The Zero Lower Bound and Endogenous Uncertainty

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Abstract

This paper documents a strong negative correlation between macroeconomic uncertainty and real GDP growth since the onset of the Great Recession. Prior to that event the correlation was weak and in many cases not statistically different from zero, even when conditioning on quarters when the economy was in a recession. At the same time, many central banks reduced their policy rate to its zero lower bound (ZLB) and continued to target that rate more than 6 years later. We contend that the constraint imposed by the ZLB has contributed to the stronger negative correlations that emerged since 2008. To test our theory, we use a model where the ZLB occasionally binds. The model has the same key feature as the data—away from the ZLB the correlation is weak but strongly negative when the policy rate is close to or at its ZLB.

Keywords: Monetary Policy; Uncertainty; Economic Activity, Zero Lower Bound, Survey Data

JEL Classifications: E32; E47; E58

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1 INTRODUCTION

There is significant interest in understanding the relationship between uncertainty and economic activity. Several papers find a negative relationship in the data using various measures of uncertainty. For example, Bloom (2009) shows that unexpected increases in uncertainty, given by stock price volatility, are associated with declines in industrial production. Bekaert et al. (2013), Bloom et al. (2014), and Pinter et al. (2013) find similar relationships in the data. While those papers focus on financial market volatility, Jurado et al. (2013) use broader proxies for uncertainty and find that spikes in uncertainty are more infrequent but more persistent and more negatively correlated with hours, employment, and industrial production than the previous literature has estimated.\footnote{Others find a negative relationship between fiscal uncertainty and economic activity [Fernández-Villaverde et al. (2013), Born and Pfeifer (2014)] and oil price uncertainty and economic activity [Elder and Serletis (2010), Jo (2014)].}

There is also a large literature that explores how uncertainty affects economic variables in theoretical models. Typically, this literature focuses on how endogenous variables respond when there is higher uncertainty about exogenous variables, such as technology. A few recent examples include Bloom (2009), Bloom et al. (2014), Basu and Bundick (2012), Christiano et al. (2014), Fernández-Villaverde et al. (2011), Justiniano and Primiceri (2008), and Mumtaz and Zanetti (2013).\footnote{There is also an older literature that examines similar research questions to those posed in the SV literature. See, for example, Leland (1968), Levhari and Srinivasan (1969), and Sandmo (1970).} These papers find a negative relationship between uncertainty and economic activity.

This article explores how uncertainty about future economic activity endogenously responds to the state of the economy. We conduct our analysis through the lens of a standard New Keynesian model that imposes a zero lower bound (ZLB) constraint on the short-term nominal interest rate. In this model, as in all rational expectations models, agents make predictions about the future values of economic variables, both exogenous and endogenous. They also make forecasts about the degree of uncertainty surrounding those predictions. The measure of uncertainty in our model is equivalent to those forecasts, which in a mathematical sense is the expected volatility of the forecast errors regarding future real GDP. We first explore how this measure varies over time in response to changes in the state of the economy in the model. With the theoretical predictions in hand, we then look to see whether the model’s predictions are consistent with features of the data.

Several important findings emerge from our analysis:

1. There is a strong negative correlation between uncertainty and real GDP growth since the onset of the Great Recession. Prior to that event the correlation was weak and in many cases not statistically different from zero, even when conditioning on quarters when the economy was in a recession.

2. The model predicts an increase in output uncertainty near and at the ZLB. When the nominal interest rate is far from its ZLB, uncertainty surrounding output is nearly constant and low.

3. The model has the same key feature as the data—away from the ZLB the correlation is weak but strongly negative when the short-term nominal interest rate is close to or at its ZLB.

4. Various measures of uncertainty that capture different parts of the economy for the U.S. and Euro area display similar conditional and unconditional correlations with real GDP growth.

Our results are important for the growing literature that links uncertainty and economic activity because they show that for particular states of the world uncertainty responds to what is happening in the economy. An increase in uncertainty occurs as the short-term nominal interest
rate approaches its ZLB due to the restriction the constraint places on the ability of the central bank to stabilize the economy. Output becomes more responsive to shocks that hit the economy and, therefore, the distribution of future realizations of output become more dispersed when compared to the same distributions that exist when the short-term nominal interest rate is far from its ZLB. This result increases the expected volatility of the output forecast errors (i.e., uncertainty rises near or at the ZLB). Of course, these results do not rule out that economic activity can respond to uncertainty. It merely shows that in at least one case, when the short-term nominal interest rate is near or at its ZLB, uncertainty is responding to an event that is endogenous to the state of the economy.

Several recent papers document that the ZLB constraint has an important effect on the economy, and that its effect is stronger in the presence of uncertainty. Gust et al. (2013) estimate a nonlinear New Keynesian model with a ZLB constraint to quantify how much of the recent decline in output was due to the binding constraint. They find the constraint accounts for about 20% of the drop in U.S. real GDP from 2008-2009 and, on average, it caused output to be 1% lower from 2009-2011 than it would have been without the constraint. Nakov (2008) finds the optimal discretionary monetary policy leads to a more negative output gap at the ZLB when households face uncertainty about the real interest rate than when they have perfect foresight. Nakata (2012) also studies the effects of uncertainty when the ZLB binds by varying the standard deviation of discount factor shocks. He finds higher uncertainty increases the slope of the policy function for output, meaning positive discount factor shocks lead to a larger reduction in output when the ZLB binds. Basu and Bundick (2012) show cost and demand uncertainty shocks cause business cycle fluctuations, which become more pronounced when the ZLB binds. Specifically, they find that a 1 standard deviation positive demand uncertainty shock causes output to decline by 0.2% when the ZLB does not bind and by 0.35% when it binds. Moreover, they calculate that demand uncertainty shocks can account for one-fourth of the drop in output in late 2008. These papers make clear that output is more sensitive to adverse shocks when the ZLB binds because the central bank is constrained.

Endogenous uncertainty is also studied in models with learning frictions. For example, Bachmann and Moscarini (2012) examine a model where uncertainty increases in recessions since it is less costly for firms to experiment with price changes to learn about their market power. Van Nieuwerburgh and Veldkamp (2006) argue that low production during a recession leads to noisy forecasts that impede learning and slow the recovery. In a related paper, Fajgelbaum et al. (2013) allow for self-reinforcing episodes of high uncertainty and low economic activity. In their model, firms learn about fundamentals by observing the investment activity of other firms. Investment is low in recessions and, since information flows slowly, uncertainty is high, which further discourages investment and causes an uncertainty trap. Gourio (2014) finds in a model without learning that the volatility of output is countercyclical because customers, suppliers, and workers expect larger losses when adverse shocks raise the probability that firms default. In our model, the constraint imposed by the ZLB reduces the effectiveness of monetary policy, which makes output more responsive to shocks that hit the economy and increases the expected forecast error volatility.

In an estimated VAR model, Leduc and Liu (2014) report that shocks to uncertainty in the post-2008 period have contributed to a much larger fraction of the observed unemployment fluctuations than in other recessions. They argue that this finding is likely attributable to the ZLB during the post-2008 period, which is consistent with the mechanism discussed in Basu and Bundick (2012).

The rest of the paper is organized as follows. Section 2 describes our measures of uncertainty in the data, computes correlations between those measures and real GDP growth, and explains how the same type of uncertainty is calculated in a theoretical model. Section 3 introduces our theoret-
icical model, its calibration, and the solution method. Section 4 presents our key result. Specifically, we find the predictions of our theoretical model are consistent with the negative correlation between uncertainty and real GDP growth that emerged in mid-2008. Section 5 concludes.

2 Empirical and Theoretical Measures of Uncertainty

This section introduces three forward looking measures of uncertainty and shows how they are correlated with real GDP growth. We also compute equivalent correlations using industrial production as a measure of economic activity. We then show similar correlations arise in a time-varying VAR with stochastic volatility. The section concludes by describing an analogous measure of uncertainty that arises in stochastic general equilibrium models and allows us to connect theory to data.

![Figure 1: Measures of economic uncertainty. Chicago Board Options Exchange Volatility Index (CBOE VXO): expected volatility in the S&P 100 over the next 30 days at an annualized rate; Business Outlook Survey Forecast Dispersion (BOS FD): dispersion in large manufacturers’ forecasts of business activity over the next 6 months; Survey of Professional Forecasters real GDP Forecast Dispersion (SPF FD): dispersion in real GDP forecasts $k$-periods ahead. The shaded regions correspond to recessions, according to the National Bureau of Economic Research.](image)

2.1 Data Description Figure 1 displays three measures of economic uncertainty: the Chicago Board Options Exchange (CBOE) S&P 100 Volatility Index (VXO), the dispersion in large manufacturers’ forecasts of business activity from the Business Outlook Survey (BOS), and the dispersion in real GDP forecasts $k$-periods ahead from the Survey of Professional Forecasts (SPF). The shaded regions correspond to recessions, according to the National Bureau of Economic Research. We focus on these data series because they are forward looking measures of uncertainty.
and are able to capture changes in people’s expectations over time, as opposed to making predictions about future uncertainty based on statistical relationships (e.g., a GARCH model). We also believe that macroeconomic uncertainty is an important factor that influences the behavior of all of these measures, despite the fact that they represent different aspects of the economy.

