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A Nonparametric Study of Real Exchange Rate Persistence over a Century

Hyeongwoo Kim* and Deockhyun Ryu†

**Auburn University*, †*Chung-Ang University*

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A nonparametric study of real exchange rate persistence over a century [*]

Hyeongwoo Kim[†] and Deockhyun Ryu[‡]

Auburn University and Chung-Ang University

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Abstract

This paper estimates the degree of persistence of 16 long-horizon real exchange rates relative to the US dollar. We use nonparametric operational algorithms by El-Gamal and Ryu (2006) for general nonlinear models based on two statistical notions: the short memory in mean (SMM) and the short memory in distribution (SMD). We found substantially shorter maximum half-life (MHL) estimates than the counterpart from linear models. Our results are robust to the choice of bandwidth with a few exceptions.

JEL Classification: C14; C15; C22; F31; F41

Keywords: Real Exchange Rate; Purchasing Power Parity; Short Memory in Mean; Short-Memory in Distribution; ϕ-mixing; Max Half-Life; Max Quarter-Life

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[†] Department of Economics, Auburn University, Auburn, AL 36849. Tel: 1-334-844-2928, Fax: 1-334-844-4615, Email: gmmkim@gmail.com.

[‡] Corresponding Author: Deockhyun Ryu, Department of Economics, Chung-Ang University, 221 Heuksuk-Dong, Dongjak-Gu, Seoul 156-756, Korea. Tel: 82-2-820-5488, Email: dhryu@cau.ac.kr.

I Introduction

This paper measures the persistence of the real exchange rate using a nonlinear nonparametric approach developed by El-Gamal and Ryu (2006) for 16 long-horizon real exchange rates of developed countries relative to the US dollar.

Taylor (2002) constructed over a hundred-year long real exchange rates for 20 countries. Implementing an array of *linear* unit root tests, he reported very strong evidence in favor of purchasing power parity (PPP), which was later questioned by Lopez, Murray, and Papell (2005) who pointed out that his results were not robust to the choice of lag selection methods. Kim and Moh (2010), however, employed a nonlinear unit root test by Park and Shintani (2005, 2012) that allowed an array of transition functions for Taylor's (2002) data, finding very strong evidence of nonlinear PPP.

Even though the current literature finds fairly strong evidence for PPP from long-horizon real exchange rates, the profession fails to find persuasive answers to the so-called PPP puzzle (Rogoff, 1996), which states that the 3- to 5-year consensus half-life, based on *linear* models, seems too large to be reconciled by highly volatile short-run exchange rate dynamics.

Furthermore, Murray and Papell (2002) and Rossi (2005), among others, report half-lives with wide confidence intervals that extend to positive infinity. Panel estimations often provide substantially shorter half-lives than the consensus half-life, however, Murray and Papell (2005) reported similarly long half-life estimates from panel models correcting for small-sample bias.[1]

[1] One related issue of aggregation bias was raised by Imbs, Mumtaz, Ravn, and Rey (2005), who point out that PPP puzzle might be caused by aggregation bias which neglects sectoral

One promising approach to understand the PPP puzzle is to employ nonlinear models of the exchange rate. As shown by Taylor (2001), half-life estimates from linear models tend to be biased upward when the true data generating process (DGP) is nonlinear. Therefore, removing the bias by adopting nonlinear models may yield reasonably short half-life estimates.

Nonlinear models have been widely used in the study of financial data including exchange rates, which are mostly motivated by market friction arguments such as transaction costs (see Dumas, 1992).[2] Examples include Sercu, Uppal, and Hulle (1995), Michael, Nobay, and Peel (1997), Obstfeld and Taylor (1997), Sarantis (1999), Taylor, Peel, and Sarno (2001), Kilian and Taylor (2003), Sarno, Taylor, and Chowdjury (2004), Kim and Moh (2010), and Lee and Chou (2013).

However, it is not straightforward how to measure the persistence from nonlinear models, because exchange rates in these researches obey state-dependent stochastic processes. That is, the half-life from these models depends upon the current state and the size of the shock.

One may estimate the persistence of the real exchange rate only in regimes outside the inaction band, that is, subsets of the full sample, which is not fully comparable to half-life measures from linear models based on the full sample. Rigorous methods include Gallant, Rossi, and Tauchen (1993), Koop, Pesaran, and Potter (1996), and Potter (2000) who proposed nonlinear analogs of impulse-response functions. See, among others, Baum, Barkoulas, and Caglayan (2001) and

heterogeneity in convergence rates, while Chen and Engel (2005), Parsley and Wei (2007), Crucini and Shintani (2008), and Broda and Weinstein (2008) have found negligible aggregation biases.

[2] Prohibitively large transaction costs may discourage economic agents from engaging in arbitrage. That is, adjustments toward the long-run equilibrium take place only when deviations from the equilibrium are big enough.

Lothian and Taylor (2008) for research work that employ such methods. Shintani (2006) also proposed a nonparametric method based on the largest Lyapunov exponent of the series to evaluate the speed of adjustment in presence of nonlinearities, finding fairly shorter half-lives than the consensus half-life.

This paper uses a nonlinear nonparametric approach proposed by El-Gamal and Ryu (2006) that employs more general time series notions of the convergence toward the long-run equilibrium: short-memory-in-mean (SMM) and short-memory-in-distribution (SMD) as an alternative to the stationarity in linear model framework (Granger and Teräsvirta, 1993; Granger, 1995). SMM and SMD nest linear models as a special case.

Our nonparametric approach does not require the knowledge on the parametric representation of transition functions nor any distributional assumptions, so our results are less likely to be influenced by specification errors. In what follows, we provide straightforward algorithms to measure the persistence not only for the first moment (SMM), but also for the entire distribution (SMD). That is, after estimating conditional and unconditional densities by kernel methods, we measure the rate of convergence by using metrics for SMM and SMD based on a worst-case scenario.

