac{1}{\alpha} - 1$ and thus per-capita consumption growth rate approaches $\frac{c_{t+1}}{c_t} = \rho(\frac{1}{\alpha} - 1)$. For $n < 0$ both the interest rate and consumption growth approach zero in the long run. Hence, hereafter we confine attention to positive population growth, $n > 0$. The resources uses constrain for each period requires

$$Y_t = C_t + I_t \Rightarrow C_t = \underbrace{A^{\frac{1}{1-\alpha}} L_t \alpha^{\frac{2\alpha}{1-\alpha}} q_t^{\frac{\alpha}{1-\alpha}} N_t}_{Y_t} - \underbrace{N_{t+1}\left(e^{\frac{\alpha}{1-\alpha}} + L_{t+1} A^{\frac{1}{1-\alpha}} \alpha^{\frac{2}{1-\alpha}} q_{t+1}^{\frac{\alpha}{1-\alpha}}\right)}_{I_t} \tag{21}$$

Applying (20) to the Euler condition (19a) yields

$$\frac{C_{t+1}}{C_t} = \frac{A^{\frac{1}{1-\alpha}} L_{t+1} \alpha^{\frac{2\alpha}{1-\alpha}} q_{t+1}^{\frac{\alpha}{1-\alpha}} N_{t+1} - N_{t+2}\left(e^{\frac{\alpha}{1-\alpha}} + L_{2+1} A^{\frac{1}{1-\alpha}} \alpha^{\frac{2}{1-\alpha}} q_{t+2}^{\frac{\alpha}{1-\alpha}}\right)}{A^{\frac{1}{1-\alpha}} L_t \alpha^{\frac{2\alpha}{1-\alpha}} q_t^{\frac{\alpha}{1-\alpha}} N_t - N_{t+1}\left(e^{\frac{\alpha}{1-\alpha}} + L_{t+1} A^{\frac{1}{1-\alpha}} \alpha^{\frac{2}{1-\alpha}} q_{t+1}^{\frac{\alpha}{1-\alpha}}\right)} = \rho\left(1 + n\right)\left(\frac{1}{\alpha} - 1\right) \tag{22}$$

Rearranging (22) we obtain

$$\frac{A^{\frac{1}{1-\alpha}} \alpha^{\frac{2\alpha}{1-\alpha}} - A^{\frac{1}{1-\alpha}}\left(1 + \widetilde{g}_N\right)\left(1 + n\right) \alpha^{\frac{2}{1-\alpha}}\left(1 + g_q\right)^{\frac{\alpha}{1-\alpha}}}{\frac{A^{\frac{1}{1-\alpha}} \alpha^{\frac{2\alpha}{1-\alpha}}}{(1+n)(1+g_q)^{\frac{\alpha}{1-\alpha}}(1+\widetilde{g}_N)} - A^{\frac{1}{1-\alpha}} \alpha^{\frac{2}{1-\alpha}}} = \rho\left(1 + n\right)\left(\frac{1}{\alpha} - 1\right) \tag{22a}$$

[9] The utility function (18) is of Millian-type. Here consumers maximize there per-capita. We pick this formulation because it is consistent with the non-altruistic preferences assumed in the OLG model, and in BS. By comparison, in Benthamite-type utility function $U = \sum_{t=0}^{\infty} \rho^t (1+n)^t Ln\left(c_t\right)$ consumers maximize per-capita consumption of the entire dynasty, hence they are altruistic toward future generations. See Strulik (2005) for further discussion.

Simplifying (22a) yields the long run variety expansion rate

$$(1+\widetilde{g}_N) = \frac{\rho\left(\frac{1}{\alpha}-1\right)}{(1+g_q)^{\frac{\alpha}{1-\alpha}}} \tag{23}$$

Proposition 3 *For $n>0$: $\frac{\partial \widetilde{g}_Y}{\partial n}>0$, in the two R&D sector model with infinitely living agents output growth is increasing with population growth.*

Proof. Applying (23) to the output growth rate (15) yields $1+\widetilde{g}_Y = \rho(1+n)\left(\frac{1}{\alpha}-1\right)$. ■

5 Conclusions

In this work we explored the implications of agents' lifetime to the predictions of R&D driven growth theory. It was shown that the standard OLG framework of finitely living agents differs from the corresponding economy of infinitely living agents, with respect to their dynamic properties and the effects of population growth rate on long run output growth.