The CBOE VXO measures the expected volatility in the S&P 100 stock market index over the next month at an annualized rate. For example, if the value on the vertical axis is \( x \)%, then people expect there is a \( 68\% \) chance the S&P 100 index will change by \( \pm x/\sqrt{12}\% \) over the next month. We average the daily series each quarter so it is consistent with the frequency of real GDP releases. The Business Outlook Survey (BOS), which is conducted monthly by the Federal Reserve Bank of Philadelphia, asks large manufacturing firms to forecast whether general business activity will increase, decrease, or remain unchanged over the next six months. Following Bachmann et al. (2013), the forecast dispersion (FD) in the responses to the survey in period \( t \) is given by

\[
BOS \text{ FD}_t = \sqrt{\text{Frac}_t^+ + \text{Frac}_t^- - (\text{Frac}_t^+ - \text{Frac}_t^-)^2},
\]

where \( \text{Frac}_t^+ \) (\( \text{Frac}_t^- \)) is the fraction of firms who forecast an increase (decrease) in business activity. We average the monthly BOS FD series each quarter and then standardize the values so the vertical axis displays the number of standard deviations from the mean response. The Survey of Professional Forecasters (SPF), which is conducted quarterly by the Federal Reserve Bank of Philadelphia, asks individuals who regularly make forecasts as part of their jobs to predict macroeconomic aggregates for the next four quarters (e.g., inflation, real GDP, interest rates). We focus on the forecasts of real GDP, which are denoted by \( y \). The inter-quartile FD is given by

\[
\text{SPF FD}_t(k) = 100 \times (\log(y_{75}^{t+k|t-1}) - \log(y_{25}^{t+k|t-1})), \quad k = 0, 1, \ldots, 4.
\]

This value is the percent difference between the 75th and 25th percentiles of the time-\( t \) forecasts of real GDP in period \( t + k \), given all historical observations dated \( t - 1 \) and earlier.\(^3\)

Since our measures of uncertainty represent different segments of the economy (i.e., the stock market, manufacturing, output), it is not surprising that they have different properties. The VXO is the least noisy of our uncertainty measures and is only modestly higher during the 1991 and 2001 recessions. The three most prominent spikes in the index correspond to Black Monday (1987), the Enron scandal (mid-to-late 2002) and the second Gulf War (early 2003), and the 2008 financial crisis.\(^4\) The dispersion in the BOS and SPF survey data is less persistent than the VXO. The SPF data also shows forecast dispersion increases over longer horizons. For example, the average of the SPF FD(1) is 0.43% and the average of the SPF FD(4) is 0.86%. The SPF measures spike around the time of Black Monday, after 9/11, and during the 2008 financial crisis. They also increase during the first Gulf War. The BOS is higher around the time of the 1991 and 2001 recessions and during the Great Recession. The Great Recession causes the largest spike in all measures.

2.2 Correlations between Output and Uncertainty

Table 1 shows conditional and unconditional correlations between real GDP growth (i.e., quarter-over-quarter log differences in real GDP) and our measures of uncertainty. The top row is based on the entire data series (1986Q1-2014Q2), whereas the second row is conditional on data before the Great Recession (1986Q1-2008Q2) and the third row is conditional on data since the onset of the Great Recession (2008Q3-2014Q2). Our data series begins in 1986Q1 rather than an earlier date because (a) there were

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\(^3\)See Croushore (1993) for more information about the history of the SPF and the other variables in the survey.

\(^4\)See Bloom (2009) for a more detailed list documenting spikes in realized and expected stock price volatility.
major changes in monetary policy beginning in the 1980s, and (b) the VXO is only available since 1986Q1 and we wanted to draw connections to this commonly used measure of uncertainty.5

Starting in 1986Q1, the unconditional correlations between the uncertainty measures and real GDP growth are all negative and statistically different from zero, except the correlation with the SPF FD(4). When we partition the sample, it becomes clear that the negative relationship has only emerged in recent data. The correlations conditional on the pre-Great Recession data are weak and none are statistically different from zero at a 5% significance level.6 In contrast, most of the correlations conditional on data since the Great Recession are statistically below zero at a 1% level. We also use the Fisher z-transformation to test whether the correlations in the pre- and post-Great Recession samples are significantly different. Those tests reveal that the correlations are statistically different at the 1% level when calculated with the BOS FD and SPF FD(1), the 5% level with the SPF FD(2), and the 10% level with the SPF FD(3) and SPF FD(4).7

<table>
<thead>
<tr>
<th></th>
<th>VXO</th>
<th>BOS FD</th>
<th>SPF FD(1)</th>
<th>SPF FD(2)</th>
<th>SPF FD(3)</th>
<th>SPF FD(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unconditional</td>
<td>-0.33***</td>
<td>-0.27***</td>
<td>-0.20**</td>
<td>-0.23**</td>
<td>-0.19**</td>
<td>-0.12</td>
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<tr>
<td>(1986Q1-2014Q2)</td>
<td></td>
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<tr>
<td>Pre-Great Recession</td>
<td>-0.04</td>
<td>-0.20*</td>
<td>-0.09</td>
<td>-0.15</td>
<td>-0.10</td>
<td>-0.03</td>
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<tr>
<td>(1986Q1-2008Q2)</td>
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<tr>
<td>Post-Great Recession</td>
<td>-0.74***</td>
<td>-0.70***</td>
<td>-0.53***</td>
<td>-0.44**</td>
<td>-0.41**</td>
<td>-0.31</td>
</tr>
<tr>
<td>(2008Q3-2014Q2)</td>
<td></td>
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<tr>
<td>Past Recessions</td>
<td></td>
<td>-0.16</td>
<td>-0.20</td>
<td>-0.15</td>
<td>0.12</td>
<td>0.08</td>
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<tr>
<td>(1968Q4-2007Q3)</td>
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Table 1: Correlations between real GDP growth (the quarter-over-quarter log difference in real GDP) and various measures of uncertainty. Values are significantly different from 0 at ***1%, **5%, and *10% levels.

The results in table 1 suggest that recessions may be a source of high uncertainty, but, perhaps counterintuitively, the bottom row shows there is little evidence for this relationship in the data. Both the SPF and BOS surveys began in 1968Q4. Between that date and the beginning of the Great Recession, the U.S. economy experienced 6 recessions, totaling 27 quarters. The correlation between real GDP growth and uncertainty in those quarters was even weaker than the unconditional correlations. Moreover, none of the values were statistically different from zero, even at a 10% significance level. These results suggest that there are characteristics unique to the Great Recession that led to a strong negative relationship between our measures of uncertainty and real GDP growth.

A major difference between the Great recession and previous recessions is that monetary policy has been constrained by the ZLB on the federal funds rate. Central banks typically conduct open market operations to stimulate demand during an economic downturn, but by 2008Q2 the Fed had cut the federal funds rate to 2% and economic conditions were sufficiently poor that a policy rate

5There is also evidence of a structural break at the start of the Great Moderation. The correlation between real GDP growth and uncertainty, as measured by SPF FD(1) or SPF FD(2), conditional on the pre-Great Moderation (1968Q4-1984Q4) is significantly different from the correlation conditional on the Great Moderation (1986Q1-2008Q2). Since interest rates were far above zero at that time, it is beyond the scope of this paper to explain that break.

6If we remove the quarters when the exogenous events identified in Bloom (2009) occurred in the pre-Great Recession sample, then the correlation between the BOS FD and real GDP growth is not significant at the 10% level.

7Sill (2012) computes an alternative measure of uncertainty based on the standard deviation of the probability distribution assigned by the forecasters. We did not use this measure because the intervals in the distribution changed in 1992Q1 and in 2009Q2. However, if we start the post-Great Recession sample in 2009Q2, then the correlations with real GDP growth are qualitatively similar to those based on the inter-quartile forecast dispersion (SPF FD).
near zero was possible. In the following months, the Fed continued to reduce the federal funds rate and in 2008Q4 it hit its ZLB. We chose to use 2008Q3 as our breakpoint in the sample rather than when the ZLB was hit, because that date is when all of the measures of uncertainty rise or remain elevated. Moreover, as we will demonstrate below, the equivalent measure of uncertainty in the model also increases before the ZLB binds. More than 6 years after the ZLB was first hit, the Fed’s target interest rate remains near 0. We contend that the constraint the ZLB places on current and future monetary policy is an important factor that contributed to the negative correlation that emerged between real GDP growth and macroeconomic uncertainty since mid-2008.