Using long-horizon real exchange rates for 16 currencies *vis-à-vis* the US dollar, we find reasonably short half-lives using notions of SMM and SMD with exceptions of Canada, Japan, and Switzerland. Especially, our maximum half-life estimates for SMM with asymptotically optimal bandwidth are substantially shorter than those from linear models (e.g., Murray and Papell, 2002, 2005; Rossi, 2005), which confirms the issue of an upward bias suggested by Taylor (2001). Our estimates for SMD add new insights to the current literature in favor of a century-long PPP, which is valid even when first moments are not well-defined. We also report maximum quarter-life estimates (Steinsson, 2008) to study monotonicity of convergence over time.

We also note that our results provide interesting contrast compared with those of El-Gamal and Ryu (2006) who used five short-horizon current float (post Bretton Woods) exchange rates relative to the US dollar. Their estimates tend to exhibit very slow convergence rates as the bandwidth parameter increases, which may imply indefinitely long half-lives, even though their half-life estimates are similar to ours when fairly wide bandwidth window is used. This may indicate that utilizing long-horizon data might be crucially important to help understand the PPP puzzle.

The remainder of the paper is organized as follows. Section 2 presents our baseline methodologies and operational algorithms for estimating convergence rates using our key statistical notions. In Section 3, we describe the data and report major empirical findings. Section 4 concludes.

II The Econometric Model

This section presents some useful definitions for our nonparametric model as an alternative to conventional linear models that are often employed in the current empirical international economics literature. We also provide our nonparametric measures of persistence for a general Markovian univariate time series models.

Let e_t be the natural logarithm nominal exchange rate as the domestic currency (US dollar) price of the foreign currency. p_t and p_t^* denote the price level in the home (US) and the foreign country, respectively, in natural logarithms.

When e_t, p_t^*, and p_t are individually integrated (nonstationary) processes, but are cointegrated with the cointegrating vector $[1,\ 1, -1]$, the real exchange rate, $x_t = e_t + p_t^* - p_t$, is a weakly stationary process, which is consistent with the conventional linear model for PPP. It is convenient to use an autoregressive process for x_t to measure the persistence of PPP deviations as follows.

$$x_{t+1} = \rho x_t + \varepsilon_{t+1},$$

where deterministic terms are omitted for simplicity and ρ is the persistence parameter bounded by 1 from above.

Alternatively, we consider the following representation for x_t which nests the previous linear representation as a special case.

$$x_{t+1} = m(x_t) + \varepsilon_{t+1}$$

Note that this equation implies $m(x_t)$ is the conditional expectation of x_{t+1} at time t given information set. The present paper extends this nonlinear representation into a general framework that extends more than the first moment.

We employ nonparametric measures of persistence for general nonlinear model, which is based on the framework proposed by El-Gamal and Ryu (2006) for a first-order Markovian univariate time series $\{x_t\}$. Abandoning linearity in time series domain, we pursue nonlinearity in density domain instead. From the Chapman-Kolmogorov equations, we define transition probability kernel and the Markov operator, which can be approximated by a finite transition matrix. We also directly apply the consistent tests of ergodicity and mixing to our real exchange rates via Domowitz and El-Gamal (1993, 1996, 2001) .

As stated in El-Gamal and Ryu (2006), we define the short memory in distribution (SMD) and the short memory in mean (SMM) as follows. The time series is said to have the *Short Memory in Distribution* (SMD) property if $F_s(x) \Rightarrow \overline{F}(x)$, as $s \uparrow \infty$ where $F_s(x) = \Pr(x_{t+s} \leq x \mid \mathrm{A}_t)$ is the cumulative distribution function of x_{t+s} conditional on the past information set $\mathrm{A}_t = \sigma(x_{t-j}; j \geq 0)$, and $\bar{F}$ be some fixed (unconditional) distribution function. The time series is said to have the *Short Memory in Mean* (SMM) property if $\| E[x_{t+s} \mid \mathrm{A}_t] - E[x_{t+s}] \| < c_s;\ c_s \xrightarrow{s\uparrow\infty} 0$.[3]

We use the asymptotic independence notion of uniform or ϕ-mixing to study SMD and SMM. As shown by El-Gamal and Ryu (2006), we can calculate measures of SMD and SMM numerically. That is, we can get the finite grid analog $\phi_n(s)$ which converges to $\phi(s)$ as the grid size $n \uparrow \infty$. Similarly, we can also get the grid $MDM_n(s)$ which converges to the Maximum Distance in Mean, $MDM(s)$, the measure of SMM, as the grid size $n \uparrow \infty$. For the detailed explanations on the numerical algorithms to compute our persistence measures and convergence arguments of finite grids of SMD and SMM, see El-Gamal and Ryu (2006).

The notion of half-life can now be replaced by the value of s at which $MDM_n(s) = 0.5 \times MDM_n(0)$, that is, the number of periods needed for the worst possible transitory shock from the unconditional mean to be cut in half. This notion may then be extended beyond half-life to consider *Max m-life* as the number of time periods before the worst possible shock would have shrunk to (1-*m*) of its original magnitude. Likewise, we define *Max quarter-life* by the number of time periods

[3] Note that SMM is equivalent to *mixing in mean* or *mixingales* as discussed in McLeish (1978) and Gallant and White (1988), while SMD shares a property of mixing.

before the worst possible shock would have shrunk to 0.25, i.e., m = 0.75 of its initial one unit shock.[4]

In the context of the real exchange rate literature, we measure the persistence of deviations of the real exchange rate by our m-life curve, which represents the time needed for a transitory deviation of the real exchange rate from its long-run PPP to be cut by $1-m$, for all $m \in (0,1)$. We calculate the m-life with a notion of the short memory-in-mean.[5] Our most general and finest measure of the persistence is the short memory-in-distribution, as measured by $\phi(s)$, which is obtained by a similar algorithm. This measure looks beyond the first moment, and can provide a general feature of the dependence structure of our time series. It should be noted that this measure can still apply even when underlying distributions do not have the first moment.[6]