We have shown that in the OLG economy long run output growth is neutral to population growth in a reference second and third generation growth models. Our results are in line with the differences in scale effect due to different saving motives explored by Dalgaard Jensen (2009), for a model with capital externalities. Nonetheless, we have shown that the dynamics of innovation cost are crucial to the relation between population and output growth rates, whereas innovation cost and population growth rate are abstracted in Dalgaard Jensen (2009).

Furthermore our direct comparative approach between corresponding frameworks of finitely and infinitely living agents confirms that the demographic structure itself affects the relations between population and output growth rates. Our results imply that the counterfactual relation between population and output growth rates derived in models of infinitely living agents are driven by their specific demographic structure, and thus are not as limiting for R&D based growth theory itself.

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Appendix: CRRA instantaneous utility function

Consider the general CRRA (instantaneous) utility function

$$u(c_t, c_{t+1}) = \frac{c_t^{1-\theta}}{1-\theta} + \rho \frac{c_{t+1}^{1-\theta}}{1-\theta} \tag{A.1}$$

The saving function modifies to

$$S_t = \frac{A(1-\alpha)N_t \alpha^{\frac{2\alpha}{1-\alpha}} L_t}{1 + \rho^{-\frac{1}{\theta}} (1 + r_{t+1})^{\frac{\theta-1}{\theta}}} \tag{A.2}$$

Where the interest rate is given by free entry condition

$$\frac{A^{\frac{1}{1-\alpha}} \left(\frac{1}{\alpha} - 1\right) \alpha^{\frac{2}{1-\alpha}} L_{t+1} + \eta_{t+1}}{\eta_t + A^{\frac{1}{1-\alpha}} L_{t+1} \alpha^{\frac{2}{1-\alpha}}} = 1 + r_{t+1} \tag{A.3}$$

The interest rate is increasing in L_t if $\alpha < \frac{1}{2}$. Replacing $(1+r_{t+1})$ in the savings equation we obtain

$$S_t = \frac{(1-\alpha)AL_t N_t \alpha^{\frac{2\alpha}{1-\alpha}}}{1 + \rho^{-\frac{1}{\theta}} \left[\frac{\left(\frac{1}{\alpha}-1\right) L_{t+1} A^{\frac{1}{1-\alpha}} \alpha^{\frac{2}{1-\alpha}} + \eta_{t+1}}{L_{t+1} A^{\frac{1}{1-\alpha}} \alpha^{\frac{2}{1-\alpha}} + \eta_t}\right]^{\frac{\theta-1}{\theta}}} \tag{A.4}$$

Equalizing saving to investment and rearranging yields variety growth rate

$$1 + g_N = \frac{A(1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}}}{\left[\frac{\eta_t}{L_t} + A^{\frac{1}{1-\alpha}} (1+n) \alpha^{\frac{2}{1-\alpha}}\right] \left[1 + \rho^{-\frac{1}{\theta}} \left[\frac{\left(\frac{1}{\alpha}-1\right) A^{\frac{1}{1-\alpha}} \alpha^{\frac{2}{1-\alpha}} + \frac{\eta_{t+1}}{L_{t+1}}}{A^{\frac{1}{1-\alpha}} \alpha^{\frac{2}{1-\alpha}} + \frac{\eta_t}{L_{t+1}}}\right]^{\frac{\theta-1}{\theta}}\right]} \tag{A.5}$$

Note that $1 + \widetilde{g_N}$ - the long run value of (A.5) - depends on value of $\lim_{t\to\infty} \frac{\eta_t}{L_t}$ just like (9a). This limit is constant iff $\gamma = n = 0$. Only then a long term scale effect presents in the model. If $\lim_{t\to\infty} \frac{\eta_t}{L_t} = 0$ equation (A.5) modifies to

$$1 + \widetilde{g}_Y = (1 + \widetilde{g}_N)(1+n) = \frac{(1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}}}{A^{\frac{\alpha}{1-\alpha}} \alpha^{\frac{2}{1-\alpha}} \left[1 + \rho^{-\frac{1}{\theta}} \left(\frac{1}{\alpha} - 1\right)^{\frac{\theta-1}{\theta}}\right]} \tag{A.6}$$

Hence long run output growth rate is independent of population growth rate for any value of θ. If $\lim_{t\to\infty} \frac{\eta_t}{L_t} \to \infty$ output growth rate approaches zero like in (9a).