We also examine the conditional and unconditional correlations for the Euro area and the U.K. because they faced constraints on monetary policy similar the Fed during the Great Recession. The overnight Libor rate was reduced to its ZLB in 2008Q4. The Euro-zone deposit rate was cut from 3.25% in 2008Q4 to 0.25% by the end of 2009Q1, and was further reduced to 0% in 2012Q3. The qualitative features of the conditional and unconditional correlations echo those for the U.S.

The European Central Bank (ECB) has conducted its own survey of professional forecasters (ECB SPF) since 1999Q1. It asks participants to forecast Euro area real GDP growth over various time horizons. For example, the survey conducted in 1999Q1 requests forecasts for 1999Q3, given the last GDP release is from 1998Q3. Similar to the U.S. SPF, we calculate the forecast dispersion as ECB SPF FD_t = |\hat{y}_{t+2}^{25%\text{-}\text{th}} - \hat{y}_{t+2}^{75%\text{-}\text{th}}|, where \hat{y}_{t+2}^{75\%|t-2}\text{-}\text{th}} \text{ and } \hat{y}_{t+2}^{25\%|t-2}\text{-}\text{th}} are the \text{ th percentile of the time- } t \text{ forecast of real GDP growth in period } t+2, \text{ given observations dated } t-2 \text{ and earlier. From 1999Q1-2014Q2, the unconditional correlation between Euro area real GDP growth and the ECB SPF FD is −0.51, which is significantly different from zero at a 1% level. The correlation conditional on pre-Great Recession data (1999Q1-2008Q2) is −0.31 and not significant, while the correlation conditional on post-Great Recession data (2008Q3-2014Q2) is −0.47 and significant at a 5% level.

The Bank of England also conducts a quarterly survey called the Survey of External Forecasters (BOE SEF), which has asked its participants to forecast real GDP growth since 1998Q1. Prior to 2006Q2, the survey asked for projections in quarter 4 of the survey year, quarter 4 1 year ahead, and the same quarter 2 years ahead. For example, the forecast dates in the 2006Q1 survey were 2006Q4, 2007Q4, and 2008Q1. Since 2006Q2, the survey asked for projections for the same quarter 1, 2, and 3 years ahead. Unfortunately, we cannot calculate unconditional correlations like we do with the ECB SPF data because the forecast horizons change. However, when we condition on post-Great Recession data, the correlation between real GDP growth and the dispersion in forecasts 1 year ahead is −0.53, which is significantly different from zero at a 5% level. These findings for the Euro area and the U.K. provide further evidence that the ZLB constraint faced by central banks led to a strongly negative correlation between real GDP growth and uncertainty.

### 2.3 Industrial Production vs. Real GDP

Our findings in table 1 are based on using real GDP as a measure of economic activity. That measure is consistent with the model in section 3, which is used to explore the theoretical relationship between macroeconomic uncertainty and real GDP.

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8The Bank of Japan has kept its policy rate near zero since 1995. There is also a monthly survey of Japanese professional forecasters conducted by the Economic Planning Association (known as the ESP) which asks for forecasts of real GDP, but the monthly survey began in mid-2004 and does not provide a large enough sample to analyze. See Komine et al. (2009) for more information about the ESP and a statistical analysis of the forecasters’ performance.

9The correlations conditional on the ZLB sample are also much stronger than the correlations conditional on data before the ZLB when we use the survey of industry from the European Commission’s Business and Consumer surveys. This survey asks manufacturers whether they expected their production to increase, decrease, or remain unchanged.

10See Boero et al. (2008) for more information about the BOE SEF and how it compares to similar surveys.
GDP. However, empirical work has often used industrial production as a measure of economic activity. A few examples include Bloom (2009), Bekaert et al. (2013), and Jurado et al. (2013).

To compare our findings with the previous literature, this section repeats the analysis in table 1 but uses industrial production growth instead of real GDP growth to compute the correlations. Those results are shown in table 2. Similar to the findings in table 1, the correlations between industrial production growth and our measures of uncertainty are stronger in the post-Great Recession period compared to the pre-Great Recession period. However, we also find an important difference in the strength of the relationship. In particular, the correlations are stronger and, in most cases, statistically different from 0 in the pre- and post-Great Recession samples.

We also examine the correlations between the SPF forecast dispersion for industrial production and real GDP growth. The forecast dispersion is analogous to the SPF FD for real GDP. We find industrial production uncertainty is also more tightly linked with changes in economic activity.

These findings suggest that there is something different about the manufacturing sector that sets it apart from the overall economy. In our paper we are interested in the connection between uncertainty and real GDP growth. It is beyond the scope of this paper and our model to formally explore why the relationship is stronger with manufacturing related variables, but we speculate that the difference may have to do with the fact that manufacturing activity is more volatile and more sensitive to changes in business conditions than other sectors of the economy. Given that the sectors in industrial production make up less than 20% of U.S. GDP in recent years, the results for industrial production are apparently not strong enough to drive the results for the whole economy.

### 2.4 Time-varying VAR with Stochastic Volatility

To provide further evidence for how the correlations between real GDP growth and macroeconomic uncertainty have changed since mid-2008, this section presents similar correlations based on an alternative measure of uncertainty that does not rely on stock market or survey data. Following Primiceri (2005), we use a time-varying VAR with stochastic volatility to estimate the volatility of real GDP growth and show how that estimate is correlated with economic activity. One difference between Primiceri (2005) and our estimation is that we use real GDP growth in place of the unemployment rate so that we can compare our results to equivalent statistics in our theoretical model. A second difference is that we use a longer sample from 1953Q1-2014Q3, which includes the Great Recession. The sample from 1953Q1-1963Q1 is used to train the prior distributions of the model parameters. Since there is a 2-quarter lag in the model, our estimates are from 1963Q3-2014Q3.

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<tr>
<th></th>
<th>VXO</th>
<th>BOS FD</th>
<th>SPF FD(1)</th>
<th>SPF FD(2)</th>
<th>SPF FD(3)</th>
<th>SPF FD(4)</th>
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<tbody>
<tr>
<td>Unconditional</td>
<td>-0.41***</td>
<td>-0.28***</td>
<td>-0.40***</td>
<td>-0.41***</td>
<td>-0.37***</td>
<td>-0.28***</td>
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<td>(1986Q1-2014Q2)</td>
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<tr>
<td>Pre-Great Recession</td>
<td>-0.09</td>
<td>-0.20*</td>
<td>-0.28***</td>
<td>-0.35***</td>
<td>-0.28***</td>
<td>-0.20*</td>
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<td>(1986Q1-2008Q2)</td>
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<tr>
<td>Post-Great Recession</td>
<td>-0.75***</td>
<td>-0.58***</td>
<td>-0.70***</td>
<td>-0.60***</td>
<td>-0.59***</td>
<td>-0.45**</td>
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<td>(2008Q3-2014Q2)</td>
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<tr>
<td>Past Recessions</td>
<td>0.01</td>
<td>-0.31</td>
<td>-0.21</td>
<td>0.02</td>
<td>0.06</td>
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<td>(1968Q4-2007Q3)</td>
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Table 2: Correlations between industrial production growth (the quarter-over-quarter log difference in the IP index) and various measures of uncertainty. Values are significantly different from 0 at ***1%, **5%, and *10% levels.
The specific model that we estimate is given by
\[ y_t = B_{0,t} + B_{1,t}y_{t-1} + B_{2,t}y_{t-2} + A_t^{-1}\Sigma_t\varepsilon_t, \quad t = 1, \ldots, T, \]
where \(y_t\) is a 3 \(\times\) 1 vector that includes real GDP growth, the inflation rate, and the T-Bill rate, \(B_{0,t}\) is a 3 \(\times\) 1 vector of time varying intercepts, \(B_{1,t}\) and \(B_{2,t}\) are 3 \(\times\) 3 matrices of time varying coefficients, and \(\varepsilon_t\) are normally distributed shocks with an identity variance-covariance matrix. \(A_t\) and \(\Sigma_t\) are the result of a triangular reduction of the variance-covariance matrix, where
\[
A_t = \begin{bmatrix}
1 & 0 & 0 \\
\alpha_{21,t} & 1 & 0 \\
\alpha_{31,t} & \alpha_{32,t} & 1
\end{bmatrix}, \quad \Sigma_t = \begin{bmatrix}
\sigma_{1,t} & 0 & 0 \\
0 & \sigma_{2,t} & 0 \\
0 & 0 & \sigma_{3,t}
\end{bmatrix}.
\]

Our parameter of interest, the volatility of real GDP growth, is \(\sigma_{1,t}\).