For non-parametric estimation of $P_{T,n}$ using a kernel estimator, we begin with the estimated $\phi(s)$ and *Max m-life* using so-called Silverman's rule of thumb $h_T = \sigma_T \; T^{-1/5}$, where σ_T is the standard deviation of our series. It turns out that this choice of bandwidth is *asymptotically* optimal.[7] We note that the estimated *Max m-life* with this bandwidth selection rule typically yield quite less persistent dynamics which is in favor of the PPP hypothesis. However, as El-Gamal and Ryu (2006) shows, such results may not be reliable because this selection rule tends to produce

[4] This metric is an extension of the quarter-life that is introduced by Steinsson (2008), which is based on linear regression models. This additional measure of persistence can be used to see if the convergence takes place monotonically.
[5] This m-life measure can be compared to the traditional linear-based half-life measure when $m = 0.5$. However, whereas traditional half-life measures are subject to specification errors, our m-life measures are free of the specification issue.
[6] For example, the Cauchy distribution does not have either the first or the second moment.
[7] Silverman's rule of thumb bandwidth is optimal if the true density is normal.

an over-smoothed estimate of the transition density in *finite samples*, which results in downward bias in the estimates of $\phi(s)$ and *Max m-life*. Therefore, one has to be careful in interpreting their empirical findings since Silverman's rule of thumb approach may not work well in small samples.

Realizing this issue, we implement estimations for an array of the choice of the level of under-smoothing, k. That is, we modify Silverman's rule of thumb as follows.

$$h_T = \left(\frac{\sigma_T}{k}\right) T^{-1/5},$$

where $k = 1$ corresponds to Silverman's rule of thumb bandwidth. We report our estimation results for k ranging 1 to 10. We note our estimates for $\phi(s)$ (or *Max m-life*) often converge each other as k approaches to 10. We interpret such results as empirical findings that support the validity of the PPP hypothesis. Likewise, the time series that fails to converge each other as k approaches to 10 provides evidence against the PPP hypothesis.

III Empirical Results

We extended Taylor's (2002) over hundred-year long real exchange rates for 16 developed countries relative to the US dollar with additional observations through 2013 for non-Eurozone countries from the IFS CD-ROM. For Eurozone countries, the sample period was extended to 2001 using official conversion rate. We omitted 4 developing countries focusing on currencies in developed countries. The data frequency is annual and all exchange rates are CPI-based rates with an exception of Portugal, which is based on the GDP deflator.

In Table 1, we first report benchmark estimates for the half-life from a linear model. We chose the number of lags by the general-to-specific rule with a maximum 6 lags. It is well-known that the least squares estimator for the persistence parameter in autoregressive models is biased when deterministic terms are present. We correct for median bias using Hansen's (1999) grid bootstrap method.

Overall, we find evidence that is consistent with the PPP puzzle from the linear model. We obtain very long half-life point estimates ranging from 2.030 years for Finland to positive infinity for Japan and Switzerland. 95% lower-bound estimates range from 1.297 to 32.596 years, while upper-bounds extend to positive infinity for 9 out of 16 currencies.

This seemingly sluggish rate of adjustment, however, does not necessarily imply strong evidence of the PPP puzzle, because as Taylor (2001) points out, if the true data generating process is nonlinear, statistical inferences based on the linear model framework are not reliable due to specification errors. In what follows, we present substantially faster convergence rates based on our nonparametric nonlinear models for the real exchange rate.

Table I around here

We next implement statistical tests for ergodicity and mixing, proposed by Domowitz and El-Gamal (2001), for our exchange rates. For this purpose, unit root processes are reformulated as a general ergodic failure in a *nonlinear* first-order Markovian univariate process. The test rejects the null hypothesis of ergodicity if the p-value of a single randomized test is smaller than a pre-specified value. We then determine the rejection of ergodicity null by the percentiles of the density of *p*-values which are less than or close to a pre-specified number, e.g., 5%.

Our randomized test fails to reject the null of mixing for all countries of which the percentile of *p*-values are substantially different from pre-specified values, which is consistent with empirical findings of nonlinear mean reversion via the inf-*t* test from Kim and Moh (2010). In contrast, the test fails to reject the null of ergodicity for 9 countries, but rejects the null for remaining 7 countries in the very restricted sense, Belgium, France, Germany, Netherlands, Portugal, Spain, and Switzerland, which may reflect the size distortion shown in Domowitz and El-Gamal (2001).

Table II around here

Next, we report our max half-life (MHL) estimates for the SMM (mixingale) and SMD properties in Tables III and IV, respectively, for the smoothing parameter (k) ranging from 1 to 10 to check how robust our estimates are to the choice of bandwidth. We also report max quarter-life (MQL) estimates for SMM and SMD in Tables V and VI, respectively. In addition, we provide graphical representations of our estimates for these properties by plotting all normalized *MDM(s)* and $\phi(s)$ for k from 1 to 10 in Figures 1 and 2, for SMM and SMD, respectively.

As we can see in Figure 1, normalized *MDM(s)* decline rapidly for all k with exceptions of Canada, Japan, and Switzerland, which imply strong evidence of SMM. Similarly, $\phi(s)$ decrease rapidly with exceptions of those three countries for all k which implies evidence in favor of SMD. Note that MHL for SMM are converging each other as k increases toward 10 where the MHL for k = 10 becomes an upper limit for most countries, while the MHL is not well-defined Canada, Japan, and Switzerland even when k = 10. Similarly, the MHL is not well-defined for these two three countries when we investigate persistence based on the SMD property, while we obtain well-defined MHLs for the rest.