The model is estimated with Bayesian MCMC methods using code that accompanies Koop and Korobilis (2010). That code implements a correction to the algorithm outlined in Appendix A of Primiceri (2005), as explained by Del Negro and Primiceri (2013). The estimate for the volatility of real GDP growth is an additional proxy for macroeconomic uncertainty. We then calculate correlations between real GDP growth and its estimated volatility conditional on various samples. All of our calculations are based on 50,000 draws from the posterior distribution.

**Figure 2:** Distributions of the correlation, \(\rho\), between real GDP growth and its estimated volatility.

Figure 2 shows the distributions of the correlation, \(\rho\), between real GDP growth and its volatility using data from 1986Q1-2014Q2. For that period, the correlation only has a 0.03\% chance of exceeding zero and a median value of \(-0.38\). For the pre-Great Recession sample, the median correlation is \(-0.24\) and exceeds zero in 2.81\% of draws. Conditional on the post-Great Recession...
sample, the median correlation is $-0.55$, which is more negative and exceeds zero in only 0.7% of draws. When conditioning on past recessions, the median correlation is less negative ($-0.14$) and exceeds zero in 7.36% of draws. The median difference between the pre-Great Recession correlation and post-Great Recession correlation is 0.32, which is less than zero in 7.2% of draws. Although these results are based on a different measure of uncertainty, they are qualitatively similar to our results in section 2.2. Most notably, the correlation is more negative since 2008.

### 2.5 Measure of Endogenous Uncertainty

A recent and growing segment of the literature introduces stochastic volatility (SV) into dynamic stochastic general equilibrium models to study the effects of exogenous changes in uncertainty. Our work differs from these papers in that we focus on how uncertainty about future macroeconomic variables endogenously responds to the state of the economy. To illustrate our measure of endogenous uncertainty, it is useful to first describe how one measures uncertainty in a model with SV. As an example, suppose a model includes an exogenous random variable $x$, such as technology or government spending, that evolves according to $x_t = \rho_x x_{t-1} + \sigma_x \varepsilon_t$, where $0 \leq \rho_x < 1$ and $\varepsilon$ is white noise. SV is introduced into the model by assuming the standard deviation of the shock is time-varying and follows an exogenous process specified by the modeler, which relaxes the common assumption of homoscedastic innovations. Given the linear process governing $x$, the expected forecast error, $FE_x$, equals

$$
\mu_{FE,t+1} \equiv E_t[FE_{x,t+1}] = E_t[(x_{t+1} - E_t x_{t+1})] = 0.
$$

Although the forecast error is mean zero, there is uncertainty about its future value. The uncertainty is measured by the expected volatility of the forecast error, which is given by

$$
\sqrt{E_t[FE_{x,t+1}^2]} = \sqrt{E_t[(x_{t+1} - E_t x_{t+1})^2]} = \sqrt{E_t[(x_{t+1} - \rho_x x_t)^2]} = \sqrt{E_t \sigma_x^2}.
$$

Models that allow for SV in various shocks are able to match features of the data that models with homoscedastic errors cannot match, but they do not explain why volatility changes over time because the uncertainty is exogenous. However, there is always uncertainty in the model that is endogenous, regardless of whether the model includes SV. We quantify the degree of endogenous uncertainty by following the logic of the SV literature. Suppose $y$ is any endogenous variable in the model, such as output. The uncertainty surrounding $y$, $k$ periods in the future, is given by

$$
\sigma_{y,t}(k) \equiv \sqrt{E_t[(FE_{y,t+k} - E_t[FE_{y,t+k}])^2]} = \sqrt{E_t[(y_{t+k} - E_t y_{t+k} - \mu_{FE,t+k})^2]},
$$

which varies over time like it does in the SV specification, except the fluctuations are now endogenous responses to the state of the economy. One example of such a state is a binding ZLB.

### 3 Economic Model, Calibration, and Solution Method

We use a conventional New Keynesian model in which the ZLB on the short-term nominal interest rate occasionally binds due to discount factor (demand) and technology (supply) shocks.

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11Furthermore, if we remove the first Gulf War from the pre-Great Recession correlation since it is an exogenous event, then the median difference between the correlations is 0.35 and is less than zero in 5.7% of draws.
3.1 HOUSEHOLDS Each household chooses \( \{c_t, n_t, b_t\}_{t=0}^\infty \) to maximize expected lifetime utility, given by, \( E_0 \sum_{t=0}^\infty \tilde{\beta}_t [\log c_t - \chi n_t^{1+\eta}/(1 + \eta)] \), where \( 1/\eta \) is the Frisch elasticity of labor supply, \( c_t \) is consumption, \( n_t \) is labor hours, \( b_t \) is the real value of a 1-period nominal bond, \( E_0 \) is an expectation operator conditional on information available in period 0, \( \tilde{\beta}_0 \equiv 1 \), and \( \tilde{\beta}_t = \prod_{j=1}^t \beta_j \) for \( t > 0 \). \( \beta \) is a time-varying discount factor that follows

\[
\beta_t = \tilde{\beta} (\beta_{t-1}/\tilde{\beta})^{\rho_\beta} \exp(\varepsilon_t),
\]

where \( \tilde{\beta} \) is the steady-state discount factor, \( 0 \leq \rho_\beta < 1 \), and \( \varepsilon_j \sim N(0, \sigma^2_\varepsilon) \). These choices are constrained by \( c_t + b_t = w_t n_t + i_{t-1} b_{t-1}/\pi_t + d_t \), where \( \pi_t = p_t/p_{t-1} \) is the gross inflation rate, \( w_t \) is the real wage rate, \( i_t \) is the gross nominal interest rate, and \( d_t \) are profits from intermediate firms. The optimality conditions to the household’s problem imply

\[
w_t = \chi n_t^\eta c_t, \quad 1 = i_t E_t [\beta_{t+1} (c_t/c_{t+1})/\pi_{t+1}].
\]

3.2 FIRMS The production sector consists of monopolistically competitive intermediate goods firms who produce a continuum of differentiated inputs and a representative final goods firm. Each firm \( f \in [0, 1] \) in the intermediate goods sector produces a differentiated good, \( y_t(f) \), with identical technologies given by \( y_t(f) = z_t n_t(f) \), where \( n_t(f) \) is the amount of employment used by firm \( f \). \( z_t \) represents the level of technology, which is common across firms and evolves according to

\[
z_t = \tilde{z} (z_{t-1}/\tilde{z})^{\rho_z} \exp(\upsilon_t),
\]

where \( \tilde{z} \) is steady-state technology, \( 0 \leq \rho_z < 1 \), and \( \upsilon_t \sim N(0, \sigma^2_\upsilon) \). Each intermediate firm chooses its labor supply to minimize its operating costs, \( w_t n_t(f) \), subject to its production function.

The representative final goods firm purchases \( y_t(f) \) units from each intermediate goods firm to produce the final good, \( y_t \equiv \int_0^1 y_t(f) \theta/(\theta - 1) df \) according to a Dixit and Stiglitz (1977) aggregator, where \( \theta > 1 \) measures the elasticity of substitution between the intermediate goods. The final goods firm maximizes its profits, which determines its demand for intermediate good \( f \),

\[
y_t(f) = (p_t(f)/p_{t-1})^{-\theta} y_t, \quad p_t = \int_0^1 p_t(f)^{1-\theta} df^{1/(1-\theta)}
\]

is the price of the final good.

Following Rotemberg (1982), each firm faces a cost to adjusting its price, \( \text{adj}_t(f) \), which emphasizes the negative effect that price changes can have on customer-firm relationships. Using the functional form in Ireland (1997), \( \text{adj}_t(f) = \varphi (p_t(f)/(\pi p_{t-1}(f))) - 1 \) scales the size of the adjustment cost and \( \pi \) is the steady-state gross inflation rate. Real profits are then given by \( d_t(f) = (p_t(f)/p_t) y_t(f) - w_t n_t(f) - \text{adj}_t(f) \). Firm \( f \) chooses its price, \( p_t(f) \), to maximize the expected discounted present value of real profits \( E_t \sum_{k=t}^\infty \lambda_{t,k} d_k(f) \), where \( \lambda_{t,t+1} \equiv 1 \), \( \lambda_{t,t+1} = \beta_{t+1}(c_t/c_{t+1}) \) is the pricing kernel between periods \( t \) and \( t+1 \), and \( \lambda_{t,k} \equiv \prod_{j=t+1}^k \lambda_{j-1,j} \).

In a symmetric equilibrium, all firms make identical decisions and the optimality condition implies

\[
\varphi \left( \frac{\pi_t}{\pi} - 1 \right) \frac{\pi_t}{\pi} = (1 - \theta) + \theta (w_t/z_t) + \varphi E_t \left[ \lambda_{t,t+1} \left( \frac{\pi_{t+1}}{\pi} - 1 \right) \frac{\pi_{t+1} y_{t+1}}{\pi} y_t \right].
\]

Without price adjustments costs (i.e., \( \varphi = 0 \)), the real marginal cost of producing a unit of output \( (w_t/z_t) \) equals \( (\theta - 1)/\theta \), which is the inverse of a firm’s markup of price over marginal cost.
3.3 Monetary Policy The central bank sets the gross nominal interest rate according to
\[ i_t = \max \{ t, \bar{i} (\pi_t / \pi^* )^{\phi_y} (y_t / \bar{y})^{\phi_y} \}, \] (6)
where \( \pi^* \) is the inflation rate target and \( \phi_y \) and \( \phi_y \) are the policy responses to inflation and output. This rule ensures \( i_t \geq \bar{i} \) and households do not expect an interest rate less than \( \bar{i} \) in the future.

3.4 Competitive Equilibrium The resource constraint is given by \( c_t = y_t - a d j_t \equiv y_t^{rdp} \), where \( y_t^{rdp} \) includes the value added by intermediate firms, which is their output minus quadratic price adjustment costs. Thus, \( y_t^{rdp} \) represents real GDP in the model. A competitive equilibrium consists of sequences of quantities \( \{c_t, n_t, b_t, y_t\}_{t=0}^{\infty} \), prices \( \{w_t, i_t, \pi_t\}_{t=0}^{\infty} \), and exogenous variables \( \{\beta_t, z_t\}_{t=0}^{\infty} \) that satisfy the household’s and firm’s optimality conditions [(2), (3), (5)], the production function \( [y_t = z_t n_t] \), the monetary policy rule [(6)], the processes for the discount factor and technology [(1), (4)], the bond market clearing condition \( [b_t = 0] \), and the resource constraint.

| Steady-State Discount Factor | \( \bar{\beta} \) | 0.995 | Monetary Policy Response to Inflation | \( \phi_y \) | 1.5 |
| Frisch Elasticity of Labor Supply | \( 1/\eta \) | 3 | Monetary Policy Response to Output | \( \phi_y \) | 0.1 |
| Elasticity of Substitution between Goods | \( \theta \) | 6 | Technology Persistence | \( \rho_\beta \) | 0.9 |
| Rotemberg Adjustment Cost Coefficient | \( \varphi \) | 59.11 | Technology Shock Standard Deviation | \( \sigma_y \) | 0.0025 |
| Steady-State Labor | \( \bar{n} \) | 0.33 | Discount Factor Persistence | \( \rho_\beta \) | 0.8 |
| Steady-State Inflation | \( \bar{\pi} \) | 1.006 | Discount Factor Standard Deviation | \( \sigma_\pi \) | 0.0025 |

Table 3: Baseline calibration.

3.5 Calibration and Solution Method We calibrate the model at a quarterly frequency using common values in the monetary policy literature. The values are summarized in table 3. The annual risk-free real interest rate is set to 2%, which implies a steady-state quarterly discount factor, \( \bar{\beta} \), equal to 0.995. That value corresponds to the ratio of the federal funds rate to the percent change in the GDP deflator from 1983-2013. The Frisch elasticity of labor supply, \( 1/\eta \), is set to 3 and the leisure preference parameter, \( \chi \), is set so that steady-state labor equals 1/3 of the available time. The elasticity of substitution between intermediate goods, \( \theta \), is calibrated to 6, which corresponds to an average markup over marginal cost equal to 20%. The costly price adjustment parameter, \( \varphi \), is set to 59.11, which is similar to a Calvo (1983) price-setting specification in which prices change on average once every four quarters. The lower bound on the quarterly gross nominal interest rate, \( \bar{\pi} \), is set to 1.00034, which is the average effective federal funds rate since December 2008.

The steady-state gross inflation rate, \( \bar{\pi} \), is calibrated to 1.006 so the annual inflation rate target is 2.4%. That value corresponds to the percent change in the GDP deflator from 1983-2013. We set the monetary response to changes in inflation, \( \phi_y \), to 1.5 and the response to changes in output, \( \phi_y \), to 0.1. Richter and Throckmorton (2014) show there is a maximum amount of time the economy can spend at the ZLB and still deliver a convergent minimum state variable solution, which bounds the persistence and standard deviation of the stochastic processes. We set \( \rho_\beta = 0.8 \) and \( \sigma_y = 0.0025 \), which are the same values used in Fernández-Villaverde et al. (2012). Steady-state technology, \( \tilde{z} \), is normalized to 1, and we set \( \rho_\sigma = 0.9 \) and \( \sigma_y = 0.0025 \). These values are calibrated so that our model is consistent with the average time the ZLB is expected to hold, rather than the duration of the current ZLB episode in the U.S. Gavin et al. (2014) and Gust et al. (2013) show that the average ZLB duration under the above calibration, which is 1.87 quarters, is roughly
consistent with forecaster expectations from the SPF and Blue Chip data. With that being said, it is possible for longer ZLB events, such as the recent episode in the U.S., to occur in our model.

We solve the model using the policy function iteration algorithm described in Richter et al. (2014), which is based on the theoretical work on monotone operators in Coleman (1991). This solution method discretizes the state space and uses time iteration to solve for the updated policy functions until the tolerance criterion is met. We use linear interpolation to approximate future variables, since it accurately captures the kink in the policy functions, and Gauss-Hermite quadrature to numerically integrate. Those techniques capture the expectational effects of going to and returning to the ZLB. For a formal description of the numerical algorithm see Richter et al. (2014).

Benhabib et al.’s (2001) finding that constrained models have two deterministic steady-state equilibria has generated considerable discussion in the literature about whether there are conditions in which a unique MSV solution exists in stochastic models with a ZLB constraint. Specifically, they find there are two nominal interest rate/inflation rate pairs consistent with the steady-state equilibrium system. In one steady state, the central bank meets its positive inflation target, whereas in the other steady state the economy experiences deflation. Richter and Throckmorton (2014) show that the numerical algorithm used in our paper converges to the inflationary equilibrium as long as there is a sufficient expectation of returning to a monetary policy rule that conforms to the Taylor principle. The algorithm, however, never converges to the deflationary equilibrium because it is unstable. That is, the algorithm does not converge to the deflationary equilibrium after a shock pushes the economy away from that equilibrium, which is similar to findings in Christiano and Eichenbaum (2012) and Wolman (2005). For further details and results see Gavin et al. (2014).

4 THEORETICAL AND EMPIRICAL RESULTS

We first build intuition on why the ZLB generates a strong negative correlation between real GDP growth and uncertainty in the model by assuming technology is fixed, so ZLB events are endogenous due to positive discount factor shocks. We also show how the central bank’s response to inflation affects this relationship. We then allow for stochastic changes in technology and show that both the discount factor and technology processes are necessary to match features of the data.

4.1 MODEL WITH CONSTANT TECHNOLOGY

We begin our theoretical analysis by assuming technology is constant (i.e., $z_t = \bar{z}$ for all $t$). In this case, the model contains only one state variable, $\hat{\beta}_{-1}$, which is exogenous and ranges from $\pm 1.85\%$ of its steady state. In all of our results, a hat denotes percent deviation from steady state (i.e., for some generic variable $x$ in levels, $\hat{x}_t \equiv 100 \times (x_t - \bar{x})/\bar{x}$) and a tilde denotes percent of steady state (i.e., $\tilde{\sigma}_{x,t}(k) = 100 \times \sigma_{x,t}(k)/\bar{x}$). Our results are based on the one-period ahead ($k = 1$) forecast error, unless specified otherwise.

The top row of figure 3 plots the decision rules for real GDP, $\hat{y}^{gdp}$ (left panel), and real GDP uncertainty, $\tilde{\sigma}_{y^{gdp}}$ (right panel), as a function of $\hat{\beta}_{-1}$ for three different values of the central bank’s response to inflation: $\phi_\pi = 1.5$ (solid line), $\phi_\pi = 2$ (dashed line), and $\phi_\pi = 2.5$ (dash-dotted line). The shaded regions indicate where the ZLB binds, which depends on the value of $\phi_\pi$. When $\phi_\pi = 1.5$ ($\phi_\pi = 2$, $\phi_\pi = 2.5$), the ZLB binds in states where $\hat{\beta}_{-1} > 0.90$ ($\hat{\beta}_{-1} > 1.02$, $\hat{\beta}_{-1} > 1.08$).