Estimated MHLs for SMM range from 0.889 to 3.262 when we use the rule of thumb $k = 1$, while we obtained much longer values when $k = 10$ is used, even though most MHLs converge as the smoothing parameter increases to k = 10. MHL estimates for SMD range from 0.940 to 4.985 when k = 1, which are longer than those for SMM. Again, with exceptions of Canada, Japan, and Switzerland, convergence was made for most countries, implying that MHLs when k = 10 serve as an upper-limit. Naturally, MQL estimates for SMM and SMD are longer than estimated MHLs, but resemble similar movements as those of MHLs. Convergence were not made only for Canada, Japan, and Switzerland.

These findings suggest strong support for a century of PPP in the sense that we find reasonably fast convergence rate toward the long-run equilibrium in a general nonlinear framework. [8]

Tables III and IV, V, and VI around here

Figures 1 and 2 around here

In addition, this paper also investigates possible non-monotonic adjustments toward the long-run equilibrium by a metric developed by Steinsson (2008) for linear models. Note that MHL should equal to MQL minus MHL if the adjustment takes place monotonically. As we can see in Tables VII and VIII V for

[8] In contrast to existing measures of the half-life, our measures are free from any parametric specification errors. Further, our SMD-based persistence measures are applicable even when the first moment is not well-defined in underlying distributions. Also, our operational algorithms provide flexible approaches to study shock dissipation processes beyond the mid-point. Murray and Papell (2005) also discussed potential advantages of looking at points other than the half-life.

SMM and SMD, respectively, mostly negative values were obtained especially when *k* is small. This implies the speed of adjustment is faster in the first half compared with that during the second half.[9]

Tables VII and VIII V around here

IV Concluding Remarks

We estimate the persistence of 16 over hundred-year long real exchange rates relative to the US dollar by a nonlinear nonparametric approach suggested by El-Gamal and Ryu (2006). We first obtain conditional and unconditional kernel density functions to acquire nonparametric measures of the speed of convergence towards the long-run equilibrium. We study not only the convergence in the first moment (SMM) but also in distribution (SMD), which might be useful when unknown underlying distributions do not have a well-defined first moment.

Our nonparametric half-life estimates obtained with asymptotically optimal bandwidth are substantially shorter than those from linear models such as Murray and Papell (2002, 2005) and Rossi (2005). Therefore, our findings confirm the existence of a potential pitfall proposed by Taylor (2001), who pointed out the existence of an upward bias in the half-life estimate from linear models. We also obtained reasonably short half-lives unless we deviate greatly from the benchmark case. Therefore, it seems that our finding of substantially shorter half-lives remains valid.

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[9] Steinsson (2008) reports mostly positive estimates using the US real exchange rate data for the post-Bretton Woods system, which may be consistent with hump-shape dynamics.

Our estimates seem robust to the choice of the smoothing parameter with exceptions of Canada, Japan, and Switzerland. The results are consistent with other research such as Kim and Moh (2010) who report strong evidence in favor of PPP by parametric nonlinear AR models for most long-horizon real exchange rates but Canada and Japan. However, as we observed in cases of Canada, Japan, and Switzerland, empirical evidence of PPP based on the rule-of-thumb bandwidth can be quite fragile. That is, our findings imply that a simple comparison between half-lives is not sufficient to check the validity of the PPP hypothesis.

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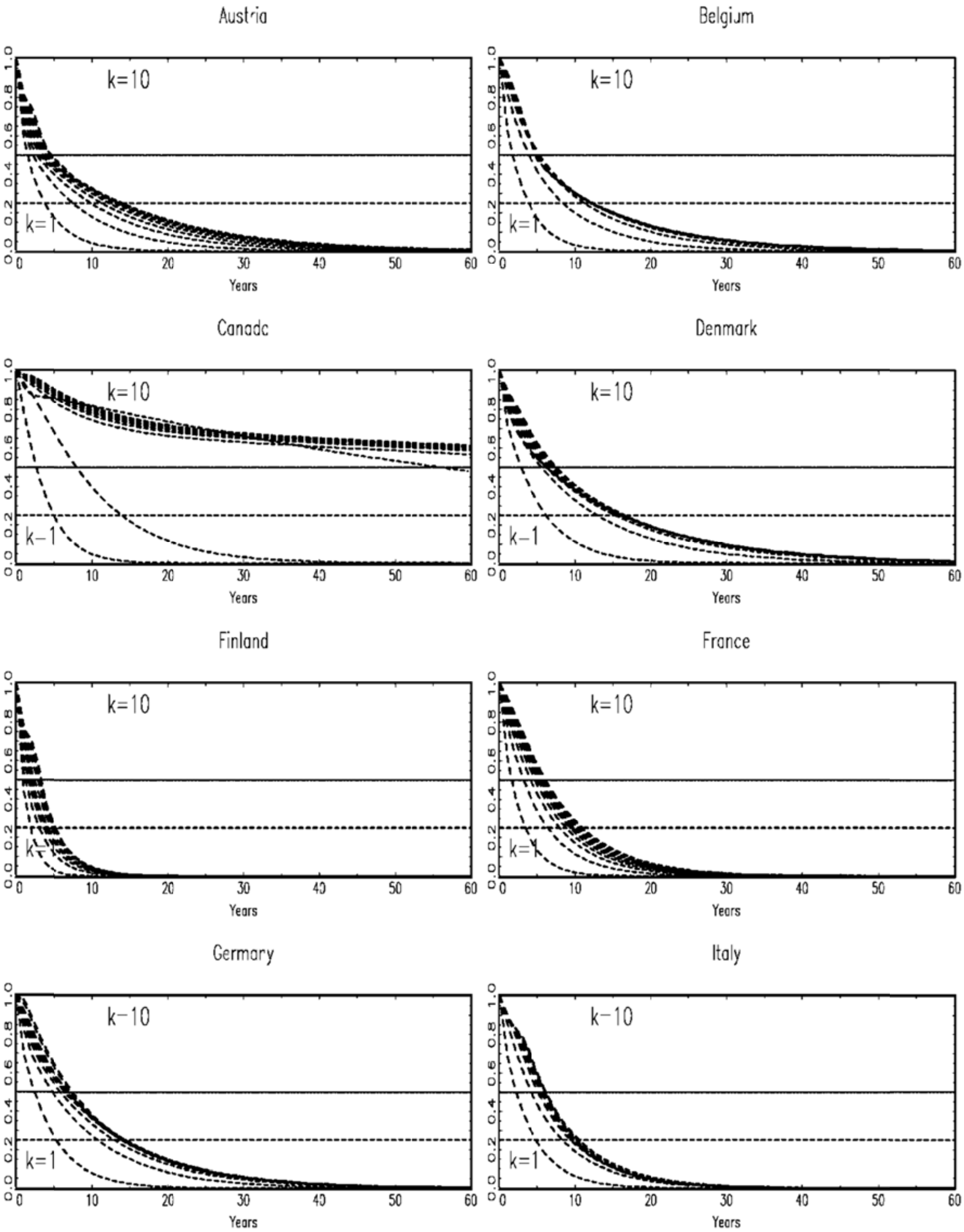
Figure 1. Short-Memory in Mean Properties



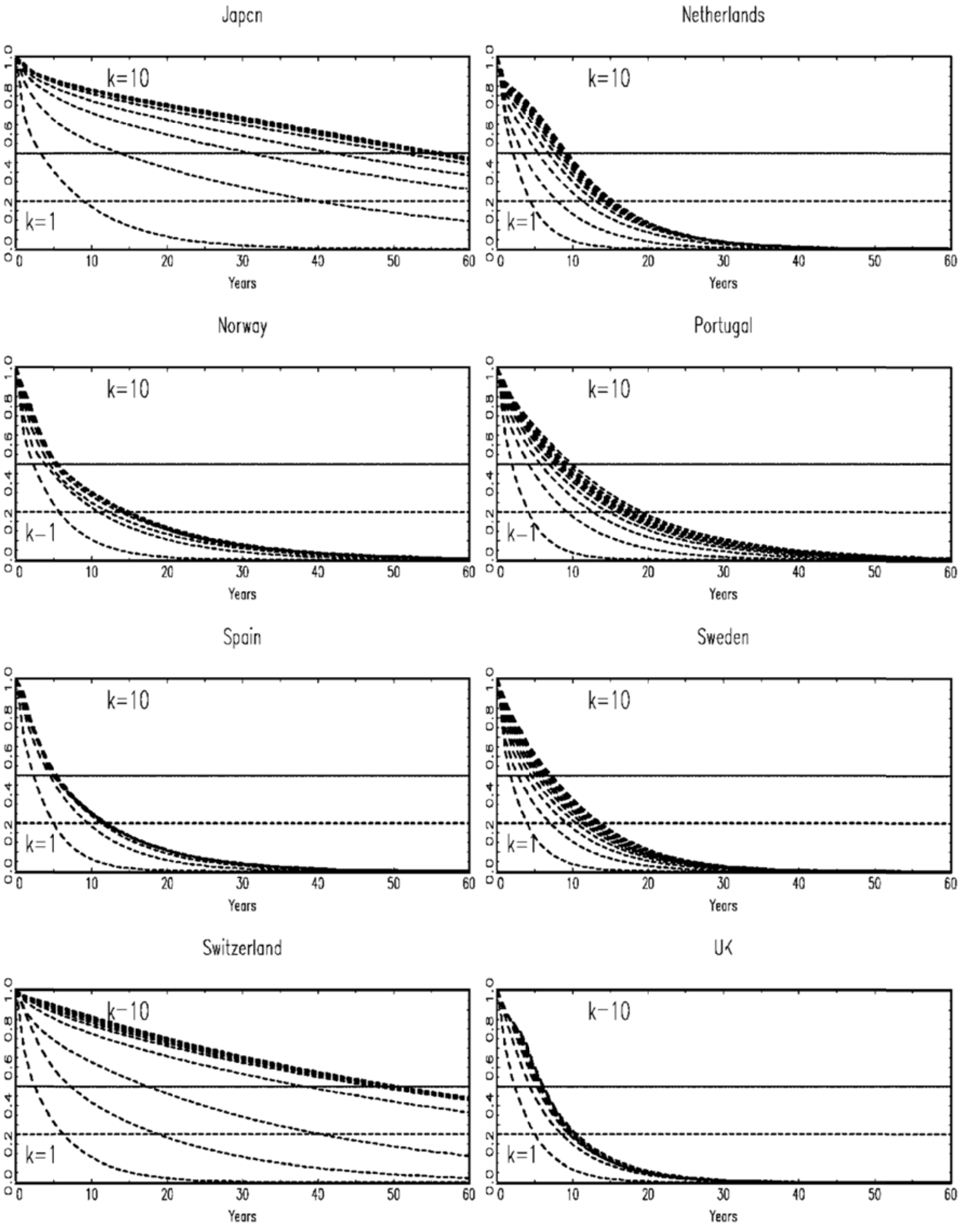
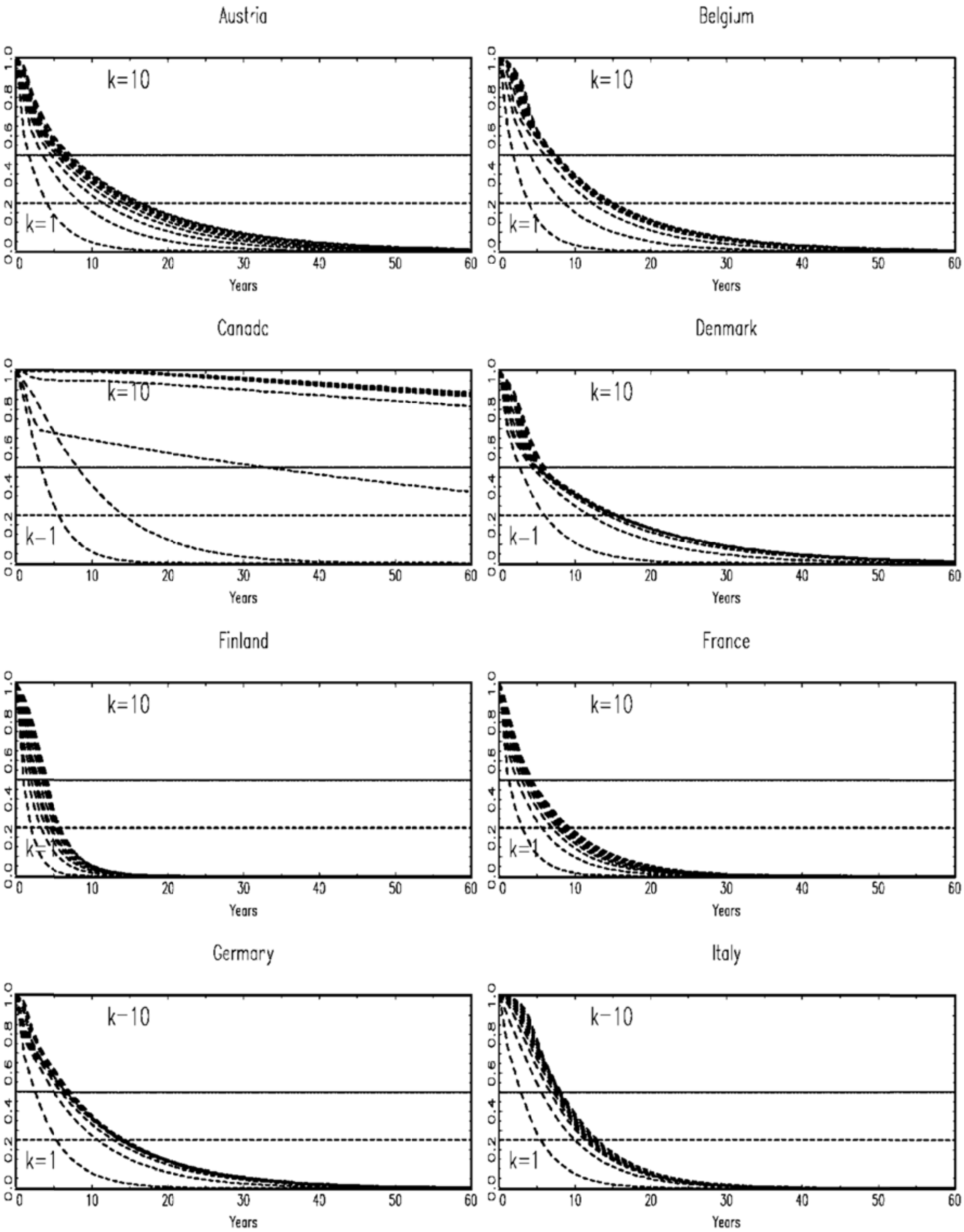