The bottom row of figure 3 plots the probability density function of future real GDP as a percent deviation from its mean, $100 \times (y_{t+1}^{gdp}/E_t[y_{t+1}^{gdp}] - 1)$. We display the density functions for three values of the initial notional interest rate—the short-term nominal interest rate that the central bank would set in the absence of a ZLB constraint: $i^*_0 = 1.1\%$ (solid line), $i^*_0 = 2.8\%$.
We begin by discussing how uncertainty changes across the discount factor states. The discount factor is a proxy for aggregate demand because it determines households’ degree of patience. When the discount factor is low (high), households are impatient (patient), and less (more) willing to postpone consumption. Firms respond to the higher (lower) demand by increasing (decreasing) their prices and output. Hence, the policy function for real GDP is downward sloping (top left panel). In discount factor states where the nominal interest rate is sufficiently far from its ZLB, the slope of the policy function for real GDP is constant. Hence, the distribution of future real GDP values is independent of the state of the economy in this region of the state space. The probability density function in these states is also narrower than in states where the short-term nominal interest
rate is close to or at its ZLB (bottom panel). Thus, real GDP uncertainty ($\tilde{\sigma}_{gdp}$) is relatively lower and nearly constant in states where the central bank is unconstrained by the ZLB (top right panel).

As the economy enters states where the short-term nominal interest rate is close to or at its ZLB, demand continues to decline and firms further reduce their prices. In states where the central bank is not constrained, it is able to respond to the lower inflation by cutting its policy rate to dampen the effects of the fall in demand. However, given a large enough decline in demand, the ZLB will bind and the central bank is unable to further reduce its policy rate. Thus, the economy becomes more sensitive to further declines in demand, which leads to lower real GDP than if the central bank was unconstrained. A steeper policy function for real GDP widens the distribution of possible real GDP values next period and skews it toward output losses. For example, when $i^*_0 = 1.1\%$, a ±1 standard deviation discount factor shock (i.e., ±0.25%) causes real GDP to move from its steady state by ±0.3%. When $i^*_0 = −0.2\%$, the same change in the discount factor can cause real GDP to decrease by −0.6 percentage points or increase by 0.3 percentage points. The broader range of future output values translates into greater forecast error volatility and hence higher uncertainty.

The policy functions in figure 3 show that when the nominal interest rate is sufficiently far from its ZLB, the uncertainty that arises endogenously in the model is low and essentially independent of the state of the economy. As the nominal interest rate declines, the probability of lower realizations of real GDP rises, which increases uncertainty even before the ZLB binds. Hence, uncertainty is state-dependent. We recognize that there are also spikes in the uncertainty data (see figure 1) when the ZLB was not a concern (e.g., 1987: Black Monday, 1990: first Gulf War, 2001: 9/11, mid-2002 to early-2003: Enron scandal and second Gulf War), but we do not view those episodes as a problem for our theory because they are due to events that are exogenous to our theoretical model.

We next consider an experiment where we vary the inflation coefficient in the monetary policy rule. We conduct this exercise for two reasons. First, it provides excellent intuition for the relationship between the slope of the policy function for real GDP and macroeconomic uncertainty. Second, it illustrates how the ability of the central bank to stabilize the economy affects macroeconomic uncertainty. When the central bank places more emphasis on inflation stability (i.e., a higher $\phi_\pi$), it affects the volatility of inflation and real GDP both away from and at the ZLB.

First consider the case where the nominal interest rate is far away from its ZLB. The top left panel of figure 3 shows that the slope of the policy functions flatten as $\phi_\pi$ increases. That reflects the central bank’s success at stabilizing inflation around its target level. Output becomes less responsive to discount factor shocks, and, as a result, the distribution of future output becomes tighter around its expected value (i.e., uncertainty is lower when $\phi_\pi$ is higher). We show this effect graphically in the top panel of figure 4, where we plot the actual density functions of future real GDP across three values of $\phi_\pi$. For each case the discount factor equals its steady state value.

Higher values of $\phi_\pi$ have similar impacts on real GDP uncertainty when the economy is at the ZLB. Greater inflation stability means the ZLB binds at higher discount factors states. It also means that when the nominal interest rate is at its ZLB, households expect inflation will be relatively more stable when the nominal interest rate rises. Since households always expect the nominal interest rate to leave its ZLB, more stable inflation away from the ZLB leads to more stable inflation and

12It would be interesting to compute how much of the uncertainty at the ZLB feeds back into real GDP. The stochastic volatility literature can isolate the effects of higher uncertainty on endogenous variables since the source of the uncertainty is exogenous. With our measure of uncertainty, we would need a way to decompose the effects of an exogenous decrease in demand on real GDP from the increased uncertainty at the ZLB. Unfortunately, there is no clear way how to compute these effects. However, the feedback channel is likely dwarfed by the effects from the ZLB.
real GDP at the ZLB. For example, if $i_0^* = -0.2$, then a ±1 standard deviation discount factor shock causes real GDP to range from $-0.6\%$ to $-1.6\%$ below its steady state when $\phi_\pi = 1.5$ and from $-0.3\%$ to $-0.7\%$ when $\phi_\pi = 2.5$. A flatter policy function for real GDP generates a narrower and more symmetric distribution for future real GDP (bottom panel), which suggests that one benefit of a higher $\phi_\pi$ is that it alleviates uncertainty near and at the ZLB. Regardless of the value of $\phi_\pi$, however, the uncertainty is unaffected by discount factor shocks when the nominal interest rate is far from its ZLB and increases sharply when it approaches and hits its ZLB.

Another way to compare the relationship between real GDP and uncertainty is with a generalized impulse response function (GIRF) of a shock to the discount factor following the procedure in Koop et al. (1996). The advantage of GIRFs is that they are based on an average of model simulations where the realization of shocks is consistent with households’ expectations over time. Figure 5 plots the responses to a 1 standard deviation positive discount factor shock at and away from the ZLB. The steady-state case (solid line) is initialized at the stochastic steady state. The ZLB case (dashed line) is initialized at the average discount factor conditional on the ZLB binding.
in a 500,000 quarter simulation. The corresponding initial notional interest rate equals $-0.23\%$.

The shock causes households to postpone consumption, which reduces output and inflation. When the economy is far from its ZLB, the drop in output is dampened by the monetary policy response. Thus, there is virtually no change in uncertainty because the household expects that future shocks will have the same effect on output regardless of the discount factor. At the ZLB, however, the central bank cannot respond by lowering the nominal interest rate, which leads to larger declines in output. Thus, uncertainty increases because the household expects a wider range of outcomes for future output when current and future monetary policy is constrained by the ZLB.

![Figure 5: Generalized impulse responses to a 1 standard deviation positive discount factor shock at and away from the ZLB. The steady-state case (solid line) is initialized at the stochastic steady state. The ZLB case (dashed line) is initialized at the average discount factor conditional on the ZLB binding in a 500,000 quarter simulation. To compute the GIRFs, we calculate the mean of 10,000 simulations of the model conditional on a random shock in the first quarter. We then calculate a second mean from another set of 10,000 simulations, but this time the shock in the first quarter is replaced with a 1 standard deviation positive discount factor shock. The vertical axis is the percentage change (or the difference in uncertainty) between the two means. The horizontal axes displays the time period in quarters.](image)

Next, we simulate the model for 500,000 quarters and plot a 100 quarter snapshot of the simulation. The ZLB binds in $1.6\%$ of quarters in the entire simulation. The top panel of figure 6 plots the paths of real GDP, $\hat{y}_{gdp}$ (left axis, solid line), and real GDP uncertainty, $\tilde{\sigma}_{y_{gdp}}$ (right axis, dashed line), which is yet another way to visualize the correlation between real GDP and uncertainty in the model. The bottom panel plots the path of the nominal interest rate (solid line) and notional interest rate (dashed line). The shaded regions indicate periods when the ZLB binds. There are four ZLB events—three that last one or two quarters and one that lasts 13 quarters. When the ZLB binds the notional rate is negative. Outside of the ZLB, the nominal and notional rates are equal.

There are three key takeaways from this simulation. One, the uncertainty surrounding future output is state-dependent. When the ZLB does not bind, uncertainty is essentially constant, except in periods when the nominal interest rate is near its ZLB. In those situations, the high probability of hitting the ZLB next period leads to persistently higher uncertainty. The closer the nominal interest is to the ZLB, the higher the uncertainty, which underscores the importance of expectational effects. When the ZLB binds, $\tilde{\sigma}_{y_{gdp}}$ is as much as four times larger than its value outside the ZLB. The degree of uncertainty depends on the notional interest rate. That value indicates how likely it is for the nominal interest rate to rise and exit the ZLB in the near-term. The lower the value of the
notional interest rate, the less likely the economy will exit the ZLB and the higher the uncertainty.

Two, there is a weak correlation between $\hat{y}_{gdp}$ and $\tilde{\sigma}_{gdp}$ when the ZLB does not bind but a strong negative correlation between those variables when it does bind. The strength of those correlations depends on the likelihood of entering and staying at the ZLB. When the nominal interest rate is sufficiently far from its ZLB, the correlation is close to zero since uncertainty is nearly constant. In periods when the nominal interest rate is near or at its ZLB due to positive discount factor shocks, uncertainty is relatively high and real GDP is well below its steady state.