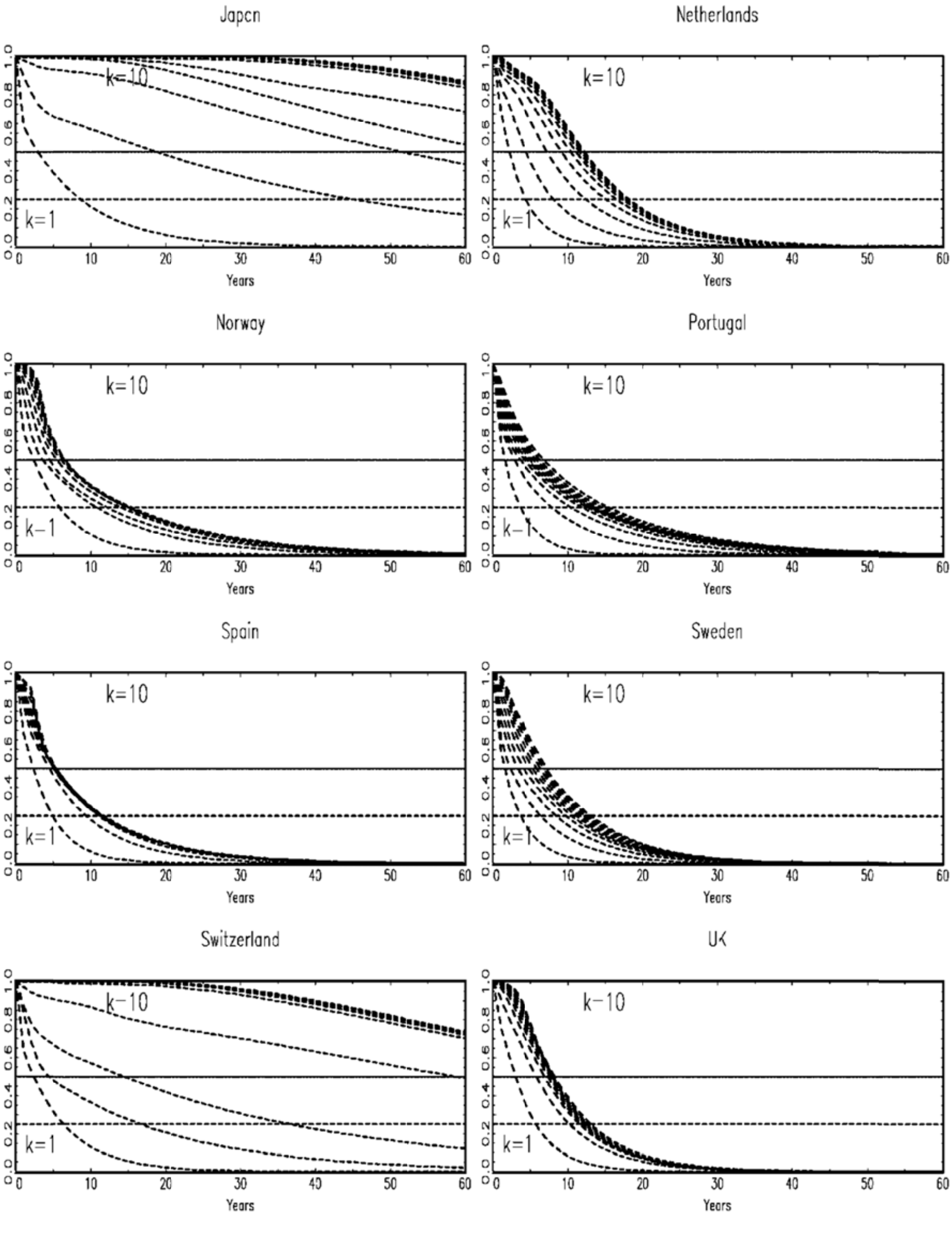

Japan
Netherlands
Norway
Portugal
Spain
Sweden
Switzerland
UK
k=10
k=1
Years
0.0
0.2
0.4
0.6
0.8
1.0
0
10
20
30
40
50
60

Figure 2. Short-Memory in Distribution Properties




Japan
k=10
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Netherlands
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Norway
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Portugal
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k=1
Years
Spain
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Years
Sweden
k=10
k=1
Years
Switzerland
k=10
k=1
Years
UK
k=10
k=1
Years

Table I. Half-Life Estimation from a Linear Model

Country	Sample Period	HL	LB	UB
Austria	1870 - 2013	10.600	4.485	∞
Belgium	1880 - 2001	3.847	2.344	14.334
Canada	1870 - 2013	9.306	4.281	∞
Denmark	1880 - 2013	12.842	5.322	∞
Finland	1881 - 2001	2.030	1.297	4.427
France	1880 - 2001	6.257	3.200	92.896
Germany	1880 - 2001	13.753	5.504	∞
Italy	1880 - 2001	3.810	2.004	9.463
Japan	1885 - 2013	∞	32.596	∞
Netherlands	1870 - 2001	10.212	4.684	∞
Norway	1870 - 2013	11.351	5.293	∞
Portugal	1890 - 2001	5.725	2.834	68.684
Spain	1880 - 2001	7.251	3.724	67.489
Sweden	1880 - 2013	6.366	3.206	∞
Switzerland	1880 - 2013	∞	28.981	∞
UK	1870 - 2013	4.754	2.600	20.659

Note: i) All real exchange rates are relative to the US dollar. Exchange rates of the Eurozone countries have been extended until 2001 using official conversion rates. ii) The point estimate and the 95% confidence interval are corrected for median bias by Hansen's (1999) grid bootstrap method. For this, 500 bootstrap simulations on each of 30 fine grid points over an interval $[\hat{\rho} - 6 \times s.e., \hat{\rho} + 6 \times s.e.]$ were implemented, where $\hat{\rho}$ and $s.e.$ are the point estimate of the persistence parameter and its standard error, respectively. iii) The number of lags was chosen by the general-to-specific rule with a maximum 6 lags.

Table II. Tests for Ergodicity and Mixing

	Ergodicity		*Mixing*	
Country	$\Pr(pv < 0.05)$	$\Pr(pv < 0.10)$	$\Pr(pv < 0.05)$	$\Pr(pv < 0.10)$
Austria	3%	7%	3%	6%
Belgium	13	21	3	7
Canada	2	6	3	8
Denmark	3	7	3	8
Finland	4	8	3	8
France	6	12	3	7
Germany	13	21	4	8
Italy	5	10	4	7
Japan	4	9	3	8
Netherlands	9	18	3	8
Norway	4	7	2	6
Portugal	9	17	4	9
Spain	11	19	5	11
Sweden	3	9	3	7
Switzerland	6	12	3	8
UK	3	7	2	6

Note: i) All real exchange rates are relative to the US dollar. ii) These are randomized tests proposed by Domowitz and El-Gamal (2001). iii) The numbers in the table are the percentage of rejections at the 5% and the 10% significance level, respectively, from 1,000 independent randomized runs.

Table III. Maximum Half Life Estimation: SMM

Country	$k = 1$	$k = 10$	Convergence
Austria	1.486	4.646	Yes
Belgium	1.713	4.765	Yes
Canada	2.645	$n.a.$	No
Denmark	2.832	7.548	Yes
Finland	0.889	3.285	Yes
France	1.597	6.098	Yes
Germany	2.385	7.493	Yes
Italy	2.169	6.116	Yes
Japan	3.262	56.551	No
Netherlands	2.021	9.353	Yes
Norway	2.227	5.504	Yes
Portugal	1.871	9.929	Yes
Spain	2.276	5.040	Yes
Sweden	1.710	7.245	Yes
Switzerland	2.427	50.821	No
UK	2.266	6.097	Yes

Note: i) All real exchange rates are relative to the US dollar. ii) Estimates are calculated by linear interpolations. iii) We estimate Max Half-Life (MHL) and Max Quarter-Life (MQL) for the smoothing parameter k raning 1 through 10. iv) We denote "Yes" in the last column when the m-life estimates converge as k approaches to 10, that is, when greater values for k produces no substantial difference in MHL and MQL estimates of the normalized Maximal Distance Measure (MDM).