Three, it is possible for uncertainty to decline while the ZLB binds, which implies that uncertainty is time-varying at the ZLB. This outcome occurs whenever the notional interest rate is below zero and there is a negative discount factor shock. A lower discount factor means the household is more optimistic about the future economy, which increases the expected nominal interest rate and reduces output uncertainty. The reason this feature of the model is important is because output uncertainty continued to fluctuate in the data even after many central banks reduced their policy rates to the ZLB in late 2008. In other words, our theory does not claim that uncertainty is always increasing when the ZLB binds, but rather that it is more strongly correlated with real GDP growth.

Our theoretical results are based on a model that does not consider the implications of unconventional monetary policies, such as quantitative easing or forward guidance. Modeling such policies is a difficult task and beyond the scope of this paper. We believe that such policies might have implications for the level of uncertainty predicted by the model. To the extent that those
policies are successful, they will most likely reduce the expected volatility of future output and therefore lower the level of uncertainty. However, we suspect that none of our qualitative findings (i.e., stronger negative correlations near and at the ZLB) would change. The reason is that unless these policies completely alleviate the constraint imposed by the ZLB, it will still be the case that output becomes more responsive to shocks near and at the ZLB than it is away from the ZLB. As a result, the correlations between uncertainty and real GDP growth will also be stronger at the ZLB.

4.2 Model with Variable Technology

Now assume technology varies across time according to the process in (4). The model contains two state variables, \( \hat{z}_{-1} \) and \( \hat{\beta}_{-1} \), which range from \( \pm 2.5 \) and \( \pm 1.85\% \) of their steady-state values, respectively. Figure 7 shows the policy function for real GDP (left panel) and real GDP uncertainty (right panel). The shaded regions indicate where the ZLB binds, which represents 28.4% of the state space. A low (high) level of technology increases (reduces) firms’ marginal cost of production. Firms respond by decreasing (increasing) their production and raising (reducing) their prices. The central bank responds by increasing (decreasing) the nominal interest rate. This means the nominal interest rate hits its ZLB given a sufficiently high level of technology. In those states, the model dynamics are similar to what occurs in high discount factor states—lower inflation raises the real interest rate, causing a sharp decline in real GDP and a large increase in uncertainty, even though technology is above steady state.\(^{13}\)

The uncertainty surrounding future real GDP is mostly unaffected by the level of technology when the nominal interest rate is far from its ZLB. In that situation, uncertainty is stable and low, even when technology and the discount factor are both below their steady states. In contrast, when the nominal interest rate is close to or at its ZLB, regardless of whether it is due to unusually high technology or a high discount factor, forecast error volatility increases. As technology and the discount factor simultaneously increase and move away from their respective steady states, real GDP rapidly declines, which drives up uncertainty at the ZLB. Thus, variable technology represents another source of uncertainty but only when the central bank is constrained by the ZLB.

Figure 8 shows a 100 quarter window of a 500,000 quarter simulation, where the ZLB binds in 3.8% of quarters. The presence of technology increases the volatility of real GDP and the likelihood of spikes in uncertainty, since variable technology is an additional source of movements in the nominal interest rate. The ZLB binds for 20 quarters at the beginning of the simulation and at three other periods in time for a shorter duration. During each ZLB event, real GDP declines and uncertainty increases. Uncertainty is highest when the notional interest rate is far below zero.

The model with variable technology preserves the simulation properties of the model with constant technology—uncertainty is time varying and strongly correlated with output near and at the ZLB. However, the correlations are weaker because variable technology adds new states of the economy where it is possible for output to remain unchanged or even fall when uncertainty rises. One example of where this occurs is along the contour where real GDP equals its steady state.

4.3 Connections between Model Predictions and the Data

Table 4 shows conditional and unconditional correlations in the model and analogous correlations in the data. The correlations in the model between real GDP growth and \( k \)-period ahead real GDP uncertainty are calculated from a 500,000 quarter simulation. The “Unconditional” correlations are based on the entire simulation, while the “Away from ZLB” correlations are conditional on \( i_t^{*} > 0.3\% \) and the

\(^{13}\)For a more thorough description of the dynamics in a model with and without capital see Gavin et al. (2014).

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Figure 7: Policy function for real GDP (left panel) and real GDP uncertainty (right panel). The horizontal (vertical) axes in these panels display technology (the discount factor), which is in percent deviations from steady state. The contours in the left panel display real GDP in percent deviations from its steady state. The contours in the right panel show real GDP uncertainty as a percent of steady-state output. The shaded regions indicate where the ZLB binds.

Figure 8: Top panel: paths of real GDP (solid line) and real GDP uncertainty (dashed line). Bottom panel: path of the nominal interest rate (solid line) and notional interest rate (dashed line). In the top panel, the left vertical axis displays real GDP in percent deviations from its steady state. The horizontal axes display the time period. The right vertical axis shows real GDP uncertainty as a percent of steady-state output. In the bottom panel, the vertical axis displays the nominal/notional interest rates as a net percent. The shaded region indicates periods when the ZLB binds.
Table 4: Comparison between conditional and unconditional correlations in the model and the data. The correlations in
the model are between real GDP growth and the standard deviation of the real GDP forecast error \( k \)-periods ahead. The
correlations in the data are between real GDP growth and various measures of uncertainty. The breakpoint between
the two samples in the model \( i_t^* \geq 0.3\% \) corresponds to where the real GDP uncertainty first shows a meaningful
increase. It is also consistent with the federal funds rate at the end of 2008Q3, which coincides with the breakpoint
in the data. The values in the data are significantly different from 0 at ***1%, **5%, and *10% levels. We did not
include the significance level for the model correlations because they are decreasing with the simulation length.

“Near ZLB” correlations are conditional on \( i_t^* \leq 0.3\% \), which is the value of the notional interest rate where real GDP uncertainty first shows a meaningful increase. The correlations in the data between real GDP growth and each measure of uncertainty are based on different data ranges. The “Unconditional” sample uses data from 1986Q1-2014Q1, while the “Away from ZLB” sample ranges from 1986Q1-2008Q2 and the “Near ZLB” sample ranges from 2008Q3-2014Q2. The breakpoint in the model is consistent with the federal funds rate at the end of 2008Q3 (annualized), which coincides with the breakpoint in the data. The 1-period ahead model correlations are compared to those based on the VXO and SPF FD(1), the 2-period ahead correlations are compared to the BOS FD, and the \( k \)-period ahead correlations are compared to the SPF FD(\( k \)), \( k > 1 \).
There are several remarkable similarities between the correlations in the model and the data. One, the correlations from both variants of the model are qualitatively similar to the data from the “Unconditional” sample and the “Near ZLB” sample. In both of these samples, the model correlations are negative, but more negative with the “Near ZLB” sample. Both of these facts are consistent with the data. Two, with the “Away from ZLB” sample, the model with constant technology fails to predict the weak correlation between the growth rate of output and uncertainty in the data, except for the BOS FD. However, when technology is stochastic the model predictions better match the weak correlations between the VXO and SPF FD$(k)$. Three, with variable technology the model qualitatively matches the decreasingly negative correlations between real GDP growth and the SPF FD$(k)$ that occur with the “Near ZLB” sample as $k$ increases. When technology is constant, the model predicts these correlations slightly increase as $k$ increases, but when technology is stochastic they decrease as they do in the data. Four, the correlations predicted by the model are closest in magnitude to those based on the SPF FD$(k)$, which is a measure of the real GDP forecast dispersion, rather than stock market volatility (VXO) or uncertainty about a particular sector of the economy (BOS FD). All of these findings provide evidence that the ZLB is a source of the negative correlations in the data since the onset of the Great Recession.

4.4 **Inflation and Interest Rate Uncertainty**

Thus far we have focused on the correlation between real GDP growth and real GDP uncertainty. This section tests our model along a different dimension by looking at the relationship between real GDP growth and the uncertainty surrounding both inflation and the nominal interest rate. We begin by examining the properties of the theoretical model and then turn to the data using SPF forecasts of inflation and the T-Bill rate.