Table IV. Maximum Half Life Estimation: SMD

Country	$k = 1$	$k = 10$	Convergence
Austria	1.654	7.169	Yes
Belgium	1.798	7.546	Yes
Canada	3.207	$n.a.$	No
Denmark	2.638	5.972	Yes
Finland	0.940	4.034	Yes
France	1.177	4.321	Yes
Germany	2.466	7.176	Yes
Italy	2.797	8.155	Yes
Japan	2.942	$n.a.$	No
Netherlands	2.123	12.226	Yes
Norway	2.341	6.575	Yes
Portugal	1.521	6.811	Yes
Spain	2.283	5.392	Yes
Sweden	1.520	7.131	Yes
Switzerland	2.329	$n.a.$	No
UK	2.943	8.218	Yes

Note: i) All real exchange rates are relative to the US dollar. ii) Estimates are calculated by linear interpolations. iii) We estimate Max Half-Life (MHL) and Max Quarter-Life (MQL) for the smoothing parameter k raning 1 through 10. iv) We denote "Yes" in the last column when the m-life estimates converge as k approaches to 10, that is, when greater values for k produces no substantial difference in MHL and MQL estimates of the normalized Maximal Distance Measure (MDM).

Table V. Maximum Quarter Life Estimation: SMM

Country	$k = 1$	$k = 10$	Convergence
Austria	3.753	17.667	Yes
Belgium	3.829	12.350	Yes
Canada	4.922	$n.a.$	No
Denmark	6.200	13.458	Yes
Finland	1.883	5.169	Yes
France	3.410	11.000	Yes
Germany	5.263	14.080	Yes
Italy	4.801	10.538	Yes
Japan	8.963	$n.a.$	No
Netherlands	4.378	15.367	Yes
Norway	5.719	17.625	Yes
Portugal	4.079	19.263	Yes
Spain	4.922	11.929	Yes
Sweden	3.908	11.094	Yes
Switzerland	6.253	45.875	Yes
UK	4.956	10.538	Yes

Note: i) All real exchange rates are relative to the US dollar. ii) Estimates are calculated by linear interpolations. iii) We estimate Max Half-Life (MHL) and Max Quarter-Life (MQL) for the smoothing parameter k raning 1 through 10. iv) We denote "Yes" in the last column when the m-life estimates converge as k approaches to 10, that is, when greater values for k produces no substantial difference in MHL and MQL estimates of the normalized Maximal Distance Measure (MDM).

Table VI. Maximum Quarter Life Estimation: SMD

Country	$k = 1$	$k = 10$	Convergence
Austria	4.002	16.486	Yes
Belgium	3.827	15.160	Yes
Canada	5.588	$n.a.$	No
Denmark	6.004	15.674	Yes
Finland	1.923	5.785	Yes
France	2.950	9.437	Yes
Germany	5.292	14.340	Yes
Italy	5.479	12.948	Yes
Japan	8.637	$n.a.$	No
Netherlands	4.336	18.153	Yes
Norway	5.813	14.922	Yes
Portugal	3.624	15.424	Yes
Spain	4.867	11.569	Yes
Sweden	3.673	13.107	Yes
Switzerland	6.328	$n.a.$	No
UK	5.699	13.062	Yes

Note: i) All real exchange rates are relative to the US dollar. ii) Estimates are calculated by linear interpolations. iii) We estimate Max Half-Life (MHL) and Max Quarter-Life (MQL) for the smoothing parameter k raning 1 through 10. iv) We denote "Yes" in the last column when the m-life estimates converge as k approaches to 10, that is, when greater values for k produces no substantial difference in MHL and MQL estimates of the normalized Maximal Distance Measure (MDM).

Table VII. Monotonic Convergence (*2MHL – MQL*): SMM

Country	$k = 1$	$k = 10$	Convergence
Austria	-0.781	-8.375	Yes
Belgium	-0.403	-2.820	Yes
Canada	0.368	$n.a.$	No
Denmark	-0.536	1.638	Yes
Finland	-0.105	1.401	Yes
France	-0.216	1.196	Yes
Germany	-0.493	0.906	Yes
Italy	-0.463	1.694	Yes
Japan	-2.439	$n.a.$	No
Netherlands	-0.336	3.339	Yes
Norway	-1.265	-6.617	Yes
Portugal	-0.337	0.595	Yes
Spain	-0.37	-1.849	Yes
Sweden	-0.488	3.396	Yes
Switzerland	-1.399	55.767	No
UK	-0.424	1.656	Yes

Note: i) All real exchange rates are relative to the US dollar. ii) Estimates are calculated by linear interpolations. iii) $2MHL - MQL$ is adopted from Steinsson (2008). Zero values for $2MHL - MQL$ imply monotonic adjustment process towards the long-run equilibrium. Negative values occur when $MHL < MQL - MHL$.

Table VIII. Monotonic Convergence ($2MHL - MQL$): SMD

Country	$k = 1$	$k = 10$	Convergence
Austria	-0.694	-2.148	Yes
Belgium	-0.231	-0.068	Yes
Canada	0.826	$n.a.$	No
Denmark	-0.728	-3.730	Yes
Finland	-0.043	2.283	Yes
France	-0.596	-0.795	Yes
Germany	-0.360	0.012	Yes
Italy	0.115	3.362	Yes
Japan	-2.753	$n.a.$	No
Netherlands	-0.090	6.299	Yes
Norway	-1.131	-1.772	Yes
Portugal	-0.582	-1.802	Yes
Spain	-0.301	-0.785	Yes
Sweden	-0.633	1.155	Yes
Switzerland	-1.670	$n.a.$	Yes
UK	0.187	3.374	Yes

Note: i) All real exchange rates are relative to the US dollar. ii) Estimates are calculated by linear interpolations. iii) $2MHL - MQL$ is adopted from Steinsson (2008). Zero values for $2MHL - MQL$ imply monotonic adjustment process towards the long-run equilibrium. Negative values occur when $MHL < MQL - MHL$.