The top panels of figure 9 show the policy function for inflation (left panel) and inflation uncertainty (right panel). The bottom panels show the policy function for the nominal interest rate (left panel) and interest rate uncertainty (right panel). The shaded regions indicate where the ZLB binds. The intuition for these results follows from our discussion about real GDP. When the discount factor state is low, households are impatient and would like to increase current consumption. Firms respond to the higher demand by increasing their price level, which raises inflation and causes the central bank to increase its policy rate. As the discount factor declines and households’ willingness to postpone consumption increases, firms cut their prices. When the central bank is unconstrained, it will reduce its policy rate to dampen the effects of the fall in demand. Given a sufficiently large decline in demand, the ZLB will bind and the central bank will be unable to respond to adverse shocks. As a consequence, positive discount factor shocks that occur at the ZLB cause relatively larger declines in inflation (i.e., a steeper policy function). The more weight households place on those outcomes in the future, the greater the dispersion in future inflation outcomes and the higher the level of inflation uncertainty. Interest rate uncertainty, in contrast, is lower at the ZLB due to the constraint faced by the central bank. Any negative demand shock that occurs at the ZLB will not change the interest rate. It is also possible that positive demand shocks will keep the interest

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\[14\] We also calculated the correlations from the model with interest rate smoothing in the monetary policy rule. The correlations conditional on the small fraction of quarters where the ZLB binds (0.15%) are similarly negative as those without interest rate smoothing. Moreover, the unconditional correlation was close to zero, since the interest rate is rarely at or near zero, which supports our finding that there is a weak correlation outside of the ZLB in the data.

\[15\] Our results are robust to alternative values of $\phi_\pi$. For example, in the model with constant technology, raising $\phi_\pi$ from 1.5 to 2 weakens the unconditional correlations and slightly strengthens them when $i^* \leq 0.3$. Decreasing $\phi_y$ from 0.1 to 0 has little effect on the correlations. The results are also robust to adding capital to the model. With constant technology, the unconditional correlation is $-0.15$, while the correlation conditional on $i^* \leq 0.3$ is $-0.46$. 

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Figure 9: Policy function for inflation (top left panel), inflation uncertainty (top right panel), the nominal interest rate (bottom left panel), and interest rate uncertainty (bottom right panel). The horizontal axes display the discount factor state in percent deviations from steady state. The vertical axes in the left panels display net rates. The vertical axis in the top (bottom) right panel shows the standard deviation of the forecast error for inflation (interest rate) as a percent of steady-state inflation (interest rate). The shaded regions indicate where the ZLB binds.

rate unchanged if the notional interest rate is sufficiently negative. Therefore, as the discount factor state increases (i.e., demand falls), households will place less weight on a positive interest rate and the uncertainty surrounding the interest rate will converge toward zero.

Table 5 displays the correlations between real GDP growth and inflation uncertainty for both models and the analogous correlation from the data using the inflation rate forecast dispersion from the SPF. The forecast dispersion is the difference between the 75th and 25th percentiles of the individual inflation rate forecasts. Relative to real GDP uncertainty, one notable difference about inflation uncertainty is that its correlation with real GDP growth in the “Away from ZLB” sample is statistically different from zero at the 5% level. However, throughout the paper we have emphasized the robustness of our finding that the correlation in the “Near ZLB” sample is more negative. That result is also holds with inflation uncertainty. The correlation conditional on the “Near ZLB” sample is far more negative, $-0.61$ compared to $-0.27$ in the “Away from ZLB” sample. Our model with both discount factor and technology shocks can explain that difference.
Table 5: Comparison between the conditional and unconditional correlations in the model and the data. The correlations in the model are between real GDP growth and the standard deviation of the inflation rate forecast error 1-period ahead. The correlations in the data are between real GDP growth and the SPF CPI inflation rate forecast dispersion. The values in the data are significantly different from 0 at ***1%, **5%, and *10% levels.

Figure 10 shows data on real GDP growth (solid line, left axis) and the forecast dispersion in the 3-month T-Bill rate (dashed line, right axis) from the SPF. The “Near ZLB” sample is indicated by the shaded region. There are two features in the data that match the predictions in the model. First, uncertainty about the T-Bill rate decreases as the economy transitions to the “Near ZLB” sample. Forecasters are aware that the short-term interest rate is constrained by the ZLB, which reduces their disagreement. In the model, the interest rate uncertainty approaches zero when the probability of exiting the ZLB declines. In other words, the longer households expect to remain at the ZLB, the more certain they are that the interest rate will remain near zero in the future. Similarly, uncertainty about the T-Bill rate has approached zero the longer the economy has remained at the ZLB. Second, uncertainty about the T-Bill rate and real GDP growth both decrease at the beginning of the “Near ZLB” sample. That feature is consistent with the model’s prediction (i.e., a drop in demand that causes the ZLB to bind decreases real GDP and simultaneously reduces uncertainty about the short-term interest rate). Furthermore, in 2011 real GDP growth increased from about 0% to 1% and at the same time uncertainty about the T-Bill also increased. Forecasters may have placed more weight on a higher T-Bill rate due to increased economic activity. The positive correlation between those two variables is explained in the model by households putting more weight on exiting the ZLB (i.e., a positive nominal interest rate) as the economy recovers.

Figure 10: Real GDP growth (solid line) and 3-month T-Bill uncertainty (dashed line). The horizontal axis displays the time period in quarters. The left vertical axis displays real GDP in quarter-over-quarter log differences. The right vertical axis shows 3-month T-Bill uncertainty, defined as the difference between the 75th and 25th percentile of the 1-quarter ahead forecast in the SPF. The shaded region indicates periods in the “Near ZLB” sample.
5 Conclusion

This paper documents a strong negative correlation between macroeconomic uncertainty and real GDP growth since mid-2008. Prior to that time, the correlation between those two variables was weak and in many cases not statistically different from zero, even when conditioning on quarters when the economy was in a recession. Why did the Great Recession lead to a stronger negative correlation compared to previous recessions? One possible answer is the ZLB on the short-term nominal interest rate. During the Great Recession many central banks around the world sharply reduced their policy rates and effectively hit the ZLB for the first time in their history. We contend that central banks’ inability to further reduce their policy rates in response to adverse economic conditions has contributed to the negative correlation between real GDP uncertainty and real GDP growth that emerged in 2008.

To test our theory, we use a model where the policy rate occasionally hits its ZLB due to technology and discount factor shocks. We find that the correlations between uncertainty and economic activity are weak when the policy rate is far away from its ZLB, but strongly negative near and at the ZLB. This result occurs in the model because real GDP becomes more responsive to shocks that hit the economy when the central bank is constrained by the ZLB, which increases the dispersion of future values of real GDP. As a result, our model has the same key feature as the data—prior to the Great Recession, which is when the policy rate was far from its ZLB, the correlations were weak, but they are strongly negative since the policy rate approached its ZLB at the onset of the Great Recession. While it may be the case that the ZLB is not the only factor causing the stronger correlation, our results provide strong evidence that it is an important factor.

References


**A. Data Sources**

This section describes the data used to calculate the correlations between real GDP and uncertainty:

**U.S. Real GDP:** Chained 2009 dollars, seasonally adjusted. Source: Bureau of Economic Analysis, National Income and Product Accounts, Table 1.1.6.

**Euro Area Real GDP:** 2010 Chained linked volumes, 12 Countries, seasonally adjusted and adjusted data by working days. Source: Eurostat, Euro-Indicators Database (EUROIND), National Accounts: ESA 2010, GDP and Main Components Table.

**U.K. Real GDP:** 2010 Chained linked volumes, seasonally adjusted. Source: Eurostat, Euro-Indicators Database, National Accounts: ESA 2010, GDP and Main Components Table.

**U.S. GDP Deflator:** Index 2009=100, seasonally adjusted. Source: Bureau of Economic Analysis, National Income and Product Accounts, Table 1.1.9.

**U.S. 3-Month T-Bill Rate:** Quarterly average, not seasonally adjusted. Source: Board of Governors of the Federal Reserve System, Selected Interest Rates (Daily) - H.15.

**U.S. SPF:** Log difference between the 75th and 25th percentiles of the individual forecasts of quarterly real GDP and industrial production. Differences between the 75th and 25th percentiles of the inflation rate (CPI) and U.S. 3 month T-Bill rate. Source: Federal Reserve Bank of Philadelphia, Survey of Professional Forecasters Measures of Cross-Sectional Forecast Dispersion.

**ECB SPF:** Individual forecasts of future quarterly real GDP growth rates (year on year percentage change of real GDP). Source: ECB Survey of Professional Forecasters individual data.
**BOE SEF:** Individual forecasts of future quarterly real GDP growth rates (year on year percentage change of real GDP). Source: Bank of England (individual data not available online).

**U.S. BOS:** Future general activity; percent reporting a decrease (GAFDSA), an increase (GAFISA) and no change (GAFNSA). Source: Federal Reserve Bank of Philadelphia, Business Outlook Survey, revised monthly data.

**U.S. VXO:** Expected volatility in the S&P 100 over the next 30 days at an annualized rate. We calculate a quarterly average of the daily observations. Source: Chicago Board Options Exchange, VIX Historical Price Data (“old methodology”).

**U.S. IP:** Index 2007 = 100, Total index, seasonally adjusted monthly data. Monthly observations are averaged to produce quarterly observations. Source: Board of Governors, Industrial Production and Capacity Utilization - G.17.